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Enhanced convex lifting for environment with complex shaped obstacles

Zhixin Zhao¹, Turan Konyalioglu^{1,2}, Sorin Olaru¹, Antoine Girard¹, Silviu-Iulian Niculescu^{1,3}

Abstract—In this paper, we revisit the path planning approach based on convex lifting and extend it to accommodate non-convex obstacles. Obstacles are modeled as overlapping unions of polyhedra. To handle complex environments, a pre-processing module is required to reconstruct the obstacles, and convex lifting is used to obtain an interconnected graph. A post-processing module will then remove graph edges that intersect obstacles, thereby extracting only feasible paths. We propose two obstacle representations that are compatible with the convex lifting framework, significantly broadening its applicability. Furthermore, we address the potential loss of connectivity resulting from the removal of obstacle-intersecting edges and propose a solution to maintain connectivity and thus ensure the preservation of collision-free paths.

I. INTRODUCTION

Navigation is a core challenge in mobile robots, manipulators, and autonomous driving, typically divided into two key sub-problems: path planning and trajectory tracking [16]. Path planning focuses on computing a collision-free sequence of waypoints that connect a given start position to a goal location, ensuring safe and feasible movement through the environment, which can have non-convex obstacles (or agents). This process enables autonomous systems to reason about their surroundings and select viable routes before executing motion.

A considerable number of classical methods developed for path planning are based on grids or graphs. The algorithms, such as Dijkstra [5] and A* family [7], are used to solve the shortest path problem along the nodes of the graph via a cost function. On the other hand, hybrid A* [17] uses certain motion primitives to also check whether a visit from one node to another is feasible, accounting for agents' dynamics. Nevertheless, creating these graphs or space partitioning is another challenging task, such as cell decomposition [4]. In the context of deterministic approaches, the Voronoi diagrams, for instance, constructs a graph that maximizes the distance from obstacles, allowing it to identify the shortest path along the graph [2]. Although they are efficient for space partitioning with the existence of non-convex obstacles, computational expansiveness in computing these diagrams and inability to consider non-holonomic constraints prevent them from being used widely. The visibility graphs, on the other hand, connect all mutually visible vertices in the environment to compute the shortest path, but they become inefficient with non-convex obstacles due to the need for complex visibility checks and increased risk of paths passing too close to obstacle boundaries [13].

To handle challenges such as non-convex obstacles, complex agent dynamics, and dynamic environments, sampling-based algorithms such as Probabilistic Roadmap (PRM) and Random Trees (RRT) have become a widely used alternative to traditional planning methods [1], [8], [11]. Whereas PRM generates an undirected graph that requires a separate graph search to find a path, RRT produces a directed acyclic graph rooted at the start configuration, inherently encoding the path without the need for an additional search algorithm. For obstacle avoidance assurance, RRT can utilize positive invariant sets during tree expansion to ensure that transitions between sets remain within safe regions [18]. However, in general, because sampling-based algorithms only guarantee path feasibility asymptotically, their computation complexity bound can be impractical and thus not suitable for applications where safety is critical.

In recent years, convex lifting-based methods have received increasing attention for routing in cluttered environments. These methods rely on convex optimization of piecewise affine functions to form polyhedral space partitions, where each convex obstacle is encapsulated within a region of the navigation space, ensuring global feasibility guarantees [10]. The resulting polyhedra are used to generate a graph, which is coarse enough to allow the construction of safe corridors along the shortest path [10]. The primary strength of the convex lifting approach lies in its ability to guarantee feasibility, rather than path optimality. To address this limitation, the authors in [12] proposed an iterative obstacle enlargement algorithm to create larger safe corridors and improve the quality of the paths. A model predictive control (MPC) strategy is then employed to track waypoints along the safe corridors, accounting for the agent's dynamics. To ensure feasibility and recursive feasibility, controlled invariant set computations are performed within these corridors as part of this MPC strategy. More recently, the method has been implemented as a mathematical toolbox [15]. A key limitation of these studies lies in the environment representation: obstacles are typically modeled as a finite set of convex, non-overlapping polyhedra.

In this paper, we revisit the problem by allowing obstacles to be modeled as overlapping polyhedra. We decompose connected, non-convex obstacles into internally disjoint convex hulls and establish neighborhood relations among them. This representation is well-suited for convex lifting. Furthermore, we propose two novel methods to solve the convex-lifting problem considering non-convex obstacles: (i) a method that imposes equality (continuity) constraints on the shared faces between neighboring convex components, and (ii) a method that introduces a cutting process along the neighborhood regions, followed by a relaxed representation and an enhanced formulation to ensure no information is lost.

The paper is organized as follows: Section II briefly

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introduces the convex lifting method and problem statement. Section III presents the non-overlapping representation as well as the enhanced convex lifting with equality constraints. Section IV introduces the relaxed representation of the union of polyhedra and their liftings. Section V provides results and a comparison of the two methods.

Notation: Let $\mathbb{N} = \{0, 1, 2, \dots\}$, $\mathbb{N}_{\leq i} = \{n \in \mathbb{N} \mid n \leq i\}$, \mathcal{I}_N denote the set of non-negative integers, $\mathcal{I}_N^2 := \{(i, j) : i \in \mathcal{I}_N, j \in \mathcal{I}_N, i \neq j\}$. Given a compact set $S \in \mathbb{R}^n$, $\text{int}(S)$ is the interior, $\text{conv}(S)$ is the convex hull of S , $\mathcal{V}(S)$ the set of its extreme points, $\text{dim}(S)$ denotes the dimension of S . Given a N sets of extreme points V_1, V_2, \dots, V_N , $\text{conv}(V_1, V_2, \dots, V_N)$ is the convex hull of $V_1 \cup V_2 \cup \dots \cup V_N$. For a polyhedron $P \in \mathbb{R}^n$, $\mathcal{V}(P)$ is the set of its vertices, $\mathcal{F}_k(P)$ is the finite collection of k -dimensional faces of the polyhedron P . If $S \in \mathbb{R}^n$ satisfies $S \in \mathcal{F}_k(P)$ then for any set $R \supseteq S$ one of the following alternatives hold: $\text{dim}(R) > k$ or $R \not\subset P$.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Path planning using convex lifting

Definition 1: Let $P = \bigcup_{i \in \mathcal{I}_N} P_i$ be a finite set of disjoint obstacles in a finite-dimensional space \mathcal{X} . A collection of sets $\{\mathcal{X}_i\}_{i \in \mathcal{I}_N}$ forms a partition of \mathcal{X} induced by the obstacles P if it satisfies:

- $\mathcal{X} = \bigcup_{i \in \mathcal{I}_N} \mathcal{X}_i$
- $\text{int}(\mathcal{X}_i) \cap \text{int}(\mathcal{X}_j) = \emptyset, \forall (i, j) \in \mathcal{I}_N^2$
- $P_i \subset \text{int}(\mathcal{X}_i)$

Furthermore, if both \mathcal{X} and \mathcal{X}_i are polyhedral, then \mathcal{X} is referred to as a polyhedral partition.

Definition 2: Given a polyhedral partition of a finite-dimensional space $\mathcal{X} = \bigcup_{i \in \mathcal{I}_N} \mathcal{X}_i$, a convex lifting is defined as a piecewise affine function $z : \mathcal{X} \rightarrow \mathbb{R}$ satisfying the following properties :

$$z(x) = a_i^\top x + b_i, x \in X_i, \quad (1)$$

and

$$z(x) > a_j^\top x + b_j, \forall x \in \text{int}(\mathcal{X}_i), (i, j) \in \mathcal{I}_N^2, \forall i \neq j \quad (2)$$

Lemma 1: A PWA lifting is continuous and, moreover, it is convex if (2) holds.

Theorem 1: [9] A piecewise affine lifting is continuous and convex if (a_i, b_i) satisfy:

$$a_i^\top v + b_i \geq a_j^\top v + b_j + \epsilon, \quad \forall v \in \mathcal{V}(P_i), \forall i \neq j, \quad (3a)$$

$$a_i^\top v + b_i \leq M, \quad \forall v \in \mathcal{V}(P_i), \forall i. \quad (3b)$$

In (a), $\epsilon, M > 0$ are suitably chosen and $\mathcal{V}(P_i)$ denotes the collection of extreme points of P_i .

Taking P_i as polyhedral sets (i.e., having a finite number of extreme points), allows to constructively obtain the lifting as the result of the following convex optimization problem:

$$\min_{a_i, b_i} \sum_{i=1}^N \left\| \begin{bmatrix} a_i & b_i \end{bmatrix}^\top \right\|_2^2 \quad \text{s.t.} \quad (3a) - (3b) \text{ hold.} \quad (4)$$

The convex lifting method constructs a polyhedral space partition such that an interconnected navigation graph $\Gamma(\mathcal{U}, \mathcal{A}, f)$ can be computed. where $\mathcal{U} = \bigcup_{i \in \mathcal{I}_N} \mathcal{V}(\mathcal{X}_i)$, $\mathcal{A} = \bigcup_{i \in \mathcal{I}_N} \mathcal{F}_1(\mathcal{X}_i)$ and a weight function $f : \mathcal{E} \rightarrow \mathbb{R}$ assigns the length of each edge.

Given a starting point x_0 and a target point x_f , if they do not already belong to the graph, we need to properly connect them to the graph. For example, for each of the two points we can compute its projection on the edges of the graph and consider the shortest connection edge that does not intersect any observed obstacle [14]. The effective path generation can be performed using Dijkstra's graph search algorithm [5].

Proposition 1: If a collision-free path exists in the continuous space from the starting point to the goal and if the optimization problem (4)-(3) is feasible, then there exists a path in the interconnected navigation graph preserving the same topological connectivity, without intersecting any obstacles.

Proof: The feasibility of the lifting procedure leads to the construction of a partition of the navigation space such that $P_i \subset \text{int}(\mathcal{X}_i)$. Any point in the collision-free path is thus located in one of the cells \mathcal{X}_i . Consequently, the respective collision-free path can be continuously projected on the frontier of the cells \mathcal{X}_i . This continuous projection is a feasible path from the starting point to the goal, which is located on the edges of these polyhedral cells (due to the PWA structure of the lifting), generating the partition, and the proof is complete. ■

B. Problem formulation

The convex lifting defined in the Section II-A builds on collection of disjoint convex regions. A natural question is whether an extension of this sort of procedure to non-convex polytopic obstacles is possible. This problem will be tackled next by extending the convex liftings to obstacles represented by non-disjoint unions of polyhedra.

In the following, in order to preserve the same mathematical formulation we will consider the obstacles to be represented by a finite union of polytopes (closed bounded polyhedra) but we will relax the non-overlapping property among its elements:

$$\mathcal{O} = \bigcup_{i \in \mathcal{I}_N} \mathcal{O}_i \quad (5)$$

The union of polyhedra \mathcal{O} will be associated to a graph $\mathcal{G}(\mathcal{O}) = (\mathcal{I}_N, \mathcal{E})$ with the edges defined as:

$$\mathcal{E} = \{(i, j) \in \mathcal{I}_N^2 \mid \mathcal{O}_i \cap \mathcal{O}_j \neq \emptyset\} \quad (6)$$

In general, the graph of interconnection $\mathcal{G}(\mathcal{O})$ for the obstacles in a cluttered but navigable environment represents a union of disconnected subgraphs. In the following, we will concentrate on the representation of each subgraph with cardinality greater than one in terms of unions of polyhedra¹, with disjoint interiors, and subsequently will integrate them in a unified convex lifting procedure.

¹Whenever the subgraph corresponds to a union of polyhedra, which is convex, this can be handled by simply replacing the obstacle with the respective polytope in the path planning. The present developments are focusing on the more interesting case of a subgraph corresponding to a non-convex obstacles.

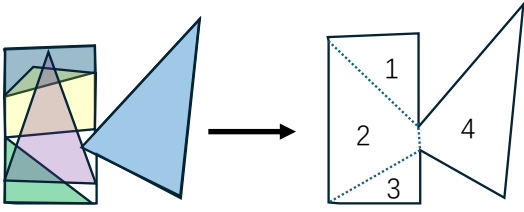


Fig. 1: Overlapping polyhedra to face to face representation

III. CONVEX LIFTING INVOLVING CONNECTED UNIONS OF POLYHEDRA

A. Non-overlapping representation

In this part we will concentrate on the collection of obstacles \mathcal{O} associated with a graph $\mathcal{G}(\mathcal{O})$ which is connected. The connectedness implies there are no subgraphs and thus the collection of obstacles need to be treated as a whole.

Lemma 2: Any connected union of polyhedra can be represented as a connected union of polyhedra with disjoint interiors.

$$\mathcal{O} = \bigcup_{j \in \mathcal{I}_N} \tilde{\mathcal{O}}_j$$

and $\text{int}(\tilde{\mathcal{O}}_i) \cap \text{int}(\tilde{\mathcal{O}}_j) = \emptyset$, $(i, j) \in \mathcal{I}_N^2$.

Proof: By defining

$$\tilde{\mathcal{O}}_i = \text{Conv} \left(\mathcal{O}_i \setminus \left(\bigcup_{j \in \mathcal{I}_N, j \neq i} \mathcal{O}_j \right) \right), \forall i \in \mathcal{I}_N$$

one obtains a new collection of polyhedra covering the same set:

$$\bigcup_{i \in \mathcal{I}_N} \tilde{\mathcal{O}}_i = \bigcup_{i \in \mathcal{I}_N} \mathcal{O}_i$$

but satisfying the non-disjoint intersection constraint. ■

The previous result shows that a connected union of polyhedra can be simplified to avoid superpositions as shown in Fig. 1. However, this constructive proof doesn't enforce a quintessential structure of the union of polytopes: *the face-to-face property among the neighbors*. In order to clarify the topological meaning, we recall here the formal definition. We denote:

$$S_{(i_1, i_2, \dots, i_n)} = \tilde{\mathcal{O}}_{i_1} \cap \tilde{\mathcal{O}}_{i_2} \cap \dots \cap \tilde{\mathcal{O}}_{i_n}$$

Definition 3 (Face-to-face property): Let $\tilde{\mathcal{O}}_i$ and $\tilde{\mathcal{O}}_j$ be two distinct n -dimensional polyhedra (i.e. $i \neq j$) in \mathbb{R}^n . These two polyhedra are said to satisfy the *face-to-face property* if whenever $\tilde{\mathcal{O}}_i \cap \tilde{\mathcal{O}}_j = S_{(i,j)}$, and $\dim(S_{(i,j)}) = d$ one has $S_{(i,j)} \in \mathcal{F}_d(\tilde{\mathcal{O}}_i)$, and $S_{(i,j)} \in \mathcal{F}_d(\tilde{\mathcal{O}}_j)$.

Lemma 3: Any connected union of polyhedra can be represented as a union of disjoint interior polyhedra, each with at least one neighbor such that the pair satisfy the face-to-face property.

Proof: The first part of the proof relies on Lemma 2. Additionally, for any neighbor regions violating the face-to-face property, a finite sub-cutting exists to transform it in a cell complex [6]. ■

Based on Lemmas 2 and 3, the entire environment containing convex or non-convex obstacles can be effectively modelled. All convex, detached obstacles are stored directly, while connected obstacles are decomposed accordingly in subgraphs

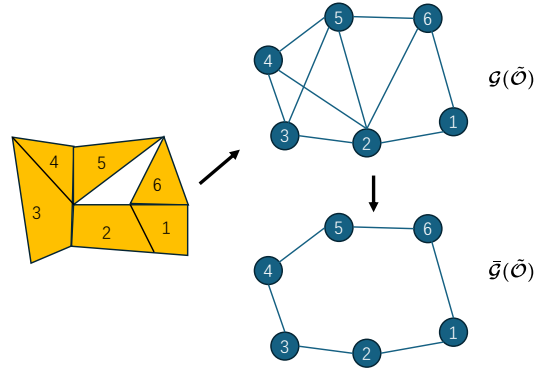


Fig. 2: Graph simplification

and brought to a regular form in light of the previous Lemma. The resulting set of N obstacles is thus recasted and can be dealt with by means of the interconnection graph. We denote by $\{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$ the collection of obstacles suitable for convex lifting and recall its graph representation.

Remark 1: The previous lemmata prove that forbidden regions defined through nonconvex obstacles can be equivalently represented by a collection of obstacles with disjoint interior and face-to-face property. However, this representation is not unique and the complexity can be reduced by invoking algorithms for the convexity recognition of union of polyhedra [3]. Next we will consider that all these pre-treatments has been applied and we deal with a an equivalent collection of n -dimensional non-overlapping polyhedra $\{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$, with face-to-face property which defines the adjacency graph $\mathcal{G}(\tilde{\mathcal{O}}) = (\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$ where $\tilde{\mathcal{N}}$ denotes the set of indices of polyhedra in the collection. We note that $\tilde{\mathcal{N}}$ and which can be different the original \mathcal{N} .

Definition 4: Given a graph $\mathcal{G}(\tilde{\mathcal{O}})$, the set of paths from the node i to the node j represent all the possible sequences of distinct edges starting in i and reaching j all by passing through distinct nodes:

$$\Pi_{(i,j)} = \left\{ (v_1, v_2, \dots, v_n) \in \tilde{\mathcal{N}}^n \mid v_1 = i; v_n = j; \right. \\ \left. (v_k, v_{k+1}) \in \tilde{\mathcal{E}}, k = 1, \dots, n-1 \right\} \quad (7)$$

In order to illustrate the concept, let us depict in Fig. 2 a cluttered environment with its associated graph $\mathcal{G}(\tilde{\mathcal{O}})$. As can be seen in this illustrations, edges exists due to redundant connections between the obstacles materialized through lower-dimensional faces. Some of these connections are indeed obstructing the navigation, while some others are superfluous. The next definition aims to formally describe an interconnection graph which retains the non-redundant interconnections.

Definition 5: Given a graph corresponding to a set of non-overlapping obstacles with face-to-face property $\mathcal{G}(\tilde{\mathcal{O}})$ its non-redundant counterpart is defined as $\bar{\mathcal{G}}(\tilde{\mathcal{O}}) = (\tilde{\mathcal{N}}, \bar{\mathcal{E}})$, with:

$$\bar{\mathcal{E}} = \{(i, j) \in \tilde{\mathcal{E}} \mid S_\pi \subsetneq S_{(i,j)}, \forall \pi \in \Pi_{(i,j)}, \pi \neq (i, j)\}$$

as simplified set of edges. □

Based on these elements, we can rely on the description of the edges. For any $(i, j) \in \bar{\mathcal{E}}$ we denote $V_{ij} = \mathcal{V}(S_{(i,j)})$ the set of vertices of the common facet shared by two neighboring polyhedra $\tilde{\mathcal{O}}_i$ and $\tilde{\mathcal{O}}_j$.

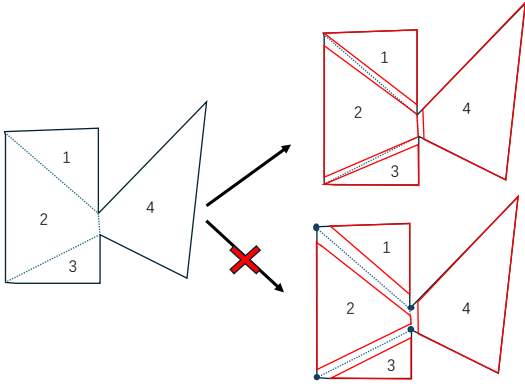


Fig. 3: Feasible cutting

Consequently for each $\tilde{\mathcal{O}}_i \in \{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$, one of the following holds:

- If $\tilde{\mathcal{O}}_i$ is separated, then there exists no j such that $(i, j) \in \tilde{\mathcal{E}}$, and we set $V_{ij} = \emptyset$.
- Otherwise, there exists at least one neighbor $\tilde{\mathcal{O}}_j$ such that $(i, j) \in \tilde{\mathcal{E}}$, and we define $V_{ij} = \mathcal{V}(S_{(i,j)})$.

B. Enhanced convex lifting with equality constraints

As mentioned before, the convex lifting has been discussed in recent years as a versatile path planning mechanism [10], [15] starting from a description of the cluttered environments as unions of disjoint convex obstacles. With the previously introduced elements, a novel convex lifting formulation allowing non-convex and overlapping obstacles can be formally introduced.

Specifically, we exploit the representation of the obstacles in terms of a union of disjoint polyhedra with facet-to-facet property. Then, the resulting interconnection graph can be constructed and will be exploited in the convex lifting procedure. Finally, as a post-treatment, the portions of the graph that intersect the original obstacle are removed to ensure valid navigation paths.

For obstacles represented as described in section 3.1 (non overlapping representation) $\{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$, the optimization problem leading to a convex lifting is formulated as follows:

$$\min_{a_k, b_k} \sum_{k=1}^N \left\| \begin{bmatrix} a_k & b_k \end{bmatrix}^\top \right\|_2^2 \quad (8a)$$

$$a_k^\top v + b_k \geq a_l^\top v + b_l + \epsilon, \forall v \in \mathcal{V}(\tilde{\mathcal{O}}_k) \setminus V_{kq}, \forall k \neq l, \forall (k, q) \in \tilde{\mathcal{E}} \quad (8b)$$

$$a_k^\top v + b_k \leq M, \quad \forall v \in \mathcal{V}(\tilde{\mathcal{O}}_k), \forall k \quad (8c)$$

$$a_k^\top v + b_k = a_q^\top v + b_q, \quad \forall v \in V_{kq}, \forall (k, q) \in \tilde{\mathcal{E}} \quad (8d)$$

As it can be noticed from the constraints in (8), each PWA component of the lifting is active (in the sense of minimality) over the vertices of one obstacle. As a particular feature, the lifting is continuous over the boundaries of convex polytopes that are connected in order to represent non-convex obstacles.

Corollary 1: If (8) admits a solution, then the feasibility of the path planning is equivalent to the construction of a path in the interconnected navigation graph.

Proof: Similar arguments as in Proposition 1. ■

IV. RELAXED REPRESENTATION OF UNION OF POLYHEDRA AND THEIR LIFTINGS

In convex lifting, inliftability refers to the infeasibility of the convex-lifting optimization problem, which may occur depending on the complexity of the environment. This concept has been studied in [10]. The formulation in (8) encounters inliftability when applied to the environment shown in Fig. 6b and Fig.6d (obstacles are colored in pink and purple).

This section explores a relaxation with respect to the obstacle representation. The goal is to approximate the non-overlapping representation by a new collection of mutually disjoint polyhedra.

A. Inner-outer approximation of connected unions of polyhedra

In practice, polyhedra are often separated by cutting along shared facets and the basic idea is to enlarge these cutting by introducing gaps between the neighboring polytopes. This process is what we refer to cutting. There exist infinitely ways to cut a connected obstacle into disjoint polyhedra. Our focus here is on constructing the neighboring relationships $\mathcal{G}(\hat{\mathcal{O}}) = (\tilde{\mathcal{N}}, \tilde{\mathcal{E}})$, and a convenient way to define the neighbor is to reuse the graph obtained from the facet-to-facet representation.

Remark 2: It should be mentioned that the inner and outer representations of a set represented by a union of polyhedra are not required to start from a facet-to-facet property. They should be simply considered as a relaxed form of decomposition that covers only a subset of the original set.

Lemma 4: Given $\{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$, a collection of polyhedra $\{\hat{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq M}}$ can be constructed such that the following conditions hold:

- $\hat{\mathcal{O}}_i \cap \hat{\mathcal{O}}_j = \emptyset$, for all $i \neq j$;
- $\bigcup_{i=1}^{N_o} \hat{\mathcal{O}}_i \subseteq \bigcup_{j=1}^N \tilde{\mathcal{O}}_j$;
- For every $i \in \tilde{\mathcal{N}}$, there exists $j \in \tilde{\mathcal{N}}$ such that $\hat{\mathcal{O}}_i \subseteq \tilde{\mathcal{O}}_j$.

These conditions imply that $M \leq N$. For clarity, we simply consider here that even if some polyhedra are removed, the indexes of the remaining polyhedra remain unchanged but the cardinality decreases. If the number of polyhedra remains unchanged during the cutting process, i.e., $M = N$, then the original graph structure continues to represent the neighboring relationships:

$$\hat{\mathcal{E}} = \tilde{\mathcal{E}}, \quad \hat{\mathcal{N}} = \tilde{\mathcal{N}}.$$

However, if the cutting process results in a reduction in the number of polyhedra, i.e., $M < N$, then the graph must be updated iteratively as follows:

For all $p \in \tilde{\mathcal{N}} \setminus \hat{\mathcal{N}}$, update:

$$\hat{\mathcal{N}} = \hat{\mathcal{N}} \setminus \{p\}$$

$$\hat{\mathcal{E}} = \left(\tilde{\mathcal{E}} \setminus \{(i, p), (p, j)\} \right) \cup \{(i, j)\}, \forall (i, p), (p, j) \in \tilde{\mathcal{E}}.$$

Next we will define the cuts that are feasible for our theoretical framework in the next section based on the graph structure.

Definition 6: Given $\{\tilde{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N}}$ we define a feasible cut as a collection $\{\hat{\mathcal{O}}_k\}_{k \in \mathbb{N}_{\leq N_o}}$ that satisfies:

$$\left\{ \tilde{\mathcal{O}}_k \right\}_{k \in \mathbb{N}_{\leq N}} \subseteq \left\{ \hat{\mathcal{O}}_k \right\}_{k \in \mathbb{N}_{\leq N_o}} \cup \left\{ \text{conv}(V_{ij}^i, V_{ij}^j) \right\}_{(i,j) \in \tilde{\mathcal{E}}} \quad (9)$$

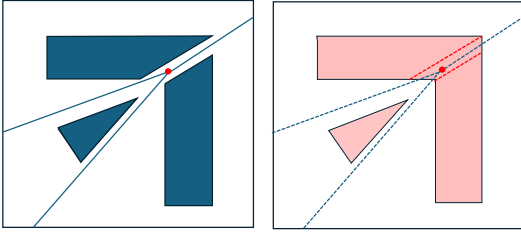


Fig. 4: Undesired partitioning in the cut region.

where

$$V_{ij}^i \subset \{x \in \tilde{\mathcal{O}}_i \mid \exists F_\ell \in \mathcal{F}_k(\tilde{\mathcal{O}}_i), (i, l, \dots, j) \in \Pi_{(i,j)} \text{ such that } x \in F_\ell \cap S_{(i,l)}\} \cap \mathcal{V}(\tilde{\mathcal{O}}_i) \quad (10)$$

$$V_{ij}^j \subset \{x \in \tilde{\mathcal{O}}_j \mid \exists F_\ell \in \mathcal{F}_k(\tilde{\mathcal{O}}_j), (j, l, \dots, i) \in \Pi_{(j,i)} \text{ such that } x \in F_\ell \cap S_{(j,l)}\} \cap \mathcal{V}(\tilde{\mathcal{O}}_j) \quad (11)$$

In practice, once a cutting is constructed, a post-verification of the condition (9) needs to be performed for validation or further refinement.

B. Inequality constraints

By applying inner and outer approximations of the obstacles, convex lifting can be directly employed. However, doing so may result in undesirable partitions, as illustrated in Fig. 4.

Consider $\tilde{\mathcal{O}}_0$ and $\tilde{\mathcal{O}}_i$, where $\tilde{\mathcal{O}}_0$ is a detached convex obstacle and $\tilde{\mathcal{O}}_i$ has a neighbor $\tilde{\mathcal{O}}_j$ such that $(i, j) \in \mathcal{E}$. Suppose a partition is obtained: $\mathcal{X} = \bigcup_{i \in \mathcal{I}_N} \mathcal{X}_i$. The region corresponding to $\tilde{\mathcal{O}}_0$ is denoted as \mathcal{X}_0 . Without any additional constraints, it is possible that $\mathcal{X}_0 \cap \text{conv}(V_{ij}^i, V_{ij}^j) \neq \emptyset$, implying that removing the intersecting parts from the interconnected graph will lose important graph information.

We propose the following enhanced convex lifting framework:

$$\min_{a_k, b_k} \sum_{k=1}^{N_o} \left\| \begin{bmatrix} a_k & b_k \end{bmatrix}^\top \right\|_2^2 \quad (12a)$$

$$a_k^\top v + b_k \geq a_l^\top v + b_l + \epsilon, \forall v \in \mathcal{V}(\tilde{\mathcal{O}}_k), \forall k \neq l \quad (12b)$$

$$a_k^\top v + b_k \leq M, \quad \forall v \in \mathcal{V}(\tilde{\mathcal{O}}_k) \quad (12c)$$

$$a_j^\top v + b_j > a_k^\top v + b_k, \quad \forall v \in V_{ij}^i, \forall (i, j) \in \hat{\mathcal{E}}, k \notin \{i, j\} \quad (12d)$$

$$a_i^\top v + b_i > a_k^\top v + b_k, \quad \forall v \in V_{ij}^j, \forall (i, j) \in \hat{\mathcal{E}}, k \notin \{i, j\} \quad (12e)$$

For the problem introduced at the beginning of this section. If (12) is feasible, then a new partition $\mathcal{X}' = \bigcup_{i \in \mathcal{I}_N} (\mathcal{X}'_i)$ can be constructed and

$$\mathcal{X}'_0 = \{x \in \mathbb{R}^n \mid a_0^\top x + b_0 > a_i^\top x + b_i + \epsilon, \forall i \in \mathcal{I}_N\} \quad (13)$$

Proposition 2: The partition \mathcal{X}' involving the region \mathcal{X}'_0 in (13) hold the property:

$$\mathcal{X}'_0 \cap \text{conv}(V_{ij}^i, V_{ij}^j) = \emptyset.$$

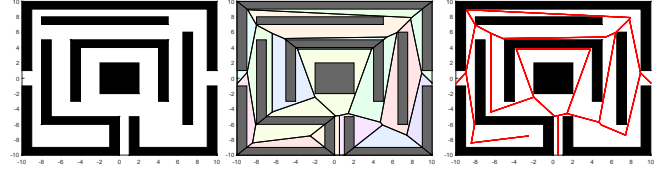
Proof: Suppose by contradiction that $x \in \mathcal{X}'_0$ and $x \in \text{conv}(V_{ij}^i, V_{ij}^j)$. Then x can be represented as :

$$x = \sum \alpha_k v, v \in (V_{ij}^i \cup V_{ij}^j)$$

where $\alpha_i \geq 0, \sum \alpha_i = 1$. According to (13) and we have :

$$a_0 \sum \alpha_k v + b_0 > a_i \sum \alpha_k v + b_i$$

$$a_0 \sum \alpha_k v_k + b_0 > a_j \sum \alpha_k v + b_j$$



(a) The maze layout. (b) Space partition. (c) Feasible edges.

Fig. 5: The polyhedral space partitioning is obtained using convex-lifting techniques in a maze-like environment with equality constraints. The space partitions are obtained by solving the optimization problem in (8) using the parameters $\epsilon = 0.1$ and $M = 0.01$.

and from constraints (12d)-(12e) we have:

$$a_i v + b_i > a_0 v + b_0, v \in V_{ij}^i; \quad a_j v + b_j > a_0 v + b_0, v \in V_{ij}^j \\ a_j v + b_j > a_0 v + b_0, v \in V_{ij}^i; \quad a_i v + b_i > a_0 v + b_0, v \in V_{ij}^j$$

which yields a contradiction with the feasibility of the lifting (12), hence the assumption is false and the proposition is proved. \blacksquare

V. IMPLEMENTATIONS, RESULTS AND DISCUSSIONS

This section presents the results of applying convex-lifting techniques to environments with the existence of non-convex obstacles. Two proposed approaches are evaluated: (i) convex-lifting with equality constraints, and (ii) relaxed representation of the non-convex regions. The effectiveness of each method is demonstrated through illustrative examples.

A. Convex lifting with equality constraints for non-convex obstacles

The first proposed method, convex-lifting with equality constraints, are tested in a maze layout containing both convex and non-convex obstacles as depicted in Fig. 5a. To facilitate convex lifting, the non-convex obstacles are decomposed into convex parts that have their face-to-face property. The convex-lifting optimization problem is constructed using (8), while imposing equality constraints on the facets that have face-to-face property. In Fig. 5b, the background shows the resulting space partition, representing each convex partition in various pastel colors. This space partition is utilized to construct the interconnected graph by considering the vertices of the partitions as the nodes and their facets as the edges of the graph, and the edges intersecting with obstacles are removed from the graph. The resulting interconnected navigation graph is illustrated in Fig. 5c.

A drawback can be noticed that some of the nodes are close to the obstacle's boundaries. This problem has been studied in [12] in the context of the obstacle enlargement algorithm, and a similar algorithm can be used to optimize the generated path in the environment.

B. Inequality constraints

In order to handle the environment as shown in Fig. 6, the inner and outer approximations of the obstacle are employed. The obstacles are partitioned into disjoint polyhedra, and the classical convex-lifting formulation is applied using (3). Nevertheless, the resulting navigation graph fails to preserve

a feasible path due to the removal of certain edges (those connected to nodes 1, 12, and 14), as discussed in the previous section and illustrated in Fig. 6a and Fig. 6b.

Subsequently, the inequality constraints (12d) and (12e) in (12) are applied to the same collection of disjoint obstacles to obtain the partition as shown in Figs. 6c. A feasible path remains after the removal of edges intersecting the obstacles as in 6d.

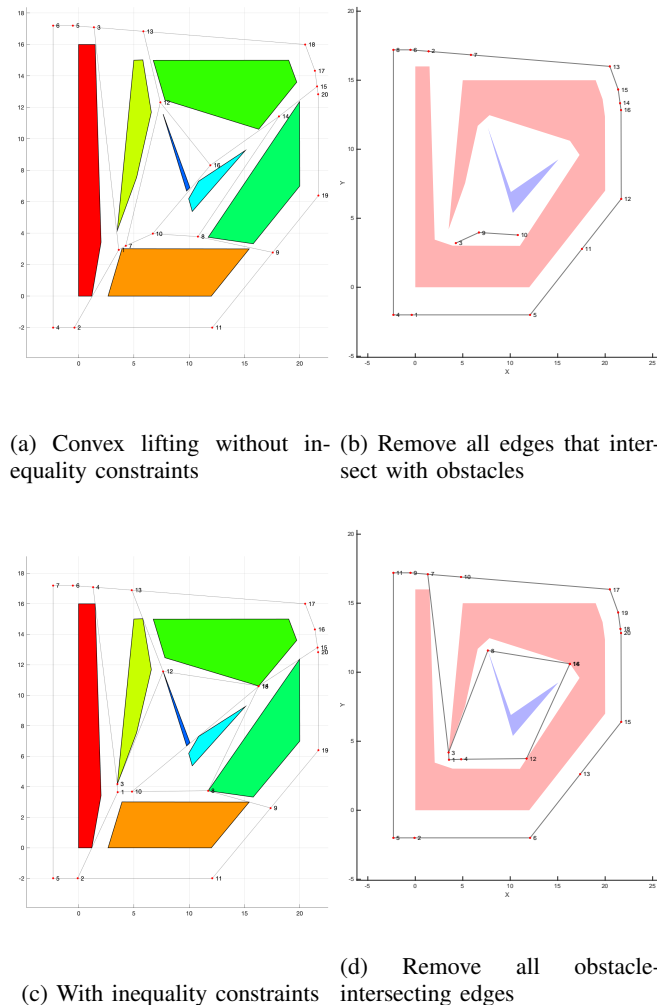


Fig. 6: Comparison of convex lifting without/with inequality constraints

VI. CONCLUSION

In this study, two novel methods have been proposed for adapting the convex-lifting technique to non-convex environments. Firstly, the traditional method has been extended with the usage of continuity constraints to be additionally imposed on the neighbouring faces of convex obstacles composing the non-convex obstacle. Secondly, we have revisited the problem of losing information on the graph after the cutting process of a non-convex obstacle to use the traditional convex-lifting method without having continuity constraints. An enhanced method with additional inequality constraints has been proposed to overcome this problem. The second method also offers relaxation as a solution to non-liftable cases. The results demonstrate that convex-lifting techniques

can effectively be applied to those environments, including non-convex obstacles, when appropriate formulations are employed.

The results also indicate that the paths generated by convex-lifting may closely approach obstacle boundaries, which can make them difficult for agents to follow. Therefore, future work will focus on improving path quality in non-convex environments by introducing an iterative obstacle enlargement algorithm.

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