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# Mathematical empowerment and participation in inclusive learning settings

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*In inclusive education, all children should work together in their diversity and develop their individual potential. With respect to mathematical difficulties, there are already numerous concepts for inclusive classrooms. In view of the many facets of diversity, however, further concepts and empirical studies are needed, e.g. in the context of mathematical potential or social skills. Social difficulties can overshadow the mathematical potential of children so that they are often unable to show or develop their full individual potential. As part of a larger research project, cooperative learning environments<sup>1</sup> for inclusive teaching have been developed to promote the mathematical potential and social participation of all learners. The article analyzes the extent to which social participation can be developed in a mathematical negotiation process using a case study. On this basis, possibilities of mathematical empowerment in three different domains are discussed.*

*Keywords: Mathematics education, inclusive education, diversity, empowerment, participation.*

## Introduction

*“Dustin needs tasks. And I think that's what he's missing in his primary school years and in his biography, that he gets tasks and can show himself.”* This quote was made by Dustin’s class teacher who was interviewed as a part of this study. The teacher describes Dustin’s family background as “difficult”. Dustin has a single mother and “difficult circumstances that don’t necessarily support him”. He was taken in relatively early by his grandparents, where he more or less grew up. In grade 3, custody was withdrawn from the grandparents and transferred back to the mother. This was followed by several changes of residence. During the interview his teacher reflects on what Dustin has already been through and summarizes his situation as follows: “From an early age, Dustin was left to his own devices and had to bite and fight his way through.” From the teacher’s point of view, this is expressed by the fact that Dustin often attacks other children while playing during breaks, when he doesn't agree with their rules or the way they play. In primary school he was not good at dealing with frustration, for example when losing in competitive games, so that “the whole thing turned into slightly aggressive behavior”. Dustin makes threats to classmates and teachers and has to learn to resolve conflicts only verbally and be more considerate to others. In addition to the description of the difficulties in social interaction, the teacher opens up a remarkable perspective in his quote, shown above, to strengthen Dustin: *“Dustin needs tasks [...] that he can show himself”*. From his point of view, such tasks are missing, to help Dustin to show his skills in learning, and in social classroom or school interaction. In this paper the case study ‘Dustin’ is used to investigate in how far a learner with

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<sup>1</sup> In this paper we understand a *learning environment* as a framework for several tasks based on a common mathematical structure (Hirt & Wälti, 2022).

social difficulties can be empowered by a special task design, so that he is not excluded but can participate in joint social and subject-related learning.

## **Empowerment**

The concept of empowerment is characterized by the experience of discrimination and the withholding of participation (Theunissen, 2016). After the concept was introduced into pedagogy, the aim was to empower people through targeted pedagogical efforts so that they gain power and become independent of support. Ernest (2001, p. 123) describes Empowerment as “the gaining of power in particular domains of activity by individuals or groups and the processes of giving power to them, or processes that foster and facilitate their taking of power”. Empowerment is aimed, among other things, at a strength that is based on the person and their individual resources, at addressing social disadvantage and discrimination, which should be overcome, and at the professional task of strengthening the vitality of people in a socially marginalized position (Theunissen, 2016, p. 114).

With regard to the empowerment of children in everyday school life, the central question is in what way empowerment can be encouraged and supported by teachers in order to offer help for self-help and at the same time enable the highest possible degree of self-determination (Lindmeier & Meyer, 2020). For this reason, the specific requirements of the learning subject must be taken into account: For the subject of mathematics the achievement of both content-related and process-related mathematical competencies and the understanding of the role of mathematics in everyday life can be understood as contributions to empowerment concerning mathematics. If children and young people are empowered in these, this not only has an impact on their school life, but also on their opportunities to participate in society. Ernest (2001) distinguishes three domains of empowerment concerning mathematics and its uses: (1) The *mathematical empowerment* includes the “Power over the language, symbols, knowledge and skills of mathematics and the ability to confidently apply this in mathematical applications within the context of schooling, and possibly to a lesser extent, outside of this context” (Ernest, 2001, p. 126). (2) When mathematics contributes to participating in a social domain, e.g. work, life and social affairs, it is called *social empowerment*. (3) An *epistemological empowerment* includes the development of personal identity and the growth of confidence in the mathematical and social senses (Ernest, 2001). It becomes clear that these three domains should not be understood separately from each other, but that the aim of mathematics teaching is to empower learners as epistemological agents, by encompassing mathematical, social and epistemological empowerment. For mathematics teaching this means that all children should be empowered to develop their individual mathematical potential using their individual strengths (mathematical empowerment). Therefore, children need needs-based resources in the form of adaptive individual access to the subject of learning combined with challenging mathematical negotiation opportunities. New mathematical knowledge is primarily developed through collaborative, subject-related negotiation processes (Häsel-Weide & Nührenbörger, 2022), so that the ability to learn socially from and with each other always has to be taken into account (social empowerment). The aim should be to offer learners adaptive access to the learning content in a way that they can engage in mathematical activities at their individual level, that challenges them, but does not make them dependent on help. By empowering individual potential, the genuine participation of all children becomes possible and can strengthen confidence in their own subject-specific and social skills (epistemological

empowerment). This orientation corresponds to the idea of inclusion, that aims to recognize and support all learners in their individuality.

## **Inclusion**

In an inclusive education system, all persons should have the chance to develop their individual potential and should be perceived, accepted, and valued in their diversity and individuality. Thus, the goal of inclusive teaching is to support all students in the best possible way (Ainscow et al., 2006). That's why the potential of all students is increasingly focused in academic and educational policy efforts (Dexel et al., 2019). Whereas in Germany, concepts for inclusion, e.g. in dealing with *learning difficulties*, are already established in everyday teaching, many concepts to support *mathematical potential* are designed rather exclusively (e.g. children's university) and offer few possibilities to cooperate, communicate and learn with and from each other (Billigen et al., 2023). Similarly, concepts for dealing with *social difficulties* tend to be exclusive rather than inclusive (e.g. Durlak et al. 2010). However, such exclusive concepts of support do not really correspond to the idea of inclusion. The present study refers to a perspective of inclusion and empowerment as an interplay of joint and individual learning processes. Every learner should develop his or her individual mathematical potential through adaptive mathematical learning opportunities as well as social participation (Krähenmann et al., 2019). In order to promote mathematical potential in inclusive learning settings and at the same time enable the social participation of all children, it is important that learning settings open up an individual access to the subject as well as joint negotiation about mathematical insights. Current studies show that processes of cooperation on a common subject in heterogeneous learning groups are particularly valuable for mathematical learning (Häsel-Weide & Nührenbörger, 2022).

## **Cooperation and participation**

The diversity and heterogeneity of learners can benefit the insights of all so that the joint engagement with a common subject can be fruitful. Therefore, it is necessary that all children *receive the same task*, which is holistic and sufficiently complex, so that it opens up *learning opportunities* for children *on different levels*. By working on a common task, a meaningful discussion about individual strategies and insights can take place. This initiates productive situations of learning with and from each other. In the German didactics of mathematics, the concept of "natural differentiation" is a well-explored concept to reach this (Scherer & Krauthausen, 2010). The children are not restricted by the teacher, but get empowered according to their individual potential concerning mathematics. Thus, this concept offers many advantages for dealing with the diversity of learners. However, looking at inclusive education from a wider perspective that includes particular mathematical difficulties as well as mathematical potential, natural differentiation is not always sufficient to create learning situations for all children. The concept of "parallelization" (Nührenbörger & Pust, 2018) builds on the idea of natural differentiation, but it can offer broader possibilities to realize the demands mentioned above. Due to target-differentiated levels, all learners can be given individual access to the subject of learning. Likewise, this opens up a variety of possibilities for a fruitful communication between the learners. They can contribute their discoveries, which they have obtained at the respective levels, so that social participation is guaranteed for all (Nührenbörger & Pust, 2018).

## Methods

The research project aims to promote and empower the individual mathematical potential of all learners even if it is not visible due to social difficulties. To achieve this, parallelized learning environments in combination with natural differentiated tasks have been developed in order to initiate social participation as well as individual learning processes. To answer the research questions, the method *case study* (Lamnek, 2010) is chosen, which allows to identify typical aspects concerning social difficulties, participation and empowerment to reconstruct a holistic picture of the case.

RQ 1: How can the *social participation* in the cooperation between Dustin and Lenny be characterized?

RQ 2: To what extent can initial references to the three *domains of empowerment* be reconstructed?

Before the results are presented, the learning environment and the analysis method is described.

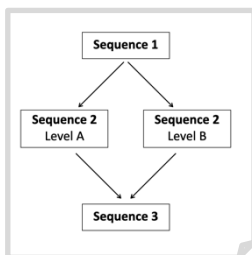
### Learning environment “Magic Squares”

The presented learning environment focuses on Magic Squares (MS). MS offer opportunities to discover and explore rich mathematical structures at different levels, which make them suitable for both natural differentiation and parallelization. In a MS the sum of each row, column, and diagonal is equal; it is called the magic number (Figure 1). In MS of odd-order the magic number is related to the odd-order and the mean number (e.g. in a 3x3 MS the magic number is three times the mean number). The current study is an ongoing PhD project, which explores two different parallelized learning environments. In this paper we refer to the learning environment MS, in which so far 31 children (aged 9 to 10 years, grade 4) were videographed

4	3	8
9	5	1
2	7	6

Magic Number: 15

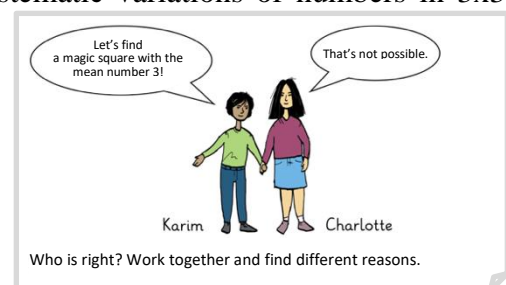
**Figure 1: Magic square (3x3)**



**Figure 2: Structure of the learning environment**

working on the different tasks. The researcher also takes the role of the interviewer. The learning environment is divided into three sequences (Figure 2). In the first sequence, children work individually in a naturally differentiating learning environment to become familiar with MS, and to explore first structures. The researcher then forms pairs of students on the basis of the students’ outcome from the first sequence. The target-differentiated classification enables adaptive individual promotion and empowerment of the individual potential of all learners. According to the concept of parallelization, some pairs work at level A and other pairs at the more complex level B in Sequence 2. Children working on level A explore relations between mean and magic numbers as well as systematic variations of numbers in 3x3 squares. Children working at level B focus on mathematical structures of different MS of size  $n \times n$ , so they are challenged to find MS of different sizes (3x3, 4x4, ...,  $n \times n$ ) with the magic number 90. Especially the comparison of odd- and even-order squares offers substantial discoveries. In Sequence 3, the students work together in new pairs composed of one learner each from level A and B (Figure 3).

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**Figure 3: Task in Sequence 3**

Both learners can participate in working on this new research task by applying knowledge gained from previous work on level A or B in sequence 2.

### Method of analysis

For the research project, the *participation analysis* (Krummheuer, 2015) represents a substantial analytical tool to reconstruct the autonomy of actions of the learners in the interaction with a social perspective on participation. In the concept of participation analysis, four functions of active speakers can be distinguished with regard to whether and to what extent (a) a *new content* is introduced and (b) this content is introduced with *new formulations* (see Table 1): (1) An “author” expresses his *own new idea* of a content in his *own new words*. (2) A “ghostee” introduces his *own new idea* of a content, but adopts the *formulation of a previous utterance*, whereas (3) a “spokesman” *reproduces the content* of a previous statement in his *own new words*. (4) A “relayer” repeats a *previous content* by using the *same formulation*.

**Table 1: Participation analysis: Function of active speakers**

Function of active speaker	New Content	Own new formulations
Author	+	+
Ghostee	+	-
Spokesman	-	+
Relayer	-	-

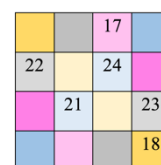
The participation analysis can show the responsibility of each learner for the *development* of the *content* and of the joint *social negotiation*. In the following, a short excerpt from the empirical data is presented, and analyzed using the “participation analysis” (Krummheuer, 2015). Building on this, references to the “domains of empowerment” (Ernest, 2001) are elaborated in the discussion.

### Results – Case study ‘Dustin’

In Sequence 1, Dustin worked on the task individually and with great interest and made some challenging mathematical discoveries. For this reason, he should work on the more complex level B in Sequence 2, together with his classmate Lenny. Before Sequence 2 begins, Dustin's school break is again characterized by social conflicts. Dustin gets into a fight with other children and, therefore, is late for Sequence 2. He enters the room, sits down besides Lenny, but then very quickly begins to immerse himself in the new discovery task: “*Find magic squares with the magic number 90. How do you do it? Give reasons together.*”

After the boys have found a 4x4 MS with Magic Number 90, they try to use the colored impulse card (Figure 4) to explain how it is possible to find such a 4x4 MS. Dustin thinks about what to write down and presumably refers back to the previous procedure by saying “We have” (turn 1). However, he is interrupted by Lenny (turn 2):

- 1 Dustin: What should we write? We have#
- 2 Lenny: #Wait, look, that is one, two, [taps on light yellow fields] one, two [taps on light blue fields].
- 3 Dustin: Yes, I wanted to say, [raises the impulse card (Figure 4)] here. We write, of each color we counted together and looked, that the result was forty-five. And then when we already had three [taps from top to bottom on



**Figure 4: Impulse card**

- the first three fields in the left column from their last previous attempt, in which they found a 4x4 MS with Magic Number 90]#*
- 4 Lenny: #Not forty-five!
- 5 Dustin: But it is!
- 6 Lenny: The half of the number. Fifty-five ehm forty-five is a half of ninety. And I can already guess that the next one we have to do, if we do a seventy MS next time, I think the#
- 7 Dustin: #Thirty-five.
- 8 Lenny: We have to calculate thirty-five. I can guess that.
- 9 Dustin: That's right, it certainly is.

The participation analysis can now be used to analyze the autonomy of actions and the type of mutual references between the two children on a content-related and social level. For this purpose, the extent to which and by whom a new content is introduced and how this content is continued or rejected is analyzed in each turn:

In turn 2, Lenny is the *author* as he contributes his own idea in his own words. He refers to two fields of the same color (light yellow, light blue) by pointing to them on the impulse card (Figure 4). Dustin responds as well as an *author* by introducing the use of colors to form the sum 45 as a new idea in his own words (turn 3). His wording indicates that he is following the previous intention to write down their done procedure (“We write”). Therefore, he possibly tries to name a strategy of their previous procedure, as he points to three numbers in a column in their found 4x4 MS (“And then when we already had three”, turn 3). In general, it would be possible in this way to determine the missing number using the magic number 90. However, Lenny interrupts him again. Lenny then takes on the role of a *ghostee* with linguistic reference to Dustin (“Not forty-five!”) by introducing a new content (turn 4). He denies that the sum is 45 and uses Dustin’s words for that. At this point a dispute arises (turn 4 & 5) and Lenny seems to have a request for clarification.

The children's statements in the transcript suggest that Lenny and Dustin are arguing on two different levels. Dustin continues to describe the *specific procedure* and states that they first determined the sum of 45 in the two fields of the series (turn 3). Lenny's statements, on the other hand, indicate that he would rather describe a *general structural relationship*: the sum of two fields in a row must always equal half of the magic number (turn 6). As a *spokesman*, Dustin refers to his own statement from turn 3 by reproducing its content in new words. These different levels of argumentation lead to a mathematical controversy between the two boys, which does not cause the breakdown of social cooperation, but leads to a meaningful clarification: In turn 6 Lenny acts as an *author* with reference to the controversy by introducing new content in his own words. To explain his statement, Lenny uses the magic number 70 as another example. Dustin takes up this new idea from Lenny and continues it as an *author* as well by introducing a new content: the sum of the pair of numbers in this case would be “Thirty-five” (turn 7). Dustin is referring directly to Lenny and develops an ambitious new idea further. As a *relayer* in turn 8, Lenny is taking content from his own statement (turn 6) as well as words from Dustin’s statement (turn 7). Dustin confirms the joint discovery in turn 9 as a *spokesman* by reflecting the content of turn 6 to 8 in his own words (“That’s right, it certainly is”). At this point, it is not yet possible to say whether Dustin has gained any real *structural* insight. However, it becomes clear that the children interact and cooperate with each other honestly on both a mathematical and social level, despite the dispute that arises.

## Discussion

The analysis of the case study provides initial indications that both children were given the opportunity to develop mathematical insights through joint negotiation, regardless of social difficulties outside of mathematics lessons. Concerning RQ 1 the numerous roles as “author” can be interpreted as an indication that the boys participate autonomously, because both Lenny and Dustin introduce new content with their own words. This leads to a deep mathematical communication and social participation in which references are made to each other's statements.

Because of the special design of the learning task (individually adaptive access and meaningful cooperative discovery task), both boys worked on a mathematically very demanding task and developed appropriate solution strategies together. The scene presented above indicates initial references to the three domains of empowerment (RQ 2): The boys manage to focus on new mathematical relations in the MS beyond summarizing their approach (*mathematical empowerment*; Ernest, 2001). These new mathematical insights are part of the social discussion. The participation analysis shows that both learners repeatedly make reference to previous statements made by the other, but develop them further in their own words and by introducing new ideas into the discourse. Even when a dispute arises, the discourse does not break off, but leads to new insights. This is remarkable because for Dustin it is often difficult to accept other opinions or rules in social situations (see interview above), which leads to conflicts with his classmates during breaks. The scene presented here provides initial indications that in this mathematical context Dustin is able to withstand both interruptions (turn 2) and counterarguments (turn 4-6) and to use them productively for his own insights (*social empowerment*; Ernest, 2001).

At this moment, further studies with additional case numbers are required to investigate and understand these initial indications in more detail. In addition, the extent to which empowerment on a mathematical and social level can contribute to strengthening children with social difficulties in such a way that the confidence in their own mathematical and social abilities is also strengthened (*epistemological empowerment*; Ernest, 2001) needs to be investigated.

## References

- Ainscow, M., Booth, T., Dyson, A., Farrell, P., Frankham, J., Gallannaugh, F., Howes, A., & Smith, R. (2006). *Improving Schools, Developing Inclusion*. Routledge. <https://doi.org/10.4324/9780203967157>
- Billigen, A.-M., Söbbeke, E., & Sprenger, L. (2023). Cooperation processes in inclusive learning settings with a special focus on mathematical potential. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 4540–4547). Alfréd Rényi Institute of Mathematics and ERME. <https://hal.science/hal-04409130>
- Dexel, T., Benölken, R., & Veber, M. (2019). Diversity, inclusion and the question of mathematics teacher education. How do student teachers reflect a potential-related view? In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *11th Congress of the European Society for*

*Research in Mathematics Education* (pp. 1862–1869). Freudenthal Group & Freudenthal Institute, Utrecht University and ERME. <https://hal.science/hal-02421247>

- Durlak, J. A., Weissberg, R. P., & Pachan, M. (2010). A meta-analysis of after-school programs that seek to promote personal and social skills in children and adolescents. *American journal of community psychology*, 45, 294–309. <https://doi.org/10.1007/s10464-010-9300-6>
- Ernest, P. (2001). Empowerment in Mathematics Education. In K. Y. Wong, H. T. Tairab, & M. A. Clements (Eds.), *Energising Science, Mathematics and Technical Education for All* (pp. 123–137). Universiti Brunei Darussalam.
- Häsel-Weide, U., & Nührenbörger, M. (2022). Inclusive math practices in primary school. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 4453–4460). Free University of Bozen-Bolzano and ERME. <https://hal.science/hal-03745438>
- Hirt, U., & Wälti, B. (2022). *Lernumgebungen im Mathematikunterricht. Natürliche Differenzierung für Rechenschwache bis Hochbegabte* [Learning environments in math lessons. Natural differentiation for the mathematically weak to the gifted]. Klett Kallmeyer.
- Krähenmann, H., Moser Opitz, E., Schnepel, S., & Stöckli, M. (2019). Inclusive Mathematics Instruction. In D. Kolloche, R. Marcone, M. Knigge, M. Godoy Penteadó, & O. Skovsmose (Eds.), *Inclusive Mathematics Education – State-of-the-Art Research from Brazil and Germany* (pp. 179–196). Springer Nature. <https://doi.org/10.1007/978-3-030-11518-0>
- Krummheuer, G. (2015). Methods for Reconstructing Processes of Argumentation and Participation in Primary Mathematics Classroom Interaction. In A. Bikner-Ahsbahr, C. Knipping, & N. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education* (pp. 51–74). Springer. [https://doi.org/10.1007/978-94-017-9181-6\\_3](https://doi.org/10.1007/978-94-017-9181-6_3)
- Lamnek, S. (2010). *Qualitative Sozialforschung* [Qualitative social research]. Beltz.
- Lindmeier, B., & Meyer, D. (2020). Empowerment, Selbstbestimmung, Teilhabe [Empowerment, self-determination, participation]. In D. Meyer, W. Hilpert, & B. Lindmeier (Eds.), *Grundlagen und Praxis inklusiver politischer Bildung* (pp. 38–56). Bundeszentrale für politische Bildung.
- Nührenbörger, M., & Pust, S. (2018). *Mit Unterschieden rechnen* [Calculating differences. Learning environments and materials for differentiated mathematics lessons]. 4. aktualisierte Auflage. Kallmeyer.
- Scherer, P., & Krauthausen, G. (2010). Natural Differentiation in Mathematics – the NaDiMa project. *Panama-Post*, 29(3), 14–26.
- Theunissen, G. (2016). Empowerment. In M. Dederich, I. Beck, G. Antor, & U. Bleidick (Eds.), *Handlexikon der Behindertenpädagogik* (pp. 114–116). Kohlhammer.