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Prescribed Exponential Convergence in Adaptive Observer Design with Applications

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Abstract: This work addresses the design of adaptive observer, where both the parameters and state variables of the system are estimated simultaneously. For this purpose, we consider a Linear Time-Invariant system with delay in the output channel, and we address the problem of the semi-autonomous adaptive cruise control system with the aim of estimating the cyberattack jointly with the system dynamics in the presence of transmission delay in the communication channel. The dynamics of the estimation errors are expressed by delay-differential equations, and its stability analysis makes use of the *multiplicity-induced-dominancy* property for spectral values of the error estimation equation. Numerical simulations are provided to show the convergence of the proposed observer in presence of delays to estimate the states and the cyber-attack jointly.

Keywords: adaptive observer, delay system, cyber-attack detection, connected vehicles, partial pole placement, frequency-domain, characteristic function.

1. INTRODUCTION

In many applications, the knowledge of system's states is crucial and aimed to design a controller or to obtain real-time information for decision-making and monitoring. However, in most of real-life scenarios, only partial information of the state is available, either due to physical constraints or the reduction in the number of sensors to reduce costs, or simply due to the absence of suitable ones. In such cases, estimation algorithms are required to process the incomplete and potentially limited data, enabling the reconstruction of a reliable estimate of the system's full state.

In this work, our first objective is to design an observer of a semi-autonomous adaptive cruise control (SA-ACC) vehicle allowing the simultaneous estimation of the states and cyber-attack on the communication channel in presence of measurement output delay. Up to now, the cyber-security problem of the intelligent vehicle networking system presents a significant security challenge, as its safety is vulnerable to various types of cyber-attacks (Ahmad et al., 2024). Therefore, identifying and reducing these attacks is essential to ensure the proper functioning of Connected autonomous vehicles (CAVs) and the maintenance of road safety (Khalil et al., 2020; Huang and Wang, 2022). To the best of the authors' knowledge, the presence of delay in such a system was not addressed in the literature. In our case, the delay occurs in the measurement

signal obtained via radar. In our opinion, such a design represents a novelty in the open literature. It is worth mentioning that the corresponding closed-loop system is infinite-dimensional (see, e.g. (Bellman and Cooke, 1963; Hale and Verduyn Lunel, 1993)), making the analysis more complicated. It is commonly known that the presence of a delay in a system model may lead to instability and bad performances (see, for instance, Niculescu (2001) and the references therein). The problem is even critical in the case of connected vehicle traffic, where safety and efficiency in road transport are qualitative criteria to be appropriately taken into account when modeling traffic flow.

In recent years, a new concept of pole placement has been proposed for controlling dynamical systems with delays, see, for instance, Mazanti et al. (2021); Boussaada et al. (2022). Entitled *partial pole placement* (PPP), this method is founded on two key spectral properties: (i) *multiplicity-induced dominancy* (MID) and (ii) *coexistent real characteristic roots induced dominancy* (CCRID), as discussed in Amrane et al. (2018); Bedouhene et al. (2020). The MID property simply asserts that, under specific conditions, a characteristic root with sufficiently high multiplicity dominates the remaining characteristic roots and, in other words, it becomes the spectral abscissa of the corresponding system defining the decay rate of the trivial solution, see Michiels and Niculescu (2014) and the references therein. By similarity, the CCRID property extends this concept by considering an appropriate number of (co-

existing) simple real roots rather than multiple ones, guaranteeing further robustness property. By focusing only on the rightmost spectral values, these properties significantly simplify the controller design, and lead to simpler control strategies that directly prescribe the dominant eigenvalue, effectively shaping the closed-loop system's exponential decay. In the sequel, we will use such an idea to address our adaptive cruise-control problem mentioned above. The use of the PPP approach in the design of observer was recently developed in Sassi et al. (2022, 2023, 2024) for the linear case and in Rojas-Ricca et al. (2022) for the nonlinear one. In this note, we extend those results with the design of adaptive observer which allows the joint estimation of the states and the unknown parameters.

The remaining of the paper is structured as follows. Some prerequisites and basic results from the literature are briefly presented in Section 2. In section 3, we introduce the SA-ACC system model. Next, in section 4, the proposed observer is developed and the stability of the estimation error is shown using a partial pole placement approach based on the MID property mentioned above. Finally, section 5 shows the numerical simulations of the proposed observer with a root location analysis of the characteristic function in function of the delay. The paper ends with conclusions. The notations are standard and explained upon first use.

2. PREREQUISITES

Consider the following linear time-invariant (LTI) delay system, represented by the following delay-differential equation (DDE):

$$\dot{\xi}(t) = A\xi(t) + A_d\xi(t-d), \quad (1)$$

under suitable initial conditions, where $\xi \in \mathbb{R}^n$ represents the state vector and d is a positive constant delay, and the constant matrices $A \in \mathbb{R}^{n \times n}$ and $A_d \in \mathbb{R}^{n \times n}$ of appropriate dimension.

It is well established that the asymptotic behavior of the solutions of (1) is determined by its spectrum (see, e.g., Bellman and Cooke (1963)), specifically the set of characteristic roots of the corresponding characteristic function. Thus, let us note $\Delta : \mathbb{C} \times \mathbb{R}_+ \rightarrow \mathbb{C}$ the characteristic function corresponding to (1) defined by

$$\Delta(s, d) = \det(sI_n - A - A_d e^{-ds}). \quad (2)$$

which can be written in a general form as

$$\Delta(s, d) = P_0(s) + P_1(s)e^{-dp}, \quad (3)$$

if the matrix A_d is of rank one. The polynomials P_0 and P_1 have real coefficients and their degrees are n and m respectively:

$$P_0(s) = s^n + \sum_{k=0}^{n-1} a_k s^k \quad \text{and} \quad P_1(s) = \sum_{k=0}^m b_k s^k \quad (4)$$

The following result from (Boussaada et al., 2024, Sec. A, p.6) gives the representation of the quasi-polynomial Δ in function of the so-called *Elimination-Produced Polynomial* (EP polynomial) denoted as $\mathbb{P}(\cdot, d)$.

Proposition 1. (Boussaada et al., 2024) Consider the quasi-polynomial Δ from (3) with $m = n - 1$. The real number s_0 is a root of multiplicity at least $n + 1$ of Δ if and only if $\mathbb{P}(s_0, d) = 0$ and

$$\Delta(s) = \frac{d}{n!} (s - s_0)^{n+1} \int_0^1 \mathbb{P}(s_0, td) e^{-(s-s_0)dt} dt$$

where \mathbb{P} is the EP polynomial given by

$$\mathbb{P}(s_0, dt) = \sum_{k=0}^n \binom{n}{k} P_0^{(k)}(s_0)$$

It should be noted that a quasi-polynomial admitting a characteristic root with intermediate (over-order) multiplicity necessarily shares its remaining roots with an appropriate linear combination of Kummer functions, as shown in Boussaada et al. (2024). Unfortunately, to the best of the authors' knowledge, there does not exist any general result in the open literature to describe the *distribution* of the *non-asymptotic zeros* of such a function combination.

A numerical-oriented approach has been proposed in Benarab et al. (2023) leading to an effective algorithm that we are explicitly using in our framework. In this sense, we first establish conditions on the system's parameters which guarantee the existence of a multiple root. Second, perform an affine change of variable in the characteristic equation $\Delta(s, d)$ of equation (3) in order to reduce the corresponding quasipolynomial to a normalized form: $\hat{\Delta}(z) = \hat{P}_0(z) + \hat{P}_1(z)e^{-z}$. Next, we derive a bound on the imaginary part of roots of the normalized characteristic function in the complex right half-plane. Lastly, a certification of the dominance of the multiple root is demonstrated. Algorithm 1, given in the appendix lists the steps to be followed to reach a suitable frequency bound (see Benarab et al. (2023); Boussaada et al. (2024)).

3. SA-ACC MODEL UNDER CYBER-ATTACK AND MEASUREMENT DELAY

In this section, we give the description of the SA-ACC system in terms of state space model as described in the literature and for which we inject the cyber-attack as unknown parameter affecting the dynamics. Then, we inject some transmission delay in the communication channel, i.e. in the measurement vector.

3.1 System model

We start by introducing the system model which was the subject of some contributions Rajamani and Zhu (2002); Jeon et al. (2020) without delay. Then, we consider that the system presents some transmission delays in the measurement output.

The considered system is a SA-ACC vehicle whose behavior is represented by the following equation when actuator dynamics are modeled as a first-order lag,

$$u_{sync} = \tau \dot{\ddot{x}}_i - \ddot{x}_i \quad (5)$$

where u_{sync} is the control input of the i^{th} vehicle, τ is a lag constant associated with the lower dynamics of the vehicle. As x_i represents the i^{th} vehicle position, then one can deduce easily that \dot{x}_i is the i^{th} vehicle velocity and \ddot{x}_i is the i^{th} vehicle acceleration (the rate of acceleration change).

If the SA-ACC vehicle consistently maintained a fixed distance from the preceding vehicle, the spacing error for the i^{th} vehicle in a string of vehicles would be given by

$$\varepsilon_i = x_i - x_{i-1} + L_i \quad (6)$$

where L_i is the constant part of the desired spacing distance between vehicles. Therefore, in the constant time-gap spacing policy, the desired spacing between vehicles is not fixed but varies linearly with respect to the speed:

$$\varepsilon_{i,desired} = L_i + h\dot{x}_i \quad (7)$$

where h is a time gap. Then, the spacing error in the constant time gap can be stated as follows

$$\tilde{\varepsilon}_i = \varepsilon_i + h\dot{x}_i = x_i - x_{i-1} + L_i + h\dot{x}_i \quad (8)$$

Hence, using equation (8), the controller is described by

$$u_{syn} = -k_1\ddot{x}_{i-1} + (k_1 + hk_1k_2)\ddot{x}_i - \frac{(1 - hk_1k_2)}{h}\dot{\varepsilon}_i - \frac{k_2}{h}\varepsilon_i - k_2\dot{x}_i \quad (9)$$

As illustrated in Fig. 1, the radar-measured gap between two vehicles can be defined as

$$\delta_i = x_{i-1} - x_i - l_{i-1} \quad (10)$$

and its derivatives are obtained as follows (l_{i-1} is constant),

$$\dot{\delta}_i = \dot{x}_{i-1} - \dot{x}_i \quad (11a)$$

$$\ddot{\delta}_i = \ddot{x}_{i-1} - \ddot{x}_i \quad (11b)$$

$$\ddot{\delta}_i = \ddot{x}_{i-1} - \ddot{x}_i \quad (11c)$$

Since the acceleration information of the preceding vehicle can be obtained via wireless communication, we assume that $\ddot{x}_{i-1} = 0$, which involves that the expression of $\ddot{\delta}_i$ in (11c) becomes $\ddot{\delta}_i = -\ddot{x}_i$. Then, equalizing equations (5) and (9) and using the expressions (6) and (11), we can easily obtain the following equation

$$\ddot{\delta}_i = \frac{1}{\tau}(1 - hk_1k_2)\ddot{x}_{i-1} - \frac{1}{\tau}(1 - k_1 - hk_1k_2)\ddot{\delta}_i - \frac{k_2}{\tau h}\delta_i - \frac{1}{\tau h}(1 - hk_1k_2)\dot{\delta}_i + \frac{k_2}{\tau}\dot{x}_i + \frac{k_2}{\tau}(L_i - l_{i-1}) \quad (12)$$

Note $x = [\delta_i \ \dot{\delta}_i \ \ddot{\delta}_i]^T$ and rewrite as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + Fa_{i-1} + \Delta \quad (13a)$$

$$y(t) = Cx(t) \quad (13b)$$

where the matrices A , B , F and Δ are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{-k_2}{\tau h} & \frac{-k_3}{\tau h} & \frac{k_1 - k_3}{\tau} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{k_2}{\tau} \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ \frac{k_3}{\tau} \end{bmatrix},$$

$$\Delta = \begin{bmatrix} 0 \\ 0 \\ \frac{k_2(L_i - l_{i-1})}{\tau h} \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

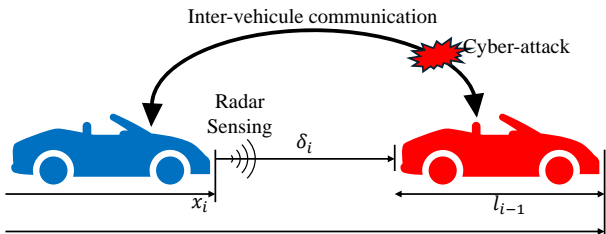


Fig. 1. Inter-vehicle communication between following and preceding vehicle.

3.2 Cyber-attacks and delay injection

In this paragraph, we incorporate transmission delays in the measurement vector and false signal resulting from any cyberattack.

Feedback Delay injection: In the system model described above, we note that the available measurements are distance δ_i and velocity $\dot{\delta}_i$, obtained through radar. Therefore, we introduce some transmission delay into the measurement vector. Then, equation (13b) becomes

$$y(t) = Cx(t - d) \quad (14)$$

where d is the transmission delay.

Cyber-attack injection: In order to inject false data attack in the model, we define the variable μ as the signal derived from inter-vehicle communication as follows

$$\mu(t) = \ddot{x}_{i-1} \quad (15)$$

It is shown in Jeon et al. (2020) that the cyber-attack can be represented by false data attack or a congestion in the inter-vehicle communication channel. Both of those cyber-attack classes can be described by the following expression.

$$\mu(t) = \ddot{x}_{i-1} + f_c \quad (16)$$

where f_c is the cyber-attacks signal which is assumed to be slowly varying or constant so that $\dot{f}_c = 0$.

Now, we remodel the SA-ACC system using the expression of μ given in (16) instead of (15) and we recalculate the expression of $\ddot{\delta}_i$ as done in equation (12). Therefore, we obtain the following equation

$$\ddot{\delta}_i = \frac{1}{\tau}(1 - hk_1k_2)\mu - \frac{1}{\tau}(1 - k_1 - hk_1k_2)f_c - \frac{1}{\tau}(1 - k_1 - hk_1k_2)\ddot{\delta}_i - \frac{1}{\tau h}(1 - hk_1k_2)\dot{\delta}_i - \frac{k_2}{\tau h}\delta_i + \frac{k_2}{\tau}\dot{x}_i + \frac{k_2}{\tau}(L_i - l_{i-1}) \quad (17)$$

Then, using the vector x , the system can be written with a state space model as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + F\mu(t) + \Delta - Gf_c \quad (18a)$$

$$y(t) = Cx(t - d) \quad (18b)$$

where the matrices A , B , F , Δ and C are the same as in system (13). The matrix G is given by

$$G^T = \begin{bmatrix} 0 & 0 & \frac{k_3 - k_1}{\tau} \end{bmatrix}$$

4. ADAPTIVE OBSERVER DESIGN

In this work, we propose to design a full order adaptive observer for the above described time delay system (18) which allows to estimate jointly the system states vector $x(t)$ and the unknown input f_c , i.e in this context the cyber-attack, while the feedback signal presents some transmission delays. The proposed observer is given by the following system.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\mu(t) + \Delta - G\hat{f}_c + L(y(t) - \hat{y}(t)) \quad (19a)$$

$$\hat{y}(t) = C\hat{x}(t - d) \quad (19b)$$

$$\dot{\hat{f}}_c(t) = Q(y(t) - \hat{y}(t)) \quad (19c)$$

where \hat{x} and \hat{f}_c are the estimates of the states vector x and the cyber-attack f_c , respectively. The matrices L and Q are the observer gains.

The aim is to determine the observer gains L and Q such that the estimation of the states \hat{x} and of the unknown parameter \hat{f}_c converges to their real values x and f_c . For

this purpose, we note e_x the states estimation error and e_f the unknown input estimation error described by

$$e_x(t) = x(t) - \hat{x}(t)$$

$$e_f(t) = f_c - \hat{f}_c(t)$$

Then, the problem becomes one of guaranteeing the convergence of the errors estimation to zero. Therefore, we compute their dynamics, and obtain:

$$\dot{e}_x(t) = Ae_x(t) - Ge_f(t) - LCe_x(t-d) \quad (20a)$$

$$\dot{e}_f(t) = -QCe_x(t-d) \quad (20b)$$

Due to the coupling effects of states and unknown inputs in the system dynamics, we combine the errors into one vector $e(t) = [e_x(t)^T \ e_f(t)^T]^T$, and we rewrite system (20) under an augmented system as follows

$$\dot{e}(t) = \mathbf{A}e(t) + \mathbf{A}_d e(t-d) \quad (21)$$

where

$$\mathbf{A} = \begin{bmatrix} A & -F \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{A}_d = \begin{bmatrix} -LC & 0 \\ -QC & 0 \end{bmatrix}$$

Thus, this work proposes conditions on the gain matrices L and Q for which the system (21) is stable.

Hypothesis 2. $\text{rank}(\mathbf{A}_d) = 1$.

Based on the above assumptions, we can analyze the stability of the delayed estimation error using the MID property. Finally, Assumption 2 provides a sufficient condition to ensure that Δ involves a single delay.

Remark 3. As the matrix \mathbf{A}_d can be rewritten as:

$$\mathbf{A}_d = \begin{bmatrix} -L \\ -Q \end{bmatrix} [C \ 0]$$

one can use the property of matrix rank so that the considered assumption becomes

$$\text{rank}(\mathbf{A}_d) = \min(\text{rank}([-L^T \ -Q^T]^T), \text{rank}([C \ 0])).$$

Then, using the above rank property, we have:

- (1) Case 1 : If the system present one measurement, so the matrix C is a one rank matrix. There is no assumptions to consider.
- (2) Case 2 : Otherwise, we need to assume that the matrix $[-L^T \ -Q^T]^T$ is of rank 1.

To study the convergence of the error estimation system given by (21), one can see that equation (21) is with the form of equation (1), where the corresponding characteristic function is as equation (2) and can be expressed as follows for the considered delayed SA-ACC system

$$\Delta(s, d) = P_0(s) + P_1(s)e^{-sd} \quad (22)$$

where

$$P_1(s) = \ell_1 s^3 - \frac{(k_1 \ell_1 - k_3 \ell_1 - \tau \ell_3)}{\tau} s^2 - \frac{(hk_1 \ell_3 - hk_3 \ell_3 - h\tau \ell_5 - k_3 \ell_1)}{\tau h} s - \frac{k_1 q_1 - k_3 q_1}{\tau},$$

$$P_0(s) = s^4 - \frac{(k_1 - k_3) s^3}{\tau} + \frac{k_3 s^2}{\tau h} + \frac{k_2 s}{\tau h}$$

Note that the one rank condition is applied on the gain matrix $[-L^T \ -Q^T]^T$ which need to be of rank one, since the matrix C is of rank 2 (> 1 , see remark 3), such that:

$$[-L^T \ -Q^T]^T = \begin{bmatrix} -\ell_1 & -\ell_3 & -\ell_5 & -q_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Remark 4. We can also opt for a gain matrix

$$[-L^T \ -Q^T]^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\ell_2 & -\ell_4 & -\ell_6 & -q_2 \end{bmatrix}$$

and we obtain a similar characteristic function with the same degree of the polynomials P_0 and P_1 . Another alternative is to consider the two column of the gain matrix $[-L^T \ -Q^T]^T$, linearly independent which also conduct to the same degree of P_0 and P_1 .

Let s_0 denote a real root of Δ given by (22) with multiplicity at least $n+1$. It is known that if $s_0 < 0$ is the rightmost root, then the zero solution of system (21) is exponential stable, implying the stability of the estimation error.

Theorem 5. Consider that Assumptions 2 holds and that Δ in (22) admits a root s_0 with multiplicity at least $n+1$. Then, s_0 is a dominant root of (22) if \mathbb{P} admits as real dominant root s_0 and $\mathbb{P}(s, t\tau)$ is of constant sign. Furthermore, if \mathbb{P} is Hurwitz then the estimation error given by (21) is exponentially stable with s_0 as decay rate and the estimation states vector and the unknown input (cyber-attack) converge to their real values.

Proof. Assumption 2 coupled with Proposition 1 allow the integral representation of the error characteristic quasipolynomial function (22) with an EP polynomial

$$\mathbb{P}(s, d) = d^4 s^4 + [4d + 16] s^3 d^3 + [6d^2 + 48d + 72] s^2 d^2 + [2d^3 + 48d^2 + 144d + 96] sd + [8d^3 + 72d^2 + 96d + 24].$$

Next, thanks to the sign constancy of $\mathbb{P}(s, dt)$ for all $t \in (0, 1)$ and the frequency bound established from Algorithm 1 results the dominance of s_0 . \square

5. NUMERICAL RESULTS

To show the effectiveness of the proposed observer, a numerical simulation is presented in this section. The numerical parameters of the SA-ACC system are $k_1 = -1$, $k_2 = 0.4$, $h = 0.5$, $L_i = 7$, $l_{i-1} = 5$, $\tau = 0.4$ and $k_3 = 1.2$. Then, matrices \mathbf{A} and \mathbf{A}_d described in (21) are given by:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & -6 & -4 & -\frac{11}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{A}_d = \begin{bmatrix} -l_1 & -l_2 & 0 & 0 \\ -l_3 & -l_4 & 0 & 0 \\ -l_5 & -l_6 & 0 & 0 \\ -q_1 & -q_2 & 0 & 0 \end{bmatrix}$$

By Assumption 2, the matrix \mathbf{A}_d has rank one. If we set l_2, l_4, l_6 and q_2 to 0, $\Delta(s, d)$ rewrites as:

$$\Delta(s, d) = s^4 + 4s^3 + 6s^2 + 2s + e^{-sd} \times \left(l_1 s^3 + (4l_1 + l_3) s^2 + (6l_1 + 4l_3 + l_5) s - \frac{11q_1}{2} \right). \quad (23)$$

Remark 6. In the lines of Remark 3, consider that the rank of the matrix $[-L^T \ -Q^T]^T$ is one due to the rank of the matrix C which is equal to 2. If the system presents only one measurement, for example, we measure only the distance δ_i via radar. So the matrix C is given as $C = [1 \ 0 \ 0]$ yielding to a matrix \mathbf{A}_d of the same form as above with rank equal to one (null gains l_2, l_4, l_6, q_2).

We note that s_0 is a root with multiplicity $n+1 = 4$ if, and only if, s_0 is a root of the EPP, defined in function of s and d as follows

$$\mathbb{P}(s, d) = d^4 s^4 + [4d + 16] s^3 d^3 + [6d^2 + 48d + 72] s^2 d^2 + [2d^3 + 48d^2 + 144d + 96] sd + [8d^3 + 72d^2 + 96d + 24].$$

Figures 3 and 4 show the evolution of the real and the imaginary part of the elimination-produced polynomial zeros as a function of the delay d . We note that the discriminant EPP vanish at $d^* = 2.87$. Then, in the interval $d = 0, \dots, d^*$, the EPP is real-rooted and hurwitz. However, for $d > d^*$, the EPP still hurwitz but admits two real roots and two complex conjugate roots accordingly to the the sign of the characteristic polynomial which becomes negative at d^* . From figure 3, we see that the dominated root s_0 (blue curve) is always negative whatever the value of the delay d .

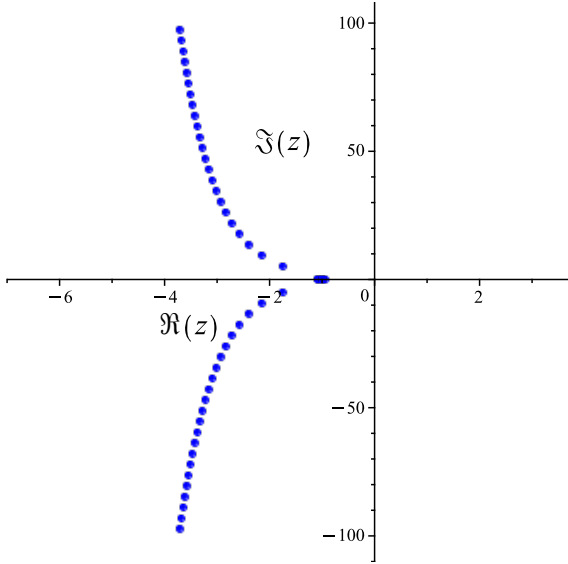


Fig. 2. The spectrum distribution of the quasipolynomial (23).

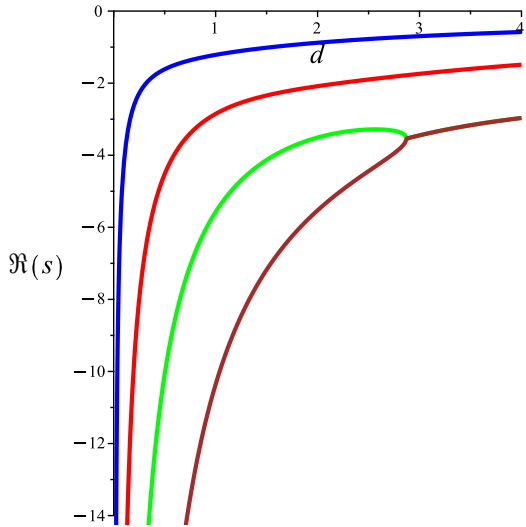


Fig. 3. The real parts of the four roots branches of the elimination-produced polynomial \mathbb{P} .

For a chosen delay $d = 3/2$ and for $s_0 = -1.01$, one gets:

$$L = \begin{bmatrix} 0.37 & 0 \\ 0.04 & 0 \\ -0.34 & 0 \end{bmatrix} \quad \text{and} \quad Q = [-0.12 \ 0]$$

Then, using a Matlab simulation, we aim to show the convergence of the states and cyber-attack estimation to their real values. The input signal, $u = x_i$ the i^{th}

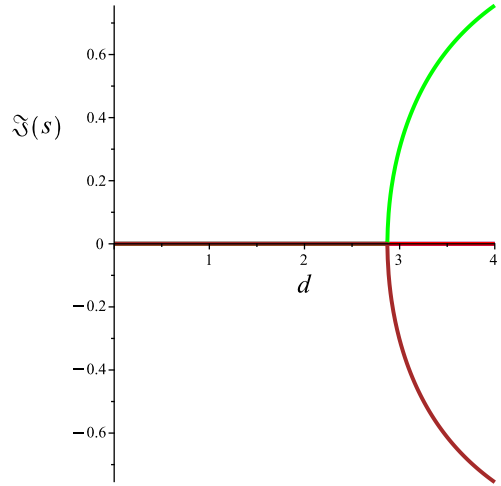


Fig. 4. The imaginary parts of the four roots branches of the elimination-produced polynomial \mathbb{P} .

vehicle position, is considered as a sinusoidal signal and the transmitted signal μ as pulse signal. The initial conditions are considered as $x(0) = [5 \ 20 \ 0]^T$ and for the observer $\hat{x}(0) = [0 \ 0 \ 0]^T$. The cyber-attack occurs after 20s and goes from 0 to 1.

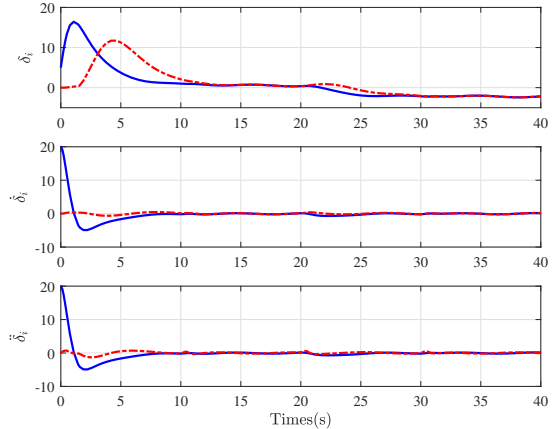


Fig. 5. Convergence of the states estimation \hat{x} in red(---) to their real values x in blue (—).

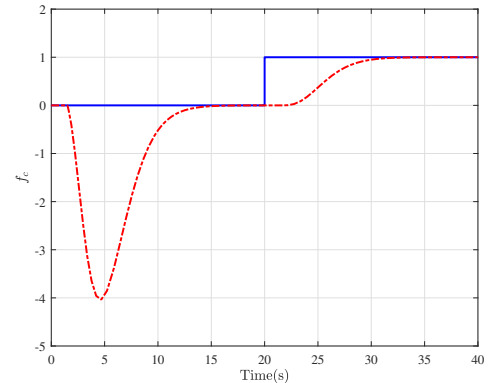


Fig. 6. Convergence of the estimation of the cyber-attack signal f_c in red (---) to its real value f_c in blue (—).

Figure 5 shows the convergence of the states estimation to their real values, and Figure 6 the evolution of the estimation of the cyber-attack occurring at $t = 20$ s and converging to its real value.

6. CONCLUSION

In this paper, an adaptive observer for linear time delay systems has been developed, allowing the joint estimation of the states and the unknown input. in the context of a SA-ACC system with a transmission delay in the measurement channel, for which the observer is able to estimate the state and the cyber-attack which can affect the communication signal between vehicles. Using a pole placement approach based on a MID property, the stability of the estimation error has been proven. Theoretical results were validated by the numerical simulations showing the effectiveness of the approach.

APPENDIX: MID FREQUENCY BOUND

Algorithm 1 Estimation of the MID frequency bound in delay-differential equations (DDEs) with single delay

Require: $\tilde{\Delta}(z) = \tilde{P}_0(z) + \tilde{P}_1(z)e^{-z}$ // Normalized quasipolynomial
// Initialization
ord = 0
// ord: order of truncation of the Taylor expansion of
// $e^{2x} = \underbrace{1}_{\text{ord}=0} + 2x + 2x^2 + \frac{4x^3}{3} + \dots$
ord=1
dominance = false
 $\exists z_0 = x + i\omega \in \mathbb{R}_+^* + i\mathbb{R}_+^*$ s.t. $\tilde{\Delta}(z_0) = 0$
 $|\tilde{P}_0(x + i\omega)|^2 e^{2x} = |\tilde{P}_1(x + i\omega)|^2$
while \sim dominance **do**
ord = ord+1
 $F(x, \omega) = |\tilde{P}_1(x + i\omega)|^2 - |\tilde{P}_0(x + i\omega)|^2 T_{ord}(e^{2x}) > 0$
// $T_{ord}(e^{2x})$: Taylor expansion of e^{2x} of order = ord
 $\omega^2 = \Omega$
 $H(x, \Omega)$ // The polynomial characterizing the real roots of F
end while
 $\Omega_k(x)$ // k^{th} real root of H , depend on free parameters
if $\max_x(\max_k(\Omega_k(x))) < \pi^2$ **then**
dominance = true
end if
return Frequency bound

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