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## COMBINING IMPORTANCE SAMPLING WITH MACHINE LEARNING TO ACCELERATE STRUCTURAL RELIABILITY ANALYSIS

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### ABSTRACT

*In reliability analysis of linear systems subjected to random excitation, an important issue is to determine the first-excursion probability. When this probability is very small, general Monte Carlo simulations (MCS) are always computationally impractical. In deterministic case where the structural properties are constant, advanced MCS such as importance sampling (IS) can be employed to improve the calculation efficiency. This results in the so-called conditional failure probability (CFP). However, when the uncertainties in structural properties are high-dimension, the IS technique still demands a large number of analysis of CFPs. Recent studies show that, machine learning (ML) theories can be used to build effective surrogates that maps the structural uncertainties into CFPs. In this study, the structural reliability, denoted as first-excursion probability, is evaluated by a more efficient technique that combines IS with ML. Besides, Karhunen–Loève (K-L) expansion is implemented to denote the stochastic excitation. The structural responses are calculated by convolution integral with respect to impulse responses. Numerical simulations show that the proposed method can effectively accelerate the evaluation process, meanwhile achieving the accuracy close to general MCS.*

## 1 INTRODUCTION

Reliability mechanical structures describes the probability that the object realizes its functions under given conditions for a specified time period [1]. For a mechanical structure subjected to stochastic excitation, an important task is to evaluate the structural failure probability. In fact, due to various factors (manufacture, environmental, fatigue...), the structural properties become uncertain. Hence, the assessment of failure probability involves a high-dimension problem considering the uncertainties in both structural parameters and the excitation. Direct MCS demand a huge number of samples to ensure a high accuracy of the estimated CFP, but this will be very time-consuming. In this aspect, surrogate models become a good alternative. Response surface method (RSM) [2] is among the most popular surrogate models. Support vector machine (SVM) has also gained much attention recently [3]. Other surrogate models such as metamodel, ANNs and Kriging have been proposed by other researchers [[4], [5], [6]]. The disadvantages in ANNs mainly include the complex architecture optimization, low robustness and enormous training time. SVM is time consuming for large-scale applications and sometimes shows large error in sensitivity calculation of reliability index. RSM has been popular however it may be time-consuming to use the polynomial function when the number of random variables is large.

In the authors' viewpoint, the evaluation of CFP can be seen as a regression problem that takes a realization of the structural uncertain properties as input and the CFP as output. To improve reliability assessment considering structural uncertainties, attention is paid to non-parametric statistical learning methods in this research. The rest of the paper is organized as follows. In Section 2, the theories pertaining to CFP and its estimation are presented. In Section 3, the framework of failure probability estimation based on ML models is introduced, and numerical simulations on different structures are applied and discussed.

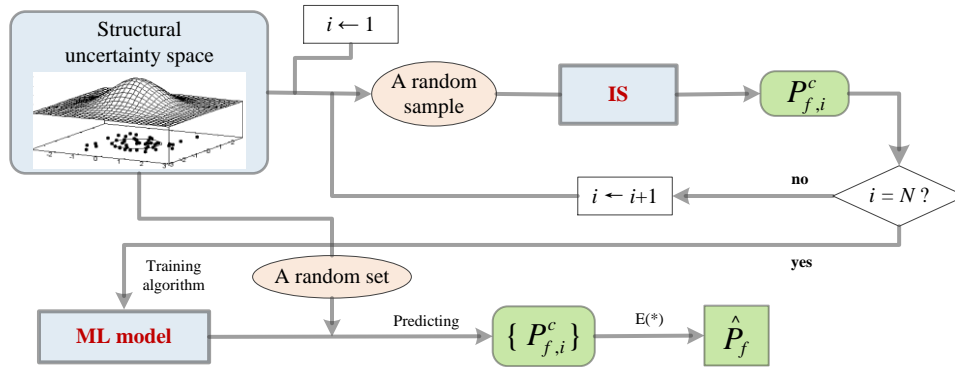


Figure 1. Theoretical framework to estimate failure probability by ML model

## 2 THEORETICAL FRAMEWORK

A basic assumption in this research is that the uncertainties of the structural properties and those of the excitation are relatively independent. Mathematically, the failure probability is determined by a multi-dimension integral over the joint distribution of all variables involved, *i.e.*

$$P_f = \int_{g(\mathbf{x}, \mathbf{z}) \leq 0} p_X(x)p_Z(z)d\mathbf{x}d\mathbf{z} = \int_{\mathbf{x} \in \Omega_X, \mathbf{z} \in \Omega_Z} I_f(\mathbf{x}, \mathbf{z})p_X(x)p_Z(z)d\mathbf{x}d\mathbf{z} \quad (1)$$

where  $I_f(\mathbf{x}, \mathbf{z})$  is the indicator function which equals to 1 when the performance function  $g(\mathbf{x}, \mathbf{z}) \leq 0$  and 0 otherwise. It is noted that the calculation in Eq. (1) considers uncertainty in

both structural parameters and excitation. To construct the CFP, eq. (1) is rewritten as

$$P_f = \int_{\mathbf{x} \in \Omega_X} \left[ \int_{\mathbf{z} \in \Omega_Z} I_f(\mathbf{z}|\mathbf{x}) p_Z(\mathbf{z}) d\mathbf{z} \right] p_X(\mathbf{x}) d\mathbf{x} = \int_{\mathbf{x} \in \Omega_X} P_f^c(\mathbf{x}) p_X(\mathbf{x}) d\mathbf{x} = E_{\mathbf{x} \in \Omega_X} [P_f^c(\mathbf{x})] \quad (2)$$

$E[\cdot]$  denotes the mathematical expectation. By eq. (2), a ML model can be built that takes the random samples of structural properties as input and CFPs as outputs. Once the ML model is built, it is employed to make predictions on the samples outside the sample set used to train the ML model. The expectation of these predictions is seen as the structural failure probability considering both kinds of uncertainties. Figure 1 illustrates the theoretical framework.

As is known that the estimations by direct MCS can be treated as a standard reference. However, it is not computationally efficient for estimating very small failure probabilities (less than 1%) since the number of samples required to achieve a given accuracy is inversely proportional to the scale of  $P_f$ . In view of this, the IS method a good choice to compute small  $P_f$  values. In principle, an IS method tries to adjust the sampling density so that more samples are obtained from the failure region  $F$ . Figure 2 shows the procedures to apply IS [7] to estimate CFPs.

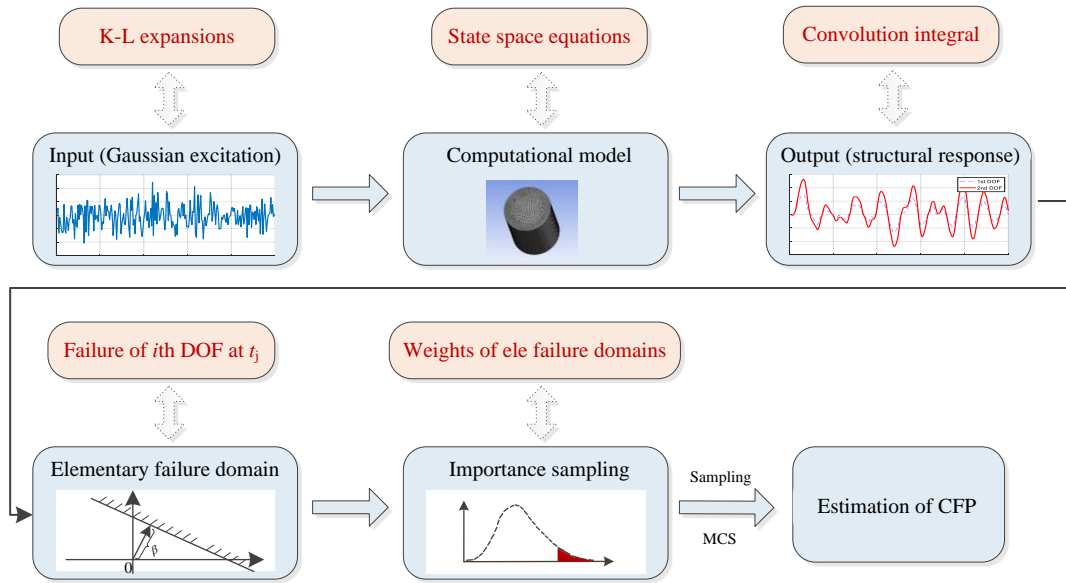


Figure 2. IS to estimate CFP

### 3 A BENCHMARK STUDY

As a benchmark problem introduced in G.I. Schueller [8], the ten-DOF Duffing type oscillator has been widely used by researchers in structural reliability domain. In this study, we focus on the linear random structures under stochastic excitation. The statistical properties of the structural parameters and their constraints are shown in [8]. In this simulation, the estimated failure probability values are very small (below  $10^{-4}$ ), therefore IS is used to estimate CFPs. Three ML models Random Forest (RF) [9], Gradient Boosting (GB) [10] and Extra-trees (ETs) [11] are employed to estimate the failure probability. The three methods are compared with standard MCS, see Table 1. In the simulations, it is found that the ensemble learning based methods all reach a convergence at the point  $n_{sample} = 500$ . This benchmark study tells that

Table 1. Reliability estimatios by different ML-based methods

Method	1stDOF	1stDOF	10thDOF	10thDOF
	$0.057m$	$0.073m$	$0.013m$	$0.017m$
<b>Standard MCS</b>	$1.06e^{-4}$	$8.07e^{-7}$	$4.88e^{-5}$	$2.52e^{-7}$
<i>num of samples</i>	$2.98e^7$	$2.98e^7$	$2.98e^7$	$2.98e^7$
<b>RF-based</b>	$7.6e^{-5}$	$1.0e^{-6}$	$4.2e^{-5}$	$1.1e^{-7}$
<i>num of samples</i>	500	500	500	500
<b>GB-based</b>	$8.48e^{-5}$	$9.15e^{-7}$	$4.24e^{-5}$	$1.06e^{-7}$
<i>num of samples</i>	500	500	500	500
<b>ETs-based</b>	$8.73e^{-5}$	$9.89e^{-7}$	$4.09e^{-5}$	$1.10e^{-7}$
<i>num of samples</i>	500	500	500	500

the combination of IS and ML models can largely improve the efficiency of failure probability estimations.

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