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Measurement bias estimation and dynamic consensus of controllable systems over directed graphs

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Abstract: Dynamic consensus pertains to the case in which a group of interconnected systems adopt a consensual dynamic behaviour; this covers in particular the more common consensus problem in which case all the systems admit a stable common equilibrium point. In this article we tackle the problem of dynamic consensus for systems under measurement bias. The latter may result, *e.g.*, from sensor faults or attacks, but it is assumed that bias is constant. Under this condition we design a nonlinear adaptive estimator that successfully estimates the bias, so the controller ensures dynamic consensus. Our results are designed for linear controllable systems, but the class of systems that we consider is broader than in the available literature since, in particular, it covers systems with singular matrices (*e.g.*, chains of integrators). The only assumption on the graph topology is that it is directed and *contains* a spanning tree. For illustration purpose, we address a case-study of formation control of a group of satellites, whose orbital relative motion is modelled as per Clohessy-Wiltshire equations, under the assumption that the measurements are biased.

Keywords: Dynamic consensus, multi-agent systems, Lyapunov stability, sensor bias

1. INTRODUCTION

Diverse consensus problems both for linear and nonlinear network-interconnected systems are available in the literature (Li and Tan, 2019), but under the assumption that the measurements are accurate or, eventually, are affected by faults (Qin et al., 2016). However, in typical engineering systems sensors are susceptible to faults. This may entail unwanted constant or time-varying measurement offsets, which in turn may cause severe accidents (Choi and Lee, 2020). These kinds of sensor faults are technically called sensor biases, and are quite common in applications involving, *e.g.*, robotic and aerospace systems— see the discussions in (Metni et al., 2005) and (Shi et al., 2020). Measurement bias, especially if time-varying, may also be caused by false injection attacks (FDI) perpetuated by malicious agents interfering in a network.

Several works on identification and compensation of sensor bias have been proposed over the last decade—see *e.g.*, (Shi et al., 2020; Sukumar et al., 2018; Dutta et al., 2024), or (Meng et al., 2020; Modares et al., 2018; Gao et al., 2019) on the related problem of false-data injection. Both leader-follower and leaderless consensus have been addressed, for systems interconnected over undirected (Shi et al., 2020) or directed networks (Meng et al., 2020; Modares et al., 2018; Gao et al., 2019; Sukumar et al., 2018; Dutta et al., 2024). Controlling multi-agent systems over directed networks is particularly challenging due to the non-symmetry. Yet, they are useful to represent several

real-world communications such as power transmission lines or social networks. Moreover, unilateral flow of information reduces the communication bandwidth of the network topology.

It is important to remark that even though some of the solutions to consensus problems available in the literature include the construction of Lyapunov functions, even in fairly general scenarii (Moreau, 2005), and it is a fairly standard technique of nonlinear control to rely on strict Lyapunov functions to redesign controllers to cope with additive disturbances (Khalil, 2002), measurement bias cannot be tackled with such techniques since the perturbation, which enters directly through the measurement, is amplified by the control gains. The bias must be estimated to be compensated for. However, for the same reason, bias estimation is not a straight forward extension of standard adaptive control techniques (Boyd and Sastry, 1986; Roy et al., 2017; Glushchenko et al., 2022; Aranovskiy et al., 2022).

In this manuscript, we propose a certainty-equivalence dynamic consensus controller for controllable linear systems that, implemented with a distributed nonlinear estimator ensures the achievement of dynamic consensus in spite of the presence of constant heterogeneous sensor biases. Even though, similar problems have been studied and are well-established, we overcome some of the limitations in the available works. Unlike the leader-follower setups in (Meng et al., 2020; Modares et al., 2018) where the

leader information is not prone to attacks, *i.e.*, they are unbiased; we assume a more realistic situation where all the exchanged information, regardless of the hierarchy of a node in the directed network, are transmitting biased data. Paper (Gao et al., 2019) covers leaderless consensus among agents interacting through directed graphs such that all the transmitted information are affected by identical multiplicative biases. The papers (Sukumar et al., 2018) and (Dutta et al., 2024) cover estimation of constant heterogenous additive biases in the measurements with the parallel requirements of leaderless dynamic consensus among linear systems which are interacting via directed network containing a spanning tree. However, they are limited in the sense that the former establishes uniform ultimate boundedness with respect to the synchronisation manifold while the class of systems in the latter is narrow since it excludes systems with singular system matrices, as for instance, in innocuous chain of integrators, which models many physical systems. Compared to the work in (Sukumar et al., 2018), we establish asymptotic stability with respect to the consensus manifold and opposed to (Dutta et al., 2024), we do not need non-singularity of the system matrix.

2. PROBLEM FORMULATION

We consider N homogeneous, linear dynamical autonomous systems modelled as per the equation

$$\dot{x}_i = Ax_i + Bu_i, \quad \forall i = \{1, 2, \dots, N\}, \quad (1)$$

where $x_i \in \mathbb{R}^n$ are the states, $u_i \in \mathbb{R}^p$ are the control inputs to be designed and A, B are constant matrices of suitable dimensions.

We are interested in establishing dynamic consensus (Panteley and Loría, 2017), *i.e.*, ensuring that

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i \neq j \in \{1, 2, \dots, N\} \quad (2)$$

—note that this does not imply that $x_i \rightarrow \text{constant}$ —for these systems via distributed control, that is, control laws $(t, y_i, y_j) \mapsto u_i(t, y_i, y_j)$ that depend on the measurement of the actual agent, y_i , and of those received by from its neighbours, y_j . These consist in the respective states of the i th and j th system, but with the *caveat* that the said measurements are affected by constant biases δ_i and δ_j respectively. That is, $y_i := x_i + \delta_i$ for all $i \leq N$; δ_j being defined similarly, for all $j \in \mathcal{N}_i$, where \mathcal{N}_i denotes the set of indexes pertaining to the agents that send information to the system i over the network.

In addition, the control input u_i is expected to depend on time because it is dynamic, it relies on a bias estimator. That is, in addition to achieving the dynamic consensus goal (2), the objective is to design distributed bias estimators that produce estimates $\hat{\delta}_i$ such that

$$\lim_{t \rightarrow \infty} \|\tilde{\delta}_i(t)\| := \lim_{t \rightarrow \infty} \|\delta_i - \hat{\delta}_i(t)\| = 0, \quad (3)$$

where $\hat{\delta}_i$ is the bias estimate for the unknown bias δ_i .

Now, it is assumed that the i th agent receives the measurements y_j from neighbouring systems through a communications network with *directed* topology. This can be defined by a directed graph with a Laplacian $L := [l_{ij}] \in \mathbb{R}^{N \times N}$, whose elements are defined as

$$l_{ii} = \sum_{j=1, j \neq i}^N a_{ij}, \quad l_{ij} = -a_{ij}, \quad i \neq j. \quad (4)$$

That is, $a_{ij} = 1$ if information flows from the j th to the i th agent and $a_{ij} = 0$ if otherwise. On the other hand, in general $a_{ij} \neq a_{ji}$.

The dynamic-consensus problem under bias measurements is reminiscent, for instance, of scenarios in which a network of cooperative agents is under false-data injection attacks (Meng et al., 2020; Modares et al., 2018; Gao et al., 2019) or the agents are physical systems equipped with faulty sensors (Shi et al., 2020; Sukumar et al., 2018; Dutta et al., 2024). In (Dutta et al., 2024) we solved the dynamic-consensus problem, with biased measurements, but under the restrictive assumption that the matrix A is full rank. This condition is clearly conservative because it excludes the common case of simple chains of integrators, such as innocuous second-order systems, which serve to model a variety of physical systems. Now, it is important to remark that even if the (A, B) is controllable, which is not an unusual property in physical systems, if A is not full rank one cannot pretend to apply a preliminary feedback $u_i := -Kx_i + \nu_i$ to transform the system (1) into one of the form

$$\dot{x}_i = (A - BK)x_i + B\nu_i, \quad \forall i = \{1, 2, \dots, N\}, \quad (5)$$

and such that, after this feedback, $(A - BK)$ is full rank, because the state x_i is not measurable; only $y_i = x_i + \delta_i$ is. Therefore, applying the feedback $u_i := -K(x_i + \delta_i) + \nu_i$ would result in a preliminary closed-loop system affected by a perturbation $K\delta_i$. Such a perturbation to (5) cannot be compensated for using classical Lyapunov redesign techniques, discontinuous control, etc. nor can it be effectively estimated, either, using, *e.g.*, unknown-input high-gain observers or classical adaptive-control methods. There is not, either, a work-around for the bias estimator proposed in (Dutta et al., 2024), which relies on the unavoidable conservative assumption that A is full rank. The estimator design that we present in this paper is completely different from (Dutta et al., 2024), but it is somewhat reminiscent of the DREM-based adaptive control (Aranovskiy et al., 2022).

The solution that we present is based on the following standing hypothesis.

Assumption 1.

- (1) The pair (A, B) is controllable and
- (2) The network's topology is described by a directed graph that contains a spanning tree¹.

The controllability assumption is little restrictive; it is a technical condition used in the design of certain auxiliary filters that make a part of a dynamic estimator. On the other hand, Assumption 1.2), which leaves considerable freedom in what the class of systems and network topology concern, is instrumental to the controller design, notably because it guarantees the following statements.

Lemma 2. (Ren and Beard, 2010, 2005) If the directed graph \mathcal{G} has a rooted spanning tree then its associated Laplacian matrix $L \in \mathbb{R}^{N \times N}$ has a simple zero eigenvalue and all of the remaining eigenvalues lie in the open right half plane. That is,

$$\lambda_1(L) = 0, \quad \Re\{\lambda_k(L)\} > 0, \quad k = \{2, \dots, N\}.$$

¹ This will naturally cover the cases in which the directed graph *consists* in a spanning tree (*i.e.*, leader-follower topology), as well as the more conservative cases of strongly-connected and balanced digraphs.

On the other hand, the right eigenvector of the zero eigenvalue is $\mathbf{1}_N = [1 \ 1 \ \cdots \ 1]^\top$ which satisfies $L\mathbf{1}_N = 0_N$ and the left eigenvector v_l satisfies $\sum_{k=1}^N v_{l_k} = 1$ and $v_l^\top L = 0_N^\top$.

Lemma 3. (Panteley et al., 2020) Let \mathcal{G} be a directed graph of order N and $L \in \mathbb{R}^{N \times N}$ be the associated non-symmetric Laplacian matrix. Then, the following statements are equivalent:

- (1) the graph \mathcal{G} has a spanning tree,
- (2) for any matrix $Q_L \in \mathbb{R}^{N \times N}$, $Q_L = Q_L^\top > 0$ and for $\alpha > 0$, there exists matrix $P = P^\top > 0$ such that

$$PL + L^\top P = Q_L - \alpha[P\mathbf{1}_N v_l^\top + v_l \mathbf{1}_N^\top P], \quad (6)$$

where v_l is the left eigenvector associated to the single zero eigenvalue of L .

The previous technical statement are fundamental to the control design and the stability analysis.

3. ESTIMATION OF THE SENSOR BIASES

In this section we develop our original bias estimator. It relies on the design of two auxiliary ‘‘filters’’ which render certain key signals exciting. The estimator design starts by pretending to introduce a preliminary state feedback Kx_i which is actually unavailable. That is, we simply rewrite (1) as

$$\dot{x}_i = (A - BK)x_i + Bu_i + BKx_i, \quad (7)$$

where $K \in \mathbb{R}^{p \times n}$ is a feedback constant matrix that is to be determined. Then, we pose the preliminary control input

$$u_i := -Ky_i + \nu_i, \quad (8)$$

where ν_i is an additional control input yet to be determined. Next, after (7) and (8), and using $y_i = x_i + \delta_i$, we obtain

$$\dot{x}_i = (A - BK)x_i + B\nu_i - BK\delta_i. \quad (9)$$

Now, with aim at defining the auxiliary filters that serve in the design of the dynamic estimator of δ_i , we solve the previous differential equation; integrating on both sides of (9), from 0 to t , we obtain

$$\begin{aligned} x_i(t) &= \Phi(t, 0)x_i(0) + \int_0^t \Phi(t, \tau)B\nu_i(\tau)d\tau \\ &\quad - \int_0^t \Phi(t, \tau)BKd\tau \delta_i, \end{aligned}$$

where $\Phi : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ is the state-transition matrix corresponding to the nominal system $\dot{z} = (A - BK)z$. Then, let us consider the following filters with null initial conditions,

$$\dot{x}_{vi} = (A - BK)x_{vi} + B\nu_i, \quad x_{vi}(0) = 0_n, \quad (10)$$

$$\dot{X}_\delta = (A - BK)X_\delta - BK, \quad X_\delta(0) = 0_{n \times n}, \quad (11)$$

and rewrite $x_i(t)$ in terms of the filters’ solutions $x_{vi}(t)$ and $X_\delta(t)$. We obtain

$$x_i(t) = \Phi(t, 0)x_i(0) + x_{vi}(t) + X_\delta(t)\delta_i, \quad i \in \{1, \dots, N\}. \quad (12)$$

Next, we proceed to find an expression for the available measurement $y_i(t) = x_i(t) + \delta_i$. To that end, we add $\delta_i - \Phi(t, 0)\delta_i$ on both sides of (12) and then, rearrange the terms to obtain

$$y_i(t) - x_{vi}(t) - \Phi(t, 0)y_i(0) = [X_\delta(t) + I_n - \Phi(t, 0)]\delta_i. \quad (13)$$

Note that the terms on the left-hand side of this equation are available, either from measurement or by design, as well as the factor of δ_i on the right-hand side. That is,

$$\Psi := X_\delta + I_n - \Phi(t, 0) \quad (14)$$

is known, so defining

$$\Upsilon_i := y_i - x_{vi} - \Phi(t, 0)y_i(0),$$

we may rewrite (13) as $\Upsilon_i = \Psi\delta_i$. This is equivalent to

$$\text{adj}(\Psi)\Upsilon_i = \text{adj}(\Psi)\Psi\delta_i = \det(\Psi)\delta_i, \quad (15)$$

where $\text{adj}(\Psi)$ denotes the adjoint matrix of Ψ . Since the terms on the left-hand side of the equation above are known, we can define the nonlinear bias estimation law as

$$\hat{\delta}_i = \gamma_i \det(\Psi) [\text{adj}(\Psi)\Upsilon_i - \det(\Psi)\hat{\delta}_i], \quad \gamma_i > 0, \quad (16)$$

which, in view of (15), is equivalent to

$$\dot{\hat{\delta}}_i = \gamma_i \det(\Psi)^2 \tilde{\delta}_i. \quad (17)$$

Remark 4. (Feasibility of the assigned bias estimate). We stress that the expression (17) is included for further development. For the purpose of implementation, we use (16), in which the matrix Ψ and the composite vector $\Upsilon_i = \text{adj}(\Psi)((x_i + \delta_i) - x_{vi} - \Phi(t, 0)(x_i(0) + \delta_i))$ are available.

Then, we have the following.

Proposition 5. (Bias estimation). Consider N identical linear systems as in (1) under Assumption 1. Consider, in addition, the bias estimator defined by (11), (14), (16) and let K be such that $\det(\Psi) \notin \mathcal{L}_2$. Then, the bias estimation errors $\tilde{\delta}_i := \delta_i - \hat{\delta}_i$ tends to zero asymptotically.

Proof: After (17) we have

$$\dot{\tilde{\delta}}_i = -\gamma_i \det(\Psi(t))^2 \tilde{\delta}_i, \quad i \in \{1, \dots, N\}, \quad (18)$$

so, solving (18), we obtain

$$\tilde{\delta}_i(t) = \exp\left[-\gamma_i \int_0^t \det(\Psi(\tau))^2 d\tau\right] \tilde{\delta}_i(0) \quad (19)$$

and, since $\det(\Psi) \notin \mathcal{L}_2$,

$$\lim_{t \rightarrow \infty} \|\tilde{\delta}_i(t)\| = 0. \quad (20)$$

Remark 6. (Regarding the used filters in (10)-(11)). It is worth remarking that unlike related literature—see, *e.g.*, (Kreisselmeier, 1977; Roy et al., 2017), the designed gain K does not need to guarantee that $A - BK$ is Hurwitz. This is to say that we design K to obtain marginally stable ($A - BK$) based ‘filters’, which in turn ensure that the signal $t \mapsto \det(\Psi(t))$ is not square integrable.

4. DYNAMIC CONSENSUS

We proceed now to design the dynamic consensus controller, which relies on the bias estimates $\hat{\delta}_i$ designed previously. To ensure the property of dynamic consensus, that is, that (2) holds, we design a distributed control law ν_i and we analyse the behaviour of the networked system, (9). To that end, we start by writing the equations in the multi-variable form

$$\dot{x} = [I_N \otimes (A - BK)]x + [I_N \otimes B]\nu - [I_N \otimes BK]\delta \quad (21)$$

where $x := [x_1^\top \ \cdots \ x_N^\top]^\top$, $\nu := [\nu_1^\top \ \cdots \ \nu_N^\top]^\top$, and $\delta := [\delta_1^\top \ \cdots \ \delta_N^\top]^\top$. Then, we apply the controller—*cf.* (Dutta et al., 2024)

$$\nu = [I_N \otimes K]y - \epsilon[L \otimes F]y + \epsilon[L \otimes F]\hat{\delta} \quad (22)$$

where $y := [y_1^\top \cdots y_N^\top]^\top$, $\epsilon \geq \lambda_{\max}(P)$, P solves (6) for $Q_L = I_N$, $F = B^\top M$, and $M = M^\top \in \mathbb{R}^{n \times n}$ is the solution to the Riccati equation

$$MA + A^\top M - MB B^\top M = -Q, \quad (23)$$

for an arbitrarily given $Q = Q^\top > 0$. The existence of the positive definite matrix M is guaranteed by the assumption that the pair (A, B) is controllable—see (Naidu, 2018).

Then, we have the following.

Proposition 7. Consider the compact form of N general linear systems given in (1) as $\dot{x} = (I_N \otimes A)x + (I_N \otimes B)u$ in closed loop with the distributed control input $u = -\epsilon[L \otimes B^\top M]y + \epsilon[L \otimes B^\top M]\hat{\delta}$ and the bias estimator defined by (16), (14), (11). Then, under the conditions of Proposition 5, (2) holds.

Proof : First, we note that Equation (21) may be rewritten as

$$\dot{x} = [I_N \otimes A]x + [I_N \otimes B]\nu - [I_N \otimes BK][x + \delta] \quad (24)$$

while Equation (22) is equivalent to

$$\nu = [I_N \otimes K]y - \epsilon[L \otimes F]x - \epsilon[L \otimes F]\hat{\delta}. \quad (25)$$

Therefore, using (24) and (25) we obtain

$$\dot{x} = [I_N \otimes A]x - \epsilon[L \otimes BF]x - \epsilon[L \otimes BF]\hat{\delta} \quad (26)$$

The dynamic consensus objective (2) is achieved if for each $i \leq N$ we have

$$\lim_{t \rightarrow \infty} \|x_i(t) - (v_\ell^\top \otimes I_n)x(t)\| = 0, \quad (27)$$

where v_ℓ is the left eigenvector corresponding to the null eigenvalue of L . Equivalently, dynamic consensus is achieved if the synchronisation errors $e = [(I_N - 1_N v_\ell^\top) \otimes I_n]x$ satisfy $e \rightarrow 0$ —cf. (Panteley and Loría, 2017). Indeed, $(v_\ell^\top \otimes I_n)x$ may be regarded as a weighted average of the states x_i .

Note that the overall closed-loop system, given by Equations (18) and (26), is in a standard cascaded form. To apply cascaded-systems theory, we rewrite the networked system (26) in terms of the synchronisation errors $e = [(I_N - 1_N v_\ell^\top) \otimes I_n]x$. Differentiating on both sides of the latter equation and using (26) we obtain

$$\dot{e} = [I_N \otimes A]e - \epsilon[L \otimes BF]e - \epsilon[L \otimes BF]\hat{\delta} \quad (28)$$

This system is also in cascade with (18). Since the interconnection term in (26) does not depend on e , and $\{\hat{\delta} = 0\}$ is asymptotically stable for (28) it is only left to prove global asymptotic stability of $\{e = 0\}$ for

$$\dot{e} = [I_N \otimes A]e - \epsilon[L \otimes BB^\top M]e, \quad (29)$$

which is obtained by replacing $F = B^\top M$. Then, after Assumption 1, we let Lemma 3 generate $P = P^\top$ for $Q_L = I_N$ and we introduce the Lyapunov function $V = e^\top (P \otimes M)e$ which is positive definite and decrescent. Its total derivative along the trajectories of (29) equates to $-\lambda_{\max}(P)e^\top (I_N \otimes Q)e$ —see the proof of (Dutta et al., 2022, Proposition 1). Global asymptotic stability of $\{e = 0\}$ follows and so does the statement of Proposition 7. ■

5. RELATIVE FORMATION CONSENSUS OF NETWORKED SATELLITES.

To illustrate the utility and performance of the distributed controller and bias estimator previously described, we

performed some numerical simulations using the case-study of a network of five satellites modelled using the Clohessy-Wiltshire equations (Schaub and Junkins, 2003). These models are linear but the drift is defined by a singular system matrix:

$$\begin{aligned} \begin{bmatrix} \dot{x}_{1i} \\ \dot{x}_{2i} \\ \dot{x}_{3i} \\ \dot{x}_{4i} \\ \dot{x}_{5i} \\ \dot{x}_{6i} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3\kappa^2 & 0 & 0 & 2\kappa & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2\kappa & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\kappa^2 & 0 \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \\ x_{3i} \\ x_{4i} \\ x_{5i} \\ x_{6i} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} \\ & \quad i \in \{1, 2, 3, 4, 5\} \end{aligned} \quad (30)$$

This equation represents the relative orbital motion in terms of Cartesian coordinates, between a common *chief* and a *deputy* satellite. More precisely, x_{1i}, x_{3i}, x_{5i} (in units of m), are the relative Cartesian position along the x, y, z direction for the i^{th} deputy satellite with respect to the chief satellite and κ being the standard orbital rate (in units of $\frac{\text{rad}}{\text{sec}}$) ($\kappa := \sqrt{\frac{\beta}{r^3}}$ where β is the standard gravitational constant (in units of $\frac{\text{m}^3}{\text{sec}^2}$) and r is the radius of the chief satellite's orbit (in units of m)). We take its value to be 0.00113 corresponding to the low earth orbit (Sampaio et al., 2014).

It is assumed that the relative state measurements between neighbours are affected by the following unknown biases due to sensor faults:

$$\begin{aligned} \delta_1 &= [50\pi \ 3\pi \ 100\pi \ 2\pi \ 140\pi \ 2.5\pi]^\top, \\ \delta_2 &= [55\pi \ 3.3\pi \ 150\pi \ 1.5\pi \ 210\pi \ 2.5\pi]^\top, \\ \delta_3 &= [45\pi \ 2.5\pi \ 90\pi \ \pi \ 120\pi \ 1.7\pi]^\top, \\ \delta_4 &= [25\pi \ 1.2\pi \ 69\pi \ 1.75\pi \ 150\pi \ 2.5\pi]^\top, \\ \delta_5 &= [30\pi \ 1.25\pi \ 30\pi \ 2.3\pi \ 60\pi \ 1.43\pi]^\top, \end{aligned}$$

where the values of the bias offsets in the measurements are somewhat comparable to the plausible actual values of the corresponding state variables they are associated with. As a result, here, the measurement still falls into the range of actual values of relative orbital position and velocity. Consequently, in such situations, the undesired bias remains imperceptible from the actual measurement and can severely affect dynamic formation and consensus.

The interaction graph, as well as the corresponding Laplacian matrix, for the five deputy satellites are given in Figure 1 below. It can be inferred from the graph that

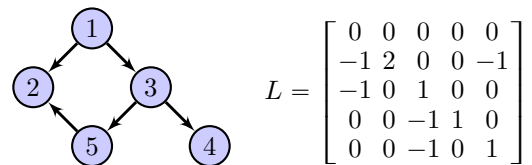


Fig. 1. Directed communication graph between five deputy satellites and corresponding Laplacian.

node 1 representing the deputy satellite 1 is the root node of the network. The intention is to artfully assign the offsets to the available state measurements which are, however, affected by constant unknown biases. This is to ensure that the deputy satellites form a desired geometric formation with respect to the common chief satellite once the unwanted biases are estimated correctly.

For instance, we assign an offset $\Delta_2 = [1000, 1000, 0]^T$, $\Delta_3 = [-1000, -1000, 0]^T$, $\Delta_4 = [1000, -1000, 0]^T$, $\Delta_5 = [-1000, 1000, 0]^T$, all in units of m, to $x - y - z$ coordinates of deputy satellite 2, 3, 4 and 5 respectively. As a result, the deputy satellite 2, 3, 4 and 5 will eventually occupy the vertices of a square with the deputy satellite 1, i.e. the root node, at the centre of this formation with common z coordinate. This is appreciated in Figure 3, Figure 2. The maintenance of desired formation is possible due to convergence of the unknown biases in the state measurements as seen in Figure 4 and Figure 5. This in turn is possible as the $\det(\Psi) \notin \mathcal{L}_2$ as seen in Figure 6.

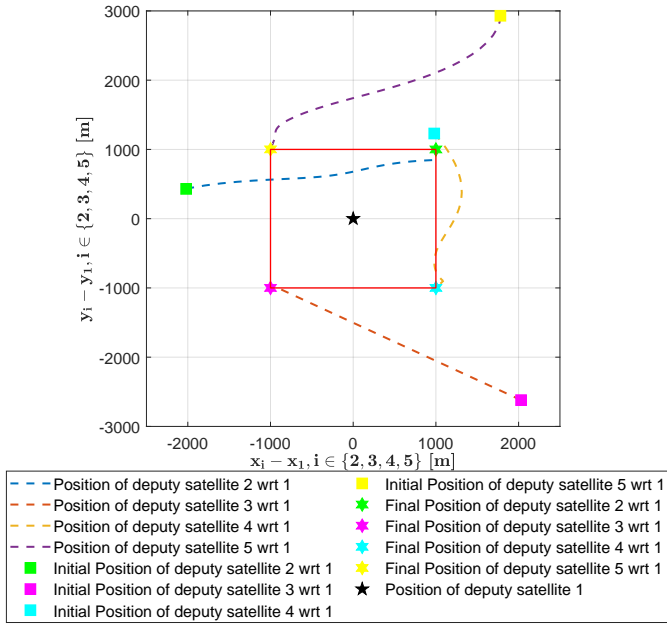


Fig. 2. Position of Deputy satellite 2, 3, 4 and 5 with respect to Deputy satellite 1.

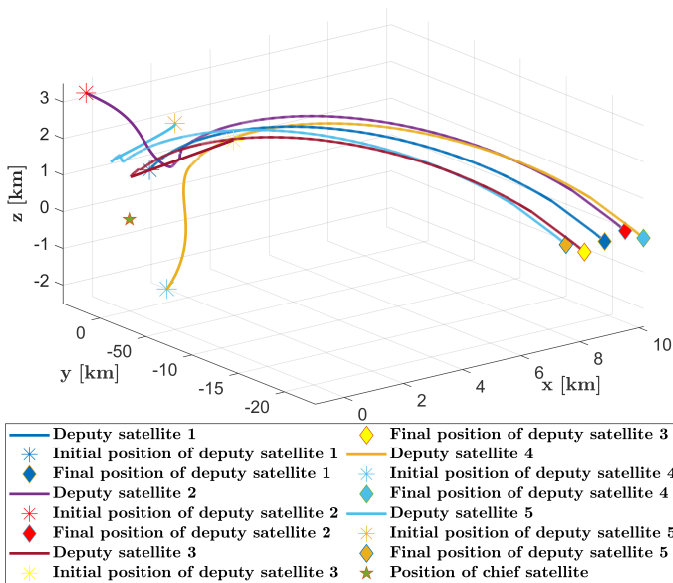


Fig. 3. 3-D Plot of Deputy satellites with respect to the chief satellite.

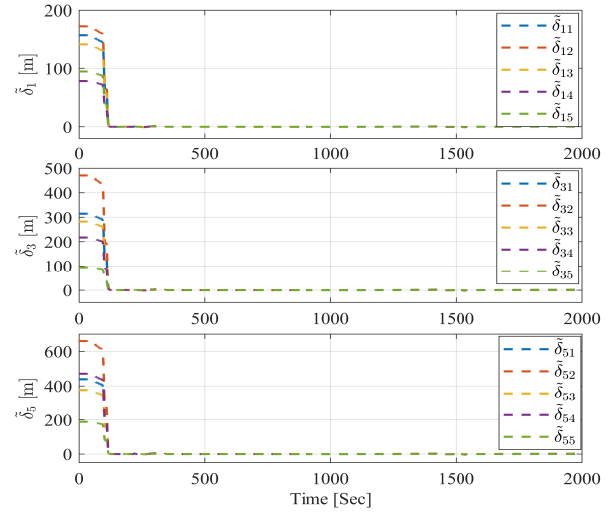


Fig. 4. Error in estimation of biases associated with the relative position measurements of C-W equations when its dynamics is given by (18).

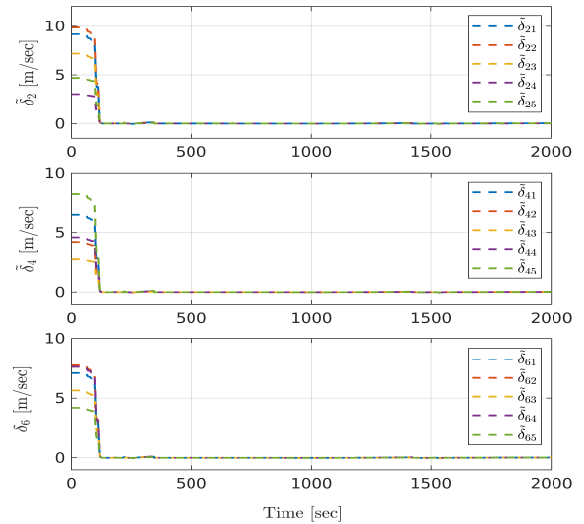


Fig. 5. Error in estimation of biases associated with the velocity measurements of C-W equations when its dynamics is given by (18).

6. CONCLUSIONS

In this manuscript, we have proposed a distributed procedure for simultaneously solving the cooperative problem of dynamic consensus in networked systems which are interacting through directed graph containing a spanning tree along with sensor bias estimation. For the latter, we converted the differential equations associated with the individual systems into algebraic equation by means of designed auxiliary filters to get the regressor-unknown bias form. A square-integral condition with respect to the determinant of the resultant regression matrix was derived. Asymptotic bias estimation ensured dynamic consensus among the closed-loop systems. Using the proposed procedure, we implemented it for formation consensus among a set of deputy satellites with respect to a chief satellite.

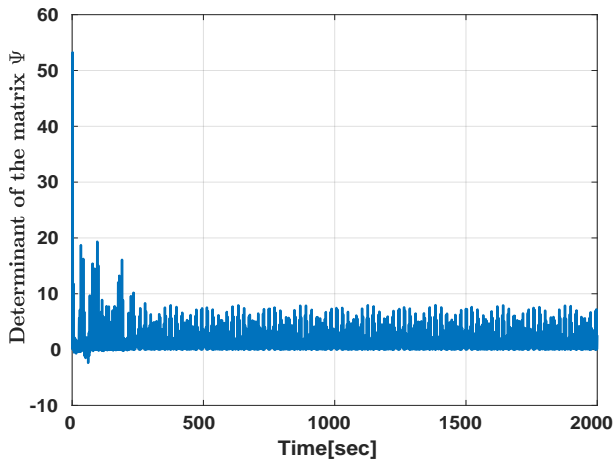


Fig. 6. Determinant of the matrix $\Psi(t) = X_\delta(t) + I_{Nn} - \Phi(t, 0)$ for the C-W equations.

Such manoeuvres are quite useful in cases where we require a cluster of satellites to fly at a specific geometry which helps in removal of financial and technical strain of building a radar of an equivalent size. Extensions of such problems to unilaterally interacting switching linear systems and general non-linear systems which are dependent on biased information are to be explored.

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