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# Robust Pole-constrained $\mathcal{H}_2$ Controller for Permanent Magnet Synchronous Motors

Hiba Houmsi, Paolo Massioni, Federico Bribiesca-Argomedo and Romain Delpoux

**Abstract**— This paper addresses the design and experimental validation of a state-feedback robust control law for electric motors with norm-bounded parametric uncertainties. The main objective is to propose a streamlined synthesis method that allows practitioners to take advantage of advanced control methods while only selecting a few, physically meaningful, tuning parameters. The synthesis is obtained by means of convex optimisation, specifically robust  $\mathcal{H}_2$  and robust pole placement within linear matrix inequality (LMI) regions. The proposed formulation can be verified to be less conservative than pre-existing results from the literature. The effectiveness of the approach is also validated experimentally on a permanent-magnet synchronous motor (PMSM) using a low-cost industrial microcontroller. A robust controller is designed to ensure that two distinct motors meet defined performance criteria. Experimental results comparing the performance of this robust controller with a non-robust design are presented.

**Index Terms**— Embedded control, linear matrix inequality (LMI), permanent-magnet synchronous motor (PMSM), robust control.

## I. INTRODUCTION

Robustness to parametric uncertainties is essential across various industrial applications, where uncertainties and nonlinearities are very common. Moreover, operational specifications may require guaranteed performance criteria such as  $\mathcal{H}_2$  or  $\mathcal{H}_\infty$  norms that are widely discussed in the literature; alternative, more intuitive, specifications might rely on transient decay rate, or damping coefficients, see for example [1]. In the context of embedded control, it is important to cope with limited computational resources; as shown in [2], it is also highly desirable to have the ability to easily reconfigure the controller without requiring an expert operator. This work continues on the same research line, aiming to propose a user-friendly approach for embedded control of PMSM, where the controller is computed in the embedded system based on a few, easy-to-understand parameters.

Several techniques are used in the literature for speed control of PMSM, such as nested speed and current controllers [3] where the speed and current dynamics are considered separately. In general, tuning a proportional-integral controller involves placing poles at fixed points in the complex plane. This approach is impossible when the system has uncertainty. Another common method is model predictive control (MPC) [4] which requires setting a cost function dependent on the

parameters of the system. However, due to the fast electrical dynamic, the optimisation horizon is extremely limited on an industrial microcontroller. Other approaches considering the PMSM as a switched system have been explored in [5], taking into consideration the switched nature of an inverter. The implementation of such methods on low-cost microcontrollers can be difficult because of high switching frequencies. Most of these methods do not consider explicitly the uncertainty of the parameters.

For this work, linear matrix inequality (LMI) techniques for multiobjective synthesis [6] are used to address controller tuning under easy-to-grasp design specifications. Pole placement in LMI regions [7] provides an explicit method to set a decay rate and a damping ratio for the transient response of a system, both an effective and an intuitive design criteria. Moreover, considering that energy consumption plays a crucial role in various onboard applications, one can consider minimising an  $\mathcal{H}_2$  energy-oriented performance criterion. Both  $\mathcal{H}_2$  and pole placement methodologies have been extended in the literature to accommodate uncertain parameters [8]. Uncertainty in a system can be modelled as structured and unstructured [9]; choosing between these options is a trade-off between computational complexity and conservatism of the solution. In the context of an embedded LMI solver [2], opting for unstructured norm-bounded parametric uncertainty reduces the computational burden, at the cost of conservatism. The main contributions of this paper are:

- the derivation of a Lemma for robust pole placement that reduces conservatism in the context of multiobjective synthesis;
- the modelling, synthesis and experimental validation of the robust controller on two distinct PMSMs using a low-cost industrial microcontroller;
- the experimental comparison between the proposed controller, and a standard (non-robust) pole-constrained  $\mathcal{H}_2$  controller.

This paper is organized as follows. Section II recalls results on robust  $\mathcal{H}_2$  control and robust pole placement. Section III, concerns the development of the proposed control law. Section IV deals with the application of this controller to the PMSM case. The effectiveness of the proposed control law is experimentally validated on two motors with distinct parameters.

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## II. PRELIMINARIES: ROBUST CONTROL

### A. Notation

We denote by  $\mathbb{R}$  the set of real numbers, by  $\mathbb{C}$  the set of complex numbers and by  $\mathbb{R}^{n \times m}$  the set of real  $n \times m$  matrices.  $A^\top$  indicates the transpose of a matrix  $A$ .  $I_n$  is the identity matrix of size  $n$ ,  $\mathbf{0}_n$  is a matrix of zeros of size  $n$ . The notation  $A \succ 0$  (resp.  $A \prec 0$ ) indicates that all the eigenvalues of the symmetric matrix  $A$  are strictly positive (resp. negative). The symbol  $\otimes$  indicates the Kronecker product, satisfying the basic properties  $(A \otimes B)(C \otimes D) = (AC \otimes BD)$ ,  $\alpha_A \alpha_B (A \otimes B) = (\alpha_A A \otimes \alpha_B B)$  (with matrices of compatible sizes). We employ the symbol  $(*)$  to complete symmetric matrix expressions, avoiding repetitions. The imaginary part of a complex number  $z$  is denoted  $\Im(z)$  and the real part  $\Re(z)$ . Finally,  $\sigma_{max}(A)$  represents the maximum singular value of  $A$ . Also,  $\langle A \rangle_S$  denotes the sum of a matrix  $A$  and its transpose,  $\langle A \rangle_S = A + A^\top$ .

### B. Problem statement

Consider a linear uncertain system, described by the state space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_u u(t) + B_w w(t) + B_\Delta w_\Delta(t), \\ z(t) &= C_z x(t) + D_z u(t), \\ z_\Delta(t) &= C_\Delta x(t) + D_\Delta u(t), \\ w_\Delta(t) &= \Delta z_\Delta(t), \quad \sigma_{max}(\Delta) \leq 1, \end{aligned} \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state of the system,  $u \in \mathbb{R}^{n_u}$  is the control input,  $w \in \mathbb{R}^{n_w}$  is an exogenous input,  $z \in \mathbb{R}^{n_z}$  is the output, and  $\Delta \in \mathbb{R}^{p \times q}$  is a feedback matrix connected between  $z_\Delta$  and  $w_\Delta$ . We are interested in designing a static state feedback controller:

$$u(t) = Kx(t), \quad (2)$$

such that:

- the  $\mathcal{H}_2$  norm of the uncertain system (1), with input  $w$  and output  $z$ , is minimised
- the poles of the uncertain closed-loop system (1) with control (2) are in  $\mathbb{D}$ , an LMI region defined as:

$$\mathbb{D} = \{z = x + iy \in \mathbb{C} \mid L + zM + \bar{z}M^\top \leq 0\}. \quad (3)$$

As shown in [10], by adequately choosing the matrices  $L$  and  $M$ , we can represent useful regions of the complex plane, notably the complex left half-plane, guaranteeing stability, or sectors to ensure desired damping rates, etc. The interconnection between (1) and (2) (dropping the explicit dependency on the time variable to simplify notations) is rewritten as:

$$\begin{cases} \dot{x} = (A + B_\Delta \Delta C_\Delta + (B_u + B_\Delta \Delta D_\Delta)K)x + B_w w, \\ z = (C_z + D_z K)x, \\ \sigma_{max}(\Delta) \leq 1. \end{cases}$$

The matrices  $A(\Delta) = A + B_\Delta \Delta C_\Delta$  and  $B_u(\Delta) = B_u + B_\Delta \Delta D_\Delta$  can be used to encode physically meaningful uncertainties over the parameters of the system.

Before showing the main theoretical result in Section III, we briefly recall the fundamental results from which it is derived.

### C. Robust- $\mathcal{H}_2$

One of the possible interpretations of the  $\mathcal{H}_2$  norm is the energy of the output  $z$  due to a zero-mean, unit-variance white noise  $w$ . With this interpretation, the objective of an optimal  $\mathcal{H}_2$  synthesis consists of minimizing this output energy for the system. The robust  $\mathcal{H}_2$  LMI problem can be put into convex form by an s-procedure [8, p. 87], yielding as result:

*Theorem 2.1:* The interconnection of system (1) and (2) is stable with a squared  $\mathcal{H}_2$  norm smaller than or equal to  $\gamma$  if and only if there exist matrices  $X = X^\top$ ,  $W, Z = Z^\top$ , and  $\mu \geq 0$  such that

$$\begin{bmatrix} \langle AX + BW \rangle_S + \mu B_\Delta B_\Delta^\top & (*)^\top & (*)^\top \\ (C_z X + D_z W) & -I & 0 \\ (C_\Delta X + D_\Delta W) & 0 & -\mu I \end{bmatrix} \prec 0, \quad (4)$$

$$\begin{bmatrix} Z & B_w^\top \\ B_w & X \end{bmatrix} \succ 0, \text{trace}(Z) < \gamma, \quad (5)$$

when such matrices are obtained, the controller can be retrieved by  $K = WX^{-1}$ .

The theorem above can be exploited as a convex optimisation problem, where the goal is to find matrices  $X$  and  $W$  and the term  $\mu$  satisfying the constraints and minimising  $\gamma$ , i.e. the squared  $\mathcal{H}_2$  norm from input  $w$  to output  $z$ .

### D. Robust pole placement in LMI regions

To guarantee that the poles of the uncertain system are within the LMI region of interest (3), we consider the robust pole placement theorem presented in [7] for model (1).

*Theorem 2.2:* The interconnection of system (1) and (2) is stable, with poles in  $\mathbb{D}$ , if there exists a matrix  $P \succ 0$  such that

$$\begin{bmatrix} N_D(A_c, P) & M_1^\top \otimes (PB_c) & (M_2^\top) \otimes C_c^\top \\ M_1 \otimes (B_c^\top P) & -I & 0 \\ (M_2) \otimes C_c & 0 & -I \end{bmatrix} \prec 0, \quad (6)$$

where

$$N_D(A_c, P) = L \otimes P + M \otimes (PA_c) + M^\top \otimes (PA_c)^\top, \quad (7)$$

the matrices  $A_c, B_c, C_c$  are the closed-loop matrices defined as:

$$\begin{cases} A_c = A + B_u K, & B_c = B_\Delta, \\ C_c = C_\Delta + D_\Delta K \end{cases} \quad (8)$$

and  $M = M_1^\top M_2$  is a full rank factorisation of  $M$ .

## III. THEORETICAL RESULT

### A. Modified robust pole placement in LMI regions

In the context of multiobjective synthesis, we are looking for a controller that both minimises the  $\mathcal{H}_2$  norm of the uncertain system and guarantees that the poles are within the intersection of multiple LMI regions. One could solve the optimisation problems stemming from Theorem 2.1 and

Theorem 2.2 together, with a common Lyapunov matrix  $X$  or equivalently  $P^{-1}$  across all LMIs. Instead, we propose an extension of Theorem 2.2 that includes an extra decision variable. The presence of this variable reduces conservatism in the context of multiobjective design, as observed in general for dilated or extended LMI formulations [11]. These considerations lead to the following lemma :

*Lemma 3.1:* The interconnection of system (1) and (2) is stable with poles in  $\mathbb{D}$  if there exists a matrix  $P \succ 0$  and  $\mu > 0$  such that

$$\begin{bmatrix} N_D(A_c, P) + \mu(M_1^\top M_1) \otimes (PB_c B_c^\top P) & M_2^\top \otimes C_c^\top \\ M_2 \otimes C_c & -\mu I \end{bmatrix} \prec 0, \quad (9)$$

with the same definition of  $N_D$ .

*Proof:* Starting from (6) and applying the Schur complement:

$$\begin{aligned} N_D(A_c, P) + (M_1^\top \otimes PB_c)(M_1 \otimes B_c^\top P) \\ + (M_2^\top \otimes C_c^\top)(M_2 \otimes C_c) \prec 0, \end{aligned} \quad (10)$$

we set  $P = \mu P$  where  $\mu \geq 0$  then we multiply both sides by  $\frac{1}{\mu}$ , equation (10) becomes,

$$\begin{aligned} N_D(A_c, P) + \mu(M_1^\top \otimes PB_c)(M_1 \otimes B_c^\top P) \\ + \frac{1}{\mu}(M_2^\top \otimes C_c^\top)(M_2 \otimes C_c) \prec 0. \end{aligned} \quad (11)$$

Using the Schur complement yields (9).  $\blacksquare$

The result (9) is still not a LMI when used in controller synthesis. The convex LMI version is below.

*Lemma 3.2:* The interconnection of system (1) and (2) is stable with poles in  $\mathbb{D}$  if there exists  $X = X^\top \succ 0, W$ , and a scalar  $\mu > 0$  such that

$$\begin{bmatrix} \Gamma + \mu(M_1^\top M_1) \otimes (B_\Delta B_\Delta^\top) & (*)^\top \\ M_2 \otimes (C_\Delta X + D_\Delta W) & -\mu I \end{bmatrix} \prec 0, \quad (12)$$

where

$$\begin{aligned} \Gamma = \Gamma(L, M) = L \otimes X + \\ M \otimes (AX + B_u W) + M^\top \otimes (AX + B_u W)^\top. \end{aligned} \quad (13)$$

*Proof:* Starting from (11), we apply the linearising changes of variables  $X = P^{-1}$  and  $W = KP$ . Finally, we apply a Schur complement to obtain (12).  $\blacksquare$

Lemma 3.2 can be applied directly in a multiple objective synthesis, as the variables  $X$  and  $W$  characterize the same Lyapunov function as the robust  $\mathcal{H}_2$ . No equality constraint should be imposed on the terms  $\mu$  associated with the constraint of pole placement and robust  $\mathcal{H}_2$ .

### B. Robust pole-constrained $\mathcal{H}_2$ controller

Our main result is then formulated after choosing the design objectives, namely 1) minimum decay rate 2) minimum damping coefficient and 3)  $\mathcal{H}_2$  norm. For 1), the minimum decay rate  $\alpha$  is obtained with Lemma 3.2 by constraining the poles to the vertical strip  $\mathbb{D}_\alpha$  LMI region (3) as

$$L_\alpha = 2\alpha, M_\alpha = 1, M_{1,\alpha} = 1, M_{2,\alpha}^\top = 1. \quad (14)$$

with  $\mathbb{D}_\alpha = \{z \in \mathbb{C}, \Re(z) \leq -\alpha\}$ . For 2), the constraint on the damping factor  $\beta$  is obtained with Lemma 3.2 by constraining the poles to the conic  $\mathbb{D}_\beta$  LMI region as:

$$\begin{cases} L_\beta = 0, M_\beta = \begin{bmatrix} \beta & 1 \\ -1 & \beta \end{bmatrix} = M_{1,\beta}^\top M_{2,\beta}, \\ M_{1,\beta} = M_\beta, M_{2,\beta} = I_2. \end{cases} \quad (15)$$

with  $\mathbb{D}_\beta = \{z \in \mathbb{C}, |\Im(z)| \leq -\beta\Re(z)\}$ . At last, 3) requires the use of Theorem 2.1 for norm minimisation. The combination of these objectives results in the following theorem:

*Theorem 3.3:* Consider the dynamical system in (1). If there exist  $X = X^\top \succ 0, W, Z = Z^\top, \mu_1 > 0, \mu_2 > 0, \mu_3 > 0$  satisfying:

$$\begin{bmatrix} \langle AX + BW \rangle_S + \mu B_\Delta B_\Delta^\top & (*)^\top & (*)^\top \\ (C_z X + D_z W) & -I & 0 \\ (C_\Delta X + D_\Delta W) & 0 & -\mu_1 I \end{bmatrix} \prec 0, \quad (16)$$

$$\begin{bmatrix} Z & B_w^\top \\ B_w & X \end{bmatrix} \succ 0, \text{trace}(Z) < \gamma, \quad (17)$$

$$\begin{bmatrix} \Gamma(L_\alpha, M_\alpha) + \mu_2(M_{1,\alpha}^\top M_{1,\alpha}) \otimes (B_\Delta B_\Delta^\top) & (*)^\top \\ M_{2,\alpha} \otimes (C_\Delta X + D_\Delta W) & -\mu_2 I \end{bmatrix} \prec 0, \quad (18)$$

$$\begin{bmatrix} \Gamma(L_\beta, M_\beta) + \mu_3(M_{1,\beta}^\top M_{1,\beta}) \otimes (B_\Delta B_\Delta^\top) & (*)^\top \\ M_{2,\beta} \otimes (C_\Delta X + D_\Delta W) & -\mu_3 I \end{bmatrix} \prec 0. \quad (19)$$

then there exists a static state feedback control law  $u = Kx$ , with square  $\mathcal{H}_2$  norm (from  $w$  to  $z$ ) smaller than or equal to  $\gamma$  and poles in  $\mathbb{D}_\alpha \cap \mathbb{D}_\beta$ . The corresponding controller gain is given by  $K = WX^{-1}$ .

This theorem can be used for controller synthesis by minimising  $\gamma$  under the constraints. Although the theorem is a conservative statement (it offers only sufficient conditions), the presence of the variables  $\mu_2, \mu_3$  is a way to reduce the conservatism of the solution.

## IV. PRACTICAL RESULTS: APPLICATION CASE

### A. Linear Uncertain model of the PMSM

The aim of this subsection is to model the PMSM as an uncertain system. Consider the classical model of the PMSM as presented in [3]:

$$\begin{cases} \frac{di_d}{dt} = \frac{v_d}{L} - \frac{R}{L}i_d + p\omega i_q, \\ \frac{di_q}{dt} = \frac{v_q}{L} - \frac{R}{L}i_q - p\omega i_d - \frac{p\phi_f}{L}\omega, \\ \frac{d\omega}{dt} = \frac{3p\phi_f}{2J}i_q - \frac{f}{J}\omega, \\ \frac{d\xi_\omega}{dt} = \omega^\# - \omega, \\ \frac{d\xi_i}{dt} = i_d^\# - i_d \end{cases} \quad (20)$$

In this model,  $v_d, v_q$  are the d-q phase voltages and  $i_d, i_q$  are the d-q phase currents,  $\omega$  is the rotor angular speed. The reference speed  $\omega^\#$  is set by the user. The reference current is imposed as  $i_d^\# = 0$  since no field weakening is necessary [12]. To ensure zero steady-state error on the speed and direct current, the classical PMSM state space representation is augmented by the two states  $\xi_\omega$  and  $\xi_i$  defined as the integral

between respectively the speed tracking error and the direct current tracking error. The model is described by the parameters:  $R$  is the phase resistance,  $L$  the phase inductance,  $\phi_f$  the peak magnetic flux of the permanent magnets seen by stator windings,  $p$  the pole pairs number,  $J$  the inertia of the system,  $f$  is the viscous frictional coefficient of the motor. The PMSM model (20) is nonlinear. This nonlinearity is split in two term, one term representing the nominal part of the nonlinearity, which can be eliminated by feedback linearization, and the remaining term to be modelled as an uncertain norm-bounded parameter. To reduce the size of the uncertainty, we first include an approximate feedback linearization that uses a nominal set of parameters for the system (not necessarily identical to the real parameters of the motor):

$$\begin{cases} v_d = u_d - \nu_0 \omega i_q, \\ v_q = u_q + \nu_0 \omega i_d. \end{cases} \quad (21)$$

Here  $\nu_0$  can be chosen such that the nonlinear term is represented with bounds of opposite signs, we bound  $\omega_{max}$  by the maximum speed attainable by the motor. The remaining nonlinear terms are then represented by an uncertain parameter:

$$\begin{aligned} \theta_1 &= (p - \frac{\nu_0}{L})\omega \in [\theta_{1,min}; \theta_{1,max}] = \\ &[\min_{\kappa} (p - \frac{\nu_0}{L})\omega_{max}; \max_{\kappa} (p - \frac{\nu_0}{L})\omega_{max}] \end{aligned} \quad (22)$$

where  $\kappa = [p_{min}, p_{max}] \times [L_{min}, L_{max}]$ . Other uncertain parameters in (20) are represented as:

$$\begin{aligned} \theta_2 &= -\frac{R}{L}, \theta_3 = -\frac{p\phi_f}{L}, \theta_4 = \frac{3p\phi_f}{2J}, \\ \theta_5 &= -\frac{f}{J}, \theta_6 = \frac{1}{L}. \end{aligned} \quad (23)$$

Each of the  $\theta_i$ , where  $i \in \{1, 2, 3, 4, 5, 6\}$ , belong to a bounded interval:

$$\theta_i \in [\theta_{i,min}; \theta_{i,max}], \quad (24)$$

this interval is equivalently represented by finding an average value  $K_i$  and  $\eta_i$  the half-length of each interval,

$$\begin{cases} \theta_i &= K_i + \eta_i \delta_i, \\ K_i &= \frac{1}{2}(\theta_{i,min} + \theta_{i,max}) \\ \eta_i &= \frac{1}{2}(\theta_{i,max} - \theta_{i,min}), \\ \delta_i &\in [-1; 1]. \end{cases} \quad (25)$$

Using the uncertain parameters  $\theta_i$ , and their average values  $K_i$ , consider the following definitions of the state  $x$ , the input  $u$  and the matrices  $A$ ,  $B_u$ ,  $B_\Delta$ ,  $\Delta$ ,  $C_\Delta$  and  $D_\Delta$  described in (1):

$$x^\top = [i_d \quad i_q \quad \omega \quad \xi_\omega \quad \xi_i], \quad u^\top = [u_d \quad u_q] \quad (26)$$

$$A = \begin{bmatrix} K_2 & K_1 & 0 & 0 & 0 \\ -K_1 & K_2 & K_3 & 0 & 0 \\ 0 & K_4 & K_5 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

$$B_u^\top = \begin{bmatrix} K_6 & 0 & 0 & 0 & 0 \\ 0 & K_6 & 0 & 0 & 0 \end{bmatrix} \quad (28)$$

$$B_\Delta = \begin{bmatrix} \eta_2 & \eta_1 & 0 & 0 & 0 & 0 & 0 & \eta_6 & 0 \\ 0 & 0 & -\eta_1 & \eta_2 & \eta_3 & 0 & 0 & 0 & \eta_6 \\ 0 & 0 & 0 & 0 & 0 & \eta_4 & \eta_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (29)$$

$$C_\Delta^\top = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

$$D_\Delta^\top = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

$$\Delta = \text{Diag}(\delta_2, \delta_1, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_6). \quad (32)$$

The matrices  $A$  and  $B_u$  represents the nominal system. The matrices  $B_\Delta$ ,  $C_\Delta$  and  $D_\Delta$  are an explicit representation of each of the uncertain parameters  $\theta_i$ .

From the above defined matrices, the PMSM model is equivalent to (1) plus some constant terms due to the presence of (possibly) non-zero reference values for  $i_d^\#$  and  $\omega^\#$ . In practice, as the system is augmented with an integrator, the states reach the references values with non steady-state error.

The stability of the system and transient behaviour are determined by the constraints on the poles of the system through the choice of  $\alpha$  and  $\beta$ . The norm of the control input can be limited through the choice of  $D_z$ . Therefore, having the minimum-energy input that satisfies pole placement criteria. In this case, we set:

$$B_w = I, \quad D_z = I, \quad C_z = \mathbf{0} \quad (33)$$

Notably, the  $\mathcal{H}_2$  performance is equivalent to the linear quadratic regulator (LQR) performance criteria [13]. Nevertheless, using the  $\mathcal{H}_2$  allows the addition of constraints on the pole-placement for the system, making the control tuning process more intuitive than that of a classical LQR control.

### B. Assessment of the modified robust pole placement

The uncertain model of the PMSM described by matrices (27) through (33) is used as an illustrative example to confirm the advantages of Theorem 3.3, namely the extra variables  $\mu_2$  and  $\mu_3$ . Using SeDuMi solver [14] and Yalmip parser [15], we solve the pole-constrained  $\mathcal{H}_2$  norm minimisation problem for the single and extra  $\mu$ . This results in finding  $\gamma$  as shown in Table I for given LMI regions  $\mathbb{D}_\alpha, \mathbb{D}_\beta$ . Table I

TABLE I:  $\gamma$  for a single and extra  $\mu$

LMI regions $\mathbb{D}_\alpha, \mathbb{D}_\beta$	single $\mu$	extra $\mu$
$(\alpha = 5, \beta = 2.5)$	1.96	1.75
$(\alpha = 10, \beta = 2.5)$	4.17	3.66

clearly confirms the advantage of the extra variables in terms

of conservatism, i.e. we achieve a lower squared  $\mathcal{H}_2$  gain ( $\gamma$ ) from  $w$  to  $z$ .

### C. Experimental validation

This subsection assesses experimentally the relevance of a robust (to parametric uncertainty) approach compared to a non-robust approach in the field of control of PMSM. To that aim, two PMSM with distinct parameters, as shown in Table II, are used to perform the experiment. We set  $\omega_{max} = 500$  rad/s. Both motors can be represented with a unique uncertain model as described in subsection IV-A.

TABLE II: Parameters of the PMSMs.

Parameter	Motor 1	Motor 2	$K_i$	$\eta_i$
$\theta_1$	83.33	-71.42	5.95	77.38
$\theta_2$	-1350	-1874	-1612	262
$\theta_3$	-127.16	-75.42	-101	25
$\theta_4$	284.98	197.21	241	43
$\theta_5$	-0.57	-0.57	0	0
$\theta_6$	3333.3	2857.1	3095	283

The hardware consists of an embedded inverter demo-board from Microchip (LVMC) based on a low-cost single core dsPIC33CK256MP508 digital signal controller (DSC). The experimental setup is shown in Fig. 1. The real-time control law is developed using a high-level embedded rapid control prototyping solution with Matlab/Simulink [16]. The sampling frequency is  $f_s = 20$  kHz. Throughout the experimental part, we set  $\alpha = 10$  and  $\beta = 2.5$ . The references are speed steps from 50 rad/s to 150 rad/s. To demonstrate the

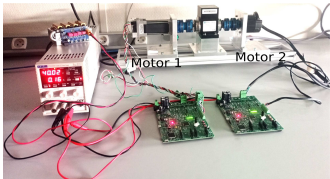


Fig. 1: Picture of the experimental test bench.

effectiveness of the robust approach, we propose a:

- **non-robust control law:** specifically designed using the parameters of motor 1 (disregarding any uncertainties). Theorem 3.3 where  $B_\Delta$ ,  $C_\Delta$ ,  $D_\Delta$ ,  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  are set to zero is used to minimise  $\gamma$  and yields the control gain  $K_c$ . This controller is applied to both motor 1 (m1) and motor 2 (m2).
- **robust control law:** considers the system in the form of (1) and characterized by matrices (27) - (33) taking into consideration the parameters of both motors. Theorem 3.3 is used to minimise  $\gamma$  and yields the control gain  $K_{unc}$ , which is subsequently applied to both motors.

The closed-loop eigenvalues of the non-robust experiment with controller  $K_c$  are depicted in Fig 2. Here,  $A_{m1}$  and

$B_{um1}$  denote the nominal model parameters for motor 1, while  $A_{m2}$  and  $B_{um2}$  represent the nominal parameters for motor 2. Using  $K_c$ , motor 1 satisfies the constraints,  $(\alpha, \beta)$  whereas motor 2 fails to satisfy these constraints. The closed-loop eigenvalues of the robust experiment using  $K_{unc}$  are shown in Fig 3. Thanks to the robust approach, both motors now satisfy the constraints. This is apparent in the transient response of the non-robust and robust experiments, depicted in Fig. 4b and Fig. 4a, respectively. Since the controller  $K_c$  is synthesised with the parameters of motor 1, the transient is within the specifications when applied to this motor. However, when using  $K_c$  with motor 2, the speed response is more than two times slower and less damped. Using the robust approach, as depicted in Fig. 4b, results in a similar speed responses for both motors. This highlights the efficiency of the proposed robust approach in the field of motor control.

Note that the robust control provides a slightly faster speed response compared to the non-robust control, which explains the higher transient currents  $i_d$ ,  $i_q$  and voltages  $v_d$ ,  $v_q$ . Figures 4d and 4c show the  $i_d$  and  $i_q$  currents for respectively the non-robust and robust experiment, the transient current  $i_q$  is higher for this particular robust synthesis, yet reaches the same value in steady state as the non-robust case. The  $i_d$  current is regulated to zero successfully in steady state. Similarly, figures 4f and 4e show the voltage command applied for both experiments, highlighting that the voltages reach their equilibrium faster in the robust case compared to the non-robust approach. The currents and voltage are within the physical limitations of the inverter and the motors, namely  $\sqrt{i_d^2 + i_q^2} \leq I_{max} = 10$  A and  $\sqrt{v_d^2 + v_q^2} \leq V_{max} = 20$  V. In this context, the robust control design proposed in this paper gives a practical result that satisfies intuitive specifications for both motors.

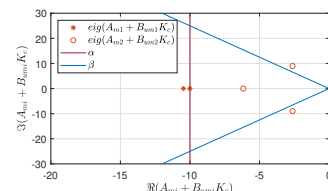


Fig. 2: Closed-loop eigenvalues for the non-robust approach.

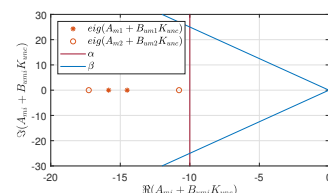
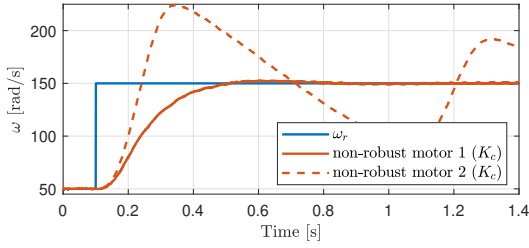


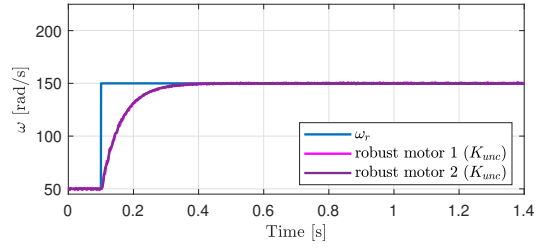
Fig. 3: Closed-loop eigenvalues for the robust approach.

## V. CONCLUSIONS

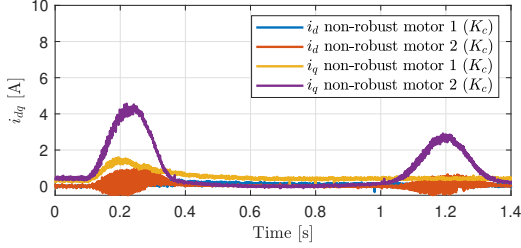
In this paper, we have modelled electrical motors as uncertain systems which allowed to synthesise a control law



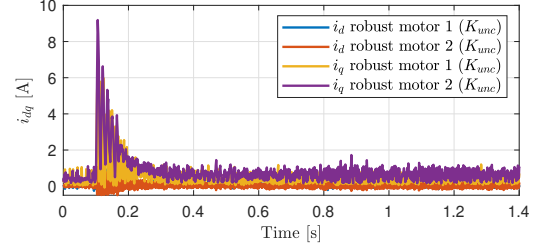
(a) Speed response with the non-robust controller for both motors.



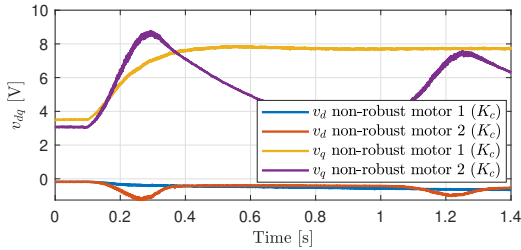
(b) Speed response with the robust controller for both motors.



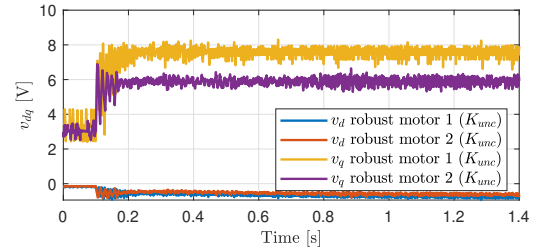
(c) Currents  $i_d, i_q$  with the non-robust controller for both motors



(d) Currents  $i_d, i_q$  with the robust controller for both motors



(e) Inputs  $v_d, v_q$  with the non-robust controller for both motors



(f) Inputs  $v_d, v_q$  with the robust controller for both motors

Fig. 4: Experimental measurements of the states and inputs of the robust and non-robust experiment.

that ensures a robust pole-constrained  $\mathcal{H}_2$  performance. To better align with experimental requirements, we proposed an extended version of the robust pole placement theorem, aiming to reduce the conservatism of the multiobjective approach. The effectiveness of the control law is validated experimentally using an industrial, low-cost microcontroller. Future work will look into a full embedded approach where the solution of the robust pole-constrained  $\mathcal{H}_2$  problem is computed as a background task by the same microcontroller, offering an easy-to-tune embedded controller.

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