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Gravity induced CP violation in mesons mixing, decay and interference experiments

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The impact of earth's gravity on neutral mesons dynamics is analyzed. The main effect of a Newtonian potential is to couple the strangeness and bottomness oscillations with the quarks zitterbewegung oscillations. This coupling is responsible for the observed CP violations in the three types of experiments analyzed here: (*i*) indirect violation in the mixing, (*ii*) direct violation in the decay to one final state and (*iii*) violation in interference between decays with and without mixing. The three violation parameters associated with these experiments are predicted in agreement with the experimental data. The amplitude of the violation is linear with respect to the strength of gravity so that this new mechanism allows to consider matter dominated cosmological evolutions providing the observed baryon asymmetry of the universe.

I. INTRODUCTION

Since the first observation of long-lived kaons decays into pairs of charged pions, reported sixty years ago by Christenson, Cronin, Fitch and Turlay [1], many complementary observables associated with flavored neutral mesons CP violation (CPV) have been identified, measured and interpreted. The canonical framework of interpretation is the standard model (SM) through the adjustment between the Kobayashi-Maskawa (KM) [2] complex phase and the experimental values of the violation parameters. In this study, we focus on the most documented and clearest experimental evidence of CPV and we demonstrate that gravity induced CPV provides a pertinent framework to interpret these experiments and to predict the violation parameters, as a function of earth's gravity, in agreement with the experimental data. As a consequence, far from any massive object, i.e. in a flat Lorentzian space-time, the Cabibbo-Kobayashi-Maskawa (CKM) [2, 3] matrix must be considered free from any CPV phase as CPV effects are just gravity induced near massive objects like earth.

Among the measured CPV observables, three types (*i*, *ii* and *iii*) of effects will be considered here: (*i*) indirect CPV in the mixing which has been observed with neutral kaons K^0/\bar{K}^0 , this CPV is described by the parameter $\text{Re } \varepsilon$ [4]; (*ii*) direct CPV in decays into one final state which has also been observed in neutral kaons decays and is characterized by the parameter $\text{Re } \varepsilon'/\varepsilon$ [5]; (*iii*) CPV in interference between decays with and without mixing, which has been observed in B^0/\bar{B}^0 decays and is described by the angle β [5].

Beside these (*i*, *ii* and *iii*) types of CPV, a fourth additional experimental evidence of CPV must be considered: (*iv*) the observed dominance of baryons over antibaryons

in our universe as CPV is one of the necessary condition to build cosmological evolution models compatible with this baryon asymmetry [6].

Despite its success to provide a framework to interpret earth based experiments such as (*i*, *ii* and *iii*), the KM mechanism, incorporated into cosmological evolution models, fails, by many orders of magnitude, to account for this (*iv*) major CPV evidence. To explain how our matter-dominated universe emerged during its early evolution we need to identify a CPV mechanism different from the KM one. Beside its potential to predict the measured parameters associated with types (*i*, *ii* and *iii*) CPV experiments, the new gravity induced CPV mechanism opens very interesting perspectives to set up cosmological models with asymmetric baryogenesis compatible with the present state of our universe as the amplitude of CPV appears here to be proportional to the strength of gravity. During the early stages of evolution of the universe, gravity/curvature was far more larger than on earth today and gravity induced CPV identified and described here, which is a linear function of the gravitational field, opens an avenue to resolve the present contradiction between the very small value of the KM mechanism and the very large CPV needed to build a pertinent model of our matter-dominated universe. To summarize, gravity induced CPV, not only explain (*i*, *ii* and *iii*) CPV effects and predict observables such as ε , ε' and β , but it also renews, in depth, the baryons asymmetry (*iv*) cosmological issue.

In this study, we demonstrate that a small secular coupling, induced by earth's gravity, between fast *quarks zitterbewegung oscillations* at the velocity of light inside the mesons and strangeness oscillations $\Delta S = 2$, or bottomness oscillations $\Delta B' = 2$, provides both a qualitative explanation of CPV and a quantitative prediction of the CPV parameters ε , ε' and β in agreement with the experimental measurements.

The new interpretation of CPV experiments presented below is based on a careful analysis of the impact of earth

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gravity on the dynamics of strangeness and bottomness oscillations. To do so we use the effective Hamiltonian of Lee, Oehme and Yang (LOY) [7, 8], completed here with Newtonian gravity. Neutral mesons oscillations such as $K^0 \rightleftharpoons \bar{K}^0$ and $B^0 \rightleftharpoons \bar{B}^0$ are very low energy oscillations ($10^{-6} - 10^{-4}$ eV). The typical earth's gravity coupling parameter $\hbar g/c \sim 10^{-23}$ eV (g is the acceleration due to gravity on earth) is very small with respect to the various energy scales involved in neutral mesons oscillations. Given the smallness of these parameters, there are no needs to rely on quantum field theory and the usual two states LOY model offers the pertinent framework to describe the interplay between two low energy quantum oscillations: quarks zitterbewegung vertical oscillations at the velocity of light in a gravitational field on the one hand and the strangeness oscillations ($\Delta S = 2$), or bottomness oscillations ($\Delta B' = 2$), on the other hand.

The three types of CPV experimental evidences are usually analyzed under the assumption of CPT conservation. The CPT theorem is demonstrated within the framework of three hypothesis: Lorentz group invariance, spin-statistics relations and local field theory. In the rest frame of a meson interacting with a massive spherical object like earth the first hypothesis is not satisfied. Thus, when earth influence is considered, we must not be surprised that CPT theorem, apparently, will no longer holds. Within the framework of a gravity induced CPV mechanism earth's gravity is described as an external field and the evolution of a meson state $|M\rangle$ alone, as a linear superposition of two flavor eigenstates $|M^0\rangle/|\bar{M}^0\rangle$, does not provide the complete picture of the dynamical system and so can not be considered as a good candidate displaying CPT invariance. But CPT might be restored for the global three bodies ($M^0/\bar{M}^0/\oplus$) evolution of the state $|M\oplus\rangle$ describing both the meson-antimeson pair and earth. In this study we consider only the evolution of $|M\rangle$ and earth's effect is described as an external static field so that CPT will appear to be violated because of this restricted two bodies (M^0/\bar{M}^0) model of the system.

The study presented below complements a previous study based on two coupled Klein-Gordon equations describing K^0/\bar{K}^0 evolution on a Schwarzschild metric [9], rather than a Newtonian framework with two coupled Schrödinger equations used here. The results on K^0/\bar{K}^0 dynamics given by the Newtonian model, presented below, are similar to those of this previous Einsteinian model [9], these results are thus model independent. Moreover, with gravity induced CPV there is no T violation at the microscopic level and, for example in the K^0/\bar{K}^0 case, the observed T violation stems from the irreversible decay of the short-lived kaons K_S continuously regenerated from the long-lived one K_L by the gravity induced coupling.

This paper is organized as follows, in the next section we briefly review the LOY model without CPV. In

section III we review the usual modifications to K^0/\bar{K}^0 and B^0/\bar{B}^0 mass eigenstates needed to accommodate CPV experimental results. The impact of earth's gravity is considered in section IV where, to describe neutral mesons oscillations $M^0 \rightleftharpoons \bar{M}^0$ on earth, the CP conserving LOY model, presented in section II, is completed with a Newtonian gravity term. We carefully analyze the nature and the impact of this additional term and discover that it contains the zitterbewegung motion at the velocity of light of the quarks inside the meson. The study of type (i), (ii) and (iii) gravity induced CPV are developed in sections V, VI and VII. We consider specifically type (i) and (ii) CPV for $K^0/\bar{K}^0 \sim (d\bar{s})/(\bar{d}s)$ and type (iii) CPV for $B^0/\bar{B}^0 \sim (d\bar{b})/(\bar{d}b)$. Section VIII provides a brief analysis of others, D^0/\bar{D}^0 and B_s^0/\bar{B}_s^0 , neutral mesons and gives our conclusions. In sections II and IV M^0/\bar{M}^0 will stand for K^0/\bar{K}^0 or B^0/\bar{B}^0 .

II. MASS AND CP EIGENSTATES WITHOUT CPV

Consider a generic neutral meson pair M^0/\bar{M}^0 , either K^0/\bar{K}^0 or B^0/\bar{B}^0 . The meson state $|M(\tau)\rangle$ is a linear superposition of the flavor eigenstates $|M^0\rangle$ and $|\bar{M}^0\rangle$ and the amplitudes (a, b) of this superposition are functions of the meson proper time τ . This state is also coupled to a set of final states $|f, \mathbf{p}, \dots\rangle$, with momentum \mathbf{p} in the M^0/\bar{M}^0 meson rest frame, described by the amplitudes w_f ,

$$|M(\tau)\rangle = a(\tau) |M^0\rangle + b(\tau) |\bar{M}^0\rangle + \sum_f w_f(\tau) |f\rangle, \quad (1)$$

where $\langle M^0 | M^0 \rangle = \langle \bar{M}^0 | \bar{M}^0 \rangle = 1$ and $\langle \bar{M}^0 | M^0 \rangle = 0$.

The Weisskopf-Wigner (WW) approximation [10] is used to describe the coupling to the final states $|f\rangle$ as an irreversible decay. Within the framework of this usual approximation we introduce a non-Hermitian decay operator $j\hat{\gamma}$ capturing the effects of the w_f amplitudes and describing $M \rightarrow f$ transitions as an irreversible decay process. It is to be noted that, as the possibilities of $f \rightarrow M$ transitions are neglected by this approximation, the use of $j\hat{\gamma}$ is thus the source of a T violation which must not be attributed to fundamental interactions but to the WW model.

The time evolution of $|M(\tau)\rangle$ can thus be restricted to a two states Hilbert sub-space: $\left[|M^0\rangle, |\bar{M}^0\rangle \right]$, at the cost of the lost of unitarity $d\langle M | M \rangle / d\tau < 0$ induced by the decay operator $j\hat{\gamma}$. This restriction of the Hilbert space to $\left[|M^0\rangle, |\bar{M}^0\rangle \right]$, allowed by the WW approximation, leads to the effective LOY Hamiltonian. The LOY Hamiltonian without CPV is the sum of the mass energy (mc^2), plus a strangeness/bottomness ($S = \pm 1$ /

$B' = \pm 1$) coupling operator ($\widehat{\delta m}c^2$), plus the WW irreversible decay ($j\hbar\widehat{\gamma}$), according to the Schrödinger equation

$$j\hbar\frac{d|M(\tau)\rangle}{d\tau} = mc^2|M(\tau)\rangle - \left[\frac{\widehat{\delta m}}{2}c^2 + j\hbar\frac{\widehat{\gamma}}{2} \right] \cdot |M(\tau)\rangle. \quad (2)$$

The coupling operator $\widehat{\delta m}$ and the decay operator $\widehat{\gamma}$ are given by

$$\widehat{\delta m} = \delta m \left[|M^0\rangle\langle\overline{M}^0| + |\overline{M}^0\rangle\langle M^0| \right], \quad (3)$$

$$\widehat{\gamma} = \Gamma \left[|M^0\rangle\langle M^0| + |\overline{M}^0\rangle\langle\overline{M}^0| \right] - \delta\Gamma \left[|M^0\rangle\langle\overline{M}^0| + |\overline{M}^0\rangle\langle M^0| \right], \quad (4)$$

where $\delta m > 0$ is the mass splitting between the heavy and light mass eigenstates and $\Gamma > 0$, $\delta\Gamma < 0$ are respectively the average and the splitting between the decay widths of the these eigenstates. These mass eigenstates are: the long-lived L and short-lived S states ($K_{S/L}$) for K^0/\overline{K}^0 , and the heavy H and light L states ($B_{L/H}$) for B^0/\overline{B}^0 . We take the convention $CP|M^0\rangle = |\overline{M}^0\rangle$. The CP eigenstates $|M_1\rangle$ and $|M_2\rangle$ are related to the flavor eigenstates by

$$|M_1\rangle = \frac{|M^0\rangle}{\sqrt{2}} + \frac{|\overline{M}^0\rangle}{\sqrt{2}} = CP|M_1\rangle, \quad (5)$$

$$|M_2\rangle = \frac{|M^0\rangle}{\sqrt{2}} - \frac{|\overline{M}^0\rangle}{\sqrt{2}} = -CP|M_2\rangle. \quad (6)$$

These CP eigenstates, M_1 and M_2 , are also mass eigenstates, thus the time evolution of the CP and mass eigenstates without CPV is given by

$$|M_1(\tau)\rangle = |M_1\rangle \exp -j\frac{c^2}{\hbar} \left[m - \frac{\delta m}{2} - j\hbar\frac{\Gamma - \delta\Gamma}{2c^2} \right] \tau \quad (7)$$

$$|M_2(\tau)\rangle = |M_2\rangle \exp -j\frac{c^2}{\hbar} \left[m + \frac{\delta m}{2} - j\hbar\frac{\Gamma + \delta\Gamma}{2c^2} \right] \tau \quad (8)$$

The above symmetric picture where CP commute with the Hamiltonian is no longer valid when the experimental results of CPV are to be taken into account.

III. MASS EIGENSTATES WITH TYPES (I) AND (III) CPV

When CPV comes into play, the Hamiltonian (2) is modified and the mass eigenstates $K_{S/L}$ or $B_{L/H}$ are no longer the CP eigenstates $K_{1/2}$ or $B_{1/2}$ (5, 6). The mass eigenvalues are not significantly changed by CPV.

For types (*i* and *iii*) CPV, experimental evidences require to modify the Hamiltonian and the resulting mass eigenstates. Type (*ii*) direct CPV in the decay to one final state is also due to earth's gravity, as will be demonstrated in section VI, but the associated ε' parameter is

not involved in the Hamiltonian describing mixing. The parameters ε and β are introduced in order to describe K^0/\overline{K}^0 type (*i*) and B^0/\overline{B}^0 type (*iii*) CPV effects.

For K^0/\overline{K}^0 type (*i*) CPV, the indirect CPV effects are described by the small complex parameter ε and the mass eigenstates $|K_{S/L}\rangle$ are related to the CP eigenstates $|K_{1/2}\rangle$ (5, 6) by

$$|K_S\rangle = |K_1\rangle + \varepsilon|K_2\rangle, \quad (9)$$

$$|K_L\rangle = |K_2\rangle + \varepsilon|K_1\rangle. \quad (10)$$

As $|\varepsilon| = 2.2 \times 10^{-3}$ we have neglect $O[10^{-6}]$ corrections associated with the normalization $\langle K_{S/L} | K_{S/L} \rangle = 1$. The quantity $\langle K_{S/L} | K_{L/S} \rangle = 2 \text{Re } \varepsilon$ is an observable.

For B^0/\overline{B}^0 type (*iii*) CPV, it is convenient to consider mass eigenstates $|B_{L/H}\rangle$ related to CP eigenstates $|B_{1/2}\rangle$ (5,6) by

$$|B_L\rangle = \cos\beta|B_1\rangle + j\sin\beta|B_2\rangle, \quad (11)$$

$$|B_H\rangle = \cos\beta|B_2\rangle + j\sin\beta|B_1\rangle. \quad (12)$$

The overlap of mass eigenstates $\langle B_L | B_S \rangle = 0$, thus there is no type (*i*) CPV with this parametrization and normalization is ensured as $\langle B_{L/H} | B_{L/H} \rangle = 1$.

In the usual KM interpretation these parameters ε and β are related to combinations of CKM matrix elements where the KM phase is adjusted to the measured CPV amplitude. Rather than adjusting a complex phase, another interpretation of the experiments is proposed below: we simply take into account the impact of earth's gravity on the experiments without the need to introduce a new parameters in a CP conserving CKM matrix which is thus free of CPV far from any massive object in a physical Lorentzian frame.

The final quantitative results predicted with this new mechanism leads to the conclusion that CPV observed in the three canonical types of flavored neutral mesons experiments (*i*, *ii* and *iii*) is (earth) gravity induced, and not fundamental at the level of the CKM matrix elements.

IV. STRANGENESS AND BOTTOMNESS OSCILLATIONS ON EARTH

The Schrödinger equation Eq.(2) is pertinent far from any massive object, but, on earth, we have to consider the very small Newtonian potential energy $mG_N M_\oplus / R_\oplus \sim 10^{-9}mc^2$. We can restrict the description of this new coupling to the first term of the Taylor expansion of $mG_N M_\oplus / (R_\oplus + X + x)$ with respect to a vertical position $X + x$ where $x \ll X \ll R_\oplus$. The position X is the vertical average position of the meson with respect to the level R_\oplus . This is an external degree of freedom: it can not enter in the τ dynamics (2) as τ is the meson proper time. The vertical position $x(\tau)$ describes the internal vertical fluctuations around this average X . This is an internal degree of freedom: it must enter the proper time

Hamiltonian (2). Thus, we consider an additional energy term $mg \hat{x}(\tau)$ in (2) with $g = G_N M_\oplus / R_\oplus^2 = 9.8 \text{ m/s}^2$,

$$j\hbar \frac{d|M(\tau)\rangle}{d\tau} = mc^2 |M\rangle - \frac{\widehat{\delta m}}{2} c^2 \cdot |M\rangle - j\hbar \frac{\widehat{\gamma}}{2} \cdot |M\rangle + mg \hat{x}(\tau) \cdot |M\rangle. \quad (13)$$

We have just applied here the *correspondence principle* between classical and quantum mechanics and this requires a careful interpretation and analysis of the vertical position operator $\hat{x}(\tau)$.

It is very important to note that, as τ is the meson proper time, the position operator $\hat{x}(\tau)$ in (13) must not be associated with the vertical position of the meson with respect to a reference vertical level in the laboratory.

As τ is the rest frame proper time of the meson, the motion described by the operator $\hat{x}(\tau)$ is associated with the (unknown) internal quark vertical motion, inside the mesons, as a function of the meson proper time τ : the zitterbewegung motion inherent to all, free or bound, spin 1/2 fermions [11].

The operator $\hat{x}(\tau)$ in (13) describes the fast fluctuating vertical motion of the quarks inside the meson with respect to the meson average position defining the rest frame of the meson. This rest frame has a proper time τ and its free fall does not affect (13) on the time scale of the experiment.

The separation between a slow average and fast fluctuations is based on a three time scales ordering of the dynamics: a very slow time scale of the meson free fall, which does not enter in the meson proper time dynamics Eq.(13), a slow time scale of the order of $\hbar/\delta mc^2$ associated with flavor oscillations, and the fast fluctuating motion associated with quarks zitterbewegung internal oscillations, with a zitterbewegung time scale of the order of the Compton wavelength divided by the velocity of light $\lambda_C/c \sim \hbar/mc^2$. This separation between: average position, fast zitterbewegung fluctuations around this average, and mixing oscillations, displays a strong ordering. The mixing and zitterbewegung time scales entering in (13) are ordered according to $\hbar/mc^2 : \hbar/\delta mc^2 \sim 10^{-15} - 10^{-13}$.

The mesons, $|M^0\rangle$ and $|\overline{M}^0\rangle$, are stationary bound states ultimately described by Dirac spinors associated with one light quark q' and one heavier quark q : $|M^0\rangle \sim |q'\overline{q}\rangle$ and $|\overline{M}^0\rangle \sim |\overline{q}'q\rangle$. The Dirac spinors are combined into a *singlet* spin zero $|q'\overline{q}\rangle$ state. In the restricted Hilbert space, $\left[|M^0\rangle, |\overline{M}^0\rangle\right]$, the internal vertical position operator $\hat{x}(\tau)$ is thus represented by the following four matrix elements $\langle |\hat{x}(\tau)| \rangle$ of the stationary Dirac spinors $|q'\overline{q}\rangle$ describing quark-antiquark bound states

$$\begin{aligned} \widehat{x}(\tau) &= \langle q'\overline{q} | \hat{x}(\tau) | q'\overline{q} \rangle |M^0\rangle \langle M^0| \\ &+ \langle \overline{q}'q | \hat{x}(\tau) | \overline{q}'q \rangle |\overline{M}^0\rangle \langle \overline{M}^0| \\ &+ \langle \overline{q}'q | \hat{x}(\tau) | q'\overline{q} \rangle |\overline{M}^0\rangle \langle M^0| \\ &+ \langle q'\overline{q} | \hat{x}(\tau) | \overline{q}'q \rangle |M^0\rangle \langle \overline{M}^0|. \end{aligned} \quad (14)$$

The internal vertical position operator $\hat{x}(\tau)$ fulfils Heisenberg's equation $j\hbar d\hat{x}/d\tau = \hat{x} \cdot \hat{H} - \hat{H} \cdot \hat{x}$ where \hat{H} is the Dirac Hamiltonian describing quarks confinement inside the meson. The values of the internal vertical position matrix elements $\langle |\hat{x}(\tau)| \rangle$ depend of the model of their confinement inside the meson.

The typical size of $\sqrt{\langle x^2 \rangle}$ is the Compton wavelength of the meson λ_C .

The instantaneous velocity operator $d\hat{x}/d\tau$ of spin 1/2 particles/antiparticles pairs, either free or bound, is well known to display the so called zitterbewegung (nonintuitive) behavior: a quiver motion on a length scale given by the Compton wavelength λ_C and an instantaneous velocity equal to the velocity light c [11].

It is important to note that the values of the instantaneous velocity matrix elements $\langle |d\hat{x}/d\tau| \rangle$ are independent of the charge and mass of the fermions as well as of the shape and strength of the effective confinement potential involved in the Dirac Hamiltonian $\hat{H}(\mathbf{x}, \mathbf{p})$. This zitterbewegung universality is a consequence of Heisenberg's equation

$$j\hbar \frac{d\mathbf{x}}{d\tau} = \left[\mathbf{x}, \hat{H}(\mathbf{x}, \mathbf{p}) \right] = j\hbar c \boldsymbol{\alpha}. \quad (15)$$

We have introduced the usual 4×4 alpha matrices: $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ [11] which can be expressed in terms of the 2×2 Pauli matrices $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Equation (15) imply that the values of the internal fluctuating velocity matrix elements are equal to c or 0 . In two previous studies, Ref. [9] and [12], we have found, with two different methods, that the normalized spinor bound states in a confining potential describing the mean field strong interactions ultimately ensuring confinement in the meson, fulfils $\langle \overline{q} | \alpha_x | \overline{q} \rangle = \langle q | \alpha_x | q \rangle = 0$ and $\langle q | \alpha_x | \overline{q} \rangle = \langle \overline{q} | \alpha_x | q \rangle = 1$. To adapt the LOY model to earth's gravity, and to describe a meson and its antimeson, we have to consider that antiparticles are just particles propagating backward in time according to the Feynman picture. Following Feynman's rule, to discriminate M^0 from \overline{M}^0 in the LOY model on earth, the internal, vertical, fluctuating, velocity operator $d\hat{x}/d\tau$, expressed on the $\left[|M^0\rangle, |\overline{M}^0\rangle\right]$ basis, is thus given by

$$\frac{d\hat{x}}{d\tau} = c |M^0\rangle \langle \overline{M}^0| - c |\overline{M}^0\rangle \langle M^0|. \quad (16)$$

This operator describes the instantaneous velocity (fast time scale) of the quarks, inside the meson with respect to the meson rest frame.

Beside the time ordering between an average and an instantaneous dynamics, the inclusion of the gravity term in (2), to give (13), introduces an energy ordering. The Compton wavelength of the meson λ_C provides an approximate maximum size of the q/\bar{q} quark matrix elements $|\langle \hat{x} \rangle|$ in (14) as quarks are bound states inside the volume of a meson. The very small numerical value of the energy $mg\lambda_C = \hbar g/c \sim 10^{-23}$ eV in front of $\delta mc^2 \sim 10^{-6} - 10^{-4}$ eV leads to the occurrence of a very strong ordering fulfilled by the four matrix elements in (14), $mg|\langle \hat{x} \rangle| \sim mg\lambda_C \ll \delta mc^2$, in front of the other LOY matrix elements. Note that, beside this energy ordering, the frequency ordering between these two terms is reversed: $\delta mc^2/\hbar \ll mc^2/\hbar$.

The very strong energy ordering identified here allows to set up a perturbative expansion of (13) with respect to the small expansion parameter $\hbar g/\delta mc^3 \sim 10^{-19} - 10^{-17}$. We define $|N(\tau)\rangle$ and $|n(\tau)\rangle$ such that the meson dynamics is described by $|N(\tau)\rangle + |n(\tau)\rangle$

$$|M(\tau)\rangle = |N(\tau)\rangle \exp -j \frac{mc^2\tau}{\hbar} + |n(\tau)\rangle \exp -j \frac{mc^2\tau}{\hbar}. \quad (17)$$

The states $|N(\tau)\rangle$ and $|n(\tau)\rangle$ are ordered according to: $|N\rangle \sim O(\hbar g/\delta mc^3)^0$, $|n\rangle \sim O(\hbar g/\delta mc^3)^1$ and the first neglected term is $O(\hbar g/\delta mc^3)^2 \sim 10^{-38} - 10^{-34}$. With this expansion scheme (17), Schrödinger's equation (13) becomes

$$j\hbar \frac{d|N\rangle}{d\tau} = -\frac{1}{2} (\widehat{\delta mc^2} + j\hbar\widehat{\gamma}) \cdot |N\rangle, \quad (18)$$

$$j\hbar \frac{d|n\rangle}{d\tau} = -\frac{1}{2} (\widehat{\delta mc^2} + j\hbar\widehat{\gamma}) \cdot |n\rangle + mg \widehat{x} \cdot |N\rangle. \quad (19)$$

To identify the dominant secular contribution of earth's gravity we introduce the inverse of the operator $\widehat{\delta mc^2} + j\hbar\widehat{\gamma}$ and then use this operator and (18) to rewrite (19)

$$j\hbar \frac{d|n\rangle}{d\tau} = -\frac{1}{2} (\widehat{\delta mc^2} + j\hbar\widehat{\gamma}) \cdot |n\rangle + 2jmg\hbar \frac{d\widehat{x}}{d\tau} \cdot (\widehat{\delta mc^2} + j\hbar\widehat{\gamma})^{-1} \cdot |N\rangle - 2jmg\hbar \frac{d}{d\tau} \left[\widehat{x} \cdot (\widehat{\delta mc^2} + j\hbar\widehat{\gamma})^{-1} \cdot |N\rangle \right] \quad (20)$$

The strong time ordering between strangeness (or bottomness) oscillations ($\hbar/\delta mc^2$) and zitterbewegung oscillations ($\sim \hbar/mc^2$) can be used to simplify (20). We are interested by the strangeness or bottomness dynamics taking place on the *slow* time scale $\hbar/\delta mc^2$, thus we introduce 2θ the period of the (unknown) *fast* periodic functions $|\langle \widehat{x}(\tau) \rangle|$ associated with the zitterbewegung oscillations. This time 2θ is such that $\hbar/mc^2 \sim \theta \ll \hbar/\delta mc^2$. We apply the averaging operator $\widehat{A}_\theta \equiv \int_{\tau-\theta}^{\tau+\theta} dt/2\theta$ on both side of (20) to average the high frequency (mc^2/\hbar) components. For any low frequency ($\delta mc^2/\hbar$) function

$f(t)$: $\widehat{A}_\theta \cdot f(t) = f(\tau)$ and $\widehat{A}_\theta \cdot df/dt = df/d\tau$ and for any high frequency function $g(t)$: $\widehat{A}_\theta \cdot dg/dt = 0$. This usual averaging methods is just *Bogolioubov-Krilov-Mitropolski* method when applied on the dynamical equations, or *Witham* method if we average directly the Lagrangian associated with the evolution equations [13].

The equations describing strangeness or bottomness oscillations of a neutral meson $|N(\tau)\rangle + |n(\tau)\rangle$ on earth are given by

$$j\hbar \frac{d|N\rangle}{d\tau} = -\frac{1}{2} (\widehat{\delta mc^2} + j\hbar\widehat{\gamma}) \cdot |N\rangle, \quad (21)$$

$$j\hbar \frac{d|n\rangle}{d\tau} = -\frac{1}{2} (\widehat{\delta mc^2} + j\hbar\widehat{\gamma}) \cdot |n\rangle + j\widehat{G} \cdot |N\rangle. \quad (22)$$

The *gravity/zitterbewegung* operator \widehat{G} , capturing the secular interplay between zitterbewegung oscillations and bottomness or strangeness oscillations, is defined as

$$\widehat{G} = 2mg\hbar \left(\frac{d\widehat{x}}{d\tau} \right) \cdot (\widehat{\delta mc^2} + j\hbar\widehat{\gamma})^{-1}. \quad (23)$$

Flavored neutral mesons pairs K^0/\bar{K}^0 and B^0/\bar{B}^0 display different m , δm , Γ and $\delta\Gamma$ and the impact of earth gravity on their oscillation behavior is to be analyzed specifically. In the following we keep the notation of Eqs. (21, 22) and (23) with an additional index K or B for these specific studies.

V. GRAVITY INDUCED TYPE (I) CPV IN THE MIXING OF K^0/\bar{K}^0

The ordering associated with the specific case of a K^0/\bar{K}^0 pair is given by: $\delta m_K/m_K \sim 10^{-15}$ and the lifetime of the K_S is 570 times shorter than the lifetime of K_L . The first step to interpret K^0/\bar{K}^0 experiments is to consider a unitary evolution and to neglect the finite lifetime of both particles, described by $j\hbar\widehat{\gamma}$, in Eqs. (21, 22) and (23). Then, as the lifetime of K_L is 570 times longer than the lifetime of K_S , we set up a steady state balance between the fast decay of the small K_1 component of a K_L , produced initially without K_1 , and its gravity induced regeneration from this K_L . Considering first a unitary evolution $\widehat{\gamma} = \widehat{0}$ ($\delta\Gamma_K = \Gamma_K = 0$), we have to solve

$$j\hbar \frac{d|N_K\rangle}{d\tau} = -\frac{1}{2} \widehat{\delta m_K c^2} \cdot |N_K\rangle, \quad (24)$$

$$j\hbar \frac{d|n_K\rangle}{d\tau} = -\frac{1}{2} \widehat{\delta m_K c^2} \cdot |n_K\rangle + j\widehat{G}_K \cdot |N_K\rangle. \quad (25)$$

The operator $\widehat{\delta m_K c^2}$ is given by (3), the operator $d\widehat{x}/d\tau$ by (16), \widehat{G}_K by (23) and $\widehat{\gamma} = \widehat{0}$. The action of \widehat{G}_K on the CP eigenstates $|K_1\rangle$ and $|K_2\rangle$ defined in (5, 6) is

$$\widehat{G}_K |K_2\rangle = \kappa \delta m_K c^2 |K_1\rangle, \quad (26)$$

$$\widehat{G}_K |K_1\rangle = \kappa \delta m_K c^2 |K_2\rangle, \quad (27)$$

where we have defined the small parameter κ

$$\kappa = 2m_K g \hbar / \delta m_K^2 c^3 = 1.7 \times 10^{-3}. \quad (28)$$

This small parameter has been identified and discussed by Fishbach, forty five years ago, as the undimensional combination matching approximately the experimental value of $\text{Re } \varepsilon$ [14, 15].

If we consider the following CP eigenstate

$$|N_{K_2}(\tau)\rangle = |K_2\rangle \exp -j\delta m_K c^2 \tau / 2\hbar, \quad (29)$$

which is the $(m_K + \delta m_K)$ mass eigenstate without CPV (6,8), it fulfils Eq. (24) and the associated solution of Eq. (25) is

$$|n_{K_2}(\tau)\rangle = j\kappa |K_1\rangle \exp -j\delta m_K c^2 \tau / 2\hbar. \quad (30)$$

Thus, on earth, the mass eigenstates $|K_2^\oplus\rangle$ is not the CP eigenstates $|K_2\rangle$ (6), but the sum of the previous solutions (29, 30)

$$|K_2^\oplus\rangle = |K_2\rangle + j\kappa |K_1\rangle. \quad (31)$$

We neglect the small correction $O[10^{-6}]$ needed for normalization and consider $\langle K_2^\oplus | K_2^\oplus \rangle = 1$. A similar result is obtained for the other $(m_K - \delta m_K)$ mass eigenstate without CPV (5,7) by taking

$$|N_{K_1}(\tau)\rangle = |K_1\rangle \exp j\delta m_K c^2 \tau / 2\hbar \quad (32)$$

as a source term on the right hand side of Eq. (25). This leads to a gravity induced correction

$$|n_{K_1}(\tau)\rangle = -j\kappa |K_2\rangle \exp j\delta m_K c^2 \tau / 2\hbar. \quad (33)$$

The other mass eigenstates on earth is not the CP eigenstates $|K_1\rangle$ (5), but the sum of the previous solutions (32, 33)

$$|K_1^\oplus\rangle = |K_1\rangle - j\kappa |K_2\rangle. \quad (34)$$

At the fundamental level of a unitary evolution, without decays, the impact of earth's gravity appears as a CPT violation, with T conservation, because the indirect violation parameter $\langle K_1^\oplus | K_2^\oplus \rangle = 2j\kappa$ is imaginary [4], rather than a CP and T violation with CPT conservation requiring a non zero real value [4].

We must now take into account the K_1 fast decay. This decay will change the picture, qualitatively: an apparent CP and T violation, with CPT conservation, is measured experimentally rather than a CPT one, and quantitatively: with the right prediction of $\text{Re } \varepsilon$ which is slightly smaller than κ .

The previous results, Eqs. (31, 34), allow to calculate the *gravity induced transition amplitude* $\Omega_{2 \rightarrow 1}$ describing the transition amplitude per unit time from $|K_2^\oplus\rangle \exp -j\delta m_K c^2 \tau / 2\hbar$ to $|K_1^\oplus\rangle \exp j\delta m_K c^2 \tau / 2\hbar$,

$$\Omega_{2 \rightarrow 1} = \left\langle \frac{dK_2^\oplus}{d\tau} | K_1^\oplus(\tau) \right\rangle = \kappa \frac{\delta m_K c^2}{\hbar} \exp j \frac{\delta m_K c^2}{\hbar} \tau. \quad (35)$$

This can be viewed as a gravity induced *oscillating regeneration* competing with the short-lived kaon irreversible decay to the set of final states $\{|f\rangle\}$. This decay takes place at a rate $\Gamma_{1 \rightarrow f} / 2 = (\Gamma_K - \delta\Gamma_K) / 2 \sim \Sigma_f |\langle f | \mathcal{T} | K_1 \rangle|^2$. Note that $|\Omega_{2 \rightarrow 1}| \sim O[10^{-3} \Gamma_{1 \rightarrow f}]$ so, starting from a pure $O[1] K_2$ population, an $O[10^{-3}] K_1$ steady state satellite will be observed.

We consider now a typical experiment dedicated to indirect CPV. Experimentally K_1 and K_2 are first produced together in equal amounts. Then, after few $1/\Gamma_{1 \rightarrow f}$ decay times, the initial content of $|K_1\rangle$ disappears and a pure $|K_2\rangle$ state is expected. In fact, the state $|K_{L \text{ exp}}(\tau)\rangle$ observed in such an experiment is not a pure $|K_2\rangle$ state. This $|K_{L \text{ exp}}(\tau)\rangle$ state is a linear superposition of $|K_2\rangle$, plus a small amount of $|K_1\rangle$,

$$|K_{L \text{ exp}}(\tau)\rangle = a_2(\tau) |K_2\rangle + a_1(\tau) |K_1\rangle, \quad (36)$$

resulting from the balance between gravity induced regeneration (35) and irreversible decay. We assume that the $|K_2\rangle$ component is stable and that the depletion of its amplitude associated with the *gravitational regeneration* of $|K_1\rangle$ is negligible so that $|a_2| = 1$ and

$$a_2(\tau) = \exp -j\delta m_K c^2 \tau / 2\hbar. \quad (37)$$

The amplitude a_1 of $|K_1\rangle$ in (36) is given by the steady-state balance between a decay at the (amplitude) rate $\Gamma_{1 \rightarrow f} / 2$ on the one hand, and a (gravity induced) transition/regeneration $\Omega_{2 \rightarrow 1}$ (35) from $|K_2\rangle$ on the other hand

$$a_2(\tau) \Omega_{2 \rightarrow 1} = a_1(\tau) \frac{\Gamma_{1 \rightarrow f}}{2}. \quad (38)$$

The solution is this equation is

$$a_1(\tau) = \frac{\delta m_K c^2}{\hbar \Gamma_1 / 2} \kappa \exp j\delta m_K c^2 \tau / 2\hbar, \quad (39)$$

where we have dropped $\rightarrow f$ in Γ_1 to simplify the notations. This short-lived $|K_1\rangle$ component is observed through its two pion decay [1]. Thus the observed long-lived mass eigenstate $|K_{L \text{ exp}}\rangle$, obtained after few $1/\Gamma_1$ decay times away from a neutral kaons source, must be represented by

$$|K_{L \text{ exp}}\rangle = |K_2\rangle + \frac{\delta m_K c^2}{\hbar \Gamma_1 / 2} \kappa |K_1\rangle. \quad (40)$$

This is the usual CPV parametrization of the kaon state Eq. (10). The value of the indirect, gravity induced, CPV parameter,

$$\text{Re } \varepsilon_{\text{exp}} = \frac{\delta m_K c^2}{\hbar \Gamma_1 / 2} \kappa = 1.66 \times 10^{-3}, \quad (41)$$

is thus in agreement with the experimental value, reported by Gershon and Nir, page 290 of Ref. [5]

$$\text{Re } \varepsilon = (1.66 \pm 0.02) \times 10^{-3}. \quad (42)$$

We have taken into account here the finite lifetime of the short-lived kaon, to complete this analysis we have also to take into account the decay of the other mass eigenstate, and this will reveal the phase of ε . Considering $\Gamma_S = \Gamma_K - \delta\Gamma_K$ for K_1 , and $\Gamma_L = \Gamma_K + \delta\Gamma_K$ for K_2 ($\delta\Gamma_K < 0$), beside the usual definition of decay rates $\Gamma_{S/L} = \sum_f |\langle f | \mathcal{T} | K_{S/L} \rangle|^2$ in terms of transition amplitudes, Bell and Steinberger [16] have demonstrated a general relation based on global unitarity starting from the evaluation of $d\langle M | M \rangle / d\tau$ at $\tau = 0$. Using the fact that, for K_S , the sum over the final states \sum_f is dominated (99.9%) by $K_S \rightarrow 2\pi$ decays, more precisely by the $K_S \rightarrow |I_0\rangle$ decays (95%) to the isospin-zero combination of $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$, the Bell-Steinberger's unitarity relations [16] can be written:

$$j \frac{\delta mc^2}{\hbar} + \frac{\Gamma_S}{2} = \frac{\langle I_0 | \mathcal{T} | K_L \rangle \langle I_0 | \mathcal{T} | K_S \rangle^*}{\langle K_S | K_L \rangle}. \quad (43)$$

The restriction of the sum $\sum_f |f\rangle$ to $|I_0\rangle$ reduces the K_S width to $\Gamma_S = \langle I_0 | \mathcal{T} | K_S \rangle \langle I_0 | \mathcal{T} | K_S \rangle^*$ so that

$$\frac{\langle I_0 | \mathcal{T} | K_L \rangle}{\langle I_0 | \mathcal{T} | K_S \rangle} = \frac{\langle I_0 | \mathcal{T} | K_L \rangle \langle I_0 | \mathcal{T} | K_S \rangle^*}{\Gamma_S}. \quad (44)$$

This expression is then substituted in (43) to obtain the final expression

$$\frac{\langle I_0 | \mathcal{T} | K_L \rangle}{\langle I_0 | \mathcal{T} | K_S \rangle} = \frac{\langle K_S | K_L \rangle}{2} \left(1 + j \frac{2\delta mc^2}{\hbar\Gamma_S} \right). \quad (45)$$

The left hand side of Eq. (45) is just the definition of the complex parameter ε and $\langle K_S | K_L \rangle / 2 = \text{Re} \varepsilon$, thus by the argument of the CPV complex parameter ε is given by

$$\arg \varepsilon = \arctan(2\delta m_K c^2 / \hbar\Gamma_S) = 43.4^\circ, \quad (46)$$

in agreement with the experimental results [5]. This last relation (46) complete (41) and confirms that gravity induced CPV provides a global pertinent framework to interpret K^0/\bar{K}^0 indirect CPV experiments.

VI. GRAVITY INDUCED TYPE (II) CPV IN THE DECAY OF K^0/\bar{K}^0

To interpret the measurements of the direct violation parameter ε' we consider the $2\pi^0$ decays of K_L and K_S .

The definition of ε' as a function of the amplitude ratio η_{00} is given by

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{T} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{T} | K_S \rangle} \equiv \varepsilon - 2\varepsilon', \quad (47)$$

where the direct violation in the decay in one final state ε' is a correction to the indirect violation in the mixing $\varepsilon \gg \varepsilon'$.

The various bra and ket in a quantum model are defined up to an unobservable phase. The arbitrary conventional phases inherent to quantum theoretical models are to be eliminated to define phase-convention-independent observables. This requirement is a consequence of the objectivity of a physical description, for example, within the framework of relativistic dynamics, all the physical observables should be expressed as combinations of Lorentz invariants and here, within the framework of quantum dynamics, all the physical observables are to be expressed as combinations of phase-convention-independent quantities. The definition of η_{00} is invariant under rephasing of the pions state $\langle \pi^0\pi^0 |$, but not with respect to the rephasing of the kaons mass eigenstates $|K_{L/S}\rangle$. We can define an amplitude ratio η_{00} which is a phase-convention-independent quantity through a multiplication with the factor φ_K

$$\varphi_K = \frac{\langle K^0 | K_S \rangle}{\langle K^0 | K_L \rangle}, \quad (48)$$

as the L/S phase of η_{00} is compensated by the S/L phase of φ_K .

If we consider the mass eigenstates (9, 10) used for the usual description of CPV

$$|K_S\rangle = \frac{1+\varepsilon}{\sqrt{2}} |K_0\rangle + \frac{1-\varepsilon}{\sqrt{2}} |\bar{K}^0\rangle, \quad (49)$$

$$|K_L\rangle = \frac{1+\varepsilon}{\sqrt{2}} |K_0\rangle - \frac{1-\varepsilon}{\sqrt{2}} |\bar{K}^0\rangle. \quad (50)$$

and those obtained at the fundamental level of an unitary evolution (31, 34) with gravity induced CPV

$$|K_1^\oplus\rangle = \frac{1-j\kappa}{\sqrt{2}} |K^0\rangle + \frac{1+j\kappa}{\sqrt{2}} |\bar{K}^0\rangle, \quad (51)$$

$$|K_2^\oplus\rangle = \frac{1+j\kappa}{\sqrt{2}} |K^0\rangle - \frac{1-j\kappa}{\sqrt{2}} |\bar{K}^0\rangle, \quad (52)$$

we obtain two different expressions of the rephasing factor φ_K .

For the usual CPV parametrization (49, 50) with CPT conservation

$$\varphi_K = \frac{\langle K^0 | K_S \rangle}{\langle K^0 | K_L \rangle} = 1. \quad (53)$$

For gravity induced CPV, we replace $|K_S\rangle$ and $|K_L\rangle$ by $|K_1^\oplus\rangle$ and $|K_2^\oplus\rangle$ and use (51, 52) to obtain

$$\varphi_K^\oplus = \frac{\langle K^0 | K_1^\oplus \rangle}{\langle K^0 | K_2^\oplus \rangle} = 1 - \langle K_1^\oplus | K_2^\oplus \rangle. \quad (54)$$

where $O[10^{-6}]$ and higher orders terms are neglected. We assume that the amplitude of $K_0 \rightarrow \pi^0\pi^0$ is equal to the amplitude of $\bar{K}^0 \rightarrow \pi^0\pi^0$ because the interaction between a $(\pi^0\pi^0)$ state and a neutral kaon state, K_0 or \bar{K}^0 , can not differentiate the K^0 from the \bar{K}^0 (a final state phase can be absorbed by a proper phase convention

between the K_0 and \bar{K}^0). The ratio of amplitude η_{00}^\oplus associated with the mass eigenstates (51, 52) describing gravity induced CPV is

$$\eta_{00}^\oplus = \frac{\langle \pi^0 \pi^0 | \mathcal{T} | K_2^\oplus \rangle}{\langle \pi^0 \pi^0 | \mathcal{T} | K_1^\oplus \rangle} = \frac{\langle K_1^\oplus | K_2^\oplus \rangle}{2}. \quad (55)$$

We conclude that the physical observable $\eta_{00}\varphi_K$ on earth is given by

$$\eta_{00}^\oplus \varphi_K^\oplus = \frac{\langle K_1^\oplus | K_2^\oplus \rangle}{2} \left[1 - 2 \frac{\langle K_1^\oplus | K_2^\oplus \rangle}{2} \right]. \quad (56)$$

When we neglect the finite lifetime of the kaons, we have demonstrated, in the previous section, that the gravity induced mixing \hat{G}_K , between $|K_1\rangle$ and $|K_2\rangle$, leads to an apparent CPT violation with $\langle K_1^\oplus | K_2^\oplus \rangle / 2 = j\kappa$. When decays are taken into account, the finite lifetime of K_1 was shown to induce a rotation from the imaginary value $j\kappa$ to the real observed value $(\delta m_K c^2 / \hbar \Gamma_1 / 2) \kappa = \langle K_S | K_L \rangle / 2$ (41). Taking into account these dissipative effects, the observed phase-convention-independent amplitude ratio $\eta_{00 \text{ exp}}^\oplus \varphi_{K \text{ exp}}^\oplus$ measured in dedicated experiments on earth [17–19] is given by the use of $\langle K_S | K_L \rangle$ instead of $\langle K_1^\oplus | K_2^\oplus \rangle$ in the phase-convention-independent observable (56)

$$\eta_{00 \text{ exp}}^\oplus \varphi_{K \text{ exp}}^\oplus = \frac{\langle K_S | K_L \rangle}{2} \left[1 - 2 \frac{\langle K_S | K_L \rangle}{2} \right]. \quad (57)$$

The definition (47) $\eta_{00}\varphi_K = \varepsilon - 2\varepsilon'$ leads to the conclusion that $\text{Re } \varepsilon' / \varepsilon = \text{Re } \varepsilon$. The gravity induced direct CPV parameter

$$\text{Re } (\varepsilon' / \varepsilon) = \frac{\delta m_K c^2}{\hbar \Gamma_1 / 2} \kappa = 1.66 \times 10^{-3}, \quad (58)$$

is in agreement with the experimental value, reported by Gershon and Nir, page 285 of Ref. [5]

$$\text{Re } (\varepsilon' / \varepsilon) = (1.66 \pm 0.23) \times 10^{-3}. \quad (59)$$

The precise definition of phase-convention-independent quantities, in order to clearly identify what is measured in an experiment, is also one of the key to interpret the experimental observation of interferences between mixing and decay in dedicated B^0/\bar{B}^0 experiments.

VII. GRAVITY INDUCED TYPE (III) CPV IN THE INTERFERENCE BETWEEN MIXING AND DECAY OF B^0/\bar{B}^0

The mass and width ordering associated with the B^0/\bar{B}^0 system is given by : $\delta m_B / m_B \sim 10^{-19}$ and $\delta m_B / \Gamma_B \sim 0.7$. The lifetime of the CP eigenstate B_1 is considered equal to the lifetime of the other CP eigenstate B_2 so that $\delta \Gamma_B = 0$. The most pronounced CPV

effects in the B^0/\bar{B}^0 system is displayed through interference experiments dedicated to the study of the phase difference between the decay path $B_0 \rightarrow f$ and the decay path $B_0 \rightarrow \bar{B}^0 \rightarrow f$ [20–22]. To set up an interpretation of these experiments we keep a finite lifetime Γ_B^{-1} for both particles and consider the diagonal decay operator

$$\hat{\gamma}_B = \Gamma_B \left[|B^0\rangle \langle B^0| + |\bar{B}^0\rangle \langle \bar{B}^0| \right], \quad (60)$$

to describe the dissipative part of the bottomness dynamics. Thus, we have to solve Eqs. (21, 22)

$$j\hbar \frac{d|N_B\rangle}{d\tau} = -\frac{1}{2} \left(\widehat{\delta m}_B c^2 + j\hbar \hat{\gamma}_B \right) \cdot |N_B\rangle, \quad (61)$$

$$j\hbar \frac{d|n_B\rangle}{d\tau} = -\frac{1}{2} \left(\widehat{\delta m}_B c^2 + j\hbar \hat{\gamma}_B \right) \cdot |n_B\rangle + j\hat{G}_B \cdot |N_B\rangle. \quad (62)$$

The operator $\widehat{\delta m}_B c^2$ is given by (3), \hat{G}_B by (23), the operator $d\hat{x}/d\tau$ by (16), and $\hat{\gamma}_B$ by (60). The action of \hat{G}_B on the CP eigenstates $|B_1\rangle$ and $|B_2\rangle$ (5, 6) is

$$j\hat{G}_B |B_2\rangle = -\delta m_B c^2 \varkappa (1 - j\chi) |B_1\rangle, \quad (63)$$

$$j\hat{G}_B |B_1\rangle = \delta m_B c^2 \varkappa (1 + j\chi) |B_2\rangle. \quad (64)$$

Where we define the real parameters χ and \varkappa associated with this gravity induced mixing of the $[|B_1\rangle, |B_2\rangle]$ CP basis

$$\chi = \delta m_B c^2 / \hbar \Gamma_B = 0.77, \quad (65)$$

$$\varkappa = 2m_B g \hbar / \delta m_B^2 c^3 (\chi + \chi^{-1}) \sim O[10^{-6}]. \quad (66)$$

In order to solve Eq. (62) and to express the mass eigenstates on earth, we consider the CP eigenstates

$$|N_{B_2}(\tau)\rangle = |B_2\rangle \exp -j \frac{\delta m_B c^2 - j\hbar \Gamma_B}{2\hbar} \tau, \quad (67)$$

which is also the $(m_B + \delta m_B)$ mass eigenstate without CPV, it fulfils (61) and the associated solution of (62) is

$$|n_{B_2}(\tau)\rangle = -\varkappa (1 - j\chi) |B_1\rangle \exp -j \frac{\delta m_B c^2 - j\hbar \Gamma_B}{2\hbar} \tau. \quad (68)$$

Then we consider the other $(m_B - \delta m_B)$ CP eigenstate as a drive on the right hand side of Eq. (62)

$$|N_{B_1}(\tau)\rangle = |B_1\rangle \exp j \frac{(\delta m_B c^2 + j\hbar \Gamma_B)}{2\hbar} \tau. \quad (69)$$

It fulfils Eq. (61) and the driven solution of Eq. (62) is

$$|n_{B_1}(\tau)\rangle = -\varkappa (1 + j\chi) |B_2\rangle \exp j \frac{\delta m_B c^2 + j\hbar \Gamma_B}{2\hbar} \tau. \quad (70)$$

Thus, on earth, the CP eigenstates $|B_1\rangle$ and $|B_2\rangle$ (5, 6) are no longer the mass eigenstates $B_{L/H}^\oplus$ which are given by the sum $|N_{B_{1/2}}\rangle + |n_{B_{1/2}}\rangle$ of (67, 69) plus (68, 70)

$$|B_L^\oplus\rangle = |B_1\rangle - \varkappa (1 + j\chi) |B_2\rangle, \quad (71)$$

$$|B_H^\oplus\rangle = |B_2\rangle - \varkappa (1 - j\chi) |B_1\rangle. \quad (72)$$

Using the bottomness basis $\left[|B^0\rangle, |\overline{B}^0\rangle\right]$ rather than the CP basis $\left[|B_1\rangle, |B_2\rangle\right]$ these mass eigenstates become

$$|B_L^\oplus\rangle = \frac{1 - \varkappa(1 + j\chi)}{\sqrt{2}} |B^0\rangle + \frac{1 + \varkappa(1 + j\chi)}{\sqrt{2}} |\overline{B}^0\rangle, \quad (73)$$

$$|B_H^\oplus\rangle = \frac{1 - \varkappa(1 - j\chi)}{\sqrt{2}} |B^0\rangle - \frac{1 + \varkappa(1 - j\chi)}{\sqrt{2}} |\overline{B}^0\rangle. \quad (74)$$

The difference between these gravity induced mass eigenstates (71, 72, 73, 74) and the usual type (iii) B^0/\overline{B}^0 parametrization (11, 12), is that gravity induced CPV requires two real number \varkappa and χ to express the eigenstates $|B_{L/H}^\oplus\rangle$ although type (iii) standard CPV parametrization (11, 12)

$$|B_L\rangle = \frac{\exp + j\beta}{\sqrt{2}} |B^0\rangle + \frac{\exp - j\beta}{\sqrt{2}} |\overline{B}^0\rangle, \quad (75)$$

$$|B_H\rangle = \frac{\exp + j\beta}{\sqrt{2}} |B^0\rangle - \frac{\exp - j\beta}{\sqrt{2}} |\overline{B}^0\rangle, \quad (76)$$

is based on a single real parameter β to interpret the experimental results.

This difference is due to the assumption of CPT invariance associated with the parametrization (11, 12) and (75, 76). There are *a priori* 4 complex numbers $\langle B^0 | B_L \rangle$, $\langle \overline{B}^0 | B_H \rangle$, $\langle B^0 | B_H \rangle$ and $\langle \overline{B}^0 | B_L \rangle$ needed to expand a mass eigenstates $|B_{L/H}\rangle$ on a bottomness basis $\left[|B^0\rangle, |\overline{B}^0\rangle\right]$, and this number is then reduced through normalization and rephasing invariance.

When the decay into one final CP eigenstate $|f\rangle$ is considered in experiments, the observable is assumed to be the combination of amplitudes λ_f defined as

$$\lambda_f = \frac{\langle \overline{B}^0 | B_L \rangle \langle f | \mathcal{T} | \overline{B}^0 \rangle}{\langle B^0 | B_L \rangle \langle f | \mathcal{T} | B^0 \rangle} = \exp -2j\beta \frac{\langle f | \mathcal{T} | \overline{B}^0 \rangle}{\langle f | \mathcal{T} | B^0 \rangle}, \quad (77)$$

which is obviously phase-convention-independent. This parameter is observable through the measurement of S_f and C_f

$$S_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad (78)$$

which can be extracted from the data obtained from interferences between the direct path $B_0 \rightarrow f$ and the mixed path $B_0 \rightarrow \overline{B}^0 \rightarrow f$.

This parameter λ_f is meaningful to characterize type (iii) CPV with the CPT invariant parametrization (75,

76) because it captures all the component of the expansion of the mass eigenstates on the bottomness basis as

$$\frac{\langle \overline{B}^0 | B_L \rangle}{\langle B^0 | B_L \rangle} = -\frac{\langle \overline{B}^0 | B_H \rangle}{\langle B^0 | B_H \rangle} = \frac{\langle \overline{B}^0 | B_L \rangle}{\langle B^0 | B_H \rangle} = -\frac{\langle \overline{B}^0 | B_H \rangle}{\langle B^0 | B_L \rangle}. \quad (79)$$

These four amplitudes ratio are different if we consider the gravity induced mass eigenstates (73, 74)

$$\frac{\langle \overline{B}^0 | B_L^\oplus \rangle}{\langle B^0 | B_L^\oplus \rangle} \neq -\frac{\langle \overline{B}^0 | B_H^\oplus \rangle}{\langle B^0 | B_H^\oplus \rangle} \neq \frac{\langle \overline{B}^0 | B_L^\oplus \rangle}{\langle B^0 | B_H^\oplus \rangle} \neq -\frac{\langle \overline{B}^0 | B_H^\oplus \rangle}{\langle B^0 | B_L^\oplus \rangle}. \quad (80)$$

Despite this difference between (79) and (80), the experimental results analyzed within a CPT invariant framework (79), can be understood and explained within the framework of gravity induced CPV (80). This situation is similar to the one encountered in section VI devoted to the study of ε' : if CPT is assumed the rephasing factors $\varphi = 1$, and the interpretation of the experimental measurements is based on the hypothesis of direct violation and imply a CPV at the fundamental level of the CKM matrix. However, if earth's gravity effects are taken into account $\varphi \neq 1$ and the very same phase-convention-independent measured quantities agree with the experiments without any additional assumptions. Earth's gravity was identified as the sole source of ε' .

The analysis below will use two different approaches to interpret the measurement of β , each providing the same final result. The two issues addressed below are: first, the invariance under rephasing of the mass eigenstates, when needed, to define an observable and second, the invariance under rephasing of the flavor eigenstates, when needed, to define an observable.

In order to accommodate the relation (77) with (79, 80), we consider a λ_f parameter constructed with the amplitude ratio $\langle \overline{B}^0 | B_L \rangle / \langle B^0 | B_H \rangle$ which is better suited to characterize the dynamics of oscillating $B_{L/S}$ as it takes into account all the eigenstates: the two flavor eigenstates and the two mass eigenstates. However, this $\tilde{\lambda}_f$ parameter reflecting the $B_{L/S}$ content of the oscillating and propagating B^0/\overline{B}^0 ,

$$\tilde{\lambda}_f = \frac{\langle \overline{B}^0 | B_L \rangle \langle f | \mathcal{T} | \overline{B}^0 \rangle}{\langle B^0 | B_H \rangle \langle f | \mathcal{T} | B^0 \rangle} = \exp -2j\beta \frac{\langle f | \mathcal{T} | \overline{B}^0 \rangle}{\langle f | \mathcal{T} | B^0 \rangle}, \quad (81)$$

is not phase-convention-independent with respect to the mass eigenstates. The decay amplitude ratio $\langle f | \mathcal{T} | \overline{B}^0 \rangle / \langle f | \mathcal{T} | B^0 \rangle$ provides invariance under rephasing of the flavor eigenstates. Both flavor eigenstates \overline{B}^0 and B^0 and both mass eigenstates B_L^\oplus and B_H^\oplus are involved in real experiments, thus the amplitude ratio $\tilde{\lambda}_f$ displaying these four states is more likely to provide the right interpretation of the experimental measurement.

In order to set up a fully phase-convention-independent interpretation of the experiments we introduce the sym-

metric rephasing factor

$$\varphi_B = \sqrt{\frac{\langle B_1 | B_H \rangle \langle B_2 | B_H \rangle}{\langle B_1 | B_L \rangle \langle B_2 | B_L \rangle}} = 1. \quad (82)$$

We have used $B_{1/2}$ states because they are the CP eigenstates like f which is also assumed to be a CP eigenstate. The amplitude ratio observed in the experimental measurement are given by phase-convention-independent product $\tilde{\lambda}_f \varphi_B$

$$\tilde{\lambda}_f \varphi_B = \frac{\langle \bar{B}^0 | B_L \rangle \langle f | \mathcal{T} | \bar{B}^0 \rangle}{\langle B^0 | B_H \rangle \langle f | \mathcal{T} | B^0 \rangle} \varphi_B = \exp -2j\beta \frac{\langle f | \mathcal{T} | \bar{B}^0 \rangle}{\langle f | \mathcal{T} | B^0 \rangle} \quad (83)$$

which is equal to λ_f (77).

When the same rephasing factor φ_B^\oplus is calculated within the framework of gravity induced CPV with (71, 72) rather than (11, 12), this gives

$$\varphi_B^\oplus = \sqrt{\frac{\langle B_1 | B_H^\oplus \rangle \langle B_2 | B_H^\oplus \rangle}{\langle B_1 | B_L^\oplus \rangle \langle B_2 | B_L^\oplus \rangle}} = \sqrt{\frac{1 - j\chi}{1 + j\chi}}. \quad (84)$$

The phase-convention-independent product,

$$\tilde{\lambda}_f^\oplus \varphi_B^\oplus = \frac{\langle \bar{B}^0 | B_L^\oplus \rangle \langle f | \mathcal{T} | \bar{B}^0 \rangle}{\langle B^0 | B_H^\oplus \rangle \langle f | \mathcal{T} | B^0 \rangle} \varphi_B^\oplus, \quad (85)$$

calculated with (73, 74, 84), becomes

$$\tilde{\lambda}_f^\oplus \varphi_B^\oplus = \exp(-j \arctan \chi) \frac{\langle f | \mathcal{T} | \bar{B}^0 \rangle}{\langle f | \mathcal{T} | B^0 \rangle} (1 + O[10^{-6}]). \quad (86)$$

To compare the interpretations based on the usual CPT eigenstates $|B_{L/H}\rangle$ (83) with the one based on gravity induced mass eigenstates $|B_{L/H}^\oplus\rangle$ (86), we must define β such as $2\beta = \arctan(0.77)$.

If $\arg(\langle f | \mathcal{T} | \bar{B}^0 \rangle / \langle f | \mathcal{T} | B^0 \rangle) = 0$ the experiments dedicated to $\bar{B}^0/B_0 \rightarrow f$ interferences between a direct and a mixed path will give measurements of the CPV parameters (78) equal to

$$S_f = \sin 2\beta = \sin[\arctan(0.77)] = 0.61, C_f = 0. \quad (87)$$

Taking in Ref. [5] a set of neutral final states $f = \{J/\psi K^{*0}, \phi K_S^0, K^0 \pi^0\}$, the reported experimental data give $S_{J/\psi K^{*0}} = 0.60 \pm 0.25$, $C_{J/\psi K^{*0}} = 0.03 \pm 0.1$, $S_{\phi K_S^0} = 0.58 \pm 0.12$, $C_{\phi K_S^0} = -0.09 \pm 0.12$ and $S_{K^0 \pi^0} = 0.64 \pm 0.13$, $C_{K^0 \pi^0} = 0.00 \pm 0.08$. These data are in good agreement with the gravity induced effect Eq. (87). For other final states f , S_f are centered around (87) but deviate from this value, for example $S_{\psi K^0} = 0.709 \pm 0.017$, $C_{\psi K^0} = 0.3 \pm 1.0$ provides a canonical reference value above 5σ . The fact that $\langle f | \mathcal{T} | \bar{B}^0 \rangle / \langle f | \mathcal{T} | B^0 \rangle \neq 1$ is

the source of the dispersion of S_f , note also that the sign of $\langle f | CP | f \rangle$ is to be considered to analyze the sign of S_f .

A clear understanding of the $\sin 2\beta$ distribution, around 0.6–0.7, requires the adoption of the mass eigenstates (73, 74) to write down the data analysis protocols of the various experimental raw data files and a precise evaluation of $\langle f | \mathcal{T} | \bar{B}^0 \rangle / \langle f | \mathcal{T} | B^0 \rangle$.

Let us adopt a second point of view. We will not consider the interpretation of interferences experiments and, rather than addressing the issue of λ_f , we address directly the issue of β . We consider the different mass eigenstates expansions on either CP or flavor eigenstates: (11, 12, 75, 76) for the CPT one, and (71, 72, 73, 74) for the gravity induced one. In order to compare the usual eigenstates parametrization (75, 76), based on a single angle β , with the gravity induced mass eigenstates (73, 74), involving two parameters \varkappa and χ , we must define β through a *gedanken* experiment providing $\exp j\beta$ as a phase-convention-independent expression.

We consider the symmetric and complete combination

$$\rho_B = \frac{\langle B^0 | B_L \rangle \langle B^0 | B_H \rangle}{\langle \bar{B}^0 | B_L \rangle \langle \bar{B}^0 | B_H \rangle}, \quad (88)$$

which takes into account the four components at work in the CPV description.

This definition of β through ρ_B takes into account all flavor and mass eigenstates but suffers from a lack of (unphysical) phase compensation with respect to the flavor eigenstates. All measured observables, independently of the interpretation of the measurement, are combinations of phase-convention-independent quantities. We introduce the coefficient φ'_B needed to provide a phase-convention-independent observable associated with ρ_B

$$\varphi'_B = \frac{\langle \bar{B}^0 | B_2 \rangle \langle B_2 | B_H \rangle \langle \bar{B}^0 | B_2 \rangle \langle B_2 | B_L \rangle}{\langle B^0 | B_1 \rangle \langle B_1 | B_L \rangle \langle B^0 | B_1 \rangle \langle B_1 | B_H \rangle}, \quad (89)$$

where we have chosen the two projection operators $|B_1\rangle\langle B_1|$ and $|B_2\rangle\langle B_2|$ because they commute with CP.

It can be checked that the product $\rho_B \varphi'_B$ is phase-convention-independent and thus can be measured in experiments.

If the usual CPT invariant parametrization of CPV effects is used (11, 12, 75, 76), this rephasing factor φ'_B changes nothing because it is equal to one

$$\rho_B = -\exp j4\beta, \quad (90)$$

$$\varphi'_B = 1, \quad (91)$$

and the product $\rho_B \varphi'_B$ can be measured and interpreted as $-\exp j4\beta$.

If CPV is gravity induced, we replace $|B_H\rangle$ and $|B_L\rangle$ with $|B_H^\oplus\rangle$ and $|B_L^\oplus\rangle$ given by (71, 72, 73, 74), and the very same observable is the product of the following fac-

tors

$$\rho_B^\oplus = -1 + O[10^{-6}], \quad (92)$$

$$\varphi_B^\oplus = \frac{1 + j\chi}{1 - j\chi} = \exp(2j \arctan \chi). \quad (93)$$

We conclude that, if gravity induced CPV is taken into account, the measurement of the phase-convention-independent observable $\rho_B \varphi_B'$ on earth gives

$$\rho_B^\oplus \varphi_B^\oplus = -\exp(2j \arctan \chi), \quad (94)$$

although if the measurement of the very same phase-convention-independent observable $\rho_B \varphi_B'$ is interpreted within the usual CPT invariant framework it defines β as

$$\rho_B \varphi_B' = -\exp j4\beta. \quad (95)$$

The conclusion of this ρ_B *gedanken* measurement with two frameworks of interpretation is that $\arctan \chi = 2\beta$ and

$$\sin 2\beta = \sin[\arctan(0.77)] = 0.61. \quad (96)$$

If we consider the entire Belle data samples in 2012 [23], plus the entire BaBar data samples in 2009 [24], with about 10^9 cumulated B^0/\bar{B}^0 pairs, the global average over the two data sets was found, at that time, to be given by $\sin 2\beta = 0.67 \pm 0.02$ [25]. The mismatch between this experimental average and (96) is of the order of 7%. Gravity induced CPV is thus a pertinent model to interpret B^0/\bar{B}^0 experiments dedicated to CPV. This last result complete the previous analysis of K^0/\bar{K}^0 experiments and confirm that the CKM matrix must be considered free from any CPV phase far from any massive object.

VIII. GRAVITY INDUCED CPV IN D^0/\bar{D}^0 AND B_s^0/\bar{B}_s^0 EXPERIMENTS AND CONCLUSIONS

The previous calculations on the impact of earth gravity on neutral mesons oscillations can be extended to $D^0/\bar{D}^0 \sim (c\bar{u})/(\bar{c}u)$ and $B_s^0/\bar{B}_s^0 \sim (s\bar{b})/(\bar{s}b)$. The framework of analysis of the experimental data on D^0/\bar{D}^0 and B_s^0/\bar{B}_s^0 is similar to the methods presented in section V, VI and VII. The parameters $m_D g\hbar/\delta m_D^2 c^3$ and $m_{B_s} g\hbar/\delta m_{B_s}^2 c^3$ for both mesons systems is very small so a type (i) indirect violations will be extremely difficult to observe. Type (ii) and (iii) can be analyzed on the basis of gravity induced CPV, presented in section VI and VII, but are also experimentally difficult to observe.

In any environment where a flavored neutral mesons $|M\rangle$, with mass m , mass splitting δm and Compton wavelength λ_C , experiences a gravity \mathbf{g} , i.e. in any curved

space-time environment, the amplitude of CP violation will be given by

$$(m/\delta m)^2 |\mathbf{g}| \lambda_C / c^2. \quad (97)$$

The first factor $m/\delta m$ is associated with electroweak and strong interactions, the second one is the product of a (wave)length, an acceleration and c , quantities related to geometry and space-time rather than to electroweak or strong interactions. The proportionality to $|\mathbf{g}|$ indicate that this new CPV mechanism allows to set up cosmological evolution models predicting the strong asymmetry between the abundance of matter and the abundance anti-matter in our present universe [6].

Beside the problem of early baryogenesis, neutrinos oscillations near a spherical massive object might be revisited to explore the impact of the interplay between gravity and mixing.

The type (i) CPV observed with K^0/\bar{K}^0 stems from a gravity induced interplay between vertical quarks zitterbewegung oscillations at the velocity of light on the one hand and the strangeness oscillations ($\Delta S = 2$) on the other hand.

The type (ii) small CPV observed with K^0/\bar{K}^0 is associated with the CPT invariant modelisation of a gravity induced CPT violation and is elucidated through a careful analysis of the rephasing invariance of the observable η_{00} .

The large type (iii) CPV observed with B^0/\bar{B}^0 is associated with the CPT invariant modelisation of a gravity induced CPT violation displaying a very small modulus and a significant phase β .

When the mesons are considered stables, the evolution is unitary and there is no T violation, T violation stems from the modelisation of the transition amplitudes w_f in Eq. (1) as irreversible decays in Eq. (2) within the framework of the WW approximation [10].

The very large type (iv) CPV observed in our universe, namely its baryon-antibaryon asymmetry, remains an open issue within the KM framework of interpretation, although gravity induced CPV displays the potential to set up cosmological evolution models in agreement with the present state of our universe.

We have demonstrated that gravity induced CPV allows to predict three experimental CPV parameters ($\varepsilon, \varepsilon', \beta$) and appears to provide the potential to explain the baryon asymmetry of the universe as its amplitude is linear with respect to the strength of gravity.

This set of new results was obtained within the canonical framework of quantum mechanics, on earth, without any speculative assumption on new coupling, or new field, or new physics. From this clear convergence of results, we can conclude that a CKM matrix free of CPV phase is to be considered as the core of the SM in a flat Lorentzian environment and earth's gravity is the sole source of $\varepsilon, \varepsilon'$ and β CPV effects in K^0/\bar{K}^0 and B^0/\bar{B}^0 experiments.

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