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### ► To cite this version:

Joannès Guichon, Sylvain Contassot-Vivier, Nazim Fatès, Massimo Amato. Temporal B2B Debt Netting: Finite Lifespans and Reimbursements of the Financer. 2025. <hal-04987059>

**HAL Id: hal-04987059**

**<https://hal.science/hal-04987059v1>**

Preprint submitted on 11 Mar 2025

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# Temporal B2B Debt Netting: Finite Lifespans and Reimbursements of the Financer

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**Abstract.** In the business-to-business (B2B) landscape, the temporal dynamics of invoice settlements are critical to the financial health of companies. Recurrent late payments create a network of interlocking debts, where each firm is simultaneously a creditor and a debtor. This situation can provoke liquidity crises and systemic risk. A method to reduce such risk is the integral netting mechanism, in which selected invoices are fully settled by injecting minimal external liquidity at well-chosen points in the network. Methods have been proposed for B2B transactions in a fixed time period. A central question is how to adapt such netting to a real-world scenario in which invoices arrive continuously over time. In this paper, we present a framework for temporal setting with daily addition of newly emitted invoices. Finally, experiments with daily netting on a real dataset of 22 million invoices allow us to evaluate the quality and interest of the proposed method.

**Keywords:** Temporal invoice graphs, B2B finance, Integral debt netting algorithm

## 1 Introduction

In the business-to-business (B2B) landscape, the temporal dynamics of invoice settlements are critical to the financial health of companies. Trade credits which allows businesses to offer differed payments to their clients is since long the primary source of short-term financing largely surpassing bank credit [3,1]. Recurrent late payments create a network of interlocking debts, where each firm is simultaneously a creditor (owed money) and a debtor (owing money). This situation can provoke liquidity crises and systemic risk [2]: if firms cannot get paid, they cannot meet their own obligations.

A method to reduce such systemic risks is the *integral netting* mechanism, in which selected invoices are fully settled (not just partially) by injecting minimal external liquidity at well-chosen points in the network. Such method has been proposed for B2B transactions in [5]. A central question is how to adapt such netting to a real-world scenario in which invoices arrive continuously over time, and do not simply exist in one static batch.

In this paper, we present a framework for *temporal* setting with daily addition (or any desired time frame) of newly emitted invoices. We also introduce the notion of a *finite-lifespan* invoice within netting: each invoice is candidate for reduction during  $D_1$  days in our netting system, after which it must revert to traditional payment channels.

Moreover, in a dynamic multi-period approach, the external financier can potentially be reimbursed in part from period to period. Whenever a firm that has been financed in a previous period ends a new reduction round with surplus (positive balance), that surplus is used to repay a portion of its debt to the financier. Such partial payback drastically reduces the total capital required from the financier over the year. Also, it lowers the debts of the companies without the need for direct monetary refunds.

Our main contributions are:

- A formal method to handle *time decomposition* of B2B invoices, specifying how to carry leftover invoices (and leftover financings) from one period to the next.
- A partial payback procedure so that netting can recycle capital efficiently.
- Large-scale experiments with daily netting on a real dataset of 22 million invoices, provided by InfoCert, a private electronic invoice service in Italy.

We begin in Section 2 by providing relevant definitions for static integral netting. Then, in Section 3 are added definitions related to the temporal context. Section 4 describes how we split the year into periods and define active invoices over multiple periods. Section 5 details the temporal netting algorithm and partial reimbursements. Section 6 reports experimental results on a full year of real data, focusing on both the global system-level benefits and the distribution of benefits across individual firms.

## 2 Background: Static Integral Netting

In order to be able to apply integral debt netting over time, we need an internal algorithm for reducing a given invoice graph  $G$ . Although any method could be used as a black-box for this inner step, without changing the temporal process, we have proposed in [5] an efficient algorithm for processing static graphs.

We recall here the important definitions introduced in this previous work that are useful in the description of our temporal process.

**Definition 1 (B2B Invoice Graph).** *Let  $V$  represent a set of companies exchanging invoices in a specific context. A B2B invoice graph is a weighted directed multi-graph  $G = (V, E)$ , where each edge  $e \in E$  is a tuple  $(s, d, w)$ :*

- $s \in V$  is the debtor (source),
- $d \in V$  is the creditor (destination),
- $w > 0$  is the amount owed from  $s$  to  $d$ .

For each edge  $e \in E$ , we define the functions  $w(e)$ ,  $s(e)$ , and  $d(e)$  associated with each element of  $e$ . For a node  $v$ , we define in graph  $G$  the set of outgoing edges  $O_G(v)$  and incoming edges  $I_G(v)$  as follows:

$$O_G(v) = \{e \in E \mid s(e) = v\}, \quad I_G(v) = \{e \in E \mid d(e) = v\}, \quad (1)$$

The *net position of node  $v$*  (or the *balance of  $v$* ) in graph  $G$  is given by:

$$B_G(v) = \sum_{i \in I_G(v)} w(i) - \sum_{o \in O_G(v)} w(o). \quad (2)$$

### 3 Temporal Graph Definition

As we are dealing with an economy problem that evolves with time, the dynamic aspect of invoices is crucial. In order to take into account this temporal aspect, we introduce the notion of time period, that corresponds to a fixed duration (day, week, month,...).

We define a *temporal graph*  $G_k = (V, E_k)$ ,  $k > 0$  where:

- $V$  is the global set of vertices (or nodes), independent from the time periods,
- $E_k$  is the set of temporal edges, where each edge is of the form  $e = (s, d, w, \tau)$  with  $s, d, w$  the same parameters as in  $G$ , and  $\tau = k$  the additional information about the creation time of the edge (invoice).

So, a temporal edge  $e = (s, d, w, \tau)$  indicates that a directed connection from vertex  $s$  to vertex  $d$  of weight  $w$  is active starting at time  $\tau$ .

For any dataset of invoices, represented by graph  $G$ , the partitioning into  $n$  discrete time periods is represented by the sequence of graphs  $\{G_k \mid k \in \{1..n\}\}$  where every edge in  $E$  belongs to exactly one  $E_k$ . This partitioning facilitates the analysis of temporal patterns.

### 4 Time Decompositions

Our work focuses on the different  $G_k = (V, E_k)$  that forms the partition of our global graph  $G$  of all invoices of all time. The initial intuition for addressing the problem of reducing invoices over time was to adopt a static approach by analysing consecutive time periods independently. For each period, we would apply the reduction separately. This straightforward method served as a preliminary step into the time-based context like in [4]. In the following sections, we present a method that takes into account the temporal links between successive periods.

#### 4.1 Active Invoices Over Multiple Periods

In reality, an invoice is almost never settled the day it is issued. It might remain pending for up to 90 days. We incorporate this by allowing each invoice to stay within our netting system for at most  $\mathcal{D}_i$  periods. Thus, on day  $k$ , the set of active invoices is:

$$I_k = G_k \cup (O_{k-1}) \setminus I_{k-\mathcal{D}_i},$$

where

- $G_k$ : newly issued invoices at day  $k$ ,
- $O_{k-1}$ : leftover invoices from day  $k-1$  that were not netted,
- $I_{k-\mathcal{D}_i}$ : the set of invoices that are now too old to remain in the netting system (they are forced to exit or be settled by classic means).

#### 4.2 Financing Lifespan

We allow each financing arrangement to remain in the netting system for up to  $\mathcal{D}_f$  periods, so that partial or complete refunds might occur if the same debtor obtains a surplus in a subsequent day. After  $\mathcal{D}_f$  days, any leftover financing is considered expired, meaning it must be paid outside of netting (the financier effectively calls it due).

### 5 Temporal Reduction Algorithm with Partial Payback

On each day  $k$ , we combine:

$$J_k = I_k \cup F_{k-1},$$

where  $F_{k-1}$  is the set of previous financings that have not yet expired or been fully refunded. Then we apply the *static integral netting* algorithm of [5] to  $J_k$ , obtaining a subgraph of cleared edges  $\mathcal{R}$  that we will here note  $\mathcal{R}(J_k)$ .

#### 5.1 New Financing Edges

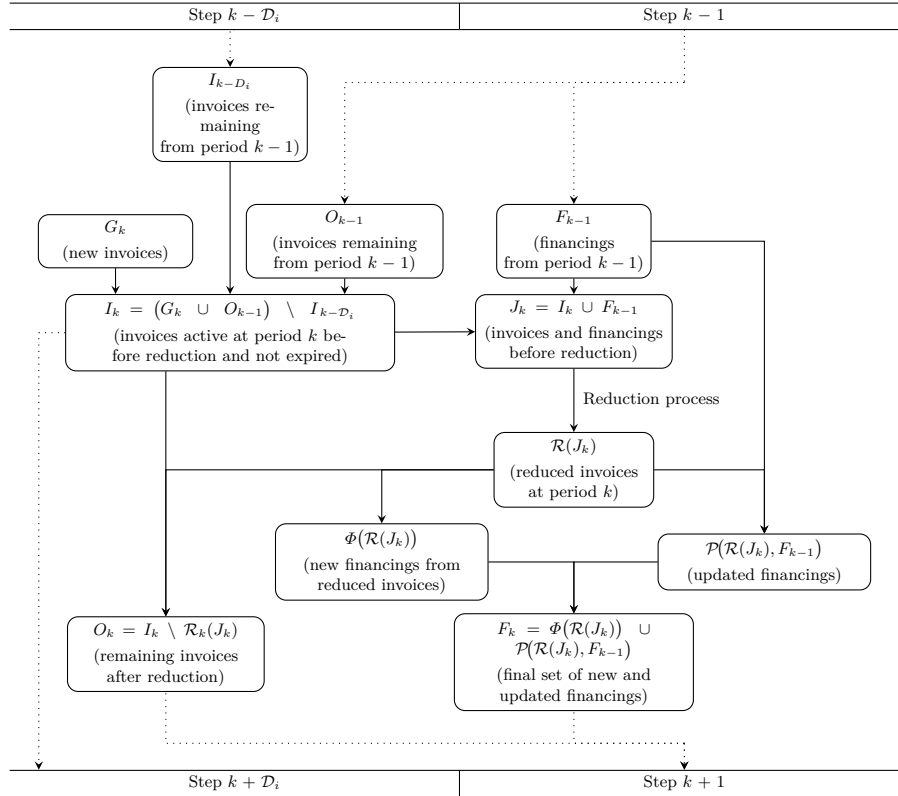
If a node  $v$  in  $\mathcal{R}(J_k)$  has negative balance inside the reduction process (not in the graph), we add a new financing edge  $(v^*, v, -B_{\mathcal{R}(J_k)}(v))$  from  $v$  to the external financier  $v^*$  and  $B_{\mathcal{R}(J_k)}(v)$  the balance of node  $v$  inside the reduction process. This financing is added inside the graph of new financings  $\phi(\mathcal{R}(J_k))$ . This financing is dated  $k$ , thus it can remain in the system until day  $k + \mathcal{D}_f$ . It represents the debt owed by the company to the financier after being financed.

## 5.2 Partial Refund Mechanism and Expelling Expired Financings

Likewise, if a node  $v$  in  $\mathcal{R}(J_k)$  finishes with  $B_{\mathcal{R}(J_k)}(v) > 0$  while it still owes money from a previous day (i.e. an edge  $(v, v^*, w)$  from  $F_{k-1}$ ), then we reduce  $w$  by  $\min\{w, B_{\mathcal{R}(J_k)}(v)\}$  inside the graph of older financings  $\mathcal{P}(\mathcal{R}(J_k), F_{k-1})$ . This effectively reimburses part or all of the old debt. If an old financing edge's weight is driven to zero, we remove it from the system. Any financing that dates from day  $t$  with  $k - t \geq \mathcal{D}_f$  is expelled from the netting graph. In this case, the leftover is settled outside our netting system by conventional means.

## 5.3 Method Summary

A visual explanation of this whole process for a given step  $k$  is proposed in Figure 1. By using this temporal construction, we can use any static reduction algorithm on those constructed graphs to have a temporal reduction process. We repeat this step for  $k = 1, \dots, n$  ( $n$  being fixed in our case but could be infinite in real exploitation context). Let  $G_i = I_i = O_i = F_i = \emptyset$  for  $i \leq 0$ .



**Fig. 1.** Flowchart of the temporal reduction process

#### 5.4 Graph partitioning inside the reduction process

An important issue when processing real data is the large size of the graph  $J_k$  at step  $k$  (Figure 1) that induces very large computation times for an entire year (several weeks of computation using daily steps). In order to speed up the calculation and to obtain acceptable processing times, at each step  $k$  we partition the initial graph  $J_k$  in two parts.

This problem was already encountered in [5] and we proposed a simple division method to apply the reduction, followed by a simple (and fast) greedy fusion. Instead of the greedy fusion we choose here to apply our complete extension process on the merged reductions of the divided parts. The results of this strategy on an entire year provide similar debt reduction and amplification as the computation without partitioning, and show only 3% loss in terms of overall returns to the financier. When comparing to the temporal process on the two first months with or without division, we obtain an acceleration of 40. These results are quite satisfying and the loss over the returns is acceptable at first sight. Still, we think that these partitioning and merging strategies deserve a deeper analysis and will be the subject of a future work.

## 6 Empirical Analysis on Real Data

We now apply the temporal netting procedure on a real dataset from an Italian e-invoice provider, containing 22 million invoices among around 2 million companies in 2019 for a total of 84 billion euros exchanged. This dataset was deeply analysed in [6].

### 6.1 Daily Aggregation and Parameters

We take  $n = 365$  days, with  $E_k$  corresponding to the invoices issued at day  $k$ . We set  $\mathcal{D}_i = 28$  days (invoices remain nettable for 28 days) and  $\mathcal{D}_f = 84$  days for financings. Hence, an invoice is expelled from netting after 28 days if not already settled, and any leftover financing is expelled after 84 days.

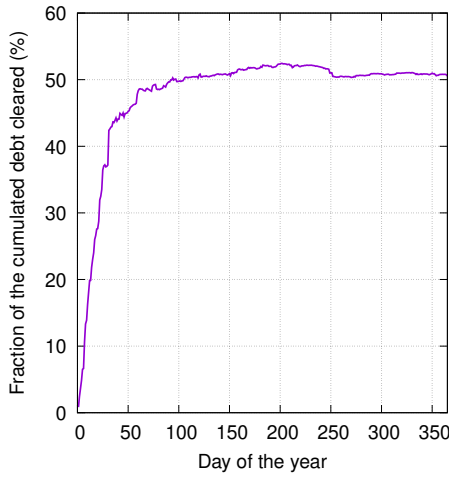
On average, each daily netting graph  $J_k$  can have around 70,000 new invoices plus leftovers from up to the past 27 days, and a handful of old financings. We use the partitioning method presented in 5.4 to split each  $J_k$  into two sub-problems to reduce computational time, merging their solutions at the end of the day. This yields manageable run times on standard hardware.

Different scales need to be observed in order to show the interest both at the global level of the network as well as at the atomic level of each company. The interest and risk of the financier is also to be looked at since we want to show that this process is viable at a long-term scale.

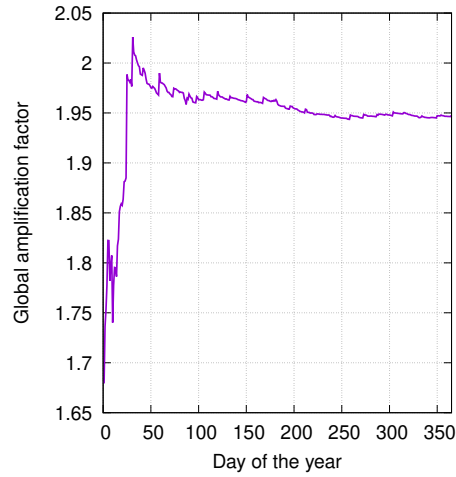
### 6.2 At a global scale

When looking at the global network, one should be able to gauge how much debt has been cleared and how well. For that we propose to look at the evolution of the amplification as time goes on as well as at the ratio of total debt cleared.

Figure 2 shows the cumulative fraction of the total 2019 invoice volume that is cleared by netting. It stabilizes around 50% of the overall debt, meaning the other half is too isolated or ill-matched for our integral netting to be efficient. Clearing half of the network’s debt is a very encouraging result and obtaining such result was not obvious at first sight.

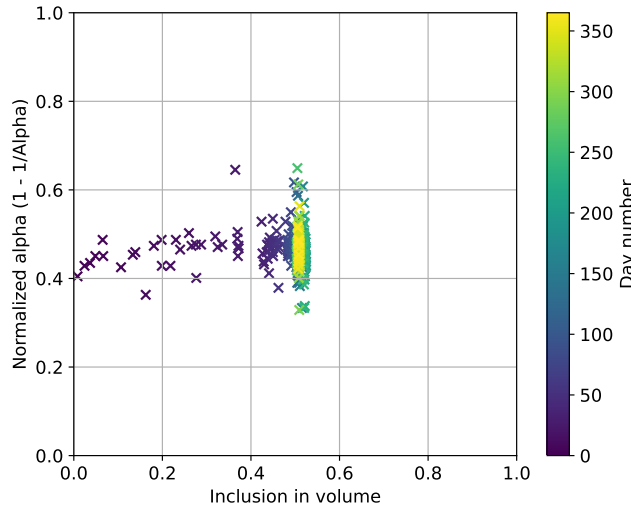


**Fig. 2.** Fraction of total yearly debt cleared over time by daily netting.



**Fig. 3.** Global amplification factor per day, ignoring refunds to the financier.

In Figure 3 we plot the amplification factor if we consider the whole debt cleared as well as the whole financing it needed. It hovers near 1.95, so about 2 euros of debt is cleared per euro injected. Note that it does *not* account for reimbursements that might effectively recycle capital. So, it is not the effective amplification, as discussed below.



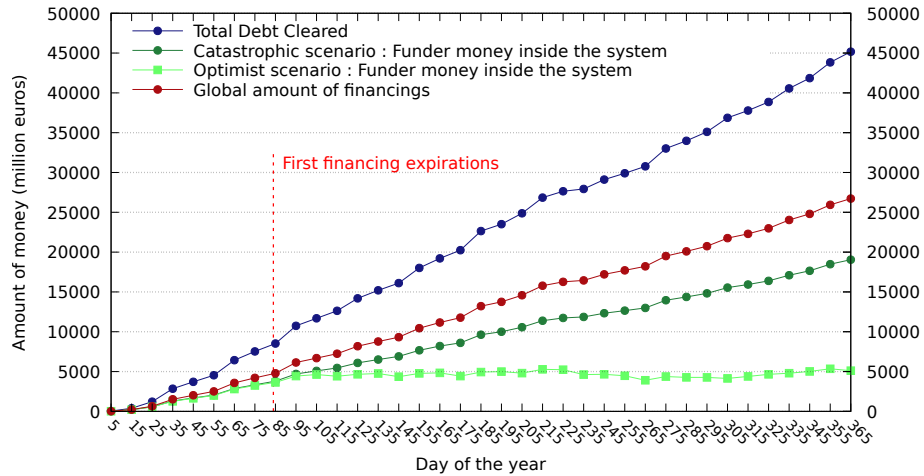
**Fig. 4.** Inclusion and normalized alpha of the global reduction at each step of the year

In Figure 4 is plotted the normalized amplification in function of the normalized inclusion. We observe that, after the 2 first months, the inclusion stabilizes around 50% according to the day number. This is quite different for the normalized amplification that varies between 0.33 and 0.68 (1.5 and 3 in terms of alpha). Still, most days are quite close to each other in terms of both amplification and inclusion.

### 6.3 Financer’s Perspective

To assess long-term viability from a financer’s standpoint, we must measure both the actual financing and the associated risk. We consider two extreme scenarios for leftover financings after their expiration date :

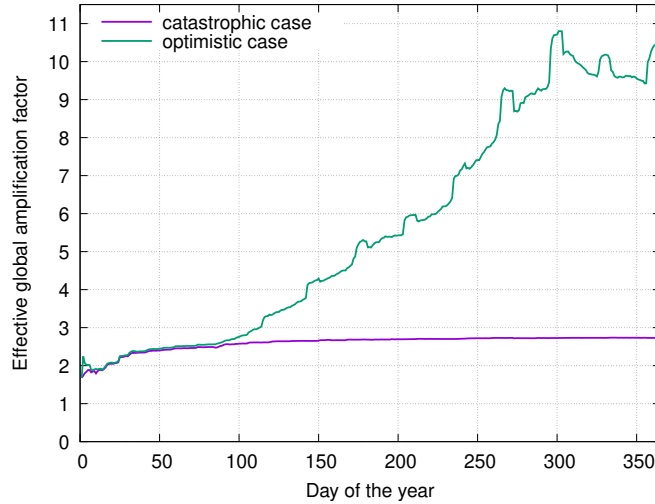
- **Catastrophic scenario:** When a financing expires, any remaining debt owed to the financer is lost. This is the worst-case view, where the financer only recovers partial reimbursements made during netting.
- **Optimistic scenario:** On expiration, the company fully repays any unpaid debt to the financer. This yields the highest possible refund.



**Fig. 5.** Evolution of debt cleared and money of the funder inside the system over the span of a year

In Figure 5, we compare these scenarios over a full year, plotting how much debt is cleared alongside the financer’s total and active capital. Under the catastrophic scenario, capital injections keep growing, but partial refunds still recover around 30% of the total outlay. In the optimistic scenario, reimbursements eventually stabilize the capital, with about 80% of the invested funds being paid back by year’s end.

Since our standard amplification factor ( $\approx 1.95$ ) does not account for partial refunds, we define an *effective amplification*: the ratio of total debt cleared to the financier’s net capital still at risk. As shown in Figure 6, the effective amplification approaches 3 in the catastrophic case and approaches 10 in the optimistic one after one year. Because refunds reduce capital exposure, netting becomes far more efficient in realistic or optimistic repayment conditions. Even modestly successful reimbursement rates could thus make the process attractive, provided actual defaults stay manageable.



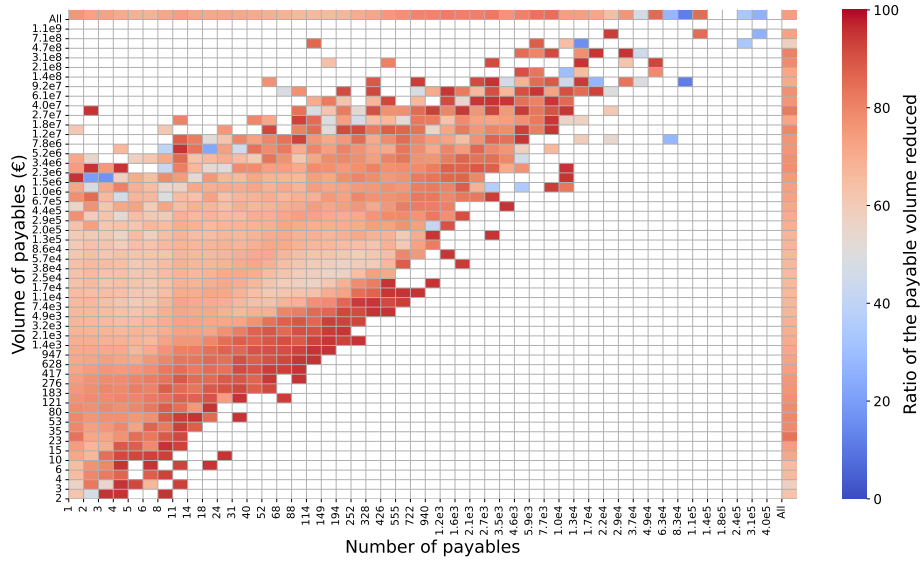
**Fig. 6.** Effective amplification factor under extreme financing scenarios.

It will be mandatory to explore more scenarios that are between those extreme cases to evaluate the stability and efficiency of the reduction in practical cases, where we could have around 2% of company failures over a year or with a given probability of default in repayment. Exploring this kind of scenario would help to understand how much loss is incurred by the financier and then explore ways to mitigate this risk.

#### 6.4 Benefiting company scope

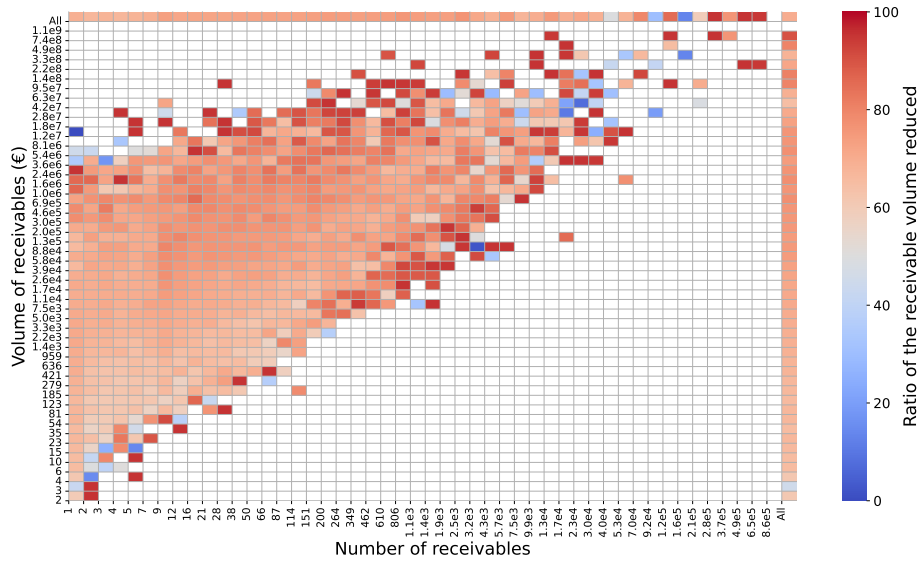
For the system to be appealing at a company level, it must offer tangible benefits to companies on an individual basis. Our reduction algorithm successfully identifies potential debt savings for 72.89% of the companies inside our data. Breaking this down further, 73.72% of companies holding receivables see benefits from our reductions, while 75.52% of those with payables also see reductions. Notably, for companies managing both payables and receivables, the efficiency of our system is even more pronounced, with 81.32% experiencing benefits.

Beyond the overall count of beneficiaries, we also examine the fraction of each company’s payables or receivables that is reduced, considering both the total amount and the number of invoices.



**Fig. 7.** Heatmap of the percentage of volume reduced for companies depending on their volume and number of payables

Figure 7 suggests that firms whose payables exhibit a simili proportional relationship between number of invoices and total volume tend to see a larger fraction of their debt reduced.



**Fig. 8.** Heatmap of the percentage of volume reduced for companies depending on their volume and number of receivables

Similarly, Figure 8 indicates that high-value receivables, especially if numerous, can be substantially netted out.

These results imply that actively submitting a broad set of invoices (rather than just a few large ones) increases netting opportunities, particularly for firms whose receivables and payables are significant enough to find counterparties in the network. Companies with unbalanced invoice patterns may see lower benefits, underscoring the importance of comprehensive participation.

## 7 Conclusion

In this paper, a temporal integral netting framework has been presented that runs repeated nettings over successive discrete periods. In this setting, non-reduced invoices from previous periods are kept until expiration date. This approach also incorporates partial reimbursements of financings, enabling capital recycling that alleviates the external financier’s exposure.

Large-scale experiments on 22 million invoices from an Italian e-invoice service show that our daily netting process consistently clears around half of the total yearly debt. The *raw* daily amplification factor hovers near 2, and, under optimistic repayment scenarios, an *effective* amplification of up to 10 is achievable, substantially improving returns for the financier. From a firm-level perspective, more than 70% of companies experience tangible debt reductions, demonstrating the broad impact of our method.

Notably, the partial reimbursement mechanism alone shows that the financier is able to get back at least 30% of the injected capital. This happened even though we did not enforce any priority for financier repayments in the reduction process. This highlights that companies benefiting from netting would have been able to pay their invoices. However, the netting gives them more flexibility in terms of their own capital management, meaning a reduction of their own failure risk.

Future work will investigate additional scenarios, including realistic default probabilities and finer reimbursement policies, to estimate the financier’s risk and reward. We also plan to enhance scalability of the process, which is critical for deployment in complex B2B ecosystems. These advancements aim to further solidify temporal integral netting as a powerful, low-risk strategy for improving liquidity management and reducing systemic risk in large-scale financial networks.

## Acknowledgment

This work is supported by the Inria’s Exploratory Action Murene.

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