



HAL
open science

Unification of Gravity and Electromagnetic interaction: from Quantum Mechanics of a Quasi-Euclidean Space with Planck Length, Rotational Symmetry and Translational Symmetry

Yan Zhou, Junyan Zhang, Ernst Meyer, Xingkai Zhang, Jing Zhang

► To cite this version:

Yan Zhou, Junyan Zhang, Ernst Meyer, Xingkai Zhang, Jing Zhang. Unification of Gravity and Electromagnetic interaction: from Quantum Mechanics of a Quasi-Euclidean Space with Planck Length, Rotational Symmetry and Translational Symmetry. *Journal of Applied Mathematics and Physics*, 2025, 13 (01), pp.302-326. <10.4236/jamp.2025.131014>. <hal-04960688>

HAL Id: hal-04960688

<https://hal.science/hal-04960688v1>

Submitted on 21 Feb 2025

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY 4.0 - Attribution - International License

Unification of Gravity and Electromagnetic Interaction: Based on Quantum Mechanics of a Quasi-Euclidean Space with Planck Length, Rotational Symmetry and Translational Symmetry

Yan Zhou^{1,2*}, Junyan Zhang^{1*†}, Ernst Meyer^{2†}, Xingkai Zhang¹ and Jing Zhang¹

¹Lanzhou Institute of Chemical Physics, Chinese Academy of Sciences, Lanzhou 730000, China.

²Department of Physics, University of Basel, 4056 Basel, Switzerland.

*Authors contributed equally to this work.

†Corresponding authors. Email: junyanzhang@licp.cas.cn (J. Z.); ernst.meyer@unibas.ch (E. M.)

Abstract: This work is focused on a quasi-Euclidean space with UV/IR cutoffs and symmetries. Mathematical analysis reveals that the UV cutoff results in the minimum structures of space. Dominated by rotational symmetry, the minimum structure should be a local one in situ or on a sphere. Investigations show that a 10D minimum structure is a non-local one with transformability between in-situ state and spherical state due to its special topology. Based on the quantum behaviors of the 10D structure controlled by translational symmetry, IR cutoff determines two long-range interactions with dimensionless constants of $\sim 1/137.036\ 082$ and $\sim 1/1.628\text{E}+38$, which are exactly equal to those of electromagnetic and gravitational forces, respectively. The result discovers that the two long-range fundamental interactions originate from the physics at the Planck scale, concretely achieving the quantization of gravity and its unification with electromagnetic interaction for the first time.

Keywords: Planck length, symmetry, high dimensional space, FSC, quantum gravity

1. Introduction

The fine structure constant (FSC, $\sim 1/137.035\ 999$), which is a dimensionless quantity characterizing the strength of the electromagnetic interaction, has fascinated innumerable scientists since it appeared in 1916 ^[1]. Its value has been measured more and more precisely in the cosmos explored by humans ^[2-5], whereas its theoretical origin remains unknown till now.

Although some interesting formulae have been proposed, such as $(137^2 + \pi^2)^{-1/2} = 1/137.036\ 016$ ^[6], $(4\pi^3 + \pi^2 + \pi)^{-1} = 1/137.036\ 034$ ^[7], *etc.*, there is never a convincing solution that has both numerical consistency and sufficient theory for FSC. It's even unknown whether FSC is calculable in principle or is a non-calculable one determined by historical or quantum mechanical accident ^[8].

Nevertheless, there are still a considerable number of scientists who insist that FSC must have theoretical derivations ^[9, 10, 11]. If FSC is really a dimensionless constant with calculability, like another fundamental constant π defined by a radius and the semicircle it determines, then two

prerequisites, including a natural object and its characteristic path relatively measured ~ 137.036 , must be both present. As for the former, some mathematical models, including the point-like one, the lattice-like one, the string-like one, *etc.*, have been assumed to be the natural objects [12, 13]. As for the latter, Euclidean spaces and manifolds have been involved to be the background for the research [14]. Although no precise solutions had been obtained directly, some attempts illuminate that the higher dimensional spaces beyond 3D/4D may be required to study some fundamental interactions [15, 16].

For the further pursuit of a calculable FSC, the present work chooses a generalized nD Euclidean space with Planck length as the background. Here the limit of Planck length turns the Euclidean space to a quasi-Euclidean space, resulting in an extremely small *space bubble*, inside which no distance is reasonably allowed. After taking such a *space bubble* as a natural object, its moving path in nD space then will be searched for. Definitely, these small *bubbles* would obey quantum mechanics and some other basic principles of physics, such as the conservation of mass and the minimum energy. Besides, we should not forget the most important thing that they satisfy rotational symmetry and translational symmetry, since they are the parts of the vacuum, which is proven by all experiments to be of absolute symmetries, no matter when and where. This means geometrical limits will be strongly involved when considering the motion of such a *space bubble*. Therefore, we set up a quasi-Euclidean space with Planck length and symmetries, trying to quantize it via the *space bubble* and explore its quantum behaviors, not only physically but also geometrically.

2. Planck units: quantization of quasi-Euclidean space \hat{P}

2.1 Quasi-Euclidean space \hat{P}

Compared to a general nD Euclidean space ${}^n\mathbf{P}$ (dimension symbols are marked in the upper left corner of a certain space \mathbf{P} in this work), a special Euclidean space noted as ${}^n\hat{\mathbf{P}}$ is established here to be a Euclidean space carrying the Planck length λ_P , which determines the minimum distance in vacuum and comes from $\lambda_P = \sqrt{\frac{G\hbar}{c^3}} = 1.616\ 252(81) \times 10^{-35}$ m, where \hbar is the reduced Planck constant, G is the gravitational constant and c is the speed of light in vacuum [17]. Therefore, space ${}^n\hat{\mathbf{P}}$ can be described as follows. On the one hand, it exists $\hat{x} \equiv x$ ($x \geq \lambda_P$), showing that ${}^n\hat{\mathbf{P}}$ behaves as same as its corresponding Euclidean space ${}^n\mathbf{P}$ on the relatively macroscopic scale $\geq \lambda_P$. On the other hand, it exists $\hat{x} = \lambda_P$ ($0 < x < \lambda_P$), showing that any 1D measure along a certain dimension of ${}^n\hat{\mathbf{P}}$ is never less than λ_P on the microscopic scale $< \lambda_P$, although ${}^n\hat{\mathbf{P}}$ remains a Euclidean space at the same time. E.g., for two adjacent points on a certain dimension of ${}^n\hat{\mathbf{P}}$, a blank interval measured λ_P occurs between them even they are originally infinitely close to each other on the macroscopic scale.

Based on the above commonalities and difference, relationship between quasi-Euclidean space ${}^n\hat{\mathbf{P}}$ and its corresponding Euclidean space ${}^n\mathbf{P}$ can be mathematically demonstrated as follows.

$$\text{Commonalities (linearity): } \hat{x} = kx + a \quad (1)$$

$$\hat{x} = kx \quad (\text{same position with } a=0) \quad (2)$$

$$\hat{x} = x + a \quad (\text{same measure with } k=1) \quad (3)$$

$$\text{Difference (Planck length): } \hat{x} = \lambda_P \cdot e^{\frac{i \cdot \hat{x}}{\lambda_P}} \quad (\bar{\lambda}_P = \frac{\lambda_P}{2\pi}) \quad (4)$$

As two equivalent spaces at macro scale, ${}^n\hat{\mathbf{P}}$ and ${}^n\mathbf{P}$ always satisfy $k=1$ and $a=0$ simultaneously in Equ. (1). However, the exclusive character of Planck length $\hat{x}_{\min}=\lambda_P$ for ${}^n\hat{\mathbf{P}}$, as shown by Euler's formula in Equ. (4), brings $\hat{x} \neq x$ (Fig. 1) and leads to the alternatively satisfied $a=0$ and $k=1$ in Equ. (2) and Equ. (3), respectively. Briefly, it exists $\hat{x}=\lambda_P$ when $x \in (0, \lambda_P]$ (Fig. 1). Besides, $\hat{x}=0$ when $x=0$ describes the special case where the minimum distance of λ_P is invalid since the two adjacent points coincide and act as one point. This special case demonstrates that it exists $\hat{x}=0$ when a single point is involved.

So quasi-Euclidean space ${}^n\hat{\mathbf{P}}$ can be mathematically defined in the space of non-negative part as

$$\hat{x}=x \quad (x \geq \lambda_P) \quad (5)$$

$$\hat{x} = \lambda_P \cdot e^{\frac{i \cdot kx}{\lambda_P}} \quad \text{or} \quad \hat{x} = \lambda_P \cdot e^{\frac{i \cdot (x+a)}{\lambda_P}} \quad (0 < x < \lambda_P) \quad (6)$$

$$\hat{x}=0 \quad (x=0) \quad (7)$$

Obviously, unless considering the minimum distance at the extremely small scale, a quasi-Euclidean space ${}^n\hat{\mathbf{P}}$ is equivalent to its corresponding Euclidean space ${}^n\mathbf{P}$. So, ${}^n\hat{\mathbf{P}}$ is basically taken as a space that not only applies the axioms and definitions of Euclid space, but also applies all the physical principles in a general Euclidean space, being a normal background for experimental or theoretical physical objects, including a particle, a field, and so on.

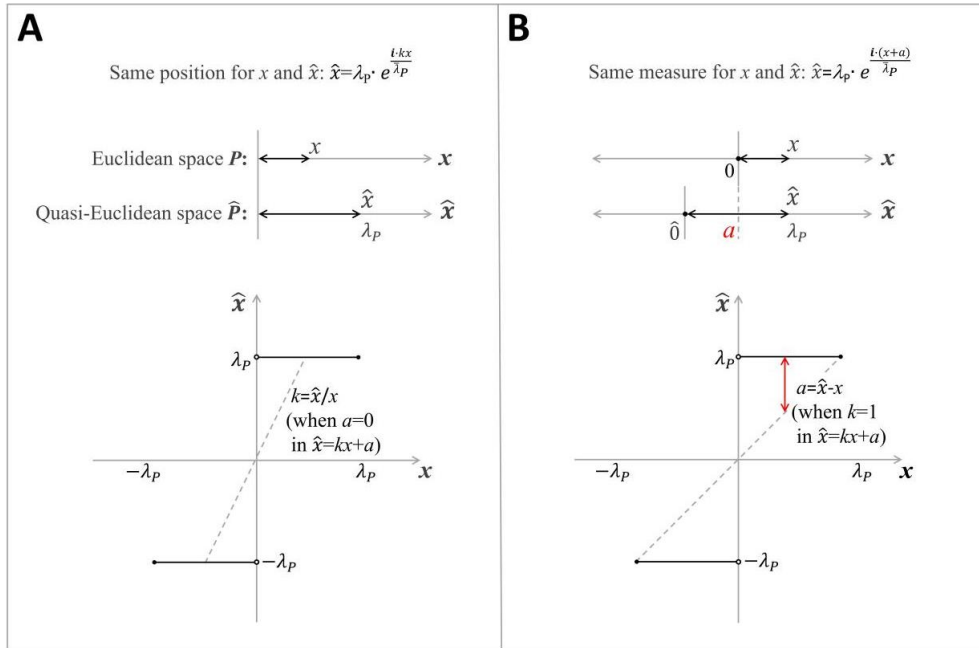


Fig. 1 Difference between Euclidean space \mathbf{P} and quasi-Euclidean space $\hat{\mathbf{P}}$ at Planck scale: $|\hat{x}|=\lambda_P$ ($0 < |x| < \lambda_P$) when x and \hat{x} share the same position (A) or the same measure (B).

2.2 Relationship between $\hat{\mathbf{P}}$ and \mathbf{P}

Based on the mathematical definition of quasi-Euclidean space $\hat{\mathbf{P}}$ in Eq. (1)-(4), the relationship between $\hat{\mathbf{P}}$ and its corresponding Euclidean space \mathbf{P} is obtained.

Non-commutativity in 1D. Eq. (2)-(4) result in

$$k = -i\bar{\lambda}_p \frac{d}{dx}; \quad k^{-1} = -i\bar{\lambda}_p \frac{d}{d\hat{x}} \quad (8)$$

$$[k, x] = -i\bar{\lambda}_p; \quad [k^{-1}, \hat{x}] = -i\bar{\lambda}_p \quad (9)$$

$$[\hat{x}, x] = -i\bar{\lambda}_p \cdot x; \quad [x, \hat{x}] = -i\bar{\lambda}_p \cdot \hat{x} \quad (10)$$

where Eq. (8) demonstrates the non-commutativity between \hat{P} and P in any one dimension (Part 1 in Supplementary information).

Commutativity in $\geq 2D$. For any a pair of local $\geq 2D$ spaces \hat{P} and P expanded respectively by orthogonal $\hat{x}_i, \hat{x}_j, \dots$ and x_i, x_j, \dots , it exists

$$\hat{P} = \prod \hat{x}_i \hat{x}_j \dots = \prod x_j x_i \dots = P \quad (11)$$

since the orthogonality results in $[x_i, \hat{x}_j] = 0$, which guarantees Eq. (11). Here ${}^n\hat{P} = {}^nP$ ($n \geq 2$) in Eq. (11) ensures the commutativity between \hat{P} and P in any $\geq 2D$ spaces.

Reciprocity. Results in Eq. (8)-(10) also determine a special relationship of reciprocity between \hat{x} and x for it always exists

$$\exists x = F(\hat{x}), \quad \forall \hat{x} = F(x) \quad (12)$$

vice versa. This reciprocity leads to the interchangeability between \hat{P} and P , demonstrating the equivalence for a certain object under two conditions, one is to measure it with reference to P when it lies in \hat{P} , the other is to do so with reference to \hat{P} when it lies in P .

Thus, the relationship between the quasi-Euclidean space \hat{P} and its corresponding Euclidean space P can be summarized into 3 points, including non-commutativity in 1D, commutativity in $\geq 2D$, and reciprocity.

2.3 Properties of quasi-Euclidean space \hat{P}

Uncertainty and UV cutoff $\bar{\lambda}_p$. According to the conclusions about non-commutativity [18], \hat{x} and x , as a pair of non-commutative quantities represented in Eq. (10), are relatively uncertain for they can't be determined simultaneously. Besides, Eq. (9) results in the same minimum 1D measure of $\bar{\lambda}_p$ (reduced Planck length $\bar{\lambda}_p = \frac{\lambda_p}{2\pi}$; Part 2 in Supplementary information) for both \hat{x} and x , since it exists

$$[k, x] = -i\bar{\lambda}_p \Rightarrow (\Delta k x)^+ \geq \frac{\bar{\lambda}_p}{2}; \quad \left[\frac{1}{k}, \hat{x}\right] = -i\bar{\lambda}_p \Rightarrow (\Delta \frac{\hat{x}}{k})^+ \geq \frac{\bar{\lambda}_p}{2} \quad (13)$$

$$|\Delta \hat{x}| = (\Delta k x)^+ - (\Delta k x)^- \geq \bar{\lambda}_p; \quad |\Delta x| = (\Delta \frac{\hat{x}}{k})^+ - (\Delta \frac{\hat{x}}{k})^- \geq \bar{\lambda}_p \quad (14)$$

As a visual understanding of the above results, the minimum measurements may always be the same $\bar{\lambda}_p$ for either of the two scenarios, one is to measure a math space with infinite scales by a physical ruler with the minimum scale $\bar{\lambda}_p$, the other is to measure a physical space with the smallest length $\bar{\lambda}_p$ by a math ruler with infinite scales. Here, the smallest 1D measure $\bar{\lambda}_p$ is named UV cutoff for both \hat{P} and P , when they are measured by each other.

Briefly, $|\Delta \hat{x}| \geq \bar{\lambda}_p$ of the UV cutoff results in a 1D blank interval, directly leading to a quantized 1D for \hat{P} .

Planck units \hat{p} . The minimum blank interval along any a dimension will naturally result in an nD minimum blank interval ${}^n\hat{p}$ or np . Algebraically, these two minimum spaces ${}^n\hat{p}$ and np share the same minimum measure $\bar{\lambda}_p^n$ because UV cutoff $\bar{\lambda}_p$ requires any local space ${}^n\hat{P}$ or nP to satisfy

$${}^n\hat{P}=\prod \hat{x}_i\hat{x}_j \dots \geq \bar{\lambda}_p^n=\hat{p}; \quad {}^nP=\prod x_ix_j \dots \geq \bar{\lambda}_p^n=p \quad (x_i, x_j, \dots > 0) \quad (15)$$

Geometrically, the boundary of an nD minimum interval ${}^n\hat{p}$ is determined by two factors, including the 1D condition and the nD condition. Regarding the 1D condition about UV cutoff $\bar{\lambda}_p$, any distance $\rho < \bar{\lambda}_p$ inside the boundary is forbidden. Let the boundary be determined by $\rho < \bar{\lambda}_p \cdot \cos\alpha \cdot \sin\beta \cdot \sin\gamma \dots$, which defines a boundary on a generalized nD sphere with center located

at $(\frac{\bar{\lambda}_p}{2}, 0, \frac{\pi}{2}, \frac{\pi}{2}, \dots)$ on the polar axis of a polar coordinates (Fig. 2A). Obviously, any distance inside

the boundary will result in $\rho < \bar{\lambda}_p$ and the violation of the 1D condition. Regarding the nD condition that the minimum interval is of nD measure ${}^n\hat{p} \succ \bar{\lambda}_p^n$, then ${}^n\hat{p}$ should be of another boundary on the surface of a generalized nD cube with side length $\bar{\lambda}_p$, and any distance outside the cube is forbidden since it will result in an nD measure $> \bar{\lambda}_p^n$ and the violation of the nD condition (Fig. 2B). These two boundaries, including the spherical one and the cubic one, determine the inner and outer boundary for the minimum interval ${}^n\hat{p}$, respectively. Moreover, any intermediate between the outer one and the inner one is also a valid boundary for ${}^n\hat{p}$ because of the linearity of ${}^n\hat{P}$. The above results indicate that it exists a series of boundaries for ${}^n\hat{p}$, and ${}^n\hat{p}$ has an uncertain boundary in nature (Fig. 2C).

Consequently, the uncertain boundary results in the uncertain structure for ${}^n\hat{p}$. E.g., to represent a 2D minimum local space ${}^2\hat{p}$ by $\hat{p}(\rho, \theta)$, it always exists certain $\rho = \bar{\lambda}_p$ when θ is completely uncertain ($0 \leq \theta \leq 2\pi$), or uncertain ρ ($\bar{\lambda}_p \leq \rho \leq \sqrt{2}\bar{\lambda}_p$) when θ is of certainty (Fig. 2D).

Here the minimum nD blank interval ${}^n\hat{p} = \bar{\lambda}_p^n$ with uncertain boundary and uncertain structure is named a Planck unit. In quasi-Euclidean space ${}^n\hat{P}$, a Planck unit ${}^n\hat{p}$ generally defines a local space with the following characteristics. Firstly, it is a blank space with constant nD generalized volume $\bar{\lambda}_p^n$ but uncertain structure varying from a sphere with a diameter of $\bar{\lambda}_p$ to a cube with a side length of $\bar{\lambda}_p$. Secondly, the longest 1D distance of the boundary varies in interval of $[\bar{\lambda}_p, \sqrt{n}\bar{\lambda}_p]$ (Fig. 2D). Obviously, the Planck unit is a natural extension of the concept of the UV cutoff. Mathematical derivation indicates that ${}^n\hat{P}$ can also be quantized by ${}^n\hat{p} = \bar{\lambda}_p^n$, just like its 1D subspace can be quantized by UV cutoff $\bar{\lambda}_p$. Considering the special case about $\hat{x} = 0$ (Section 2.1), an iD Planck unit ${}^i\hat{p}$ ($0 < i < n$) is also allowed when $\Delta\hat{x}_j = 0$ and $\hat{x}_j = c$ ($i < j < n$), demonstrating that ${}^n\hat{P}$ can also be quantized in subspaces (Fig. 2E). Therefore, quasi-Euclidean space ${}^n\hat{P}$ can be redefined as such a special Euclidean space, which behaves as same as Euclidean space nP at scale $\geq \lambda_p$ on the one hand, but does differently from nP for its iD measure always satisfies ${}^i\hat{P} \geq \bar{\lambda}_p^i$ at scale $< \lambda_p$ on the other hand, although it remains a Euclidean space at the same time. This redefinition takes the original definition about 1D blank interval as a special case of 1D and results in a phenomenon that a serials of *space bubbles* with various dimensional structures exist in ${}^n\hat{P}$ at the extremely small scale (Fig. 2E).

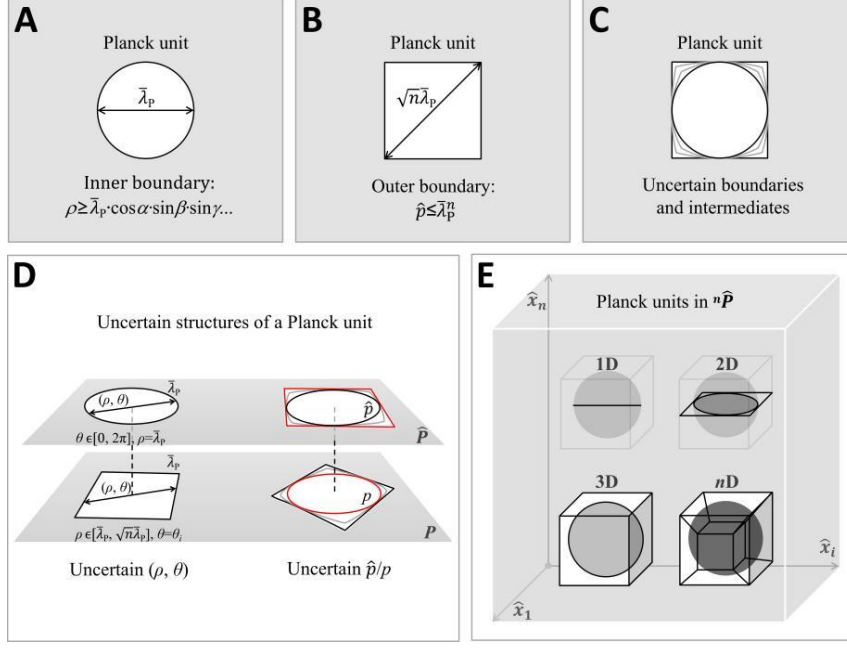


Fig. 2 The uncertain boundary of a Planck unit: it can be a generalized circle without any distance shorter than $\bar{\lambda}_p$ inside (A), a generalized cube without any distances longer than $\sqrt{n}\bar{\lambda}_p$ outside (B), or an intermediate between them (C). The uncertain structure of a Planck unit and the uncertain relationship between \hat{p} and p (D). *Space bubbles* of Planck units $i\hat{p}$ located in subspaces of quasi-Euclidean space \hat{P} (E).

IR cutoff L . For a generalized nD local space ${}^n\hat{P} > {}^n\hat{p}$ ($n \geq 2$), its nD generalized volume is of a constant measure of ${}^n\hat{P} \equiv {}^n P > \bar{\lambda}_p^n$, according to Equ. (15). Besides, linearity of ${}^n\hat{P}$ results in

$$(\hat{x})_{\max} = (\sum_1^n \Delta \hat{x}_i)_{\max} = n\bar{\lambda}_p \mid_{n \rightarrow \infty} \rightarrow +\infty \quad (\Delta \hat{x}_i = \bar{\lambda}_p, \hat{x} > 0) \quad (16)$$

when \hat{x} is taken as the linear summation of innumerable UV cutoffs between any adjacent point pairs. Obviously, $(\hat{x})_{\max} \rightarrow +\infty$ satisfies uncertainty of ${}^n\hat{P}$, when only one dimension along \hat{x} is involved. Whereas the nD condition about the commutative ${}^n\hat{P}$ and ${}^n P$, as shown in Equ. (11), should be involved when \hat{x} is included in a certain local space ${}^n\hat{P}$ ($n \geq 2$). Considering the requirement aroused by the UV cutoff in Equ. (17), the possible maximum and the minimum for 1D condition can be obtained as $+\infty$ and $\bar{\lambda}_p$ (the mathematically reasonable solution about $|\hat{x}_i|_{\min}=0$ is prohibited to ensure compliance with the physical principle of conservation of mass in the current 1D space), respectively, as shown in Equ. (18). When the nD condition about the commutative volume is involved in Equ. (19) as

$$\Delta \hat{x}_i \geq \bar{\lambda}_p \text{ or } \Delta \hat{x}_i \leq -\bar{\lambda}_p \quad (17)$$

$$|\hat{x}_i|_{\max} \rightarrow +\infty, \quad |\hat{x}_i|_{\min} = \bar{\lambda}_p \quad (18)$$

$$\prod_1^n \hat{x}_i \equiv {}^n P \quad (19)$$

the allowed maximum of a certain \hat{x} included in a local space ${}^n\hat{P}$ is of

$$\text{IR cutoff: } (\hat{x}_i)_{\max} = \frac{{}^n P}{\bar{\lambda}_p^{(n-1)}} \quad (\text{when } \hat{x}_j, \hat{x}_k, \dots = \bar{\lambda}_p) \quad (20)$$

demonstrating that a certain IR cutoff L concretely exists for a general local space ${}^n\hat{P}$. To

transform Equ. (20) into $L = \frac{nP}{\bar{\lambda}_p} \cdot \bar{\lambda}_p$, the physical or geometrical meaning for IR cutoff can be discovered to be the longest path for a Planck unit when its motion path covers the entire local space ${}^n\hat{P}$ uniformly and without overlap (Part 3 in Supplementary information).

Thus, properties of the quasi-Euclidean space \hat{P} can be summarized into 4 points, including uncertainty, UV cutoff $\bar{\lambda}_p$, Planck unit \hat{p} with certain measure $\bar{\lambda}_p^n$ and uncertain structure varying from a generalized cube to a generalized sphere, and IR cutoff $L = \frac{nP}{\bar{\lambda}_p^{(n-1)}}$ as the longest 1D measure for a local space ${}^n\hat{P}$.

Section 2 defines a generalized quasi-Euclidean space \hat{P} with Planck length. Mathematical study discovers its uncertainty and clarifies the minimum structures of Plank units in it. As a micro-object with measure $\bar{\lambda}_p$, a Planck unit might play the role of a quantum object when its motion inside \hat{P} is investigated at scale $\geq \bar{\lambda}_p$, where \hat{P} behaves as a normal Euclidean background space. Consequently, a series of physical principles, including quantum mechanics, the minimum energy, conservation of mass, conservation of energy, *etc.*, should be obeyed by a moving Planck unit. Besides, rotational symmetry and translational symmetry should be strictly satisfied by a Planck unit, since it is also part of the background space. Next, motion for such a Planck unit should be pursued, assuming that it can distinguish itself from the quasi-Euclidean space \hat{P} of the macro background. And IR cutoff L is expected to help in determining the ground state when the object is confined to a certain local space.

3. Planck units under control of rotational symmetry

3.1 A general Planck unit controlled by rotational symmetry

As part of space, a Planck unit \hat{p} should satisfy rotational symmetry (abbreviated as RS).

RS states (I, R and T) and RS spaces (P_I , P_R and P_T). Strict RS bans Planck unit \hat{p} from any radial displacement ($dr=0$) by infinite potential barrier V_r^∞ , requiring $r=c$ mathematically. The solution results in two types of states, one is in-situ ($c=0$), named state **I** (Fig. 3A), the other is revolving around the in-situ position in the surface of a certain sphere ($c \neq 0$), named state **R**. Geometrically, the nD surface of an $(n+1)D$ sphere provides the simplest spherical space for an nD Planck unit ${}^n\hat{p}$ (Fig. 3B). Considering the UV cutoff of quasi-Euclidean space \hat{P} , the sphere nearest to state **I** is determined by $r=\bar{\lambda}_p$, since a closer sphere is prohibited by the definition of UV cutoff $\bar{\lambda}_p$. After normalizing the system by $\bar{\lambda}_p=1$, the simplest and nearest space for the revolving state **R**, symbolized as P_R , should be

$$P_R: \sum_1^{n+1} \hat{x}_i^2 = 1 \quad (21)$$

Here, the curved nD surface P_R is embedded in the $(n+1)D$ flat space, so state **R** actually takes the $(n+1)D$ space as its background, breaks away from the original nD background space of state **I**, and violates space conservation directly. Space conservation requires that ${}^n\hat{p}$ always moves in the same flat nD background space, and this then requires a flattened P_R , which is a finite plane P_T tangent to P_R (to show it more clearly, P_T 's tangent point is set up to be at the bottom of P_R to distinguish P_T from P_I in Fig. 3C, which represents the original nD background by a parallel space). Let m be the generalized nD volume of an nD sphere with radius $r=1$ (Part 5 in

Supporting information), its generalized surface should be $(n-1)$ D space measured $(^n m)'$, and the flattened surface should be measured $\int_0^{2\pi} dx_1 \cdots \int_0^{2\pi} dx_{n-1} = (2\pi)^{n-1}$.

Logically, RS states **I**, **R** and **T** for ${}^n \hat{p}$ should be of spaces measured

$$P_I=1 \quad (22)$$

$$P_R=(^{n+1}m)' \quad (23)$$

$$P_T=(2\pi)^n \quad (24)$$

According to Equ. (20), their IR cutoffs are $L_I=1$, $L_R=(^{n+1}m)'$ and $L_T=(2\pi)^n$, respectively. It should be noted that these spaces are of n D measurement while their IR cutoffs are of 1D measure, and they share the same algebra expression only because of the normalization of $\bar{\lambda}_p=1$.

So rotational symmetry requires Planck unit ${}^n \hat{p}$ exist at in situ state **I**, revolving state **R** or flattened revolving state **T** at the extreme condition, and these states are confined to certain spaces P_I , P_R and P_T with certain IR cutoff L_I , L_R and L_T , respectively. Generally, there exists infinite potential V_r^∞ to separate state **I** from the topologically connected P_R/P_T , resulting in localized Planck units with no transformability (Fig. 3C). And such a background space \hat{P} without any non-local Planck units is taken to be a trivial one (a general case is shown in Part 4, Supplementary information).

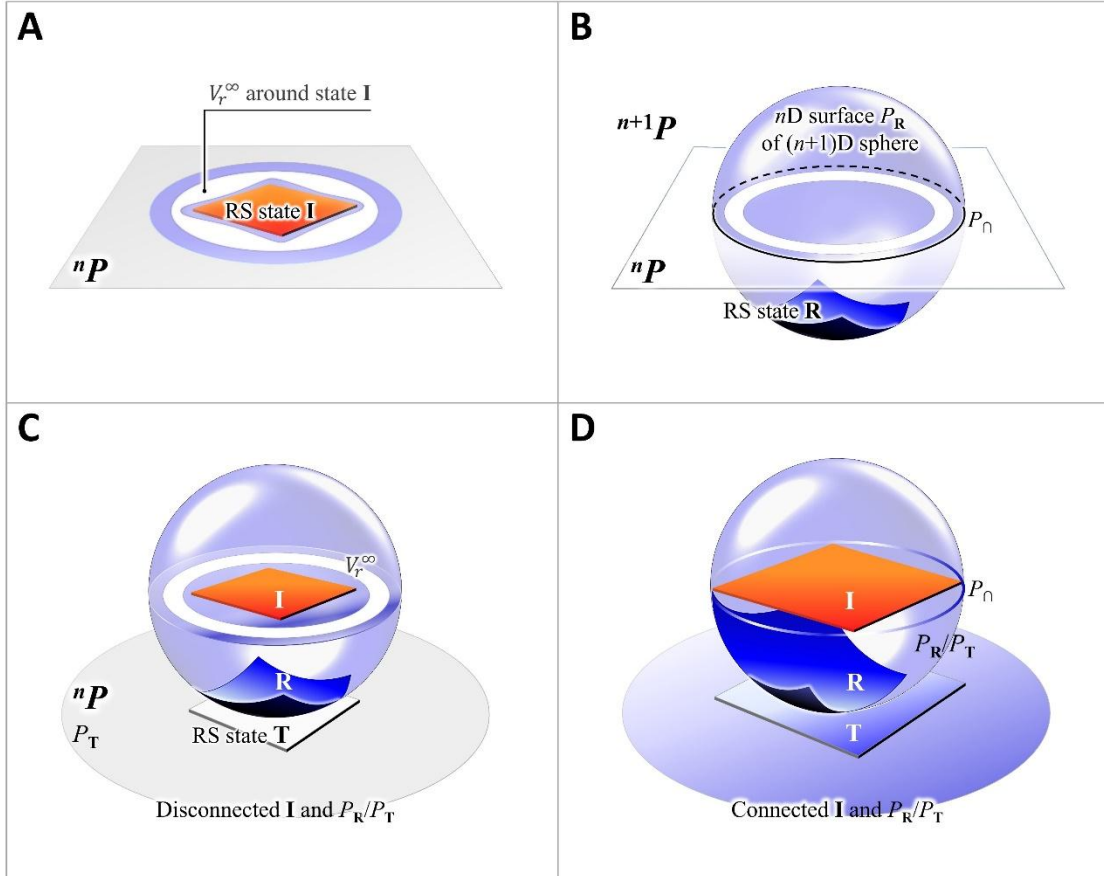


Fig. 3 RS spaces of ${}^n \hat{p}$ at RS states of **I** (in situ state, **A**), **R** (revolving state, **B**) and **T** (tangent state, **C**). Transformable ${}^n \hat{p}$ with topologically connected **I** and P_R/P_T (**D**).

3.2 Non-triviality: transformable Planck units in high-dimensional spaces

If state **I** and space P_R/P_T are topologically connected, V_r^∞ will be invalidated, transformation between **I** and **R/T** will be possible, and the background space \hat{P} will become a non-trivial one since non-local Planck units in it are allowed (Fig. 3D).

The topology of the RS spaces was investigated here. Assuming that a Planck unit at state **I** takes a generalized cube as its uncertain structure and takes diagonal \sqrt{n} as its maximum 1D measure, the increasing \sqrt{n} will make itself possible to reach P_R/P_T in higher dimensional spaces. The possible points connecting **I** and P_R/P_T should belong to the intersecting space P_n determined by $P_I (\subset {}^n P)$ and $P_R (\sum_1^{n+1} \hat{x}_i^2=1)$ as

$$P_n: \sum_1^n \hat{x}_i^2=1 \quad (25)$$

which is an $(n-1)$ D spherical surface enclosing an n D sphere (Fig. 3D). Logically, the longest 1D measurement enclosed by P_n is ${}^n m$, since the enclosed sphere measured ${}^n m$ is of IR cutoff $L={}^n m$. Thus, state **I** will share points with P_n when $\sqrt{n} \geq {}^n m$, meaning that the cube or some other intermediate reaches P_R/P_T topologically (Fig. 3D).

Calculation demonstrates the relationship between \sqrt{n} and ${}^n m$ in n D space (Fig. 4), showing topology for **I** and P_R/P_T as follows. Firstly, 1-9D spaces are all trivial ones for $\sqrt{n} < {}^n m$, resulting in that their state **I** is always separated from P_R/P_T by V_r^∞ and state transformation is always forbidden. Secondly, 10D, 11D and 12D spaces are three non-trivial spaces with $\sqrt{n} > {}^n m$, where topological connection between **I** and P_R/P_T exists, potential V_r^∞ can be invalidated, and non-local Planck units are allowed. Lastly, ${}^n m < 1 < \sqrt{n}$ in ≥ 13 D spaces makes these solutions meaningless for ${}^n m < 1$ violates the minimum local space ${}^n \hat{p} \geq 1$ (Fig. 4).

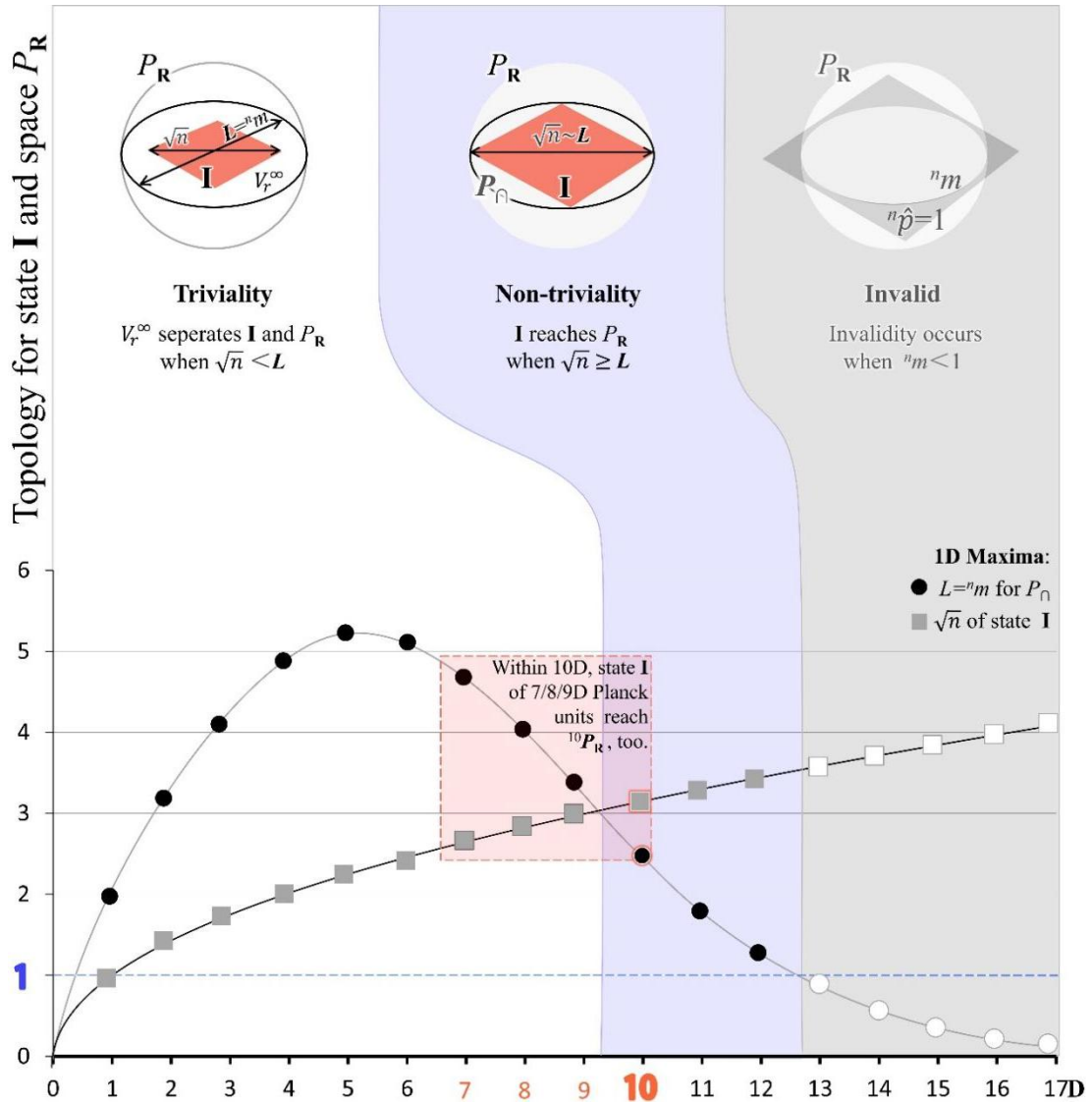


Fig. 4 Topological relationship for state **I** and P_R/P_T in nD space.

Thus, Planck units ${}^{10}\hat{p}$, ${}^{11}\hat{p}$ and ${}^{12}\hat{p}$ are proven to be of non-trivial topology connections between their state **I** and P_R/P_T , discovering that transformation of a Planck unit is allowed in 10D, 11D and 12D spaces. Planck units ${}^{>12}\hat{p}$ inside quasi-Euclidean spaces $>12D$ involve threshold space ${}^n m \leq 1$, e.g., ${}^{13}m=0.910$, which violates the definition of a Planck unit, resulting in the loss of reasonability for the current case. According to the monotonic decrease trend of the generalized formulae for ${}^n m$, spaces with higher dimensions are reasonably ignored (Part 5 in Supplementary information).

3.3 ${}^{10}\hat{p}$: RS space structures and two transforming paths

${}^{10}\hat{p}$ is the first non-trivial Planck unit with transformability, since its in-situ state **I** has the possibility to reach the space of its revolving state **R**, resulting in the probability for itself to transform into the corresponding tangent state **T**. This possible transformation is determined by the geometrical characters of these different states, heavily depending on that **I** shares same point(s) with P_R . Investigation in section 3.2 discovers that there is no possibility for such a

sharedness in a quasi-Euclidean space ${}^n\hat{\mathbf{P}}$ when $n \leq 9$. Although any a Planck unit has the possibility to behave as a spherical one with completely uncertain θ but certain $\rho = \bar{\lambda}_p$, which keeps itself always isolated from \mathbf{P}_R , a possible sharedness still exists when it satisfies $\sqrt{n} \geq {}^n m$, where \sqrt{n} is the possible maximum 1D distance for a Planck unit and ${}^n m$ equals to the maximum 1D distance inside the threshold space P_\cap . Essentially, algebraic relationship between $\sqrt{10}$ and ${}^{10}m$ determines that ${}^{10}\hat{p}$ is a transformable Planck unit with the lowest dimensional structure (Fig. 4). Its non-triviality, however, changes motion of ${}^{10}\hat{p}$ in turn, since the topological connection changes its RS space structures and the transformation changes its movements.

Changed P_I and P_T . Topological connection invalidates the infinitely potential V_r^∞ , which is originally separating in-situ state \mathbf{I} from the revolving space P_R , and then the disappeared V_r^∞ makes state \mathbf{I} exist actually in a space enclosed by P_\cap , which is the 10D sphere measured ${}^{10}m$ (Fig. 3D). Besides, the topological connection makes P_R be flattened from a certain point on 9D space of P_\cap and the corresponding P_T becomes $P_T = \int_0^{2\pi} x^8 dx \cdot \int_0^{2\pi} dx$. The above changes result in

$${}^{10}P_I = {}^{10}m = \frac{\pi^5}{120} \quad (26)$$

$${}^{10}P_T = \frac{(2\pi)^{10}}{9} \quad (27)$$

Two paths for $\mathbf{I} \rightleftharpoons \mathbf{R}/\mathbf{T}$: the highest rotational symmetry (\mathbf{RS}^\dagger) and the minimum energy (\mathbf{E}_l). Accordingly, spaces P_I and P_T of 10D Planck unit ${}^{10}\hat{p}$ are of IR cutoff $L_I = \frac{\pi^5}{120}$ and $L_T = \frac{(2\pi)^{10}}{9}$, respectively. And these two IR cutoffs might provide quantum paths for ${}^{10}\hat{p}$ at ground states. But as an RS state, movement of state \mathbf{I} should also be measured equally along each dimension, meaning that P_I should be an equally expanded space of $\prod_1^{10} x_i = \frac{\pi^5}{120}$ where $x_1, x_2, \dots, x_{10} = (\frac{\pi^5}{120})^{\frac{1}{10}}$ and its observable IR cutoff should be $L_I = (\frac{\pi^5}{120})^{\frac{1}{10}}$. As for state \mathbf{R} , dimensional equivalence would not work here since curved P_R is of no dimensional equivalence intrinsically. Consequently, ${}^{10}P_R$ remains the same with $L_R = ({}^{11}m)'$, and so does ${}^{10}P_T$ with $L_T = \frac{(2\pi)^{10}}{9}$.

Thus, ${}^{10}\hat{p}$ at in-situ state \mathbf{I} has two ways for its movement, one takes $L_I = \frac{\pi^5}{120}$ as its half wavelength when it satisfies the minimum energy, the other takes $L_I = (\frac{\pi^5}{120})^{\frac{1}{10}}$ as its half wavelength when it satisfies the highest RS. The two principles, one is the minimum energy noted as \mathbf{E}_l , the other is the highest rotational symmetry noted as \mathbf{RS}^\dagger , then determine the two different transforming paths for ${}^{10}\hat{p}$.

$\mathbf{I} \rightleftharpoons \mathbf{T}$: space structures for a transforming ${}^{10}\hat{p}$. Considering that ${}^{10}\hat{p}$ should remain in 10D space to satisfy conservation, its \mathbf{R} state in P_R , which is a 10D generalized surface embedded in 11D background space (Fig. 3B), is unobservable in ${}^{10}\hat{\mathbf{P}}$ for it violates the conservation. In fact, only state \mathbf{I} and \mathbf{T} for ${}^{10}\hat{p}$ are RS states with physical legality inside ${}^{10}\hat{\mathbf{P}}$, whereas state \mathbf{R} is a virtual one just bridging \mathbf{I} and \mathbf{T} mathematically. Accordingly, the 9D space P_\cap ($\sum_1^{10} \hat{x}_i^2 = 1$) in ${}^{10}\hat{\mathbf{P}}$

actually determines the topological connection for **I** and **T**, instead of $P_R (\sum_1^{11} \hat{x}_i^2=1)$ embedded in $^{11}\hat{\mathbf{P}}$ (Fig. 3B).

Taking 10D quasi-Euclidean space $^{10}\hat{\mathbf{P}}$ as the simplest background space with non-locality, critical space P_\cap plays the role of threshold not only for $^{10}\hat{p}$, but also for its substructures in each subspace of $^{10}\hat{\mathbf{P}}$. Based on the maximum length $^{10}L_I=\frac{\pi^5}{120}$ allowed by P_\cap , transformability for each substructure in 1-9D proper subspaces had been also investigated. Results show that the 1D maximum $^{10}L_I=\frac{\pi^5}{120}=2.55$ allows Planck units $^7\hat{p}$, $^8\hat{p}$ and $^9\hat{p}$ to be transformable ones besides $^{10}\hat{p}$, since it exists $\sqrt{6}<2.55<\sqrt{7}$. This means that any ≥ 7 D substructure is of transformability for its state **I** can exceed the threshold and connect to the corresponding state **T**. In other words, any ≤ 6 D substructure never reaches the threshold space of P_\cap , being prohibited from transformation and maintaining triviality.

Let triviality be of priority to nontriviality and let x_i expand the i^{th} dimension of the i D subspace ($1 \leq i \leq 10$), the maximum space with triviality, noted as $^i\mathbf{P}$, should be 6D space expanded by x_1, x_2, \dots, x_6 , and its complement space involving non-triviality, noted as $^m\mathbf{P}$, should be 4D space expanded by x_7, x_8, x_9 and x_{10} . Then a map of space structures for $^{10}\hat{p}$ can be obtained for both state **I** and state **T** in each of the 1-10D subspaces (Fig. 5).

of ${}^8\hat{p}$ as an example, its P_I should be expanded by $x_1=x_2=\dots=x_6=1$ and $x_7=x_8=(\frac{\pi^4}{24})^{\frac{1}{2}}$, and its P_T should be of IR cutoffs $(2\pi)^6$ and $\frac{(2\pi)^2}{7}$ in 6D trivial subspace and the other 2D non-trivial subspace, respectively. Similarly, P_I for ${}^{10}\hat{p}$ should be expanded by $x_1=\dots=x_6=1$ and $x_7=x_8=x_9=x_{10}=(\frac{\pi^5}{120})^{\frac{1}{4}}=1.26$, and its corresponding P_T should be of IR cutoffs $(2\pi)^6$ and $\frac{(2\pi)^4}{9}=173$ in ${}^t\mathbf{P}$ and ${}^n\mathbf{P}$, respectively (Fig. 5B).

Regarding the principle the minimum energy (E_l), ${}^n\hat{p}$, as an n D Planck unit or n D substructure, would take IR cutoff L as its motion spaces in each of its own subspaces, acquiring $P=\prod_1^n iL$ be its largest motion space. Taking 2D Planck unit or 2D substructure of ${}^2\hat{p}$ as an example, its proper substructure ${}^1\hat{p}$ takes ${}^1L_I=1$ and ${}^1L_T=2\pi$ as its longest paths for state **I** and **T**, respectively, and there must be ${}^1L_I//x_1$ and ${}^1L_T//x_1$. Besides, ${}^2\hat{p}$ itself takes ${}^2L_I=1$ and ${}^2L_T=(2\pi)^2$ as its longest paths for state **I** and **T**, respectively, and there must be ${}^2L \subset {}^2\mathbf{P}$. Obviously, 2P_I and 2P_T have the maximum values of ${}^2P_I={}^1L_I \times {}^2L_I$ and ${}^2P_T={}^1L_T \times {}^2L_T$, respectively, if and only if ${}^2L \perp {}^1L$ and ${}^2L // x_2$. Similarly, it exists $P_I=\prod_1^{10} iL_I=\frac{8\pi^{16}}{4465123}=161$ and $P_T=\prod_1^{10} iL_T=\frac{(2\pi)^{55}}{3024}=2.63 \times 10^{40}$ for ${}^{10}\hat{p}$, when ${}^{10}\hat{p}$ is dominated by Principle E_l .

2 dimensionless constants. For ${}^{10}\hat{p}$ satisfying Principle RS^\dagger , it conserves triviality in ≤ 6 D space of ${}^t\mathbf{P}$, meaning that the non-locality aroused by transformation only occurs in the 4D complement space of ${}^n\mathbf{P}$. According to the map of the space structure (Fig. 5B), ${}^{10}\hat{p}$ would take $L_I=(\frac{\pi^5}{120})^{\frac{1}{4}}$ and $L_T=\frac{(2\pi)^4}{9}$ as its half wavelengths for its ground states, and ${}^{10}\hat{p}$ would be observed to be with wavelength change of $\alpha_1=(\frac{L_T}{L_I})^{-1}=1/137.036\ 082$ times during its transformation in 4D space ${}^n\mathbf{P}$.

Similarly, ${}^{10}\hat{p}$ satisfying Principle E_l would be observed to be with wavelength change of $\alpha_2=(\frac{L_T}{L_I})^{-1}=1/(1.628\ 008 \times 10^{38})$ times during its transformation in global 10D space.

Section 3 investigated on a Planck unit dominated by rotational symmetry (RS). RS strictly defines the spaces for a Planck unit in-situ (state **I**) or in flattened spherical surface (state **T**), and these two states are generally isolated from each other, meaning that a Planck unit is always localized. Geometry study discovered that a 10D Planck unit acquired its transformability and non-locality when its two states **I** and **T** were topologically connected. Following the two principles, one was the highest rotational symmetry (RS^\dagger), the other was the minimum energy (E_l), ${}^{10}\hat{p}$ presented two dimensionless constants of $\sim 1/137.036\ 082$ and $\sim 1/(1.628 \times 10^{38})$ for its two transforming paths, respectively.

4. 10D Planck unit under control of translational symmetry

As part of 10D quasi-Euclidean space, Planck unit ${}^{10}\hat{p}$ should satisfy translational symmetry (TS) besides rotational symmetry (RS).

RS State T'. Besides P_R , RS also defines other concentric surfaces with $r > \bar{\lambda}_P$ and results in innumerable state R' in infinite space. However, transformation between anyone of these R' states and the in situ state I is forbidden because R' is not so close to I , which causes them to be separated by potential barrier V_r^∞ . Consequently, it also exists innumerable state T' without transformability into state I (Fig. 6). To satisfy principle of least action [19], a Planck unit at state T' should be of action $S=h$ for its ground state.

Constant velocity. Because of the translational symmetry, Planck units $^{10}\hat{p}$ in 10D background space should be identical objects, and their T states should be of constant velocity v .

Sharing-coupling effect. A system containing two centers A and B had been built up to explore the interaction between any two $^{10}\hat{p}$. Because of the constant velocity v of state T , one center of state I_A shares T_B as its state T'_A at distance r ($r \gg 1$), since T_B and T'_A are indistinguishable for they are identical objects sharing the same velocity v . So are T_A and T'_B for another center of I_B . The two shared states T and T' spontaneously bring about energy difference of

$$\Delta E = \frac{hv}{2L_T} - \frac{hv}{2L_{T'}} = \frac{hv}{2L_T} - \alpha \frac{hv}{2L_I} \frac{1}{r} \quad (28)$$

between them when they both have the least action h and take IR cutoffs as their half wavelengths at ground states. This effect caused by the shared T and T' states is named sharing-coupling effect (Fig. 6).

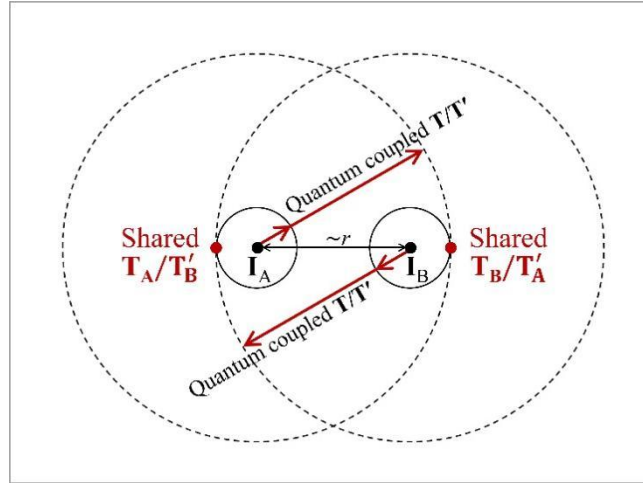


Fig. 6 Shared state T/T' : sharing-coupling effect for a 2-center system.

Two long range interaction fields. The velocity v for state T or T' leads to their constant mass of 0 since $\frac{dv}{dk} = \frac{\hbar}{m_T} = 0$ requires $m_T \equiv 0$ when $dv=0$. So kinetic energy of T or T' should be protected

by their constant velocity and mass, which results a kinetic energy difference of $\Delta T_{T-T'} = \frac{hc}{\lambda_T} - \frac{\alpha hc}{2\pi r}$.

Considering that the energy for each point in a vacuum should be always equal, there must be a potential energy different $\Delta U = -\Delta T_{T-T'}$ between I_A and I_B to balance the energy of the system, resulting in two long-range potential fields

$$\begin{aligned}
\text{RS}^\dagger: \quad F &= \left(\frac{1}{2L_1}\right) \cdot \frac{hc}{137.036 r^2} \\
\text{E}_\perp: \quad F &= \left(\frac{1}{2L_1}\right) \cdot \frac{hc \times 10^{-38}}{1.628 r^2}
\end{aligned} \tag{29}$$

when the two dimensionless constants of $\alpha_1=1/137.036\ 082$ for RS^\dagger process or $\alpha_2=1/(1.628\ 008 \times 10^{38})$ for E_\perp process had been brought in and state \mathbf{T} took the light speed c as its constant velocity (Part 6 in Supplementary information).

Section 4 discovered the sharing-coupling effect determined by translational symmetry, obtaining a spontaneous potential field between any two shared \mathbf{T}/\mathbf{T}' states in 10D space. According to the two transforming paths with dimensionless constants $\alpha_1 \sim 1/137.036$ and $\alpha_2 \sim 1/(1.628 \times 10^{38})$, two long-range interacting fields had been obtained accurately.

5. Conclusion and discussion

Based on its three initial settings, including Planck length, rotational symmetry and translational symmetry, a quasi-Euclidean space is quantized by the extremely small structures of Planck units, and a 10D Planck unit is discovered to be the simplest Planck unit with two different transforming paths, resulting in two long-range interactions in a 4D subspace and in a 10D global space, respectively. The two constants, $1/137.036\ 082$ for the transformation dominated by the highest rotational symmetry (RS^\dagger) and $1/(1.628 \times 10^{38})$ for the transformation dominated by the minimum energy (E_\perp), are exactly equal to the fine structure constant ($\text{FSC}=1/137.035\ 999$) and approximately equal to the dimensionless gravitational constant ($\frac{Gm_p^2}{\hbar c} = \frac{1}{1.693 \times 10^{38}}$) [17], with deviations of $\sim 10^{-6}$ and $\sim 4\%$, respectively.

But the two corresponding forces between any 2 Planck units are neither the electromagnetic interaction between any two static electrons nor the gravitation between any two protons, and the difference between them is $(2L_1/2\pi)$ times. Obviously, it suggests another pseudo space with clear geometrics. Besides the pseudo space, the 4D non-trivial space expanded by the 7th, 8th, 9th and 10thD, the positive or negative nature of the two long range interactions, the obvious deviation between $\alpha_2 \sim 1/(1.628 \times 10^{38})$ and the dimensionless gravitational constant, *etc.*, are also puzzling. This work distinguishes the 10th dimension x_{10} from the other 3 ones of x_7 , x_8 and x_9 since ${}^7\hat{p}$, ${}^8\hat{p}$ and ${}^9\hat{p}$ all have their own subspaces to conserve their trivality but ${}^{10}\hat{p}$ has not. The relationship between the non-trivial 4D subspace and the current 4D space-time need to be explored further.

If ${}^{10}\hat{p}$ plays the role of an electron, its metastable states can be obtained when replacing some of its non-trivial structure by a trivial one, E.g., to replace the non-trivial value 4.72 in ${}^7\mathbf{P}$ subspace by 1, or to replace the non-trivial value 2.01 in ${}^8\mathbf{P}$ subspace by 1. In fact, a researcher could get a series of particles with familiar masses, if he can bear the symbols of this work (Part 7 in Supporting information) and try to finish the space structure maps for ${}^{11}\hat{p}$ or ${}^{12}\hat{p}$. But the more important thing is to answer what are the hidden principles for nature to screen these candidates.

The above conclusion will show its natural rationality further when being reviewed from an opposite perspective. Gravitation and electromagnetic force both have their potentials with form

of $U = \alpha \frac{hc}{r}$, which is consistent with $E = \frac{hc}{\lambda}$ in quantum mechanics. If the coefficient α is observed to be $\alpha = 1$ or $\alpha = \frac{1}{2}$, a boson carrying force can be recognized directly and a quantized long-interaction can be obtained immediately. If $\alpha = \frac{1}{2\pi}$, another interaction might be understood by an intelligence in 2D space after the rotational symmetry being realized. However, these two simplest cases are denied by the above results and the reality is proved to be about the more complicated $\alpha = \frac{9 \times (\frac{\pi^5}{120})^{\frac{1}{4}}}{16\pi^4}$ ($\sim \frac{1}{137.036\ 082}$) or $\alpha = \frac{\pi^9}{23625 \times (2\pi)^{48}}$ ($\sim \frac{1}{1.628\ 008 \times 10^{38}}$), which demands quantum solutions in a space far beyond the current 3D one. It might partly explain why the developed quantum mechanics had not produced a quantized long-range interaction for decades when a physically concrete high-dimensional space was absent. In fact, *Wylers's* constant of $\frac{9}{8\pi^4} (\frac{\pi^5}{2^4 \cdot 5!})^{\frac{1}{4}}$, which is equal to $\alpha_1 = \frac{9 \times (\frac{\pi^5}{120})^{\frac{1}{4}}}{16\pi^4}$ in above, had ever tried to solve FSC in 1969, with all of the relevant works stopped abruptly here almost at the same time [20]. But today is so different. Now the high-dimensional space has increasingly affected the scholars' thoughts, although it has never been proved by experiments. It must be admitted that the string theory is the first one to systematically explore the vacuum beyond 3D, and to illuminate modern sciences by the adequate high-dimensional spaces.

Finally, let's go back to the centenary goal of finding out a natural object with a characteristic path relatively measured ~ 137.036 . It's time to declare that the special path has been found, besides another much longer one measured $\sim 1.628 \times 10^{38}$. Such a result should be owned to the presentation of the *space bubbles*, which are naturally existing in the vacuum and are given name of Planck units by the current work. Obviously, these *bubbles* are bringing hopes to modern physics.

Acknowledgement. Yan Zhou would like to express the gratitude for support from Special Funding for Talents of West and Northeast China, Chinese Academy of Sciences. Special tribute is made to the mathematician *Armand Wylers*, who was pioneering this work.

References

- [1] Eckert, M. How Sommerfeld extended Bohr's Model of the Atom (1913–1916). *The European Physical Journal H*, **39**, 141-156 (2014). <https://doi.org/10.1140/epjh/e2013-40052-4>
- [2] Webb, J. K. *etc.* Further Evidence for Cosmological Evolution of the Fine Structure Constant. *Physical Review Letter*, **87**, 091301 1-3 (2001). <https://doi.org/10.1103/physrevlett.87.091301>
- [3] Webb, J. K. *etc.* Indications of a spatial variation of the fine structure constant. *Physical Review Letter*, **107**, 191101 1-5 (2011). <https://doi.org/10.1103/physrevlett.107.191101>
- [4] Wilczynska, M. R. *etc.* Four direct measurements of the fine-structure constant 13 billion years ago. *Science Advances*, **6**, e9672 1-9 (2020). <https://doi.org/10.1126/sciadv.aay9672>

- [5] Webb, J. K. & Lee, C. Convergence properties of fine structure constant measurements using quasar absorption systems. *Monthly Notices of the Royal Astronomical Society*, **528**, 6550-6558 (2024). <https://doi.org/10.1093/mnras/stae306>
- [6] Mellen, W. R. Bulletin of the American Physical Society, **20**, 492 (1975).
- [7] Yee, J. The relationship of the Fine Structure Constant and π , *arXiv* (2019). <https://doi.org/10.13140/RG.2.2.30832.92162>
- [8] Gross, D. Millennium Madness: Physics Problems for the Next Millennium, Strings 2000 conference at University of Michigan (2000).
- [9] Adler, S. L. Theories of the Fine Structure Constant. Publication 72/059-T. Batavia, IL: Fermilab (1972).
- [10] Sherbon, M. A. Fine-structure constant from Sommerfeld to Feynman. *Journal of Advances in Physics*, papers.ssrn.com (2019). <https://doi.org/10.2139/ssrn.3148761>
- [11] Stakhov, A. & Aranson, S. The Fine-Structure Constant as the Physical Mathematical MILLENNIUM PROBLEM. *Physical Science International Journal*, **9**(1), 1-36 (2016). <https://doi.org/10.9734/psij/2016/21966>
- [12] Craig, B. I. The formation of 3-dimensional Euclidean space and the fine structure constant. hal.science., HAL Id: hal-03677871f (2022).
- [13] Polchinski, J. String Theory. Cambridge University Press, Cambridge (1998). <https://doi.org/10.1017/cbo9780511618123>
- [14] Marek-Crnjac, L. On the unification of all fundamental forces in a fundamentally fuzzy Cantorian $\varepsilon^{(\infty)}$ manifold and high energy particle physics. *Chaos, Solitons & Fractals*, **20**, 669-682 (2004). <https://doi.org/10.1016/j.chaos.2003.10.013>
- [15] Veselov, A. I. & Zubkov, M. A. 10D Euclidean dynamical triangulations. *Physics Letters B*, **591**, 311-317 (2004). <https://doi.org/10.1016/j.physletb.2004.04.047>
- [16] Sabra, W. A. & Vaughan, O. 10D to 4D Euclidean supergravity over a Calabi-Yau three-fold. *Classical and Quantum Gravity*, **33**, 015010 (2016). <https://doi.org/10.1088/0264-9381/33/1/015010>
- [17] Mohr, P. J., Taylor, B. N. & Newell, D. B. CODATA recommended values of the fundamental physical constants: 2006. *Journal of Physical and Chemical Reference Data* **80**, 633-1284 (2008). <https://doi.org/10.1103/revmodphys.80.633>
- [18] Robertson, H. The Uncertainty Principle. *Physical Review* **34**, 163-164 (1929). <https://doi.org/10.1103/physrev.34.163>
- [19] Feynman, R. P. The principle of least action in quantum mechanics. (1942).
- [20] Wyler, A. L'espace symetrique du groupe des equations de Maxwell. *Comptes Rendus de l'Académie des Sciences*, Paris **269**, 743-745 (1969).

Supplementary information

Unification of Gravity and Electromagnetic Interaction: Based on Quantum Mechanics of a Quasi-Euclidean Space with Planck Length, Rotational Symmetry and Translational Symmetry

Yan Zhou^{1,2*}, Junyan Zhang^{1*†}, Ernst Meyer^{2†}, Xinkai Zhang¹ and Jing Zhang¹

¹Lanzhou Institute of Chemical Physics, Chinese Academy of Sciences, Lanzhou 730000, China.

²Department of Physics, University of Basel, 4056 Basel, Switzerland

*Authors contributed equally to this work.

†Corresponding authors. junyanzhang@licp.cas.cn (J. Z.); ernst.meyer@unibas.ch (E. M.)

Contents

1. Non-commutativity between \hat{x} and x
2. On $|\Delta k \cdot \Delta x| \geq \frac{\bar{\lambda}_P}{2}$ for $[k, x] = -i\bar{\lambda}_P$
3. On physical meaning of IR cutoff L
4. A general case for Planck unit " \hat{p} " with or without transformability
5. Calculation on generalized volume " m " for a general sphere
6. Detailed solution about the coupling field
7. Symbol list

1. Non-commutativity between \hat{x} and x

Based on the mathematical definition of quasi-Euclidean space $\hat{\mathbf{P}}$ in Eq. (2)-(4), relationship between \hat{x} and its corresponding x in Euclidean space could be obtained.

To substitute Eq. (2) into Eq. (4) and to take derivative with respect to x on the both sides, it obtains

$$\hat{x} = \lambda_P \cdot e^{\frac{ik \cdot x}{\lambda_P}} \quad (S1)$$

$$\frac{d\hat{x}}{dx} = 2\pi i k \cdot e^{\frac{ik \cdot x}{\lambda_P}} = \frac{2\pi i k \cdot \hat{x}}{\lambda_P} \quad (S2)$$

And Eq. (S2) leads to

$$k = -i\bar{\lambda}_P \frac{d}{dx} \quad (S3)$$

Then to substitute Eq. (2) and (3) into Eq. (4) and to take derivative with respect to \hat{x} on the both sides, it obtains

$$kx = \lambda_P \cdot e^{\frac{i \cdot \hat{x}}{\lambda_P}} \quad (S4)$$

$$\frac{dx}{d\hat{x}} = \frac{d(x+a)}{d\hat{x}} = \frac{2\pi i}{k} \cdot e^{\frac{i \cdot (x+a)}{\lambda_P}} \quad (S5)$$

And Eq. (S5) leads to

$$k^{-1} = -i\bar{\lambda}_P \frac{d}{d\hat{x}} \quad (S6)$$

Furthermore, Eq. (S3) results in

$$\begin{aligned} [k, x]\hat{x} &= k(x \cdot \hat{x}) - x \cdot (k\hat{x}) \\ &= k(x) \cdot \hat{x} + x \cdot (k\hat{x}) - x \cdot (k\hat{x}) \\ &= -i\bar{\lambda}_P \hat{x} \end{aligned} \quad (S7)$$

which then leads to the non-commutativity of

$$[k, x] = -i\bar{\lambda}_P \quad (S8)$$

Similarly, Eq. (S6) results in another non-commutativity of

$$[k^{-1}, \hat{x}] = -i\bar{\lambda}_P \quad (S9)$$

Based on Eq. (S8) and (S9), relationship between x and \hat{x} can be obtained as

$$[\hat{x}, x] = [k, x] x = -i\bar{\lambda}_P x \quad (S10)$$

$$[x, \hat{x}] = [k^{-1}, \hat{x}] \hat{x} = -i\bar{\lambda}_P \hat{x} \quad (S11)$$

demonstrating the non-commutativity between \hat{x} and x along any a dimension.

2. On $|\Delta k \cdot \Delta x| \geq \frac{\bar{\lambda}_P}{2}$ for $[k, x] = -i\bar{\lambda}_P$

For $[k, x] = -i\bar{\lambda}_P$ in Equ. (S8), let $|g(x)|^2$ be Gaussian for $\Delta x = a$, then it should exist

$$g(x) = \frac{1}{(2\pi)^{\frac{1}{4}} \sqrt{a}} e^{-\frac{x^2}{4a^2}} \quad (S12)$$

with $|g(x)|^2$ satisfying normalization condition of

$$\int_{-\infty}^{+\infty} |g(x)|^2 dx = 1 \quad (\text{S13})$$

Let expansion coefficient c_p be

$$\begin{aligned} c_p &= \frac{1}{\sqrt{\lambda_p}} \cdot \int g(x) e^{\frac{i(k-k_0)x}{\lambda_p}} dx \\ &= \frac{1}{\sqrt{\lambda_p}} (2\pi)^{\frac{1}{4}} \int_{-\infty}^{+\infty} \exp\left[-\frac{(x)^2}{4a^2} + \frac{i(k-k_0)x}{\lambda_p}\right] dx \end{aligned} \quad (\text{S14})$$

And let $\zeta = x + 2a^2 i(k-k_0)/\lambda_p$, Equ.(S14) leads to

$$\begin{aligned} c_p &= \frac{1}{(2\pi)^{\frac{1}{4}} \sqrt{\lambda_p a}} e^{\frac{-a^2(k-k_0)^2}{\lambda_p^2}} \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{\xi^2}{4a^2}\right) d\xi \\ &= \sqrt{\frac{2a}{\lambda_p}} (2\pi)^{\frac{1}{4}} \exp\left[-\frac{a^2(k-k_0)^2}{\lambda_p^2}\right] \end{aligned} \quad (\text{S15})$$

showing that c_p is also Gaussian distribution with $\bar{c}_p = k_0$. So it exists

$$\overline{(\Delta k)^2} = \int_{-\infty}^{+\infty} (k - k_0)^2 c_p^2 dp \quad (\text{S16})$$

Based on

$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2} e^{-\frac{3}{2}} \quad (\text{S17})$$

it could be obtained that

$$\overline{(\Delta k)^2} = \frac{\bar{\lambda}_p^2}{4a^2} = \frac{\lambda_p^2}{4(\Delta x)^2} \quad (\text{S18})$$

Equ. (S18) leads to $|\Delta k \cdot \Delta x| = \frac{\bar{\lambda}_p}{2}$, which then results in $\Delta \hat{x} = 2 \times |\Delta k \cdot \Delta x| = \bar{\lambda}_p$ given the even symmetry for Gaussian. Considering Gaussian with the narrowest contribution, it generally exists $\Delta \hat{x} \geq \bar{\lambda}_p$.

3. On physical meaning of IR cutoff L (Fig. S1).

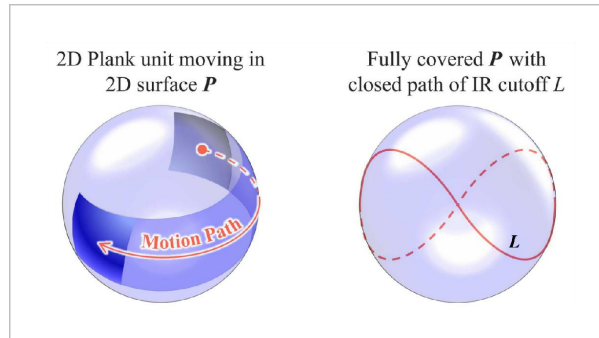


Fig. S1 Physical meaning for IR cutoff L : the theoretically longest path for a Planck unit $2\hat{p}$ confined to a local space $2\hat{P}$ on a surface of a 3D sphere.

4. A general cases for Planck unit with or without transformability.

Generalized surface of an $(n+1)$ D sphere is also an n D space, since it is expanded by orthogonal $X_1, X_2, X_3, \dots, X_n, X_{n+1}$, where $X_{n+1}=0$ and $X_{n+1} // r$, according to the Frenet frame shown in Fig. S2A. Accordingly, an n D Planck unit ${}^n\hat{p}$ can be located in both the curve surface \hat{P}_s and the flat space ${}^n\hat{P}$ (Fig. S2B).

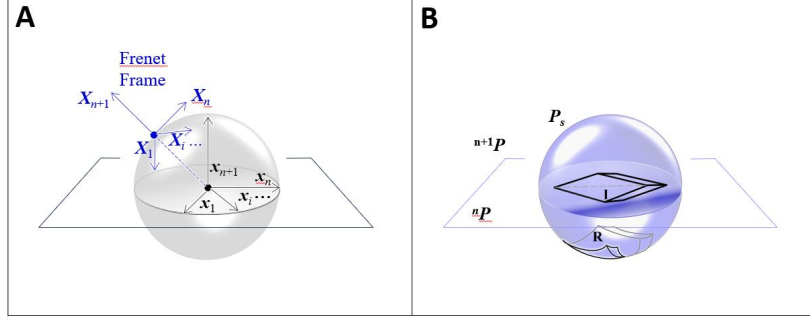


Fig. S2 A general case for a Planck unit ${}^n\hat{p}$ at state **I** and state **R** (without transformability).

5. Calculation on generalized volume ${}^n m$ for a generalized sphere

For a general sphere determined by $\sum x_i^2=1$ ($i=1, \Lambda, n$) in n D Euclidean space, its n D measurement ${}^n m$ should be

$${}^1 m = \int_{-1}^1 dx = 2$$

$${}^2 m = {}^1 m \cdot \int_{-1}^1 \sqrt{1-x^2} dx = \pi = 3.141\ 592\ 654$$

$${}^3 m = {}^2 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^2 dx = \frac{4\pi}{3} = 4.188\ 790\ 205$$

$${}^4 m = {}^3 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^3 dx = \frac{\pi^2}{2} = 4.934\ 802\ 201$$

$${}^5 m = {}^4 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^4 dx = \frac{8\pi^2}{15} = 5.263\ 789\ 014$$

$${}^6 m = {}^5 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^5 dx = \frac{\pi^3}{6} = 5.167\ 712\ 780$$

$${}^7 m = {}^6 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^6 dx = \frac{16\pi^3}{105} = 4.724\ 765\ 970$$

$${}^8 m = {}^7 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^7 dx = \frac{\pi^4}{24} = 4.058\ 712\ 126;$$

$$({}^8 m)^{\frac{1}{2}} = \left(\frac{\pi^4}{24}\right)^{\frac{1}{2}} = 2.014\ 624\ 562$$

$${}^9 m = {}^8 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^8 dx = \frac{32\pi^4}{945} = 3.298\ 508\ 903;$$

$$({}^9 m)^{\frac{1}{3}} = \left(\frac{32\pi^4}{945}\right)^{\frac{1}{3}} = 1.488\ 581\ 281$$

$${}^{10} m = {}^9 m \cdot \int_{-1}^1 (\sqrt{1-x^2})^9 dx = \frac{\pi^5}{120} = 2.550\ 164\ 040;$$

$$({}^{10} m)^{\frac{1}{4}} = \left(\frac{\pi^5}{120}\right)^{\frac{1}{4}} = 1.263\ 694\ 308$$

$${}^{11} m = {}^{10} m \cdot \int_{-1}^1 (\sqrt{1-x^2})^{10} dx = \frac{64\pi^5}{10395} = 1.884\ 103\ 879$$

$${}^{12}m = {}^{11}m \cdot \int_{-1}^1 (\sqrt{1-x^2})^{11} dx = \frac{\pi^6}{720} = 1.335\ 262\ 769$$

$${}^{13}m = {}^{12}m \cdot \int_{-1}^1 (\sqrt{1-x^2})^{12} dx = \frac{128\pi^6}{135135} = 0.910\ 628\ 755$$

$${}^{14}m = {}^{13}m \cdot \int_{-1}^1 (\sqrt{1-x^2})^{13} dx = \frac{\pi^7}{5040} = 0.599\ 264\ 529$$

$${}^{15}m = {}^{14}m \cdot \int_{-1}^1 (\sqrt{1-x^2})^{14} dx = \frac{256\pi^7}{2027025} = 0.381\ 443\ 281$$

...

And the general formulae for ${}^n m$ are

$${}^n m = \frac{\pi^{(n/2)}}{(n/2)!} \quad (n=2N)$$

$${}^n m = \frac{2^{[(n+1)/2]} \cdot \pi^{[(n-1)/2]}}{1 \cdot 3 \cdot 5 \dots n} \quad (n=2N+1)$$

showing obviously the monotonic decrease trend aroused by $({}^{n+2}m / {}^n m) < 1$ for cases of $n > 12$. Considering both the monotonic decrease and ${}^n m > 0$, ${}^n m$ converges to ${}^n m |_{n \rightarrow \infty} \rightarrow 0$.

Table S1 shows the longest 1D measurement $(x_m)_{\max} = {}^n m$ for an n D Planck sphere and diagonal $(x_{1b})_{\max} = \sqrt{n}$ for an n D Planck unit in Fig. 4.

Table S1. IR cutoffs $L = x_{\max}$ for an n D Planck sphere ($L = {}^n m$) and the maximum length for an n D Planck unit at state **I** ($L = \sqrt{n}$)

Dim. (n)	1	2	3	4	5	6	7	8	9	10	11	...
${}^n m$	2.00	3.14	4.19	4.93	5.26	5.17	4.72	4.06	3.30	2.55	1.88	...
\sqrt{n}	1	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{8}$	$\sqrt{9}$	$\sqrt{10}$	$\sqrt{11}$...

6. Detailed solutions about the coupling fields

The maximum wavelengths for the two **T** states should be

$$RS^\dagger: \quad \lambda_T = 2 \times {}^m L \times \bar{\lambda}_P = 8.91 \times 10^{-34} \text{ m (in 4D } {}^m \mathbf{P}) \quad (S19)$$

$$E_l: \quad \lambda_T = 2 \times {}^{10} L \times \bar{\lambda}_P = 1.35 \times 10^5 \text{ m (in 10D } {}^{10} \mathbf{P}) \quad (S20)$$

The wavelength $\lambda_T = 1.35 \times 10^5$ m for E_l in ${}^{10} \mathbf{P}$ should be of $L = \frac{(2\pi)^{34}}{6 \times 7 \times 8 \times 9} = 4.544\ 998\ 428 \times 10^{23}$

in ${}^m \mathbf{P}$, with the maximum wavelength being

$$E_l \text{ of } {}^m \mathbf{P}: \quad \lambda_T = 2 \times {}^m L \times \bar{\lambda}_P = 2.34 \times 10^{-12} \text{ m (in 4D } {}^m \mathbf{P}) \quad (S21)$$

For a 2-body system, TS requires parity between the two symcenters of **I**₁ and **I**₂ (Fig. 6), resulting in the shared **T**₁ ~ **T**'₂, **T**₂ ~ **T**'₁ and **T**'₁ ~ **T**'₂ at the tangent point(s) and the interacting point(s) (**T**₁ and **T**'₂ are identical when $m_T = 0$ and $dv_T = 0$, and so do **T**₂ ~ **T**'₁ and **T**'₁ ~ **T**'₂), respectively (Fig. S3).

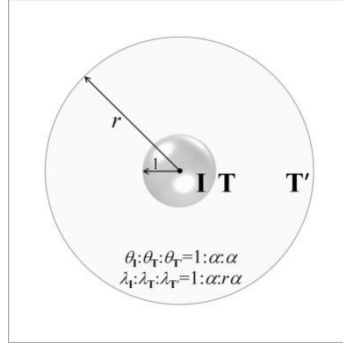


Fig. S3 A potential field aroused by quantum coupled state \mathbf{T}/\mathbf{T}'

So it should be $U_{\mathbf{T}_1}=U_{\mathbf{T}_2}=U_{\mathbf{T}'_1}=U_{\mathbf{T}'_2}=0$ and $\frac{dU_{\mathbf{T}}}{dr}=0$ for potential U of state \mathbf{T} . Based on $S_{\mathbf{T}}=S_{\mathbf{T}'}=\hbar$, kinetic energy for state \mathbf{T} satisfies

$$\Delta T_{\mathbf{T}}=T_{\mathbf{T}'}-T_{\mathbf{T}}=\frac{\hbar c}{\lambda_{\mathbf{T}'}}-\frac{\hbar c}{\lambda_{\mathbf{T}}} \quad (\text{S22})$$

In the flat background space for state \mathbf{I} and state \mathbf{T} , the conserved energy requires $d(E_{\mathbf{I}}+E_{\mathbf{T}})=0$ and $d(U_{\mathbf{I}}+T_{\mathbf{I}})+d(U_{\mathbf{T}}+T_{\mathbf{T}})=0$, resulting in

$$\Delta U_{\mathbf{I}}=-\Delta T_{\mathbf{T}}=\frac{\hbar c}{\lambda_{\mathbf{T}'}}-\frac{\hbar c}{\lambda_{\mathbf{T}}}=\frac{\hbar c}{\lambda_{\mathbf{T}}}-\frac{\alpha \hbar c}{2\pi r} \quad (\text{S23})$$

between \mathbf{I}_1 and \mathbf{I}_2 since $dU_{\mathbf{I}}=0$ ($U_{\mathbf{I}}=U_{\mathbf{I}'}\equiv 0$) and $dT_{\mathbf{I}}=0$. Consequently, it exists

$$F_{\mathbf{I}}=\frac{d\Delta U_{\mathbf{I}}}{d\rho}=\frac{\alpha \hbar c}{2\pi \rho^2} \quad (\rho=r\bar{\lambda}_{\mathbf{P}}\gg \bar{\lambda}_{\mathbf{P}}) \quad (\text{S24})$$

Then a coupling field should be

$$F_{\text{RS}^\dagger}=\frac{2.307\ 076\times 10^{-28}}{\rho^2} \text{ N} \quad (\alpha_1=1/137.036\ 082) \quad (\text{S25})$$

for the two $^{10}\hat{p}$ satisfying principle RS^\dagger in $^m\mathbf{P}$.

And another coupling field should be

$$F_{\text{E}_l}=\frac{1.941\ 959\times 10^{-64}}{\rho^2} \text{ N} \quad (\alpha_2=1/(1.628\ 008\times 10^{38})) \quad (\text{S26})$$

for two $^{10}\hat{p}$ satisfying principle E_l in $^x\mathbf{P}$.

7. Symbol list

Symbols	Explanations
RS	rotational symmetry
TS	translational symmetry
$^n\hat{p}$	the minimum nD local space in quasi-Euclidean space, nD Planck unit with uncertainty, where the uncertainty forbids \hat{p} to be with certain measure and certain position simultaneously (\hat{p}_θ and $\hat{\theta}$ can't be of determinacy simultaneously)

I	in-situ state for a Planck unit \hat{p} with RS, with uncertain structures, such as a generalized sphere with completely uncertain azimuth $0 \leq \Delta\theta \leq 2\pi$ but certain 1D measure $x_i \equiv 1$, a generalized cube with certain azimuth $\Delta\theta$ but completely uncertain x_i varying within $[\bar{\lambda}_P, \sqrt{n}\bar{\lambda}_P]$, or any an intermediate between the sphere and the cube
R	revolving state for a RS unit in Planck surface determined by $r = \bar{\lambda}_P$
T/T'	state for an RS unit in tangent plane of spherical surface determined by $r \geq \bar{\lambda}_P$
${}^n P$	n D orthogonal background space, also the super set of ${}^1 P, \Lambda, {}^n P$
${}^n P$	a local n D space
${}^1 P$	≤ 6 D subspace in 10D background, without non-triviality for any a Planck unit or a substructure expanded by x_1, Λ, x_6
${}^m P$	4D non-trivial subspace in 10D background, expanded by x_7, Λ, x_{10}
V_r^∞	infinite barrier to forbid an object from any radial displacement (demanding $dr=0$)
L	IR cutoff of $(x_i)_{\max} = \frac{{}^n P}{\bar{\lambda}_P^{(n-1)}}$ for a local space measured ${}^n P$
λ	wavelength of a Planck unit limited in a certain local space, where the unit always takes IR cutoff L as its motion path; when the path is not a closed loop, it exists $\lambda=2L$ for the unit in ground state
${}^m m$	generalized volume for an n D sphere with normalized radius $r=1$
E_\downarrow	the minimum energy principle, results in a state with motion space as large as possible.
RS^\uparrow	the highest RS principle, results in a state with RS as high as possible
α	ratio $\frac{L_\downarrow}{L_\uparrow}$ during transformation $\mathbf{I} \rightarrow \mathbf{T}$ for a Planck unit \hat{p} in 10D background space, with value $\sim 1/137.036$ in ${}^m P$ when it satisfies Principle RS^\uparrow , or $\sim 1/(1.628 \times 10^{38})$ when it satisfies Principle E_\downarrow
