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The Sound of Silence in Social Networks

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Abstract. We generalize the classic multi-agent DeGroot model for opinion dynamics to incorporate the Spiral of Silence theory from political science. This theory states that individuals may withhold their opinions when they perceive them to be in the minority. As in the DeGroot model, a community of agents is represented as a weighted directed graph whose edges indicate how much agents influence one another. However, agents whose current opinions are in the minority become *silent* (i.e., they do not *express* their opinion). Two models for opinion update are then introduced. In the *memoryless opinion model* (SOM^-), agents update their opinion by taking the weighted average of their *non-silent* neighbors' opinions. In the *memory based opinion model* (SOM^+), agents update their opinions by taking the weighted average of the opinions of *all* their neighbors, but for silent neighbors, their most recent opinion is considered.

We show that for SOM^- convergence to consensus is guaranteed for clique graphs but, unlike for the classic DeGroot, not guaranteed for strongly-connected aperiodic graphs. In contrast, we show that for SOM^+ convergence to consensus is not guaranteed even for clique graphs. We showcase our models through simulations offering experimental insights that align with key aspects of the Spiral of Silence theory. These findings reveal the impact of silence dynamics on opinion formation and highlight the limitations of consensus in more nuanced social models.

Keywords: Opinion Dynamics, Spiral of Silence, DeGroot Model, Social Networks, Consensus, Agent-Based Modeling, Social Influence

1 Introduction

Social networks have played a significant role in shaping users' opinions, often influencing democratic processes and contributing to social polarization. Broadly, the dynamics of opinion formation in social networks involve users expressing their opinions, being exposed to the opinions of others, and potentially adapting their own views based on these interactions. Modeling these dynamics enables us to glean insights into how opinions form and spread within social networks.

The DeGroot model [12] is one of the most prominent formalisms for opinion formation and consensus-building in social networks. In this model, a social network is represented as a weighted directed graph, where edges denote the degree

to which individuals (referred to as *agents*) influence one another. Each agent holds an opinion, expressed as a value in $[0, 1]$, indicating their level of agreement with an underlying proposition (e.g., “*AI is a threat to humanity*”). Agents repeatedly update their opinions by taking the weighted average of their opinion differences with those who influence them (i.e, their *neighbours* or *contacts*). There is empirical evidence validating the opinion formation through averaging of the model in controlled sociological experiments (e.g., [8]).

Consensus, i.e., convergence to a common opinion, is a central property in models of social learning and opinion formation [16]. In fact, difficulties in achieving consensus are a sign of a polarized society. A fundamental result in the DeGroot model shows that agents converge to consensus if the influence graph is strongly connected and aperiodic. The DeGroot model is recognized for its mathematical simplicity, derived from its associations with matrix powers and Markov chains. It continues to be focus of research for constructing frameworks for understanding opinion formation dynamics in social networks (e.g., [16,5,15,20,2,11,10,27,26,13,9,4,3,22]).

Nevertheless, the DeGroot model makes an assumption that could be overly constraining within social network contexts. It assumes that *all agents express their opinions at each time unit*. This assumption, which renders the model tractable, may hold in some controlled scenarios as participants are often encouraged to express their views freely and consistently. However, in many real-world situations, some individuals may choose not to express their opinions due to personal choice or social pressure.

Indeed, the Spiral of Silence [21] is a well-established social theory that describes how individuals may be unwilling to express their opinions when they perceive themselves to be in the minority. This reluctance can lead to the *reinforcement of dominant views* within a social network. The theory asserts that individuals have a natural tendency to avoid social isolation and seek acceptance within their social groups. When people believe their opinions are unpopular or likely to be met with disapproval, they may opt to remain silent.

In this paper, we generalize the classic DeGroot model into a framework where agents may choose to remain silent at a given time unit in line with the Spiral of Silence theory. We consider two possibilities, leading to the two models described below.

Memoryless framework SOM^- . In this model, silent agents are excluded from the opinion updates of the agents they would typically influence. Additionally, agents become silent at a given time (thus withholding their opinions) if their views do not align with the majority of their non-silent contacts. This framework is called the *memoryless silence opinion* model (SOM^-), as the previous opinions of silent agents are not retained. This corresponds to a social scenario in which opinions (expressed as messages, posts, etc.) are removed once they have been accessed.

Notice that ignoring silent agents at a given time unit amounts to removing certain edges from the underlying influence graph at that time. Thus, a fun-

damental distinction from the DeGroot model is that SOM^- exhibits *dynamic influence*; i.e., edges may disappear and reappear during opinion evolution.

Memory-based framework SOM^+ . In this model, agents choose to be silent if their opinion does not align with the most recent public opinions of *the majority of* their contacts. In addition, when their current opinion is unknown, their most recent public opinion is taken into account in the update. Furthermore, silent agents are not excluded from the opinion updates of the agents they influence. This framework is called the *memory-based silence opinion* model (SOM^-), as some previous opinions are retained.

A property that distinguishes SOM^+ from the DeGroot model (and SOM^-) is that the latter is a *Markovian* process: The next state depends on the current state but not the past states. Thus, SOM^+ is a *history-dependent* model but with limited memory; only the most recent public opinions are retained. We will show that this limited memory will have an impact in opinion formation and consensus.

Contributions. The main contributions in this paper are the following: (1) We generalize the DeGroot model to account for relevant social phenomena described by the theory of the Spiral of Silence. To the best of our knowledge, this is the first extension of the DeGroot model that incorporates this theory. (2) We show that in SOM^- , convergence to consensus is guaranteed in clique graphs (i.e., graphs where each pair of agents influences each other) with more than two agents. This intuitively means that in fully connected communities, consensus can be achieved under the Spiral of Silence assumptions, even if silent agents are excluded from the update process. (3) We show that in SOM^- , unlike in the DeGroot model, convergence to consensus is not guaranteed for strongly connected aperiodic graphs. (4) We demonstrate that in SOM^+ , convergence to consensus is not guaranteed, even for cliques. This shows that, under the Spiral of Silence Theory, even the limited memory in SOM^+ can significantly increase the complexity of opinion dynamics, making consensus more difficult to achieve. Finally, (5) we also demonstrate our model with examples and computer simulations that provide insights into opinion formation under the Spiral of Silence. We provide examples that align with assertions of the theory of Spiral of Silence: In particular, *reinforcement of dominants view in social networks*. The open code for these simulations can be provided upon request.

All in all, this paper highlights the impact of silence dynamics and memory on opinion formation and highlight the limitations of consensus in more nuanced social models.

The paper is organized as follows: The new silence opinion models are introduced in Section 2. The study of consensus for these models is presented in Section 3. Some case studies emerging from our models and highlighting the spiral of silence effect are presented in Section 4. The concluding remarks are given in Section 5.

2 The Model

In the DeGroot model [12], each agent updates their opinion by taking the weighted average of the opinions of those who influence them. This model, however, does not account for the social phenomenon known as the Spiral of Silence [21], where some agents may choose to become or remain silent if their opinion does not align with the majority. As a result, their *current* opinion may not influence their contacts.

In this section, we generalize the DeGroot model to take into account the Spiral of Silence. If an agent j decides to be silent, there are at least two natural options when updating the opinions of the agents having j as a contact: (1) agent j is simply ignored in the update since their current opinion is unknown (or not public), or (2) the most recent opinion when j was not silent is taken into account in the update. The former corresponds to a scenario where, for privacy purposes, opinions (messages) are removed once they have been accessed. The latter represents a typical scenario in social networks where previous opinions are kept and thus continue to influence others despite the agent's current silence.

The above options lead us to the two generalizations of the DeGroot model studied in this paper: the *memoryless* silence opinion model SOM^- , where previous opinions are forgotten, and the *memory-based* (or history-dependent) silence opinion model SOM^+ , where previous opinions are remembered. In both generalizations, agents become silent by a majority rule appropriate for each case.

In the next sections, we introduce the elements of the models.

2.1 The Influence Graph

In social learning models, a *community/society* is typically represented as a directed weighted graph with edges between individuals (agents) representing the direction and strength of the influence that one carries over the other. This graph is referred to as the *Influence Graph*.

Definition 1 (Influence Graph). *An (n -agent) influence graph is a weighted directed graph $G = (A, E, I)$, where $A = \{1, \dots, n\}$, $E \subseteq A \times A$, and $I : A \times A \rightarrow [0, 1]$ a weight function s.t. such that $I(i, j) = 0$ iff $(i, j) \notin E$ and for each $i \in A$, $\sum_{j \in N(i) \cup \{i\}} I(j, i) = 1$ where $N_i = \{j \in A \setminus \{i\} : (j, i) \in E\}$.*

The vertices in A represent n agents of a given community or network. The set of edges $E \subseteq A \times A$ represents the (direct) influence relation between these agents; i.e., $(i, j) \in E$ means that agent i (*directly*) *influences* agent j . The value $I(i, j)$, for simplicity written I_{ij} , denotes the strength of the influence: 0 means no influence, and a higher value means stronger influence. The normalization condition ensures that the total influence on each agent sums to 1. The set $N(i)$ represents the *neighbors (or contacts)* of agent i .

Let us recall some notions from graph theory [14]. A sequence in E of the form $(i, i_1)(i_1, i_2) \dots (i_{m-1}, j)$ is a path (of length m) from i to j . The graph G is said to be *strongly connected* if for every pair (i, j) of distinct nodes in A , there

is a path from i to j . A graph G is a *clique* if for every pair (i, j) of distinct nodes in A , $(i, j) \in E$. A cycle is a path of the form $(i, i_1)(i_1, i_2) \dots (i_{m-1}, i)$ with all i_1, \dots, i_{m-1} being distinct. Finally, G is *aperiodic* if the greatest common divisor of the lengths of its cycles is one.

2.2 Silence Opinion Models

To incorporate the Spiral of Silence into the DeGroot framework, we will model the evolution of agents' opinions alongside their decisions to remain silent about a given underlying *statement* or *proposition*. Such a proposition could include controversial statements like, for example, “*AI poses a threat to humanity*” or “*pineapple belongs on pizza*”. Therefore, the state of the agents (system) with respect to the proposition involves both the *state of opinion* and the *state of silence*.

The *state of opinion* of all agents is represented as a vector in $[0, 1]^n$. If \mathbf{B} is a state of opinion, then \mathbf{B}_i denotes the opinion of agent i with respect a given proposition. If $\mathbf{B}_i = 0$ ($\mathbf{B}_i = 1$) agent i complete disagrees (agrees) with the proposition. The higher the value, the stronger the agreement.

The *state of silence* is represented as a vector in $\{0, 1\}^n$. If \mathbf{S} is a state of silence, $\mathbf{S}_i = 0$ ($\mathbf{S}_i = 1$) indicates agent i is silent (is not silent)⁴.

At each discrete time unit $t \in \mathbb{N}$, every agent $i \in A$ updates their opinion and their silence state. We shall use \mathbf{B}^t and \mathbf{S}^t to denote the state of opinion and silence at time t . We can now define a general Silence DeGroot opinion model as follows.

Definition 2 (Silence Opinion Model). *A Silence Opinion (SO) Model is a tuple $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ where $G = (A, E, I)$ is an n -agent influence graph, \mathbf{B}^0 the initial state of opinion, \mathbf{S}^0 the initial state of silence, $\mu_G : [0, 1]^n \times \{0, 1\}^n \times \mathbb{N} \rightarrow [0, 1]^n \times \{0, 1\}^n$ the state-transition function, called (state) update function. For every $t \in \mathbb{N}$, the state of the system at time $t + 1$ is $(\mathbf{B}^{t+1}, \mathbf{S}^{t+1}) = \mu_G(\mathbf{B}^t, \mathbf{S}^t, t)$.*

The update functions can be used to express any deterministic and discrete transition from one state to the next, possibly taking into account the influence graph, the current and even previous states. These functions are typically expressed by means of equations between states. In what follows, we will define particular update functions that take the spiral of silence into account.

2.3 Spiral of Silence Models

To build some intuition, we recall the opinion update of the DeGroot model, which can be expressed as in the following equation:

$$\mathbf{B}_i^{t+1} = \mathbf{B}_i^t + \sum_{j \in N_i} I_{ji} \cdot (\mathbf{B}_j^t - \mathbf{B}_i^t) \quad (1)$$

⁴ Some readers may feel that we should have used $\mathbf{S}_i = 1$ to indicate agent i is silent. Our choice, however, simplifies the update equations in the next sections

for each $i \in A$, $t \in \mathbb{N}$. Thus, in the DeGroot model each agent updates their opinion by taking the weighted average of the opinion differences (disagreements) of those who influence them.

We now generalize the above DeGroot update (Eq. 1) as opinion update functions that depend not only on the current opinion state but also on the state silence and, possibly, on previous states.

Memoryless Update Our first update corresponds to Option (1) mentioned earlier in Section 2: The opinions of silent contacts are ignored in the update. This can be easily realized by modifying Eq. 1 as shown in the following opinion update equation:

$$\mathbf{B}_i^{t+1} = \mathbf{B}_i^t + \sum_{j \in N_i} I_{ji} \cdot \mathbf{S}_j^t \cdot (\mathbf{B}_j^t - \mathbf{B}_i^t) \quad (2)$$

We now define the corresponding silence update function following the Spiral of Silence Theory. First, we need some notation. Let $x, y, \tau \in [0, 1]$. The τ -proximity relation $x \sim_\tau y$ holds true iff $|x - y| \leq \tau$, i.e., if x and y are within a tolerance radius τ . Also let $N_i^t = \{j \in N_i : \mathbf{S}_j^t = 1\}$ be the sets of non-silent neighbours of i at time t . The silence update function is given as follows:

$$\mathbf{S}_i^{t+1} = \begin{cases} 1 & \text{if } \left\lceil \frac{|N_i^t|}{2} \right\rceil \leq |\{j \in N_i^t \mid \mathbf{B}_i^t \sim_{\tau_i} \mathbf{B}_j^t\}| \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where $\tau_i \in [0, 1]$ is a tolerance radius constant for agent i .

Intuitively, an agent i considers the opinion of j to be close enough to theirs if it is within their tolerance radio. The agents decide to be silent if and only if the opinions of the majority of their *non-silent* contacts are not close enough to theirs.

We can now define the memoryless model for the spiral of silence.

Definition 3 (SOM⁻). Let $M = (G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SO model with $G = (A, E, I)$. Then M is said to be a memoryless SO model (SOM⁻) if for each $i \in A$ and $t \in \mathbb{N}$, $\mu_G(\mathbf{B}^t, \mathbf{S}^t, t) = (\mathbf{B}^{t+1}, \mathbf{S}^{t+1})$ where \mathbf{B}_i^{t+1} and \mathbf{S}_i^{t+1} are determined by Eq .2 and Eq .3, respectively.

Clearly, we can recover the DeGroot update (Eq .1), and hence the model, by setting each tolerance radius constant τ_i in Eq .3 to 1 and the initial state of silence \mathbf{S}^0 to the unit vector $\mathbf{1}_n = (1, 1, \dots, 1)$ of size n .

Remark 1. It is worth noting that the dynamic nature of the influence graph in SOM⁻ models sets them apart from the static influence in the DeGroot model. Silencing an agent j at a given time amounts to removing all edges $(j, i) \in E$ from the graph at that moment. This allows for more complex opinion formation behaviors.

Notice also that we could normalize the sum in Eq .2 by dividing it by $\sum_{j \in N_i^t} I_{ji}$ when non-zero. While this would not impact our technical results in

Section 3, it would amplify the influence of the non-silent neighbors of i at time t , which may seem unnatural.

Instead, notice that since the right-hand side of Eq. 2 is equivalent to $(1 - \sum_{j \in N_i} I_{ji} \cdot \mathbf{S}_j^t) \cdot \mathbf{B}_i^t + \sum_{j \in N_i} I_{ji} \cdot \mathbf{S}_j^t \cdot \mathbf{B}_j^t$ then the influence I_{ji} of a silent agent j at time t may be seen as increasing the weight of agent i 's opinion at that time. This can be interpreted as agent i increasing confidence in their own opinion in the absence of external influence from agent j .

Memory-based Update We now introduce the model corresponding to Option (2) mentioned in Section 2: If a contact j is silent at time t , the opinion update takes into account the opinion they had the last time unit u (where $u \leq t$) when they were not silent. For this to be well-defined, we assume that initially all agents are not silent; i.e., $\mathbf{S}^0 = \mathbf{1}_n$.

Let $\bar{t}_j = \max\{u \leq t \mid \mathbf{S}_j^u = 1\}$. The *public state of opinion* at time t is a state of opinion \mathbf{pB}^t such that $\mathbf{pB}_j^t = \mathbf{B}_j^{\bar{t}_j}$ for each $j \in A$. The following opinion update equation captures the above intuition:

$$\mathbf{B}_i^{t+1} = \mathbf{B}_i^t + \sum_{j \in N_i} I_{ji} \cdot (\mathbf{pB}_j^t - \mathbf{B}_i^t) \quad (4)$$

The corresponding silence update tells us that an agent i becomes or remains silent at time $t+1$ precisely when the public opinion of the majority of *all* their contacts are not close enough to their own. More precisely:

$$\mathbf{S}_i^{t+1} = \begin{cases} 1 & \text{if } \left\lceil \frac{|N_i|}{2} \right\rceil \leq |\{j \in N_i \mid \mathbf{B}_i^t \sim_{\tau_i} \mathbf{pB}_j^t\}| \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where $\tau_i \in [0, 1]$ is a tolerance radius constant for agent i .

The memory-based models are defined thus:

Definition 4 (SOM⁺). Let $M = (G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SO model where $G = (A, E, I)$ is an n -agent influence graph. Then M is said to be a memory-based SO model (SOM⁺) if $\mathbf{S}^0 = \mathbf{1}_n$ and for each $i \in A$ and $t \in \mathbb{N}$, $\mu_G(\mathbf{B}^t, \mathbf{S}^t, t) = (\mathbf{B}^{t+1}, \mathbf{S}^{t+1})$ where \mathbf{B}_i^{t+1} and \mathbf{S}_i^{t+1} are determined by Eq .4 and Eq .5, respectively.

The DeGroot update (Eq .1) is a particular case of the SOM⁺ opinion update (Eq. 4): We only need to set each tolerance radius constant τ_i in Eq. 5 to 1 since \mathbf{S}^0 is already required to be the unit vector of ones $\mathbf{1}_n$ in SOM⁺ models.

Remark 2. The main difference between SOM⁺ and the DeGroot model (and SOM⁻) is that the latter is a *Markovian* process: The next state depends on the current state but not past states. In fact, much of the tractability of the DeGroot model derives from its connection to Markov chains. Nevertheless, the next state in SOM⁺ does not depend on the entire state history but just on the most recent public opinions. In the next sections, we will see the impact of this minimal amount of memory on opinion evolution.

3 Results on Consensus

Consensus is a central problem in social learning models. Often, an inability to reach a consensus serves as an indicator of polarization within a social network. In our model, consensus represents convergence to the same opinion value over time.

Definition 5 (Consensus). *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SO model with $G = (A, E, I)$. We say that the agents in A converge to consensus if there exists a value $v \in [0, 1]$ such that for all $i \in A$, $\lim_{t \rightarrow \infty} \mathbf{B}_i^t = v$.*

Conversely, we refer to the lack of (convergence to) consensus as *dissensus*, which occurs when agents fail to converge to a single opinion value.

In this section, we explore the different results on consensus for both our models on two different graph topologies *clique* and *strongly connected*. We find that consensus can only be guaranteed for the SOM^- model on clique graphs. In the remaining 3 cases, consensus cannot be guaranteed, leading to potential dissensus due to the existence of *perpetual silence* for SOM^- and public opinions for SOM^+ . We show the counterexamples via simulations.

3.1 SOM^- Properties

A key property of the SOM^- model is that if all agents become silent at time t , they will all speak up at the very next round $t + 1$. The following lemma formalizes this property:

Lemma 1. *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with $G = (A, E, I)$. For any $t \in \mathbb{N}$, if $\mathbf{S}_i^t = 0$ for all $i \in A$ then for all $i \in A$, $\mathbf{S}_i^{t+1} = 1$.*

Proof. Let \mathbf{S}^t such that $\mathbf{S}_i^t = 0$ for all $i \in A$. Then, by applying Eq. 3, we can conclude that $\mathbf{S}_i^{t+1} = 1$ for all $i \in A$, given that $|N_i^t| = 0$ for all $i \in A$,

As a result of Lem. 1, it follows that in SOM^- we cannot have all agents silent forever.

Corollary 1. *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with $G = (A, E, I)$. For any $t \in \mathbb{N}$, there exist $i \in A$ such that $\mathbf{S}_i^t = 1$ or $\mathbf{S}_i^{t+1} = 1$.*

We shall now prove that the sequences of maximum and minimum opinion values, $\{\max(\mathbf{B}^t)\}_{t \in \mathbb{N}}$ and $\{\min(\mathbf{B}^t)\}_{t \in \mathbb{N}}$, are (bounded) monotonically non-increasing and non-decreasing, respectively, so they must converge to some opinion values, say U and L with $L \leq U$.

First, we show that the opinion values in a state are bounded by the extreme opinions in the previous state.

Lemma 2 (Bounds of Opinion Evolution). *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with $G = (A, E, I)$. For any $t \in \mathbb{N}$, $\min(\mathbf{B}^t) \leq \mathbf{B}_i^{t+1} \leq \max(\mathbf{B}^t)$ for all $i \in A$.*

Proof. From Definition 3 and Eq. 2 we know that:

$$\mathbf{B}_i^{t+1} = \mathbf{B}_i^t + \sum_{j \in N_i^t} I_{ji} \cdot \mathbf{S}_j^t \cdot (\mathbf{B}_j^t - \mathbf{B}_i^t) = \sum_{j \in N_i^t \cup \{i\}} I_{ji} \cdot \mathbf{S}_j^t \cdot \mathbf{B}_j^t$$

Now, as $\mathbf{B}_j^t \leq \max(\mathbf{B}^t)$, $\mathbf{S}_j^t \in \{0, 1\}$, and $\sum_{j \in N_i^t \cup \{i\}} I_{ji} = 1$ by Definition 1, we can conclude that:

$$\mathbf{B}_i^{t+1} \leq \sum_{j \in N_i^t \cup \{i\}} I_{ji} \cdot 1 \cdot \mathbf{B}_j^t \leq \sum_{j \in N_i^t \cup \{i\}} I_{ji} \cdot 1 \cdot \max(\mathbf{B}^t) = 1 \cdot 1 \cdot \max(\mathbf{B}^t)$$

As wanted. The proof that $\min(\mathbf{B}^t) \leq \mathbf{B}_i^{t+1}$ is similar.

Notice that monotonicity does not necessarily hold for the opinion values of agents. Nevertheless, it follows from Lem. 2 that $\max(\mathbf{B}^t)$ is monotonically non-increasing and $\min(\mathbf{B}^t)$ is monotonically non-decreasing with respect to t .

Corollary 2 (Monotonicity of extremes). *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with $G = (A, E, I)$. For all $t \in \mathbb{N}$, $\max(\mathbf{B}^{t+1}) \leq \max(\mathbf{B}^t)$ and $\min(\mathbf{B}^{t+1}) \geq \min(\mathbf{B}^t)$.*

Monotonicity and boundedness of extremes, together with the Monotonic Convergence Theorem [24], lead us to the existence of limits for the opinion values of extreme agents.

Theorem 1 (Limits of extremes).

Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with $G = (A, E, I)$. There exist $U, L \in [0, 1]$ such that $\lim_{t \rightarrow \infty} \{\max(\mathbf{B}^t)\} = U$ and $\lim_{t \rightarrow \infty} \{\min(\mathbf{B}^t)\} = L$.

Notice that if the limits for the opinion values of extreme agents are the same (i.e., $U = L$) and by the squeeze theorem [24], we know the following:

$$\lim_{t \rightarrow \infty} \max(\mathbf{B}^t) = \lim_{t \rightarrow \infty} \min(\mathbf{B}^t) = \lim_{t \rightarrow \infty} \mathbf{B}_i^t \quad \forall i \in A$$

Hence, if $U = L$, the model converges to consensus.

3.2 Consensus in SOM^- Cliques

In this section we show that consensus is guaranteed for SOM^- models whose influence graphs are cliques with at least three agents.

The proof strategy is based on the following key observations: 1. From any time t onwards, there will always be non-silent agents with an opinion value greater or equal to U (Lem. 3). 2. The maximum and minimum opinion values, which are monotonically non-increasing and non-decreasing respectively, must converge to some value, say U and L respectively, with $L \leq U$ (Th. 1). 3. As the graph is a clique, the high opinion non-silent agents (i.e., those non-silent agents with opinion value $\geq U$) will influence all other graph agents infinitely

often (Lem. 3. 4. Consider any clique with size $n \geq 3$, if $L < U$, the difference between the maximum and minimum opinions will be reduced by a constant factor infinitely often, such that eventually the decrement would be greater than the difference between the maximum and minimum opinions; a contradiction. Hence, $L = U$; which guarantees convergence to consensus.

Remember that in an SOM^- model we cannot have all the agents becoming silent forever (Cor. 1). Therefore, at least one agent will be non-silent in the future from any time; specifically, we can also show that from any time onwards, in a clique, at least an agent with an opinion value greater or equal to U will be non-silent infinitely often.

Lemma 3 (silence and maximal opinions). *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be a SOM^- model with $G = (A, E, I)$ where G is a clique. For any $t \in \mathbb{N}$, there exists $k \in \mathbb{N}$ and $i \in A$, such that $\mathbf{B}_i^{t+k} \geq U$ and $\mathbf{S}_i^{t+k} = 1$.*

Proof. Assume, by contradiction, that from some time t onwards, no agent with an opinion value greater or equal to U will speak; this implies that all the agents that speak at time t and after time t will have opinion values lower than U . In fact, from Eq. 2, we know that the highest opinion of any non-silent agent from time t onwards will be at most $\max(\mathbf{B}_i^t \mid \mathbf{B}_i^t < U \text{ for any } i \in A)$.

From Cor. 1 and the contradiction assumption, at least one of these non-silent agents, whose opinion is less than or equal to $\max(\mathbf{B}_i^t \mid \mathbf{B}_i^t < U \text{ for any } i \in A)$, will speak infinitely often.

Since the graph is a clique, the opinion values of all agents with opinion values greater than or equal to U at time t will decrease, moving, progressively, closer to a value lower or equal to $\max(\mathbf{B}_i^t \mid \mathbf{B}_i^t < U \text{ for any } i \in A)$; therefore, the opinion values of these agents will not converge to U ; which contradicts Th. 1.

From the above, to prove consensus (i.e., $U = L$), we will now show how bounded the distance between the maximum and minimum opinion values is at different time units.

For the following lemma, we need to consider the minimum influence of the graph, defined as $I_{min} = \min_{(i,j) \in E} I(i, j)$ and the difference between the maximum and minimum opinion value at time t , defined as $R_t = \max(\mathbf{B}^t) - \min(\mathbf{B}^t)$, notice that $R_t \in [0, 1]$ at any time t .

Lemma 4 ($m - \epsilon$ decrement).

Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM^- model with an n -agent influence graph $G = (A, E, I)$ where G is a clique with $n \geq 3$. For all $m \in \mathbb{N}$, there exists a $t \in \mathbb{N}$ such that

$$R_0 - R_t \geq m * \epsilon$$

with $\epsilon = I_{min} \cdot (U - L)$.

Proof. It can be proved by induction on m for all $m \in \mathbb{N}$ by using Lem. 3; notice that in each time unit t where there is at least a non-silent agent with an opinion

greater or equal to U , the distance between the maximum and minimum opinions will be reduced by a value greater or equal to a constant factor, specifically $R_{t+1} - R_t \geq \epsilon$ with $\epsilon = I_{min} \cdot (U - L)$.

Now, we can state our consensus result for cliques with at least three agents.

Theorem 2 (Consensus in SOM⁻ Cliques). *Let $(G, \mathbf{B}^0, \mathbf{S}^0, \mu_G)$ be an SOM⁻ model with an n -agent influence graph $G = (A, E, I)$ where G is a clique. If $n \geq 3$, then the agents in A converge to consensus.*

Proof. From Th. 1, there exists $U, L \in [0, 1]$ such that $U = \lim_{t \rightarrow \infty} \max(\mathbf{B}^t)$ and $L = \lim_{t \rightarrow \infty} \min(\mathbf{B}^t)$.

We now prove $U = L$ by contradiction.

Suppose, by contradiction, that $U \neq L$. As U can not be lower than L , therefore, $U - L > 0$. As $I_{min} > 0$, ϵ (i.e. $I_{min} \cdot (U - L)$) is a constant greater than zero. Thus, there exist some $m \in \mathbb{N}$ such that $m * \epsilon > 1$, however, from Lem. 4 there must exist a time t where $R_0 - R_t \geq m * \epsilon > 1$, therefore, $R_0 > 1 + R_t$, but it is not possible as $R_0, R_t \in [0, 1]$, which is a contradiction.

Therefore, $U = L$ and the model converges to consensus.

Remark 3. Notice that for 2-agent cliques, consensus is not guaranteed; let us consider a clique with two agents: agent 1 and agent 2, where the opinions are $\mathbf{B}_1^0 = 1$ and $\mathbf{B}_2^0 = 0$, the influences are $I(1, 2) = I(2, 1) = 1$ and the tolerance radii are $\tau_1 = \tau_2 = 1$. Suppose that $\mathbf{B}^0 = (1, 1)$.

In this case, the opinion evolution of agent 1, starting with opinion 1, and agent 2, starting with opinion 0, always alternate between the values 1 and 0. Figure 1 illustrates the opinion evolution of this clique.

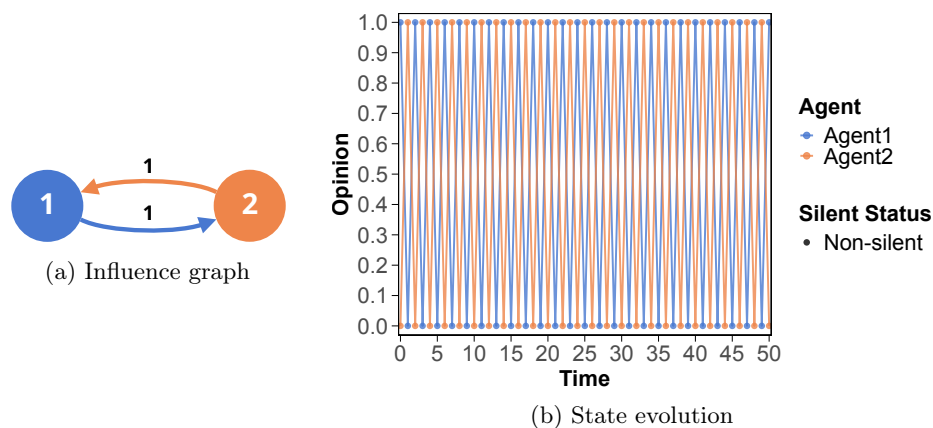


Fig. 1: Opinion evolution in a 2-agent clique with influence 1, with $\mathbf{B}^0 = (1, 0)$.

3.3 Dissensus in SOM^- Models

In strongly connected aperiodic graphs, the SOM^- model differs from the DeGroot model by no longer guaranteeing consensus. Agents can enter a state of perpetual silence, effectively disrupting opinion propagation as if severing connections from the graph. This phenomenon is particularly critical when silent agents form bridges between connected components. Their opinions, influenced by opposing connected components, may remain below the 50% non-silent threshold indefinitely. As a result, they prevent opinion exchange between components, obstructing the possibility of achieving consensus. The following example illustrates this scenario through by showing the agents state evolution and influence graph (see Figure 2).

Remark 4. For visual clarity, self-influences are omitted from the influence graphs. As a result, the visible incoming influences for each agent may not sum to one. Nevertheless, self-influences are implicitly present to ensure conformity with Definition 1. Readers can infer an agent’s self-influence by subtracting the sum of its visible incoming influences from one.

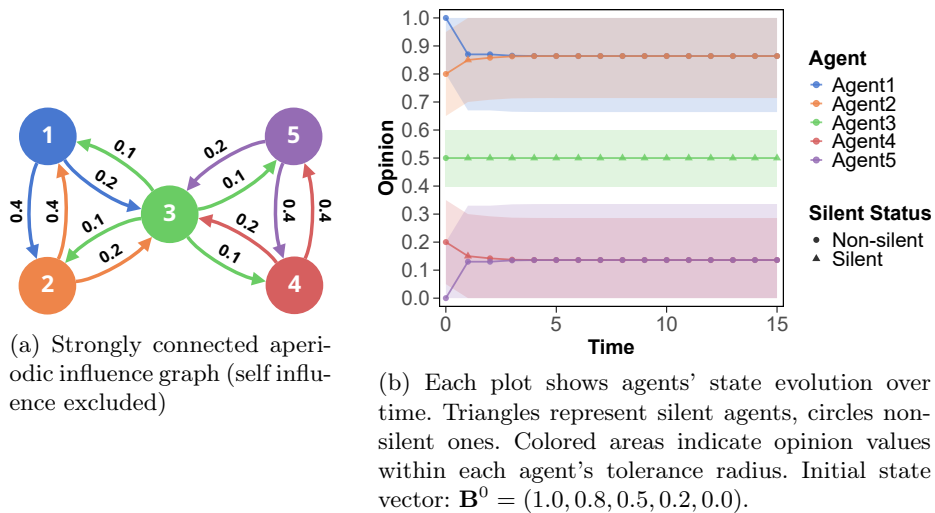


Fig. 2: Examples of Dissensus in SOM^- Models

In this example(Figure 2), agent 3 acts as a critical bridge between two graph components: agents 1 and 2, and agents 4 and 5. Initially, all agents have enough opinions within their tolerance radii, except for agent 3’s. This causes only agent 3 to become silent at $t = 1$, effectively severing the only link between the two components. As the components converge to their local consensus values, agent 3’s opinion remains static due to equal influences from

both components. These evolving component opinions consistently fall outside agent 3’s tolerance radius, trapping it in perpetual silence. Consequently, the two components remain permanently disconnected, preventing global consensus and exemplifying how the SOM^- model can lead to dissensus in strongly connected graphs.

3.4 Dissensus in SOM^+ Models

For the SOM^+ models, consensus cannot even be guaranteed for clique graphs. While this model shares similarities with SOM^- regarding perpetual silence, it introduces a unique phenomenon where the entire graph can enter and indefinitely remain in a silent state. This distinction stems from how silent agents influence the opinion update in SOM^+ .

Recall from Section 2.3 that $p\mathbf{B}^t$ is the public state of opinion at time t and represents the most recent public opinion of each agent. Henceforth we will refer to $p\mathbf{B}_i^t$ as the *public opinion of agent i* .

In SOM^+ , non-silent agents influence the opinions of their neighbors as in SOM^- . But, silent agents influence the opinions of their neighbors with their public opinion instead of excluding their opinion from the updates as in SOM^- .

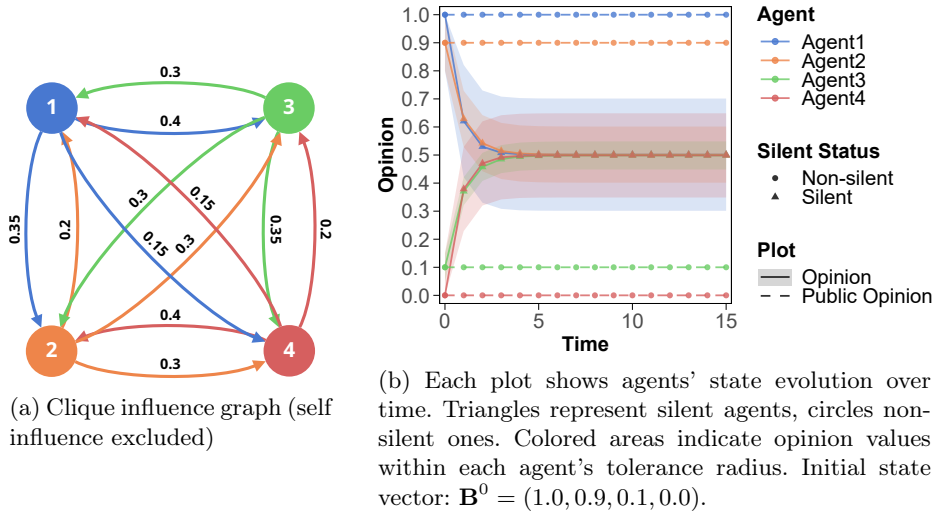
Consequently, if an agent’s opinion converges to a value that places the last public opinions of more than half of its neighbors outside its tolerance range, the agent will remain perpetually silent unless its neighbors change their public opinions. This scenario can lead to a state where intuitively some or all agents become “tired” of the discourse and cease participation (i.e., become silent), while the remaining active agents continue to consider the public opinions of these silent agents.

Unlike SOM^- , where opinions of silent agents are disregarded, SOM^+ allows for the persistence of unchanged public opinions indefinitely. This characteristic can result in dissensus due to the formation of public opinions that no longer reflect the current opinions of silent agents.

To illustrate these dynamics, we present a dissensus example (Figure 3) that with a clique influence graph where all agents initially have more than half of their neighbors’ opinions outside their tolerance radius. Consequently, all agents become silent at $t = 1$. At this point, each agent’s updated opinion places all other agents’ opinions outside its tolerance radius. The agents’ opinions then converge to distinct values, determined by the initial opinions \mathbf{B}^0 , which perpetuates the condition for silence. This state persists indefinitely, as the convergence values maintain the silence condition for all agents. The result is dissensus. This example demonstrates how the SOM^+ model can lead to complete silence and opinion divergence in a clique, preventing the possibility of consensus.

4 Experimenting with the spiral of silence

To explore the implications of our proposed models, we developed a simulator, which generated all the examples presented in this paper. This section illustrates

Fig. 3: Example of Dissensus in SOM⁺ Model

specific scenarios providing insights into opinion dynamics under the spiral of silence.

We begin by demonstrating how our SOM⁻ model reflects the phenomenon of *reinforcement of dominant views*, a key aspect of the spiral of silence theory introduced earlier. This example illustrates, as predicted by the theory, how a vocal minority can disproportionately influence overall opinion in a network despite being outnumbered.

Next, we present a noteworthy case from our SOM⁺ model where agents' opinions converge to consensus while their public opinions remain divergent. This scenario mirrors situations in social media where individuals may unknowingly share common views, yet perceive a state of disagreement due to the persistence of outdated public opinions as discussed [17].

Finally, we outline our current experimental capabilities, showcasing how our simulator can model opinion evolution in networks that more closely approximate real-world social structures in both topology and scale.

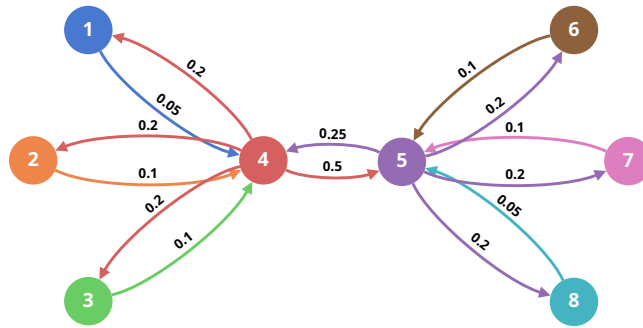
4.1 Reinforcement of Dominant Views

Figure 4 illustrates how our SOM⁻ model captures the essence of the spiral of silence theory. In this strongly connected influence graph, a vocal minority effectively dominates the discourse, causing the silent majority to converge towards the perceived majority opinion.

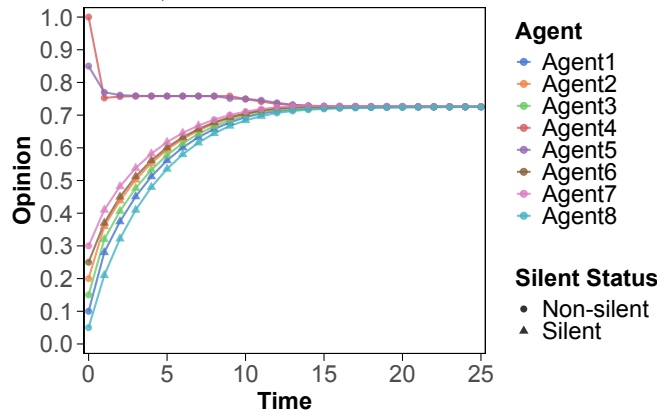
The network comprises two groups: Group 1 (Agents 1, 2, 3, 6, 7, and 8) with opinions in the lower half of the spectrum, and Group 2 (Agents 4 and 5) with opposing views near the higher end. Despite Group 1 being the actual majority, the network topology and differing tolerance radii τ lead to Group 2 dominating

the opinion dynamics. Group 1 agents lack intra-group connections but are all linked to Group 2 agents. Moreover, Group 2’s significantly larger tolerance radii allow them to remain non-silent more frequently.

This configuration results in the vocal minority (Group 2) disproportionately influencing the network. The silent majority (Group 1) remains quiet for most of the update process, only becoming active when opinions have already shifted closer to the perceived majority view. This example demonstrates how the SOM^- model can simulate scenarios where a minority opinion, through strategic positioning and persistent vocalization, can shape the overall opinion landscape, even when numerically outnumbered.



(a) Aperiodic strongly connected influence graph (self influence excluded)



(b) Each plot shows agents’ state evolution over time. Triangles represent silent agents, circles non-silent ones. Colored areas indicate opinion values within each agent’s tolerance radius. Initial state vector: $\mathbf{B}^0 = (0.1, 0.2, 0.15, 1.0, 0.85, 0.25, 0.3, 0.05)$ and $\tau = (0.1, 0.05, 0.1, 0.85, 0.6, 0.05, 0.1, 0.05)$.

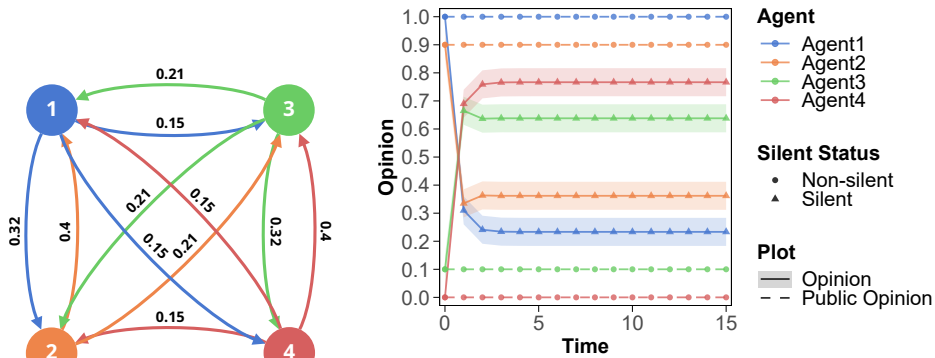
Fig. 4: Silent Majority vs. Vocal Minority: Opinion Dynamics in SOM^-

4.2 Hidden Consensus in SOM⁺ Model

The SOM⁺ model reveals a noteworthy phenomenon: the possibility of reaching a consensus that remains undetected by the agents themselves. Figure 5 illustrates this scenario using a clique graph with four agents.

Initially, the agents hold diverse opinions ($\mathbf{B}^0 = (1.0, 0.9, 0.1, 0.0)$). However, due to the graph’s influence structure, all agents become silent after $t = 0$. Despite this silence, their private opinions converge to a common value (approximately 0.5) over time. Crucially, this convergence occurs without any further public expression of opinions, leaving each agent unaware of the emerging consensus.

This hidden consensus phenomenon mirrors real-world scenarios in social media where individuals may unknowingly share common views while perceiving disagreement due to outdated public expressions [17].



(a) Influence graph self influence excluded

(b) Each plot shows agents’ state evolution over time. Triangles represent silent agents, circles non-silent ones. Colored areas indicate opinion values within each agent’s tolerance radius. Initial state vector: $\mathbf{B}^0 = (1.0, 0.9, 0.1, 0.0)$.

Fig. 5: Silent Consensus Scenario

4.3 Scaling to Large-Scale Network Simulations

While the small-scale examples presented earlier are valuable for demonstrating mathematical properties, they fail to capture the complexity of real-world social networks. To address this limitation, we developed a simulation platform capable of modeling networks with over a million agents, providing insights into how our models behave in more realistic scenarios.

Our approach utilizes a highly optimized version of the preferential attachment algorithm [1] to generate networks exhibiting small-world properties [25]

and power-law degree distributions. These networks are commonly for experimentation because they reflect real-world social networks. To manage the computational demands of large-scale simulations, we employ parallel processing via Scala and Akka Actors [19,18], with results stored in a PostgreSQL database for efficient analysis.

Simulations of these large-scale networks reveal intriguing dynamics in both the SOM^+ and SOM^- models. In the SOM^+ model, we observe that increased network connectivity hinders consensus formation. Conversely, less dense networks facilitate consensus more readily. The SOM^- model exhibits opposite behavior: higher connectivity promotes the formation of a global spiral of silence, driving silent agents' beliefs towards the perceived majority opinion. In contrast, sparser networks tend to foster local spirals of silence or echo chambers, often interconnected by "bridge" agents that may become perpetually silent, impeding global consensus.

These findings align with previous research [23,7], which similarly identified the emergence of local spirals of silence in lower-density networks and global spirals of silence in denser networks. Our large-scale simulations provide a computational framework for exploring the intricate relationship between network structure and opinion dynamics under the influence of the spiral of silence.

5 Conclusions and Related Work

In this work, we have extended the classical DeGroot model to incorporate key social dynamics described by the Spiral of Silence. Our contributions highlight how the addition of memory, even in a limited form as in SOM^+ , introduces significant complexity to consensus-building processes. Through theoretical results and simulations, we demonstrated that consensus, while achievable in fully connected networks (i.e., cliques) in SOM^- , is no longer guaranteed, in contrast with the DeGroot model, in more general strongly-connected aperiodic graphs. This points to the nuanced impact that silence and memory can have on opinion formation in social networks. It also offers insights into the challenges of converging to consensus in real-world scenarios. We also discuss simulations reflecting predictions of the Spiral of Silence such as the reinforcement of dominant views and hidden consensus. Finally we discussed the simulation in our models of large-scale networks.

We are not aware of any prior work that extends DeGroot-based models to incorporate the Spiral of Silence. However, some recent studies have explored the Spiral of Silence theory within agent-based networks. In [23], the authors analyzed the impact of manipulative actors in social networks by building an agent-based model grounded in Spiral of Silence theory and complex adaptive systems. In this model, agents hold a binary opinion (agree/disagree) that remains fixed over time but may choose whether or not to express it, depending on the prevailing opinion climate. In [7], authors investigate how the number of communities in a network and connectivity between them affects the per-

ceived opinion climate. Nevertheless, these works do not deal with convergence to consensus, opinion updates or memory as done in this paper.

The exclusion of silent agents in SOM^- amounts to having edges (influences) disappearing and reappearing during opinion evolution which reflects the dynamic influence nature of this model. There are several works studying dynamic influence in opinion formation. The work [13] introduces a version of the DeGroot model in which self-influence changes over time while the influence on others remains the same. The works [9,10] explore convergence and stability, respectively, in models where influences change over time. The work [6] demonstrates how asynchronous communication, when combined with dynamic influence, can prevent consensus. None of this work deals with the dynamics derived from the Spiral of Silence.

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