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Eric Veron, Jean-François Sigrist, Daniel Broc. Implementation of a Structural-Acoustic Homogenized Method for the Dynamic Analysis of a Tube Bundle With Fluid Structure Interaction Modeling in ABAQUS: Formulation and Applications. ASME 2014 Pressure Vessels and Piping Conference, Jul 2014, Anaheim, United States. 10.1115/PVP2014-28357 . hal-04949501

HAL Id: hal-04949501

<https://hal.science/hal-04949501v1>

Submitted on 23 Feb 2025

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PVP2014-28357

Implementation of a structural-acoustic homogenized method for the dynamic analysis of a tube bundle with fluid structure interaction modeling in abaqus: formulation and applications

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ABSTRACT

The present paper deals with the dynamic analysis of a tube bundle with Fluid Structure Interaction (FSI) modeling using a structural acoustic homogenized method. Such a coupled problem leads to many degrees of freedom [a system of very large matrices] to compute tube displacements and pressure in the acoustic domain, it is therefore irrelevant to use standard coupled methods in industrial cases. Instead, specific modelings have to be used, such as structural acoustic homogenized method. Implementation and applications of such a technique within the general finite element code ABAQUS are performed using the so-called UEL Fortran subroutine. Firstly, general theoretical aspects on the homogenized method proposed by Broc & Sigrist are revisited. Then, subroutines developments are validated comparing results from the homogenized method to those of a standard approach on the representative case of a 10X10 tube bundle in two-dimensional and three-dimensional configurations subjected to seismic loadings. Results show that: (i) homogenized elements can easily be used as standard elements from the ABAQUS elements library, (ii) the homogenized approach is accurate on a physical point of view and (iii) considerably reduces modeling effort and computational time compared to a standard structural acoustic method.

Keywords: Tube bundle, Fluid Structure Interaction (FSI), Homogenization method, ABAQUS.

INTRODUCTION

Dynamic analysis of tube bundles immersed in a fluid and subjected to seismic or shock loadings is of paramount importance to the safety assessment of nuclear equipments: (i) power production and (ii) naval propulsion. Such structure-type can be come across in steam generators, tube heat exchangers, spent fuel storage racks or nuclear reactor cores (see Figure 1).

Since the fluid is highly confined in tube bundles, the dynamic behavior is strongly influenced by its presence and Fluid

Structure Interaction has to be taken into account in order to render the significant inertial coupling effect. The use of standard FSI formulations in most general finite element codes [11] to assess the dynamic behavior of such complex geometries with thousands of tubes leads to many degrees of freedom and cannot always be operated. In that case, specific modelings have to be used, such as structural acoustic homogenized method based on the homogenization of the periodic fluid-structure media.



Figure 1 – Tube heat exchanger (meretmarine.com)

Homogenization techniques, based on a multi-scale method, have been developed [1] and used in many fields for systems with periodic patterns [13, 14] such as stiffened plates and corrugated core sandwich panels [4], fluid in porous media [10] or fluid structure interaction [1, 3, 5, 6, 8, 9, 12, 19, 20]. Regarding fluid structure interaction problems, homogenization methods have been applied to analyze nuclear reactor core or tube bundle (ex: tube heat exchangers, steam generator).

The present paper deals with the implementation of a homogenized fluid-structure method based on the formulation proposed by Broc & Sigrist [mean pressure (p)/tube displacement (u_s) homogenized formulation] (we will formalize a specific treatment of out-of-plane displacement in the present paper) [15, 16, 17] in the general finite element software ABAQUS, using the so-called UEL Fortran subroutine. This formulation is based on the following assumptions: (i) only inertial coupling effect is taken into account, thus implying that tube displacements are assumed to be small and the fluid incompressible and inviscid (that assumption can reasonably be made if the eigenfrequencies of the coupled system are much lower than the eigenfrequencies of the associated acoustic problem) and (ii) the behavior between neighboring patterns is assumed to undergo small variations (periodicity condition). Formulation and subroutine developments are validated comparing results from the homogenized method to those of a standard approach on the representative case of a 10X10 tube bundle in two-dimensional and three-dimensional configurations subjected to seismic loadings. For the two-dimensional configuration, tubes are modeled using equivalent spring/mass systems calibrated to the first flexural eigenmode properties of the tubes.

I. HOMOGENIZATION IN TUBE BUNDLE

1. Acoustic fluid model

The acoustic fluid model describes the small disturbance of an irrotational homogeneous inviscid newtonian fluid at rest with no body force. Under these assumptions, the fluid is governed by the following set of equations:

$$\ddot{p} + \rho_F c_F^2 \nabla \cdot \ddot{u}_F = 0 \quad (1)$$

$$\rho_F \ddot{u}_F + \nabla p = 0 \quad (2)$$

The first equation describes the acoustic conservation of mass and the second equation translates the acoustic momentum conservation.

2. Inertial coupling effect – Elementary cell

The inertial coupling effect is described on an elementary cell. Figure 2 describes the main notations use in the further theoretical developments.

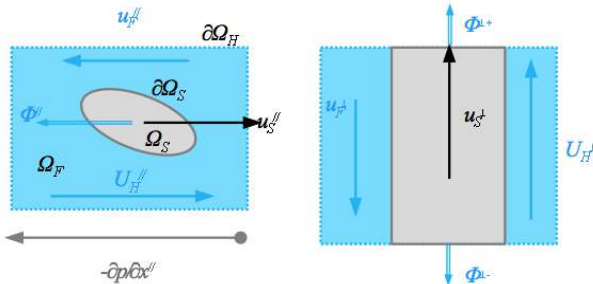


Figure 2 – Elementary cell (left: in-plane, right: out-of-plane)

^{//} and ^{perp} superscripts denote in-plane and out-of-plane displacements, respectively. The homogenized domain and the

fluid domain boundary state as $\Omega_H = \Omega_F \cup \Omega_S$ and $\partial\Omega_F = \partial\Omega_S \cup \partial\Omega_H$ respectively.

i. Darcy acceleration

The homogenization method is based on the average acceleration of a cell, called Darcy acceleration in reference to the Darcy velocity used for the analysis of porous media. The Darcy acceleration is calculated as follows:

$$\ddot{U}_H = \int_{\Omega_H} \ddot{u} d\Omega = \frac{|\Omega_F|}{|\Omega_H|} \ddot{u}_F + \frac{|\Omega_S|}{|\Omega_H|} \ddot{u}_S \quad (3)$$

Using the divergence theorem, it appears that \ddot{U}_H also stands for the mean acceleration of the fluid flow at the cell boundary.

ii. Forces on tubes

Fluid force on the tube is the resultant of the pressure at the tube/fluid interface:

$$\Phi = - \int_{\partial\Omega_S} p n_S d\Gamma \quad (4)$$

Fluid force applied on a single tube is not directly dependent on the behavior of neighboring tubes. However neighboring cells have an indirect effect on the current tube because of the continuity of fluid flow at the cell interface.

• In-plane fluid force

For the in-plane displacement, the fluid force on the tube is written using the Darcy acceleration and the tube acceleration. The Fritz problem gives the force on a rigid body immersed in a rigid tank [7], therefore the in-plane fluid force is given by:

$$\Phi^// = -m_A^// \ddot{u}_S^// + (m_A^// + \rho_F |\Omega_S|) \ddot{U}_H^// \quad (5)$$

In-plane fluid force depends on the tube acceleration $\ddot{u}_S^//$ and the cell acceleration $\ddot{U}_H^//$. $m_A^//$ is numerically computed for each in-plane direction with a quasi-static structural-acoustic analysis on the elementary cell with relevant boundary conditions (see Figure 3).

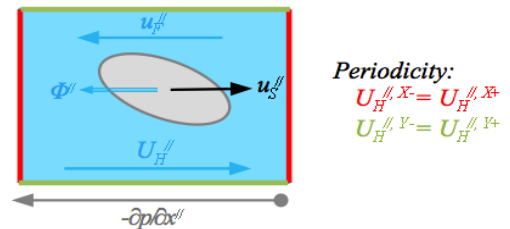


Figure 3 – Periodic boundary conditions

In finite element codes, structural-acoustic problems can only be computed with dynamic analysis. In order to avoid numerical oscillation during the quasi-static analysis, it can be more convenient to use the standard analogy between heat

conduction equations (T, φ_T) and inviscid incompressible fluid flow equations (p, \ddot{u}_F):

Values	Incompressible fluid	Heat conduction
Primal unknown	p	T
Dual unknown	\ddot{u}_F	φ
Balance equation	$\nabla \cdot \ddot{u}_F = 0$	$\nabla \cdot \varphi = 0$
Behavior law	$\ddot{U}_F = -1/\rho_F \nabla p$	$\Phi = -\lambda \nabla T$
Boundary condition	$p - p_0 = 0$	$T - T_0 = 0$
Flux loadings	$\ddot{u}_F \cdot \mathbf{n} = -1/\rho_F \partial p / \partial \mathbf{n}$	$\varphi \cdot \mathbf{n} = -\lambda \partial T / \partial \mathbf{n}$

Table 1 – Incompressible fluid vs Heat conduction

- *Out-of-plane fluid force*

Regarding the out-of-plane displacement, the fluid force on the tube is expressed as follows:

$$\Phi^\perp = - \int_{\partial \Omega_S} p n_S^\perp d\Gamma \quad (6)$$

3. Homogenized equations

The use of Darcy acceleration enables the switching from a local point of view to a global point of view when writing the equations of conservation.

i. Conservation of mass

Integrating the equation of mass balance (1) over the fluid volume $|\Omega_F|$, the homogenized equation of conservation of mass takes the following form:

$$\frac{\lambda}{\rho_F c_F^2} \ddot{p} + \nabla \cdot \ddot{U}_H - \frac{|\Omega_S|}{|\Omega_H|} \nabla \cdot \ddot{u}_S = 0 \quad (7)$$

The former equation is obtained by using the Darcy acceleration expression (3) combined with the porosity factor expressed as $\lambda = |\Omega_F| / |\Omega_H|$.

Assuming that the tube cross-section is rigid (i.e. $\nabla \cdot \ddot{u}_S = \nabla \cdot \ddot{u}_S^\perp$) the equation of conservation of mass can be reworded as follows:

$$\frac{\lambda}{\rho_F c_F^2} \ddot{p} + \nabla \cdot \ddot{U}_H - B_S^\perp \nabla \cdot \ddot{u}_S^\perp = 0 \quad (8)$$

In which B_S^\perp is the out-of-plane confinement factor, as follows $B_S^\perp = |\Omega_S| / |\Omega_H|$.

ii. Momentum conservation

The homogenized momentum conservation equation is obtained by integrating (2) over the fluid volume and by using the divergence theorem.

- *In-plane momentum conservation equation*

The momentum conservation equation gives the following form to the in-plane homogenized momentum balance equation:

$$\rho_F |\Omega_F| \ddot{U}_H'' = -|\Omega_H| \nabla'' p - \Phi'' \quad (9)$$

The in-plane fluid force Φ'' is replace by its expression equation (5) to rewrite the in-plane homogenized momentum balance equation (9) under the following form:

$$[\rho_F (|\Omega_H| + |\Omega_S|) + m_A''] \ddot{U}_H'' = -|\Omega_H| \nabla'' p + (\rho_F |\Omega_S| + m_A'') \ddot{u}_S'' \quad (10)$$

- *Out-of-plane momentum conservation equation*

Based on equations (2) and (3), the out-of-plane homogenized equation is deduced and is written as follows:

$$\rho_F \ddot{U}_H^\perp = -(1 - B_S) \nabla^\perp p + \rho_F B_S^\perp \ddot{u}_S^\perp \quad (11)$$

Finally, considering the in-plane confinement factor $B_S'' = (\rho_F |\Omega_S| + m_A'') / [\rho_F (|\Omega_H| + |\Omega_S|) + m_A'']$, the homogenized momentum conservation equation for the in-plane and the out-of-plane displacements is reworded under the following form:

$$\rho_F \ddot{U}_H = -(1 - B_S) \nabla p + \rho_F B_S \ddot{u}_S \quad (12)$$

Equations (8) and (12) describe the mechanical behavior of the homogenized tube/fluid domain.

iii. Fluid forces on the tube

- *In-plane fluid force*

The in-plane density fluid force on the tube is obtained by dividing the in-plane fluid force applied on the tube (5) by the cell volume $|\Omega_H|$:

$$\varphi'' = \frac{\Phi''}{|\Omega_H|} = \rho_F (|\Omega_S| / |\Omega_H| - B_S'') \ddot{u}_S'' - B_S'' \nabla'' p \quad (13)$$

- *Out-of-plane fluid force*

The out-of-plane fluid force on the tube is described by the following equation:

$$\Phi^\perp = -|\Omega_S| p n^\perp \quad (14)$$

Φ^\perp is the resultant force exerted by the fluid on the tube section at its extremities.

4. Variational formulation

The general fluid/tube bundle interaction problem is depicted in Figure 4.

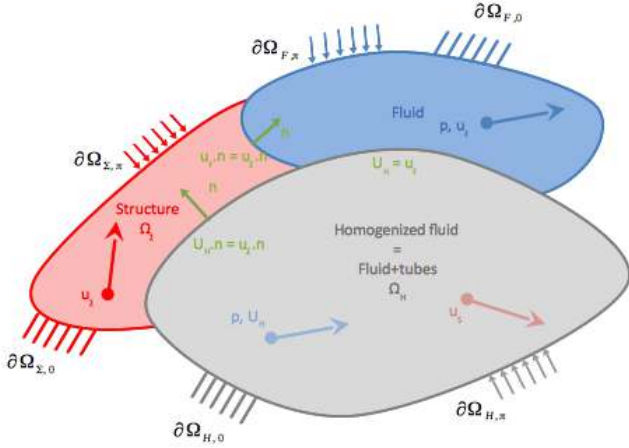


Figure 4 – Homogenized fluid with inner structures

Let Ω_H be the homogenized fluid (fluid with inner structures) described by the average pressure p in the fluid and Ω_S be the inner structure described by the tube displacement u_s (which is supposed to undergo small variations between two neighboring cells). $\partial\Omega_{H,0}$ and $\partial\Omega_{H,\pi}$ stand for the fluid part with an imposed pressure (in most applications $p=0$ at the free surface) and for the homogenized fluid boundary with imposed fluid displacements (in most applications $U_H=0$ for rigid wall), respectively. $\partial\Omega_{S,0}$, $\partial\Omega_{S,\pi}$ and $\partial\Omega_{HS}$ are the tube part with imposed displacements, the tube part with imposed forces and the tube interface between the tube and the homogenized fluid, respectively. Let Ω_Σ be a structure in contact with the homogenized fluid Ω_H and the fluid Ω_F described with a field displacement u_Σ . $\partial\Omega_{\Sigma,0}$ and $\partial\Omega_{\Sigma,\pi}$ denote the structure part with prescribed displacements and the structure boundary part with an imposed force, respectively. Let Ω_F the fluid domain described with the field pressure p . $\partial\Omega_{F,0}$ and $\partial\Omega_{F,\pi}$ are the fluid part with a prescribed pressure (in most applications $p=0$ at the free surface) and the fluid boundary with imposed fluid displacements (in most applications $u_F=0$ for rigid wall), respectively.

A variational formulation of the homogenized tube/fluid problem is obtained using the test function method.

i. Homogenized fluid

The following strong form of the homogenized problem is obtained by premultiplying equation (7) by the test function δp (which complies with the Dirichlet boundary conditions) and integrating over the fluid domain:

$$\forall \delta p \quad \frac{\lambda}{\rho_F c_F^2} \int_{\Omega_H} \bar{p} \delta p d\Omega + \int_{\Omega_H} \nabla \cdot \bar{U}_H \delta p d\Omega - \int_{\Omega_H} B_S^+ \nabla^+ \cdot \bar{u}_S^+ \delta p d\Omega = 0 \quad (15)$$

The former expression gives the variational formulation of the homogenized fluid by using the homogenized momentum

conservation equation (12), the divergence theorem and the product differentiation:

$$\forall \delta p \quad \frac{\lambda}{\rho_F c_F^2} \int_{\Omega_H} \bar{p} \delta p d\Omega + \int_{\Omega_H} \frac{(1-B_S)}{\rho_F} \nabla p \nabla \delta p d\Omega - \int_{\Omega_H} B_S^{II} \bar{u}_S^+ \cdot \nabla^+ \delta p d\Omega - \oint_{\partial\Omega_{HS}} B_S^+ \bar{u}_S^+ \delta p d\Gamma = - \oint_{\partial\Omega_{HS}} \bar{U}_H \cdot n \delta p d\Gamma \quad (16)$$

ii. Tube in the homogenized fluid-structure

The strong form of the tube problem in the homogenized fluid is formulated using the method of weighted residuals:

$$\forall \delta u_s \quad \int_{\Omega_S} \text{div}(\sigma) \cdot \delta u_s d\Omega + \int_{\Omega_S} (\varphi + f_v) \cdot \delta u_s d\Omega - \rho_s \int_{\Omega_S} \ddot{u}_s \delta u_s d\Omega = 0 \quad (17)$$

The former equation is verified for any virtual displacement δu_s that complies with the boundary conditions on $\partial\Omega_{S,0}$. Using the divergence theorem and considering the fluid force expression, the former equation yields the variational formulation for the tubes:

$$\forall \delta u_s \quad \rho_s \int_{\Omega_S} \ddot{u}_s \delta u_s d\Omega + \int_{\Omega_S} \sigma : \varepsilon(\delta u_s) d\Omega - \rho_F \int_{\Omega_S} (|\Omega_S|/|\Omega_H| - B_S) \ddot{u}_s \delta u_s d\Omega + \int_{\Omega_H} B_S^{II} \nabla^+ p \cdot \delta u_s^+ d\Omega + \oint_{\partial\Omega_{HS}} B_S^+ p \delta u_s^+ d\Gamma = \oint_{\partial\Omega_{HS}} \sigma n \cdot \delta u_s d\Gamma \quad (18)$$

The in-plane fluid-structure interaction is described by volume integrals $-\int_{\Omega_H} B_S^{II} \bar{u}_S^+ \cdot \nabla^+ \delta p d\Omega$ and $\int_{\Omega_H} B_S^{II} \nabla^+ p \cdot \delta u_s^+ d\Omega$ whereas the out-of-plane fluid-structure interaction is calculated with surface integrals $-\oint_{\partial\Omega_{HS}} B_S^+ \bar{u}_S^+ \delta p d\Gamma$ and $\oint_{\partial\Omega_{HS}} B_S^+ p \delta u_s^+ d\Gamma$.

II. IMPLEMENTATION OF THE HOMOGENIZED METHOD WITHIN ABAQUS

Equations (16) and (18) are the variational formulation of the homogenized fluid-tubes coupled problem. The discretization is performed with the finite element method and yields the fluid, structure and fluid-structure interaction matrices. U_s and P are the displacement of the tubes and the average pressure in the homogenized field, respectively.

1. Numerical discretization of the homogenized fluid-tubes problem

- Tube stiffness and mass account for the strain and kinetic energies in the tube bundles, respectively. Discretization of the integral terms in the left-hand side yields (18):

$$\int_{\Omega_S} \sigma : \varepsilon(\delta u_s) d\Omega \approx \delta U_s^T K_s U_s \quad (19)$$

$$\rho_s \int_{\Omega_S} \ddot{u}_s \delta u_s d\Omega \approx \delta U_s^T M_s \ddot{U}_s \quad (20)$$

- Fluid stiffness matrix (mobility matrix) and fluid mass matrix (named compressibility matrix as it account for compressibility effects) account for the potential and kinetic

energies in the homogenized fluid, respectively. Discretization of the integral terms in the left-hand side yields (16):

$$1/\rho_F \int_{\Omega_H} (1-B_S) \nabla p \nabla \delta p d\Omega \approx \delta P^T K_F^H P \quad (21)$$

$$\frac{\lambda}{\rho_F c_F^2} \int_{\Omega_H} \ddot{p} \delta p d\Omega \approx \delta P^T M_F^H \ddot{P} \quad (22)$$

• Fluid-tubes coupling matrix account for energy exchanges between the homogenized fluid and the tube bundle. Discretization of the integral terms in the left-hand side (16) and (18) yields:

$$-\int_{\Omega_H} B_S^H \ddot{u}_s^H \cdot \nabla \delta p d\Omega - \oint_{\partial\Omega_{HS}} B_S^+ \ddot{u}_s^+ \delta p d\Gamma \approx -\delta P^T C_{FS} B_S \ddot{U}_s \quad (23)$$

$$\int_{\Omega_H} B_S^H \nabla \delta p \cdot \delta u_s^H d\Omega + \oint_{\partial\Omega_{HS}} B_S^+ \delta p \delta u_s^+ d\Gamma \approx \delta U_s^T (C_{FS} B_S)^T P \quad (24)$$

• Inertial coupling effect on tubes (which is due to confinement in the tube bundle) is highlighted by added mass term in equation (18). Discretization of the integral terms in the left-hand side yields:

$$-\rho_F \int_{\Omega_s} (|\Omega_s|/|\Omega_H| - B_S) \ddot{u}_s \delta u_s d\Omega \approx \delta U_s^T M_S^H \ddot{U}_s \quad (25)$$

Finally, the discretization of the homogenized coupled problem (16) and (18) leads to the following matrix system:

$$\begin{bmatrix} M_F^H & -C_{FS} B_S \\ 0 & M_S + M_S^H \end{bmatrix} \begin{Bmatrix} \ddot{P} \\ \ddot{U}_s \end{Bmatrix} + \begin{bmatrix} K_F^H & 0 \\ (C_{FS} B_S)^T & K_S \end{bmatrix} \begin{Bmatrix} P \\ U_s \end{Bmatrix} = \begin{Bmatrix} \Psi \\ F \end{Bmatrix} \quad (26)$$

where Ψ and F are the nodal concentrated fluxes in the acoustic domain and the nodal concentrated forces on the tube, respectively.

The former discrete system has the exact same form as a standard coupled acoustic-structural system in non-symmetric pressure/displacement formulation. Beam elements from the general finite element code library are employed to model tubes (M_S and K_S matrices), hence the implementation of the homogenized method consists only in implementing a homogenized element which stands for:

$$\begin{bmatrix} M_F^H & -C_{FS} B_S \\ 0 & M_S^H \end{bmatrix} \begin{Bmatrix} \ddot{P} \\ \ddot{U}_s \end{Bmatrix} + \begin{bmatrix} K_F^H & 0 \\ (C_{FS} B_S)^T & 0 \end{bmatrix} \begin{Bmatrix} P \\ U_s \end{Bmatrix} = \begin{Bmatrix} \Psi \\ F \end{Bmatrix} \quad (27)$$

The homogenized element accounts for: (i) potential and kinetic energies of the homogenized fluid (K_F^H and M_F^H), (ii) inertial effect due to confinement in the tube bundle (M_S^H) and (iii) homogenized fluid/tube coupling ($-C_{FS} B_S$ and $(C_{FS} B_S)^T$).

2. ABAQUS implementation

In the general finite element software ABAQUS, the so-called UEL (or UELMAT) Fortran subroutine enables the implementation of additional element formulations by defining the element contribution in discretized operators.

i. ABAQUS coupled acoustic-structural formulation

In ABAQUS, dynamic coupled acoustic-structural problems are solved with the non-symmetric pressure/displacement formulation (however, modal analysis is performed with the symmetric pressure-displacement potential/displacement using the displacement potential as an additional variable) [18]:

$$\begin{bmatrix} -M_F^H D_a & C_{FS} B_S / D_a \\ 0 & M_S^H \end{bmatrix} \begin{Bmatrix} \ddot{P} \\ \ddot{U}_s \end{Bmatrix} + \begin{bmatrix} -K_F^H / D_a & 0 \\ (C_{FS} B_S)^T & 0 \end{bmatrix} \begin{Bmatrix} P \\ U_s \end{Bmatrix} = \begin{Bmatrix} -\frac{\Psi}{D_a} \\ F \end{Bmatrix} \quad (28)$$

where $D_a = \delta \ddot{u} / \delta u = \delta \ddot{p} / \delta p$ can be expressed with the time integration scheme parameters and the time increment [18].

The UEL Fortran subroutine enables us to define the contributions of a user-defined element to the left-hand side (matrices stored in *AMATRIX* variable) and to the right-hand side (residual vector stored in *RHS* variable). Values of degrees of freedom have to be stored as an internal state vector in *SVARS* variable. *SVARS* variable can also be used to store values of interest such as values used for post-processing or values stored in order to reduce computing time (storage of matrices for linear elements). These variables must be defined for each calculation step of the dynamic procedure (initial acceleration, velocity jump, half time increment step used during the time integration with the Hilbert Hugues Taylor scheme and the normal time increment step) [18].

ii. Two-dimensional homogenized element

In a first approach, a two-dimensional equivalent model can be used to model tube bundles. Tubes are modeled using an equivalent spring/mass system calibrated to the tube first flexural eigenmode properties. A two-dimensional five-nodes element has been developed and corresponds to an enriched two-dimensional four-nodes acoustic element.

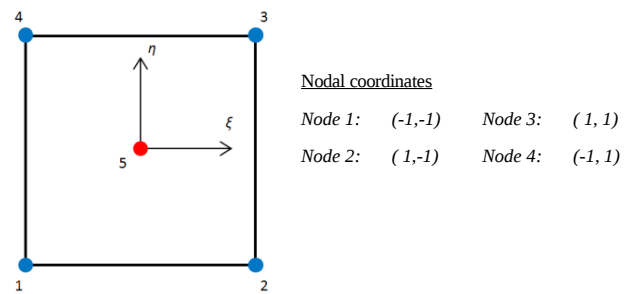


Figure 5 – Two-dimensional homogenized element

The fifth node enables the connection and coupling of structural elements with the fluid. Degrees of freedom of nodes

1, 2, 3 and 4 are pressure-type. The fifth node has x-axis and y-axis translational degrees of freedom:

DOF Number	Node	DOF type
1	1	8: Acoustic pressure
2	2	8: Acoustic pressure
3	3	8: Acoustic pressure
4	4	8: Acoustic pressure
5	5	1: x-axis translation
6	5	2: y-axis translation

Table 2 – DOF type of the two-dimensional element

Node 5 lies in the element but does not take part in the element shape description that relies on nodes 1, 2, 3, and 4:

$$x(\xi, \eta) = \sum_{i=1,4} T_i(\xi, \eta) x_i \quad y(\xi, \eta) = \sum_{i=1,4} T_i(\xi, \eta) y_i \quad (29)$$

where the shape functions T_i are linear and are defined as follows [21]:

$$\begin{aligned} T_1(\xi, \eta) &= \frac{1}{4}(1-\xi)(1-\eta) & T_2(\xi, \eta) &= \frac{1}{4}(1+\xi)(1-\eta) \\ T_3(\xi, \eta) &= \frac{1}{4}(1+\xi)(1+\eta) & T_4(\xi, \eta) &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \quad (30)$$

The fifth node in the two-dimensional homogenized element is only used to add displacement degrees of freedom about x-axis and y-axis. Spring and mass elements modeling the tube are directly connected to node 5. Pressure field in the element is a linear combination of pressure at the element nodes:

$$p(\xi, \eta) = \sum_{i=1,4} N_{p,i}(\xi, \eta) p_i \quad (31)$$

Weights are determined with pressure linear interpolation functions $N_{p,i}$, which are exactly the same as T_i :

$$\begin{aligned} N_{p,1}(\xi, \eta) &= \frac{1}{4}(1-\xi)(1-\eta) & N_{p,2}(\xi, \eta) &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_{p,3}(\xi, \eta) &= \frac{1}{4}(1+\xi)(1+\eta) & N_{p,4}(\xi, \eta) &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \quad (32)$$

Displacement is only dependent on the displacement of the fifth node ($N_{U,1} = 1$ and $N_{U,2} = 1$):

$$u_x = N_{U,1} u_{x,5} = u_{x,5} \quad u_y = N_{U,2} u_{y,5} = u_{y,5} \quad (33)$$

iii. Three-dimensional homogenized element

A three-dimensional ten-nodes element has been developed in order to model tube bundles immersed in a quiescent inviscid fluid. This user-defined element corresponds to an enriched three-dimensional eight-nodes acoustic element:

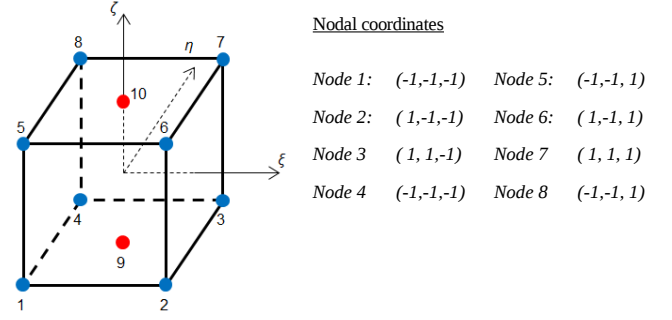


Figure 6 – Three-dimensional homogenized element

The ninth and the tenth nodes enable the connection and coupling of structural elements with the fluid. Degrees of freedom of nodes 1 to 8 are pressure-type. The ninth and tenth nodes have x-axis, y-axis and z-axis translational degrees of freedom.

DOF Number	Node	DOF type
1 to 8	1 to 8	8: Acoustic pressure
9	9	1: x-axis translation
10	9	2: y-axis translation
11	9	3: z-axis translation
12	10	1: x-axis translation
13	10	2: y-axis translation
14	10	3: z-axis translation

Table 3 – DOF Type of the three-dimensional element

Nodes 9 and 10 lie on faces 1234 and 5678, respectively, but do not take part in the element shape description that relies on the eight first nodes:

$$\begin{aligned} x(\xi, \eta, \zeta) &= \sum_{i=1,8} T_i(\xi, \eta, \zeta) x_i & y(\xi, \eta, \zeta) &= \sum_{i=1,8} T_i(\xi, \eta, \zeta) y_i \\ z(\xi, \eta, \zeta) &= \sum_{i=1,8} T_i(\xi, \eta, \zeta) z_i \end{aligned} \quad (34)$$

where the shape functions T_i are linear and are defined as follows:

$$\begin{aligned} T_1(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta) & T_2(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta) \\ T_3(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta) & T_4(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta) \\ T_5(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta) & T_6(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta) \\ T_7(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta) & T_8(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta) \end{aligned} \quad (35)$$

Nodes 9 and 10 in the three-dimensional user-defined element are only used to add displacement degrees of freedom about x-axis, y-axis and z-axis. A beam element modeling the tube is directly connected to these two nodes. Pressure field in the

element is a linear combination of pressure at the element nodes:

$$p(\xi, \eta, \zeta) = \sum_{i=1,4} N_{p,i}(\xi, \eta, \zeta) p_i \quad (36)$$

Weights are determined with pressure linear interpolation functions $N_{p,i}$, which are exactly the same as T_i :

$$\begin{aligned} N_{p,1}(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta) & N_{p,2}(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta) \\ N_{p,3}(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta) & N_{p,4}(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta) \\ N_{p,5}(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta) & N_{p,6}(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta) \\ N_{p,7}(\xi, \eta, \zeta) &= \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta) & N_{p,8}(\xi, \eta, \zeta) &= \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta) \end{aligned} \quad (37)$$

Displacement in the beam element is only dependent on the displacement of nodes 9 and 10:

$$\begin{aligned} u_x(\zeta) &= N_{U,1}(\zeta) u_{x,9} + N_{U,4}(\zeta) u_{x,10} \\ u_y(\zeta) &= N_{U,2}(\zeta) u_{y,9} + N_{U,5}(\zeta) u_{y,10} \\ u_z(\zeta) &= N_{U,3}(\zeta) u_{z,9} + N_{U,6}(\zeta) u_{z,10} \end{aligned} \quad (38)$$

As the displacement field of Timoshenko beam element type B31 in ABAQUS is interpolated by linear functions (for further details see [18]), the displacement field of the homogenized element is linearly interpolated between the two nodes with the following interpolation functions:

$$\begin{aligned} N_{U,1}(\zeta) &= \frac{1}{2}(1-\zeta) & N_{U,2}(\zeta) &= \frac{1}{2}(1+\zeta) \\ N_{U,3}(\zeta) &= \frac{1}{2}(1-\zeta) & N_{U,4}(\zeta) &= \frac{1}{2}(1+\zeta) \\ N_{U,5}(\zeta) &= \frac{1}{2}(1-\zeta) & N_{U,6}(\zeta) &= \frac{1}{2}(1+\zeta) \end{aligned} \quad (39)$$

Remark: if Euler-Bernoulli element type has been used to model tubes, it would have been more relevant to interpolate the field displacement with cubic functions.

Two-dimensional and three-dimensional elements are exactly numerically integrated with a Gauss quadrature. Numbers of integration points are chosen for each integral term (see §II.1). Unlike ABAQUS acoustic elements, mass matrix of homogenized user-defined elements is a consistent mass matrix (linear elements in ABAQUS have diagonal condensed mass matrices).

iv. Input data file and Post-processing

The acoustic-structural homogenized user-defined element is employed in the input data file as a standard element of the ABAQUS element library. For homogenized elements, properties such as fluid density, bulk modulus, directional added mass factors and cell ratio have to be defined.

Visualization of user-defined elements requires dedicated methods: for instance overlaying user-defined elements with

standard elements with low properties ($\rho_F^* \gg \rho_F$). Hence, homogenized user-defined elements are overlaid with acoustic elements (two-dimensional problem: AC2D4, three-dimensional problem: AC3D8) in order to visualize pressure field in the homogenized domain.

III. APPLICATION ON THE REPRESENTATIVE CASE OF A 10X10 TUBE BUNDLE

Homogenized user-defined elements implementation is then validated on a representative industrial case of a 10X10 tube bundle subjected to seismic loading. Computations are performed on a two-dimensional and a three-dimensional problems. The homogenized solutions are compared to results obtained with a standard structural-acoustic approach.

1. Inertial coupling effect on an elementary cell

The inertial coupling effect, highlighted by m_A'' , is numerically computed on an elementary cell to take into account the fluid confinement in the tube bundle. Focusing on one directional displacement problem, the elementary cell problem is reduced to a system with two degrees of freedom: \ddot{u}_s and \ddot{U}_H , which stand for the tube and the mean cell accelerations, respectively. Any solution would be a linear combination of two independent trivial solutions respecting the periodic conditions. Since two independent solutions are constructed, elementary solutions (case 1: $\ddot{u}_s = 1 \text{ m/s}^2$ and $\ddot{U}_H = 0 \text{ m/s}^2$, case 2: $\ddot{u}_s = 0 \text{ m/s}^2$ and $\ddot{U}_H = 1 \text{ m/s}^2$) can therefore be determined.

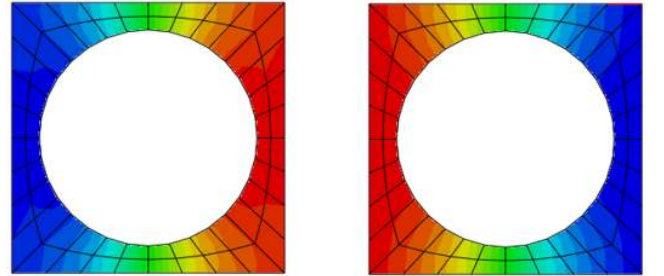


Figure 7 – Elementary solutions (left: $\ddot{u}_s=1\text{m/s}^2$ and $\ddot{U}_H=0\text{m/s}^2$, right: $\ddot{u}_s=0\text{m/s}^2$ and $\ddot{U}_H=1\text{m/s}^2$)

m_A'' is identified by post-processing the resultant force on the tube due to pressure field at the fluid/tube interface. Theoretically, elementary solutions must be computed with a fine mesh to model accurate boundaries and well assess inertial coupling effects. Regarding the current validation, as the dynamic behavior is strongly influenced by the inertial coefficient, the tube discretization for the elementary model is exactly the same as the tube discretization employed for the reference model. As the two-dimensional model discretization differs from the three-dimensional model discretization, m_A'' is computed in both cases:

Case	Number of nodes	m_A''
2D	32	3.4726e-01 kg/m
3D	16	3.1873e-01 kg/m

Table 4 – Inertial coupling factor m_A''

Element cell mesh refinement has a major influence on m_A'' and on the representation of tube and fluid volumes, which is of paramount importance when evaluating the elementary force on a tube (13).

2. Two-dimensional case

As aforementioned, a two-dimensional modeling can be adopted in a first approach to model tube bundle immersed in a fluid. Tubes are then represented with spring/mass systems calibrated on the first flexural eigenmode properties ($f_0 = \sqrt{k/m}/(2\pi)$). Seismic acceleration $\gamma(t)$ is applied to one extremity of the spring and the mass element is connected to the other one. Two-dimensional modeling principles are depicted in Figure 8:

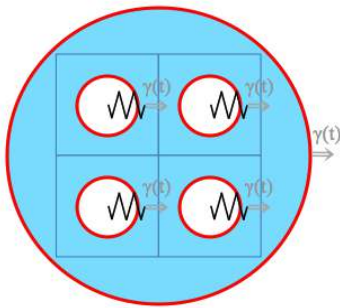


Figure 8 – Two-dimensional modeling

Regarding the current development validation, first flexural eigenfrequency is calibrated at the peak of the Shock Response Spectrum (or Response Spectrum used to characterize dynamic excitation). Figure 9 presents the seismic acceleration applied to the tube bundle on the x direction and the associated SRS:

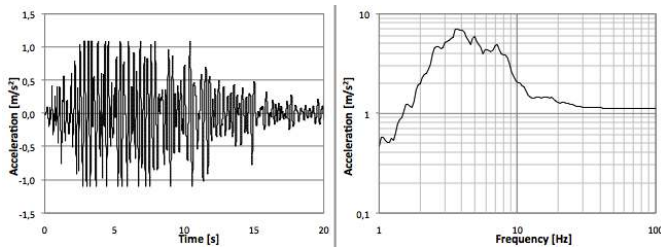


Figure 9 – Seismic loading (left: acceleration, right: SRS)

Remark: Although seismic loading is only applied in x direction, two-dimensional coupling effect, and therefore the two-dimensional homogenization, has to be taken into account.

Figure 10 provides a view of finite element meshes of the fluid domain used for the dynamic analysis of the tube bundle with both methods. As tube interfaces are not explicitly meshed in the homogenized model, the homogenized method considerably reduces modeling efforts and the number of degrees of freedom. Tube geometrical characteristics are set as a parameter to the user-defined homogenized elements.

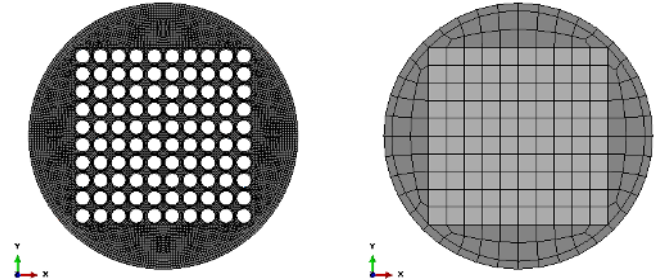


Figure 10 – Finite Element Models (left: standard FEM, right: homogenized FEM)

Figure 11 shows the time-history of the displacement of one tube in the tube bundle computed with the standard and the homogenized methods, alternatively. The homogenization method handles well both the displacement amplitude and the frequency content of the signal.

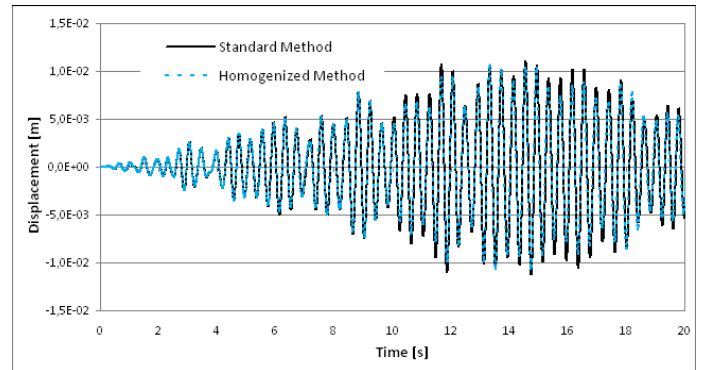


Figure 11 – X displacement history – Comparison with the two methods

Figure 12 shows the pressure field at a given moment in the fluid domain. The homogenization method enables the representation of pressure fluctuations generated by the tubes displacement.

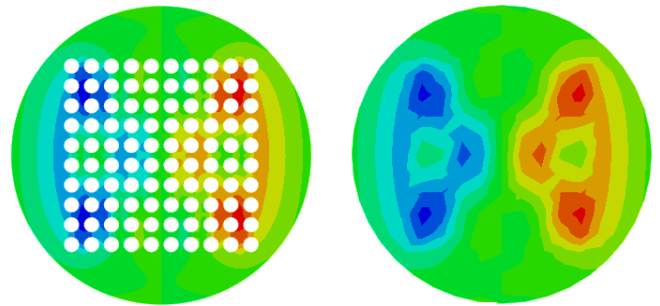


Figure 12 – Pressure field (left: standard FEM, right: homogenized FEM)

Table 5 gives the 2D computing performances for both the standard method and the homogenized method:

Characteristics	Standard Method	Homog. Method
Number of nodes	14 294 [-]	434 [-]
Number of DOF	57 265 [-]	1 465 [-]
Computation time	9 230 s	500 s

Table 5 – Computing performances for 2D case

Homogenized method substantially reduces modeling effort and is cost effective since it lowers considerably the number of degrees of freedom.

3. Three-dimensional case

The 10X10 tube bundle is totally immersed in a fluid at rest that is embedded in a flexible cylindrical tank. One extremity of the tube and the tank bottom are subjected to seismic loading in three directions. The other extremity of the tubes is not constrained. Figure 13 shows seismic accelerations applied to the system and the associated SRS.

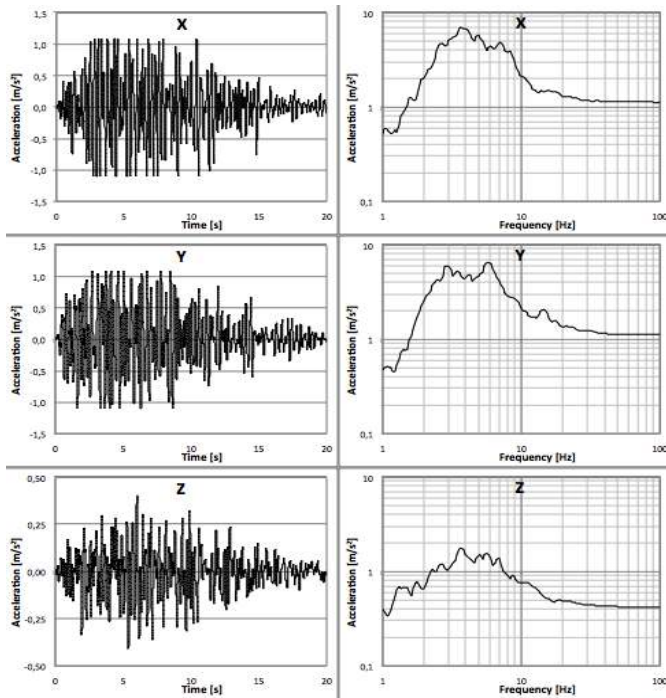


Figure 13 – Seismic loading (left: acceleration, right: SRS)

Regarding the three-dimensional simulation, tubes are modeled with beam elements.

Figure 14 gives a view of finite element meshes of the fluid domain used for the dynamic analysis of the tube bundle with the two methods. Since the tubes are not explicitly meshed, the geometrical characteristics used for the calculation of the confinement ratio B_s are set as parameters in the user-defined elements.

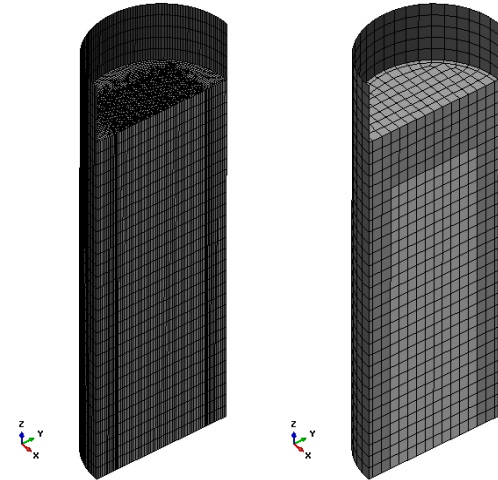


Figure 14 – Finite Element Models (left: standard FEM, right: homogenized FEM)

Figure 15 shows the time-history of the displacement of one tube extremity in the tube bundle computed with the standard and the homogenized methods, alternatively. The homogenization method handles well both the displacement amplitude and the frequency content of the signal.

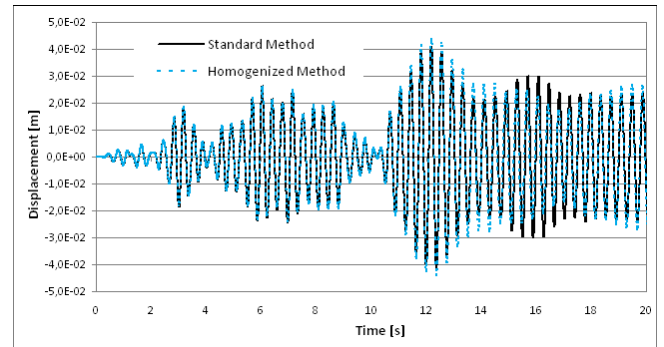


Figure 15 – X displacement history – Comparison with the two methods

Figure 16 compares the maximum displacement at the upper extremity of the tubes computed with the standard and the homogenized methods.

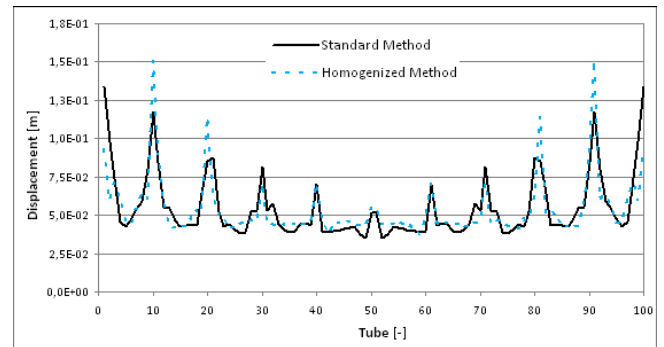


Figure 16 – Maximum X displacement – Comparison with the two methods

Figure 17 shows the pressure field at a given moment in the fluid domain at a given altitude. The homogenization method handles pressure fluctuations generated by the tubes displacement:

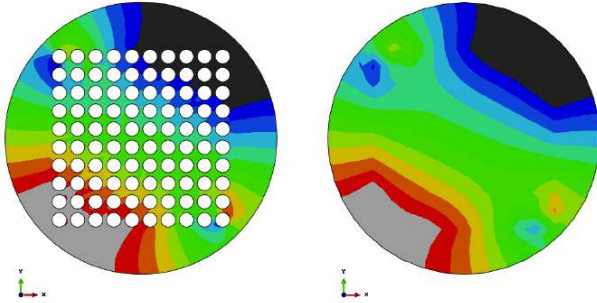


Figure 17 – Pressure field (left: standard FEM, right: homogenized FEM)

Figure 18 renders the pressure field at a given moment in the fluid domain at a given Y value.

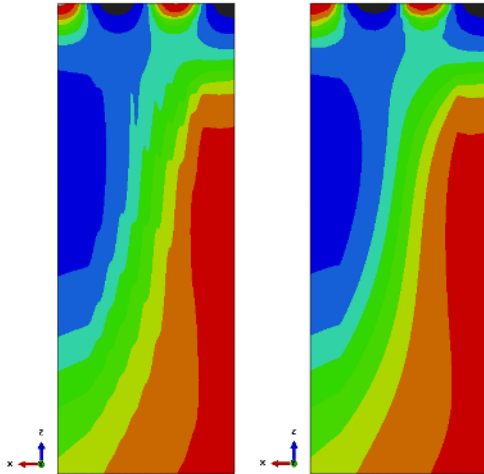


Figure 18 – Pressure field (left: standard FEM, right: homogenized FEM)

Wave elements have been implemented in ABAQUS to take into account sloshing modes due to free surface displacement (for further informations see [11]). Localized pressure fluctuations seen at the free surface are therefore due to the wave condition imposed as $\ddot{u}_F = \ddot{p}/(\rho_F g)$. Since tube interfaces are not explicitly described with the homogenized finite element mesh, specific fluid-structure coupling elements have also been developed to couple homogenized element to the bottom of the flexible tank.

Table 6 gives the 3D computing performances for both the standard method and the homogenized method. Since the homogenized method considerably decreases the number of degrees of freedom conserving the sparse property of global matrices, this method substantially reduces computation time (factor x20 between the standard method and the homogenized method).

Characteristics	Standard Method	Homog. Method
Number of nodes	164 416 [-]	10 024 [-]
Number of DOFs	807 442 [-]	49 418 [-]
Computation time	897 716 s	50 676 s
Output Data Base	34 Go	4 Go

Table 6 – Computing performances for 3D case

CONCLUSION

The homogenization method proposed by Broc & Sigrist has been revisited (specific treatment of the out-of-plane direction) and implemented in ABAQUS in order to perform the dynamic analysis of coupled tube bundles subjected to dynamic loadings such as seismic or shock accelerations. First, theoretical aspects have been exposed: the homogenized method lies on a set of equations that enables the description of the fluid behavior from a global point of view. The numerical implementation in the general finite element code ABAQUS has been detailed: we have developed and implemented homogenized user-defined elements in ABAQUS with the so-called UEL Fortran subroutine for both the two-dimensional and the three-dimensional problems. Then, these developments have been validated on an industrial representative case of a 10X10 tube bundle with a two-dimensional and a three-dimensional points of view: results validate the implementation and show that (i) homogenized elements can easily be used as standard elements from the ABAQUS elements library, (ii) the homogenized approach is accurate on a physical point of view and (iii) cost effective in modeling efforts and computing time compared to the standard structural acoustic method.

ACKNOWLEDGEMENT

The authors wish to thanks Mr. Quesnel at PRINCIPIA for his contribution to this work through scientific exchanges with DCNS Research and CEA Saclay.

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