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M Raghunathan, Y Marandet, H Bufferand, G Ciraolo, Ph Ghendrih, et al.. Corrigendum: Multi-temperature generalized Zhdanov closure for scrape-off layer/edge applications (2022 Plasma Phys. Control. Fusion 64 045005). Plasma Physics and Controlled Fusion, 2023, 66 (1), pp.019501. 10.1088/1361-6587/ad05d5 . hal-04948130

HAL Id: hal-04948130

<https://hal.science/hal-04948130v1>

Submitted on 14 Feb 2025

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Fusion* **64** 045005)

To cite this article: M Raghunathan *et al* 2024 *Plasma Phys. Control. Fusion* **66** 019501

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Corrigendum: Multi-temperature generalized Zhdanov closure for scrape-off layer/edge applications (2022 *Plasma Phys. Control. Fusion* **64** 045005)

M Raghunathan^{1,*} , Y Marandet¹ , H Bufferand², G Ciraolo², Ph Ghendrih² , P Tamain² and E Serre³ 

¹ Aix-Marseille Univ., CNRS, PIIM, Marseille, France

² IRFM-CEA, F-13108 Saint-Paul-Lez-Durance, France

³ Aix-Marseille Univ., CNRS, M2P2, Marseille, France

E-mail: madhusudan.raghunathan@univ-amu.fr

Received 28 July 2023, revised 2 October 2023

Accepted for publication 23 October 2023

Published 17 November 2023



CrossMark

Abstract

Corrigendum for the manuscript (Raghunathan *et al* 2022 *Plasma Phys. Control. Fusion* **64** 045005).

The authors would like to correct the following errors in their original manuscript.

1. Corrections in subsection 6.2

The passage in subsection 6.2, starting from ‘Generally, in the drift approximation...’, up until ‘...which is included in the momentum equation’, has been rewritten to correct for some errors and edited for better clarity. Firstly, the drift-kinetic ordering scheme has been made more explicit, along with their definitions better reflecting neoclassical terminology. Furthermore, there are two additional terms from equation (C8) which enter the RHS of the reduced equation for $\sigma_{\alpha,|||}$, arising from the terms being of low knudsen number, and therefore surviving. The first of these extra terms was pointed out by Sergei Makarov (Max-Planck Institut für Plasmaphysik, Germany), whom we would like to thank, in addition to suggesting changes for clarifying the ordering scheme. The equation after equation (64) has now been

split into two equations numbered equations (65) and (66), for the reduced equations for $r_{\alpha 0,||}$ and $\sigma_{\alpha 0,|||}$ respectively. Furthermore, with the clarified ordering, we also find the first-order terms (in the drift ordering scheme) need to be explicitly written on the RHS of equations (63)–(66). The indicated portion should now read as follows:

Following neoclassical theory, we can separate out flows as $\mathbf{u}_{\alpha} = \mathbf{u}_{\alpha 0} + \mathbf{u}_{\alpha 1}$, where the zeroth order flow $\mathbf{u}_{\alpha 0}$ consists of the individual species flow and $\mathbf{E} \times \mathbf{B}$ flow. The common flow \mathbf{w}_{α} now would be calculated by the density weighted average of $\mathbf{u}_{\alpha 0}$. In the drift approximation of Hinton and Hazeltine^{35,36}, especially for tokamak regions with strong pressure gradients, it may so happen that the total first-order flow velocity $\mathbf{u}_{\alpha 1} = \mathbf{u}_{\alpha 1,\perp} + \mathbf{u}_{\alpha 1,||}$, where $\mathbf{u}_{\alpha 1,\perp} = \mathbf{B} \times \nabla p_{\alpha} / n_i Z_i e B^2$ is the diamagnetic flow velocity and $\mathbf{u}_{\alpha 1,||}$ is its parallel return flow, is comparable to the main particle flow and the $\mathbf{E} \times \mathbf{B}$ flows. Accounting for this flow is important, since it often is the main contributor to the augmented friction force that leads to impurity accumulation in steady state with absence of turbulence^{37,38}. Furthermore, this diamagnetic flow is usually accompanied by the corresponding first-order neoclassical heat-flux³⁹ $\mathbf{q}_{\alpha 1} = \mathbf{q}_{\alpha 1,\perp} + \mathbf{q}_{\alpha 1,||}$, where the

* Author to whom any correspondence should be addressed.

diamagnetic heat-flux is $\mathbf{q}_{\alpha 1, \perp} = (5/2)\rho_{\alpha} \mathbf{B} \times \nabla p_{\alpha} / n_i Z_i e B^2$ and $\mathbf{q}_{\alpha 1, \parallel}$ the corresponding neoclassical parallel heat-flux of the first-order. The gradients of this first-order heat flux may indeed be significantly larger than the gradients of particle fluid flow velocity and the $\mathbf{E} \times \mathbf{B}$ flow velocity. And indeed any further gradient of the gradient of the diamagnetic heat-flux would be an order higher, and hence can be neglected. Furthermore, the Knudsen number ordering scheme of the Zhdanov closure, other than for the first-order heat-flux and flow, is compatible with the drift ordering. Thus, in such a situation, one only needs to incorporate the diamagnetic heat-flux in equations (56) and (58). In case of this, the expressions can be modified as follows to find

$$-\frac{5kT_{\alpha}}{2m_{\alpha}} \nabla_{\parallel} \cdot (\pi_{\alpha 0} + \pi_{\alpha 1}) + \frac{5k}{2m_{\alpha}} n_{\alpha} k T_{\alpha} \nabla_{\parallel} T_{\alpha} = \sum_{\beta} \left(R_{\alpha\beta 0, \parallel}^{11} + M_{\alpha\beta}^{110} u_{\beta 1, \parallel} + M_{\alpha\beta}^{111} h_{\beta 1, \parallel} + M_{\alpha\beta}^{112} r_{\beta 1, \parallel} \right), \quad (63)$$

and

$$\frac{4}{5} \left(\{ \nabla \mathbf{q}_{\alpha 0} \}_{\parallel\parallel\parallel} + \{ \nabla \mathbf{q}_{\alpha 1} \}_{\parallel\parallel\parallel} \right) + 2n_{\alpha} k T_{\alpha} (\epsilon_{\parallel\parallel\parallel} + \epsilon_{\alpha 1, \parallel\parallel\parallel}) = \sum_{\beta} \left(R_{\alpha\beta \parallel\parallel\parallel}^{20} + M_{\alpha\beta}^{200} \pi_{\beta 1, \parallel\parallel\parallel} + M_{\alpha\beta}^{201} \sigma_{\beta 1, \parallel\parallel\parallel} \right), \quad (64)$$

where $\epsilon_{\alpha 1, \parallel\parallel\parallel} = \{ \nabla \mathbf{u}_{\alpha 1} \}_{\parallel\parallel\parallel}$. The equations for \mathbf{r}_{α} is similarly modified,

$$0 = \sum_{\beta} \left(R_{\alpha\beta \parallel}^{12} + M_{\alpha\beta}^{120} u_{\beta 1, \parallel} + M_{\alpha\beta}^{121} h_{\beta 1, \parallel} + M_{\alpha\beta}^{122} r_{\beta 1, \parallel} \right), \quad (65)$$

but the equation for σ_{α} contains a new term, as follows

$$\frac{4}{5} \left(\{ \nabla \mathbf{s}_{\alpha 0} \}_{\parallel\parallel\parallel} + \{ \nabla \mathbf{s}_{\alpha 1} \}_{\parallel\parallel\parallel} \right) + \frac{n_{\alpha} k}{m_{\alpha}} \frac{14}{5} \{ (\mathbf{q}_{\alpha 0} + \mathbf{q}_{\alpha 1}) \nabla T_{\alpha} \}_{\parallel\parallel\parallel} = \sum_{\beta} \left(R_{\alpha\beta \parallel\parallel\parallel}^{21} + M_{\alpha\beta}^{210} \pi_{\beta 1, \parallel\parallel\parallel} + M_{\alpha\beta}^{211} \sigma_{\beta 1, \parallel\parallel\parallel} \right), \quad (66)$$

where \mathbf{s}_{α} is a moment given by $\mathbf{s}_{\alpha} = \int (m_{\alpha} c_{\alpha}^4 / 4) \mathbf{c}_{\alpha} f_{\alpha} d\mathbf{c}_{\alpha}$. A simplification of equation (66) can be performed by Grad's closure on the moment \mathbf{s}_{α} therefore setting $\mathbf{s}_{\alpha} = (7/2\gamma_{\alpha}) \mathbf{h}_{\alpha}$. The first-order parallel quantities can be substituted for the ones in the relevant neoclassical regime. A possible method of solution for such a system can be found in the solution of a similar system in reference 13, in which, the equations are simplified so as to make the rank-1 equations solvable first by

neglecting the stress-tensor divergence term, and then rank-2 equations are solved including the heat-flux term and by Grad's closure on \mathbf{s}_{α} .

2. Corrections in appendix C

Equation (C5) should read as follows

$$\begin{aligned} & \frac{d\pi_{\alpha}}{dt} + \pi_{\alpha} \nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{x}_{\alpha} + \frac{4}{5} \left\{ \frac{\partial}{\partial x_s} \left(\frac{5}{2} n_{\alpha} k T_{\alpha} w_{\alpha r} + h_{\alpha r} \right) \right\} \\ & - \rho_{\alpha} \left\{ w_{\alpha r} \left(\frac{X_{\alpha s}}{m_{\alpha}} + \frac{Z_{\alpha} e E_{\alpha s}}{m_{\alpha}} + \{ \mathbf{u} \times \boldsymbol{\omega}_{\alpha} \}_s - \frac{du_s}{dt} \right) \right\} \\ & + 2 \left\{ \pi_{\alpha r l} \frac{\partial u_s}{\partial x_l} \right\} + 2n_{\alpha} k T_{\alpha} \left\{ \frac{\partial u_r}{\partial x_s} \right\} - 2 \{ \pi_{\alpha s l} \epsilon_{rst} \omega_{\alpha t} \} \\ & = \sum_{\beta} R_{\alpha\beta}^{20}. \end{aligned} \quad (C5)$$

The general balance equation for rank-1, equation (C7), has a correction for a typographical error in its second line

$$\begin{aligned} & \frac{d}{dt} (n_{\alpha} b_{\alpha}^{1n}) + \frac{2n+5}{5} n_{\alpha} b_{\alpha}^{1n} \nabla \cdot \mathbf{u} \\ & + \frac{\partial}{\partial x_r} \left(n_{\alpha} b_{\alpha r}^{2,n} + \frac{n}{\gamma_{\alpha}} n_{\alpha} b_{\alpha r}^{2,n-1} \right) \\ & + \frac{2}{3} \left\{ \frac{\partial}{\partial x_r} \left(\frac{2n+3}{2\gamma_{\alpha}} n_{\alpha} b_{\alpha}^{0,n} + n_{\alpha} b_{\alpha}^{0,n+1} \right) \right\} \\ & - n n_{\alpha} b_{\alpha l}^{2,n-1} \left(\frac{X_{\alpha l}}{m_{\alpha}} + \frac{Z_{\alpha} e E_{\alpha l}}{m_{\alpha}} + \{ \mathbf{u} \times \boldsymbol{\omega}_{\alpha} \}_l - \frac{du_l}{dt} \right) \\ & - \frac{2n+3}{3} n_{\alpha} b_{\alpha}^{0,n} \left(\frac{\mathbf{X}_{\alpha}}{m_{\alpha}} + \frac{Z_{\alpha} e \mathbf{E}}{m_{\alpha}} + \mathbf{u} \times \boldsymbol{\omega}_{\alpha} - \frac{d\mathbf{u}}{dt} \right) \\ & - n_{\alpha} \epsilon_{rst} \omega_{\alpha t} \delta_r b_{\alpha s}^{1n} + n_{\alpha} \left[n b_{\alpha rs}^{3,n-1} + \frac{n(n-1)}{\gamma_{\alpha}} b_{\alpha rs}^{3,n-2} \right] \left\{ \frac{\partial u_r}{\partial x_s} \right\} \\ & + n_{\alpha} \left(\frac{2n+5}{5} b_{\alpha s}^{1n} \frac{\partial u_r}{\partial x_s} + \frac{2n}{5} b_{\alpha t}^{1n} \frac{\partial u_t}{\partial x_r} \right) \\ & + \frac{2n(2n+3)}{5} \frac{n_{\alpha}}{\gamma_{\alpha}} b_{\alpha s}^{1,n-1} \left\{ \frac{\partial u_r}{\partial x_s} \right\} \\ & + \left(\frac{n(2n+3)}{5} + \frac{2}{3} \right) \frac{n_{\alpha}}{\gamma_{\alpha}} b_{\alpha}^{1,n-1} \nabla \cdot \mathbf{u} \\ & + \frac{n_{\alpha} k n (2n+3)}{m_{\alpha} 2} b_{\alpha}^{1,n-1} \frac{dT_{\alpha}}{dt} \\ & + \frac{n_{\alpha} k n (2n+3)}{m_{\alpha} 2} \left[b_{\alpha r}^{2,n-1} + \frac{n-1}{\gamma_{\alpha}} b_{\alpha r}^{2,n-2} \right] \frac{\partial T_{\alpha}}{\partial x_r} \\ & + \frac{n_{\alpha} k n (2n+3)}{m_{\alpha} 3} \left(\frac{2n+1}{2\gamma_{\alpha}} b_{\alpha}^{0,n-1} + b_{\alpha}^{0,n} \right) \frac{\partial T_{\alpha}}{\partial x_r} \\ & = \sum_{\beta} R_{\alpha\beta}^{1n}. \end{aligned} \quad (C7)$$


The general moment equation for rank-2 tensorial moments b^{2n} , equation (C8), should read as

$$\begin{aligned} & \frac{d}{dt} (n_\alpha b_\alpha^{2n}) + \frac{2n+7}{7} n_\alpha b_\alpha^{2n} \nabla \cdot \mathbf{u} \\ & + \frac{\partial}{\partial x_r} \left(n_\alpha b_{\alpha r}^{3,n} + \frac{n}{\gamma_\alpha} n_\alpha b_{\alpha r}^{3,n-1} \right) \\ & + \frac{4}{5} \left\{ \frac{\partial}{\partial x_s} \left(\frac{2n+5}{2\gamma_\alpha} n_\alpha b_{\alpha r}^{1,n} + n_\alpha b_{\alpha r}^{1,n+1} \right) \right\} \\ & - n n_\alpha b_{\alpha l}^{3,n-1} \left(\frac{X_{\alpha l}}{m_\alpha} + \frac{Z_\alpha e E_{\alpha l}}{m_\alpha} + (\mathbf{u} \times \boldsymbol{\omega}_\alpha)_l - \frac{du_l}{dt} \right) \\ & - \frac{2(2n+5)}{5} \left\{ n_\alpha b_{\alpha r}^{1,n} \left(\frac{X_{\alpha s}}{m_\alpha} + \frac{Z_\alpha e E_{\alpha s}}{m_\alpha} + (\mathbf{u} \times \boldsymbol{\omega}_\alpha)_s \right. \right. \\ & \left. \left. - \frac{du_s}{dt} \right) \right\} + n_\alpha \left[n b_{\alpha rs}^{4,n-1} + \frac{n(n-1)}{\gamma_\alpha} b_{\alpha rs}^{4,n-2} \right] \left\{ \frac{\partial u_r}{\partial x_s} \right\} \\ & + n_\alpha 2 \left[\frac{2n+7}{7} \left\{ b_{\alpha rl}^{2n} \frac{\partial u_s}{\partial x_l} \right\} + \frac{2n}{7} \left\{ b_{\alpha rl}^{2n} \frac{\partial u_l}{\partial x_s} \right\} \right] \\ & + n_\alpha \frac{n(2n+5)}{7\gamma_\alpha} \left[2 \left\{ b_{\alpha rl}^{2,n-1} \frac{\partial u_s}{\partial x_l} \right\} + 2 \left\{ b_{\alpha rl}^{2,n-1} \frac{\partial u_l}{\partial x_s} \right\} \right] \end{aligned}$$

$$\begin{aligned} & + b_\alpha^{2,n-1} \nabla \cdot \mathbf{u} \Big] + n_\alpha \frac{2(2n+5)}{15} \left(2b_\alpha^{0,n+1} + \frac{2n+3}{\gamma_\alpha} b_\alpha^{0,n} \right) \\ & \times \left\{ \frac{\partial u_r}{\partial x_s} \right\} - 2n_\alpha \{ b_{\alpha sl}^{2n} \epsilon_{rst} \omega_{\alpha t} \} + \frac{n_\alpha k}{2m_\alpha} n(2n+5) b_\alpha^{2,n-1} \frac{dT_\alpha}{dt} \\ & + \frac{n_\alpha k}{m_\alpha} \frac{n(2n+5)}{2} \left[b_{\alpha r}^{3,n-1} + \frac{n-1}{\gamma_\alpha} b_{\alpha r}^{3,n-2} \right] \frac{\partial T_\alpha}{\partial x_r} \\ & + \frac{n_\alpha k}{m_\alpha} \frac{2n(2n+5)}{5} \left\{ \left(\frac{2n+3}{2\gamma_\alpha} b_{\alpha r}^{1,n-1} + b_{\alpha r}^{1,n} \right) \frac{\partial T_\alpha}{\partial x_s} \right\} \\ & = \sum_\beta R_{\alpha\beta}^{2n} \end{aligned} \tag{C8}$$

Notice the typographical correction on lines 2 and 11. Notice also an algebraic correction on line 4, for which we would like to thank Jason Hamilton (Los Alamos National Laboratory, USA) for pointing out.

ORCID iDs

- M Raghunathan  <https://orcid.org/0000-0003-2772-786X>
- Y Marandet  <https://orcid.org/0000-0002-9970-538X>
- Ph Ghendrih  <https://orcid.org/0000-0002-1505-6512>
- E Serre  <https://orcid.org/0000-0002-3174-7727>