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Uncovering group theory in French secondary school curricula

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INTRODUCTION

In France, future mathematics teachers usually study group theory. This advanced knowledge is not only source of difficulties, it may also seem irrelevant for students, with regard to the content they will have to teach in secondary school. This has been coined by Klein (1908/2016) as a “second discontinuity”.

The aim of my master’s thesis (Rolland, 2023), supervised by Nicolas Grenier-Boley, was to uncover the links between group theory and French secondary school (*collège*, age 11-12 to 14-15 and *lycée*, age 15-16 to 17-18). These links are not explicit in the curricula, and require a thorough investigation to be brought to light. This poster endeavours to present a visual summary of this work.

The research question addressed in this poster is: In which ways is group theory linked to secondary school mathematics in the official curricula?

METHODOLOGY AND THEORETICAL FRAMEWORK

Wasserman thoroughly studied the content of the *Common Core Mathematics Standards from the United States* (CCSS-M), and identified associations between school mathematics and abstract algebra that could be relevant for teachers (2016). Building on his work, my aim was to uncover all the associations between abstract algebra and school mathematics, not only those deemed useful for teachers.

The theoretical framework used in this study is the Anthropological Theory of the Didactic (ATD). My study was focused on external didactic transposition, from scholarly knowledge to knowledge to be taught (Chevallard & Johsua, 1991). I used the 2022 *collège* and *lycée général* curricula. My work was focused on group theory, so whenever I identified a connection with a ring or a field, I only considered its underlying group(s).

A praxeological study of the curricula’s items allowed me to gauge whether each item was backed-up by group theory notions (Chevallard, 1992). I treated each item as a type of task, and determined a corresponding technique. I then assessed the presence of group theory notions in the technology justifying the technique, or in the theory justifying the technology. An example of praxeology built on the type of task “add two fractions with different denominators” will be provided in the poster.

RESULTS

I identified the same four content areas as Wasserman (“Arithmetic properties”, “Inverses”, “Solving equations”, “Structure of sets”). The latter originally

encompasses structures that gradually get enriched (for example $(\mathbb{N}, +)$ gets enlarged to $(\mathbb{Z}, +)$ in middle school), but since I focus only on groups, in my analysis it means abstract groups, without a specified set or law.

I identified three extra content areas. The area “Geometric transformations” encompasses the Euclidean plane isometries (translation, rotation, symmetries), which, endowed with the composition, is a group. “Group homomorphism” includes exponential and logarithm functions. This area could be linked to “Structure of sets”, but since this notion plays an important role in group theory, I isolated it. The area “Iterated elements and order” gathers the operations which, iterated a given number of times, give the original element back. For example, the proof of Fermat’s little theorem relies on the notion of order of an element.

CONCLUSION

The study of the French secondary school curricula showed that group theory is implicitly involved in seven content areas. These links offer a potential for a capstone course designed for prospective teachers. The aim of this course would be to draw their attention on some concepts of group theory, work on them to fix potential misconceptions, and show of these concepts could be beneficial for teaching.

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