



**HAL**  
open science

# Students' conceptions on the notion of a polynomial

Juliette Veuillez–Mainard

► **To cite this version:**

Juliette Veuillez–Mainard. Students' conceptions on the notion of a polynomial. Fifth conference of the International Network for Didactic Research in University Mathematics, Centre de Recerca Matemàtica [CRM], Jun 2024, Barcelona, Spain. hal-04944240

**HAL Id: hal-04944240**

**<https://hal.science/hal-04944240v1>**

Submitted on 13 Feb 2025

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Students' conceptions on the notion of a polynomial

Juliette Veuillez--Mainard<sup>1</sup>

<sup>1</sup>IMAG, University of Montpellier, CNRS, France,  
juliette.veuillez-mainard@umontpellier.fr

*Polynomials are essential objects in mathematics, and hold a fundamental place in education, particularly at the secondary-tertiary transition. Our research focuses on students' conceptions about polynomials, and we present the results of a questionnaire covering three main themes: definition and recognition of a polynomial, equations solving and roots of polynomials, polynomials functions and changes of semiotic register. We identify schemes of high-school and higher education students, and highlight potential underlying difficulties.*

*Keywords: Teaching and learning of linear and abstract algebra, Transition to, across and from university mathematics, Curricular and institutional issues concerning the teaching of mathematics at university level, Teachers' and students' practices at university level, Polynomials and polynomial functions.*

## **POLYNOMIALS AT THE SECONDARY-TERTIARY TRANSITION**

The transition between secondary and tertiary education is an issue highlighted by mathematics education research. This transition has been identified as a source of numerous challenges, and potential ruptures. Research pointed out some recurring difficulties faced by students, along with epistemological and didactical obstacles (Gueudet, 2008). Thus, general patterns have been identified, complemented by research on specific subjects such as linear algebra (Dorier, 1997), probabilities (Doukhan, 2020), or functions (Vandebrouck, 2011). However, we found limited research on the teaching and learning of the notion of polynomial, although it is a fundamental concept in mathematics, being at the crossroad of multiple fields.

Initially underlying polynomial equations, polynomials then become elements of polynomial algebras. In analysis, polynomial functions are used as reference functions. They are important tools in numerical analysis, due to their computational properties and regularity that make them easier to study and implement. The universal property of polynomial algebras provides a specialization morphism that allows to consider matrix or endomorphism polynomials for instance. These aspects were highlighted in our previous work (Veuillez-Mainard, 2022), as well as in the epistemological analysis carried out by Pleština (2023). These works stress the duality between the formal and the functional aspects of polynomials. This diversity of points of view makes polynomials a central concept in the teaching of analysis and algebra. Students first encounter polynomials in junior high school, through the manipulation of first-degree algebraic expressions. Then, in tertiary education, formal polynomials are defined and studied, which will lead later to formal series as well as to algebraic geometry.

However, even though they may seem basic, students often struggle to understand and use polynomials, which complicates various tasks for them. Building upon this

observation, we aim to identify potential continuities and ruptures in the teaching of polynomials, by examining students' conceptions about them. We address this question by presenting an experiment carried out in 2023 (Veuillez--Mainard, 2023). We first provide a brief overview of prior research related to polynomials, and sum up the stakes identified in a curricular analysis we conducted. Then, we describe the methodology of our experiment and present an *a priori* analysis of the questionnaire we designed. Finally, we share the results of the experiment.

## **PREVIOUS WORKS ON POLYNOMIALS**

Previous studies highlighted that the teaching of polynomials is likely to involve challenges stemming from various domains. Most of studies on polynomials either focus on the teaching of specific families of polynomials (tangent lines (Montoya Delgadillo et al., 2016), second-degree polynomials (Chaachoua et al., 2022)), or on the relations between these families (Buck, 1995). Polynomials are also encountered in literature on other subjects: expansion and factorisation of algebraic expressions (in the beginning of high-school), real functions, calculus, Taylor expansions ... This implies a wide array of potential difficulties and obstacles in the teaching of this notion. Specific studies on these objects and their teaching are therefore necessary.

Bolondi et.al (2020) showed that the very definition of a polynomial can be ambiguous in textbooks, by analysing the *definition schemes* used for terms like variable, algebraic expression, literal equation, etc. Dede and Soybas (2011) examined the *concept images* of polynomials for preservice mathematics teachers. They identified several conflicts with the formal definition of a polynomial, e.g. with regard to the definition domain of coefficients, or to the degree of a polynomial. They also showed that some students define polynomials as equations. In her PhD, Pleština (2023) conducted an epistemological and didactical study of the teaching and learning of polynomials. She describes the genesis and development of the notion, carries out an analysis of the knowledge to be taught in several institutions and examine students' understanding and difficulties, in particular at the secondary-tertiary transition, in Croatia.

Following on from these studies, we aim to identify students' conceptions of this subject at the transition from secondary to tertiary education.

## **CONTEXTUAL ELEMENTS ON THE FRENCH CURRICULUM**

The French secondary high school is divided in three classes: seconde (grade 10), première (grade 11), terminale (grade 12). In tertiary education, mainly two institutions offer advanced mathematics courses: university and "classes préparatoire" (preparatory classes), the latter leading students to engineering schools. In high school and "classes préparatoires", the curriculum is national whereas each university chooses its own syllabus. We summarize in Table 1 the contexts where students meet polynomials, from grade 11 to the beginning of tertiary education.

It should be noted that in high school, only the notion of a polynomial function is introduced, and called a "polynomial":

“The notion of a polynomial function can be freely used, more simply called polynomial”  
(Ministère de l’Éducation nationale et de la Jeunesse, 2019)

grade 11 - mathematics major	grade 12 - mathematics major	grade 12 - experts mathematics option	Scientific “classes préparatoires” (first year of higher education)
- Solving in $\mathbb{R}$ of second-degree equations - Reference functions for derivation and table of variation	- Reference functions for limits, continuity, convexity and primitives	- Solving in $\mathbb{C}$ of second-degree equations - Definition of any real polynomial function	- Formal polynomial, arithmetic of polynomial rings - Example of vector space - Reference function in calculus - Reference approximation function (Lagrange polynomials, series expansions...)

**Table 1: curriculum regarding polynomials**

We sum up here the stakes of the teaching of polynomials that we identified (Veuillez-Mainard, 2022). In high school, all the textbooks’ tasks deal with polynomials of degree 2 or 3, and exceptionally of degree up to 6. The properties at stake are mostly specific to second-degree polynomials, e.g. in quadratic-equation solving. In tertiary education, students encounter high-degree polynomials, explicit polynomials of arbitrary degrees, and arbitrary polynomials. Moreover, polynomials appear through new mathematical theories, such as polynomial arithmetic, vector spaces or series expansions. Formal polynomials are introduced in higher education, sometimes with different definitions in different classes. Finally, we noticed that in grade 12 “spécialité mathématiques”, no new specific work on polynomials is initiated. Polynomial functions are supplanted by other reference functions, and it is not explicit in the textbooks that these new functions are not polynomial functions. This situation is closed to the one in Croatia (Pleština & Milin-Sipus, 2022).

## METHODOLOGY AND RESEARCH QUESTIONS

### Theoretical framework and research questions

Our research aims to investigate student’s conceptions on polynomials. For this purpose, we use the *theory of Conceptual Fields* (Vergnaud, 2009). It offers a cognitivist perspective on didactic questions, which appears to be relevant for studying individuals’ conceptions. We rely on the concept of *scheme*, that emphasizes the operational aspect of knowledge, as “the invariant organization of activity for a certain class of situations” (Vergnaud, 2009, p. 88). In our study, we investigate schemes through the search of operational invariants, such as theorems-in-action (propositions held as true by the students in their activity) or action rules (implicit rules that guide the action of the student). In this framework, to address the various representations of polynomials, we will draw on the *semiotic registers* (Duval, 2017).

This leads us to investigate three research questions: What are the students’ schemes on polynomials? In what way do these schemes evolve during the secondary-tertiary transition? What specific challenges do students face in the learning of polynomials?

## Methodology

In order to identify students' conceptions, we designed two questionnaires, one for high school and one for higher education, which allowed us to collect a great number of data to analyse. The questionnaires address similar tasks, considering the academic contexts, so as to be able to compare the answers. We hypothesize that contrasting the answers to the two questionnaires is relevant to understand some of the issues of the learning of polynomials at the secondary-tertiary transition. The themes addressed were guided by the issues raised by literature, the epistemological analysis of the subject, and the analysis of the French curriculum. We conducted an *a priori* analysis of the questionnaires, in order to identify which operational invariants are likely to appear, and then compared this analysis with the collected answers.

For high school, the questionnaires were submitted to grade 11 and 12 students following advanced mathematics options. For tertiary education, it was given to students in first year of bachelor of mathematics and to students in economics "classe préparatoire", who also learn advanced mathematics. Overall, the sample is composed of 31 high-school students and 87 higher education students.

### Design of the questionnaire

The questionnaire was designed around 3 themes: definition of a polynomial, solving of polynomial equations and roots, and polynomial functions<sup>1</sup>. The final questionnaire for university students can be found in the appendix of this document.

In the first theme, we investigate the student's proposal for a definition of a polynomial, and the recognition of polynomials. In order to identify students' operational invariants in the second task, we selected polynomial and non-polynomial expressions with various characteristics: linear combinations involving square roots; reciprocal and rational functions; polynomials with integer, rational, and irrational coefficients; polynomials of low, high and arbitrary degree; and polynomial equations. We also varied the forms of the proposed polynomials: factored, expanded, or hybrid forms; and a polynomial written with the summation symbol  $\Sigma$ .

For the second theme (equations and roots), we first asked students to solve quadratic equations. We chose 4 equations: a zero product of factors, a zero difference of two squares, an equality of a square with a negative number, and the expanded form of the equation  $(3x - 1)^2 = 0$ . The subsequent questions allow to examine the connections between roots, specialization, and factorization: one question focuses on the existence of roots of various polynomials, and another asks to describe the set of polynomials (or second-degree polynomials in high school) with a given root.

The last theme (polynomial functions) explores the properties of polynomial functions graphs. This allows us to observe changes of semiotic registers that students may use. We first asked to draw the graphs of monomial functions, based on the parity of the

---

<sup>1</sup> Our questionnaires have questions in common with or close to those in the questionnaire designed by Pleština. We were not aware of this other questionnaire when we designed and carried out our experimentation.

exponent. Then, we focused on the recognition of graphs of polynomial functions. We selected graphs of polynomial functions of degree 1,2,3 and 4, as well as the graphs of the exponential, sine, reciprocal and rational function. This type of task is uncommon for students, as we have not observed any textbook or exercise sheet offering a similar task, neither in high school nor in higher education. Finally, students were asked to study the optimum of a polynomial function of degree 2.

We detail in the following section some of the analyses of our experiment.

## RESULTS

We will focus on the results regarding three questions, selected to represent the three themes addressed. In this way, we will highlight some students' difficulties regarding the understanding of polynomials. All the percentages given in this section are expressed in relation to the number of students who addressed the question.

### Definition of a polynomial

Various definitions of a polynomial are given in high school and higher education. As a result, we did not expect high-school students and economics "classes préparatoires" students to give a definition of a formal polynomial, whereas university students are supposed to know this notion. We distinguished three groups of definitions in students' responses, characterized by the nature of the defined polynomial: polynomial as a null sequence from a certain rank, polynomial as a function, polynomial as an "algebraic expression" satisfying certain properties.

In the first group of definitions, students characterize a polynomial as a function, and most of them provide an algebraic expression of the form " $P = \sum_{k=0}^n a_k x^k$ ", or written in expanded form " $P = a_0 + a_1 x + \dots + a_n x^n$ ". This definition is consistent with the one given in grade 11 for quadratic polynomials and with the one given in grade 12 (option "mathématiques expertes") for polynomials of any degree. The term "function" appears for 56% of grade 11 students, 29% of grade 12 students and a third of higher education students. Several higher education students specify in this definition that polynomials are continuous, differentiable functions.

In a second group, students use a null sequence from a certain rank to define a polynomial, and provide an algebraic expression of the form " $P = \sum_{k=0}^{+\infty} a_k X^k$ ". This definition is given by 28% of university students. Within this group, two types of use of a null sequence can be distinguished. Some answers define a polynomial as an expression of the form " $P = \sum_{k=0}^{+\infty} a_k X^k$ " where  $(a_k)$  is a real sequence that vanishes from a certain rank. In other responses a polynomial is defined as a sequence, and the expression " $P = \sum_{k=0}^{+\infty} a_k X^k$ " is a notation. This definition is the closest to the expert definition of a polynomial as an element of a polynomial ring. However, these definitions aren't operational for some of the other tasks of the questionnaire.

In the third group (43% of high-school students, 40% of higher education students), the form of a polynomial is described in natural language, in particular the operations of the variable needed to obtain a polynomial. The nature of the polynomial is neither

a function nor a sequence; it can be a “mathematical object”, “an expression”, or may not be defined. Thus, few students resort to the formal definition of a polynomial: most only mention the general expression of a polynomial, and/or consider it as a function. These results are consistent with (Pleština & Milin-Sipus, 2022). The authors identify similar categories of answers: polynomial as an algebraic equation, as an algebraic expression, as a function or a formal polynomial.

We have also identified definitions that exhibit characteristics from several of the groups described above. For instance, some students provide an algebraic expression of a polynomial without explicitly stating that it is a function, and some add that the expression is continuous and differentiable. Some university students define a polynomial as a formal object but without employing the notion of a null sequence: they use the uppercase  $X$ , sometimes specifying that it is called the indeterminate.

To conclude, we’d like to point out that the work on definitions is at the heart of the secondary-tertiary transition. Indeed, very few high-school students specified the nature of all variables involved in their definition: the unknown  $x$ , its exponents, and the coefficients. In higher education, 37% of students still did not specify that the coefficients are real, and 20% did not explicitly consider the case of integer powers.

### **Equation solving**

In the textbooks, most of the tasks focus on the use of the discriminant for solving quadratic equations. However, all the equations that we selected for the questionnaire can be solved without employing this technique. We noted that when it is possible to compute the discriminant after only one expansion, it is often used by students. Indeed, for the equation  $(x - 4)(x - 5) = 0$ , 30% of students used this technique, and over 70% for the other equations. This seems to indicate a lack of connection among students between roots and factorization, and the study of high-school textbooks supports this hypothesis too. These results are consistent with previous studies on the teaching of second-degree polynomials (Chaachoua et al., 2022). Some students also used the discriminant outside of its field of validity, for example on equations of degree 3, 4, and even of arbitrary degree. This suggests that some students, even in higher education, engage in purely syntactic work when solving equations.

Another technique of equation solving (present in grade 10 textbooks) is the factorization of the polynomial expression to reduce it to a product of first- or second-degree factors. It works for two of the equations of the questionnaire, using binomial squares formulas. In high-school textbooks, the use of those formulas for equation solving is systematically guided, leaving little room for initiative. Consequently, on the non-factored equation we proposed, no high-school student used this technique. In comparison, 30% of university students factored the polynomial  $(x^2 + 2)^2 - 9$  to find its roots. Note that some students both factored and calculated the discriminant to find the solution of  $9x^2 - 6x + 1 = 0$ . This may be because the discriminant technique is highlighted in high school, which could make it the only valid technique for students.

Finally, for the equation  $(x^2 + 2)^2 - 9 = 0$ , 80% of grade 12 students (i.e. 4 out of the 5 who tackled the question) and 24% of university students transform it into an equality of two squares. This kind of equation, worked since grade 10, poses difficulties for students. Most of them use erroneous theorems-in-action, such as:

1. The solutions of the equation  $x^2 = \alpha$  where  $\alpha \in R^-$  are  $x = \pm\sqrt{|\alpha|}$
2. The solution of the equation  $x^2 = \alpha$  where  $\alpha \in R^+$  is  $x = \sqrt{\alpha}$
3. If  $(\alpha, \beta) \in R^{+ \times R}$ , the equations  $(x^2 + \beta)^2 = \alpha$  and  $x^2 + \beta = \sqrt{\alpha}$  are equivalent.

These theorems-in-action sometimes coexist with the valid theorems during the same task. For instance, in higher education, out of the 9 students who transformed the equation into an equality of two squares, 6 used the third theorem-in-action to deduce that  $x^2 + 2 = 3$ , then all of them concluded with the implication  $x^2 = 1 \Rightarrow x = \pm 1$ . Such mistakes may arise from classical misconceptions about the square root.

### **Properties of polynomial function and semiotic registers**

In the third theme, we presented tasks involving polynomial functions. One of these tasks is the recognition of curves of polynomial functions. This provides a context for observing the semiotic registers that students implement in their answers. Indeed, they can mobilize algebraic properties of polynomial functions (number of zeros) and functional properties (limits, continuity) to justify some attributes of the graphs. They can also use the register of algebraic expressions if they provide an expression of the given function. This is an unusual task for students, and consequently 52% of high-school students and 35% of higher education students answer at least one of these questions without providing justifications.

We identified that some students identify graphs as polynomial graphs only if they have the characteristics of monomial graphs. This leads them to use criteria like symmetry to identify polynomial graphs. Besides, one student states that “a polynomial is either a parabola (if the degree of the polynomial is even) or of this form [...]” and draws the graph of an even-degree polynomial: this student does not differentiate between monomial and polynomial graphs.

Students were more successful in demonstrating that a graph does not represent a polynomial function. For the graph of the exponential function, 31% of grade 11 students, 86% of grade 12 students, and 53% of university students explicitly identify the exponential. However, only 3 of those students give a justification that the exponential is not a polynomial function. Among the other students, 6 higher education students (16%) justify that the graph is not polynomial by examining its limits at infinity. The continuity of polynomial functions is mentioned by a greater number of students, who succeed in justifying that the graph of a function with a discontinuity cannot represent a polynomial function. Finally, 7 students (18%) manage to justify that the graph of the sine function is not that of a polynomial function (noticing that a nonzero polynomial function has a finite number of zeros).



## CONCLUSION AND OUTLOOKS

Our experiment gave some answers to the research questions we introduced. We identified operational invariants regarding the recognition of a polynomial expression and the recognition of the graph of a polynomial function. For the first point, students use invariants that apply either to the overall form of the expression (“a polynomial is not factored” or “an expression is a polynomial if a degree can be identified”), to the operations of the variable (“the variable cannot be in the denominator of a fraction”, “an expression containing a power is a polynomial”), or to the coefficients in front of the variable (“the coefficients of a polynomial cannot be fractions”, or “should not contain a square roots”). While some of these operational invariants are valid, they are not sufficient to identify polynomial expressions in general. For the recognition of polynomial graphs, some students seem to identify graphs of polynomial functions with those of monomial functions. When solving a polynomial equation, most students aim to identify the coefficients of a second-degree polynomial, and then calculate the discriminant. This rule is sometimes extended to polynomials of degrees greater than 2, provided that the expression comprises 2 or 3 terms, which they identify as the coefficients of a polynomial. Specific theorems-in-action have also been highlighted in solving equations of the form  $x^2 = \alpha$  or  $(x + a)^2 = \alpha$ .

We have also noticed that the action rule “to answer a question about a polynomial, I start by expanding it” is used by students beyond the context of equation solving, for instance for setting up a table of variation. We also observed it in the question “is  $-2$  a root of  $(x + 2)^2 - 1$  ?” that 45% of grade 12 students expanded the expression. For most of the questions, this method is not effective, and leads to more errors.

We then question the evolution of students' schemes during the secondary-tertiary transition. The emergence of a new definition (formal polynomial) at university leads students to produce new types of definitions, including hybrid ones that combine a functional and an algebraic vision of polynomials. We also observed that higher education students are better able to define all the parameters involved in the definitions they provide. Higher education students performed significantly better on the recognition tasks, having more properties and tools available on polynomial functions. For example, more than half of high-school students answer that  $x + 1$  is not a polynomial, and 38% answer that the graph of an affine function is not a polynomial graph. Most of higher education students correctly handle these questions. Both secondary and higher education students are familiar with quadratic equation solving, but only higher education students managed to handle the 4<sup>th</sup>-degree equation.

These analyses provide clues about potential difficulties students face during the transition from secondary to higher education. The definition of a polynomial seems complex for students, especially since only the definition of a real polynomial function is provided in high school. In contrast, in higher education, the definitions of a formal polynomial, a real polynomial function, and sometimes a polynomial function in any ring coexist. Furthermore, few tasks involve non-real polynomials, which may latter

confuse students when regarding the utility of the concept of a formal polynomial. Similar difficulties have been observed in Croatia by Pleština (2023). For equation solving, the systematic use of the second-degree discriminant makes computation errors more likely and could prevent polynomial arithmetic comprehension. Moreover, this technique cannot be directly used in higher-degree equations, while students sometimes try to. Finally, we have noted difficulties regarding the graphical representation of monomial and polynomial functions, both at the high school and university levels. Students seem to be familiar with the general shapes of monomial graphs, but face challenges in determining their relative positions. This raises questions about their ability to mobilize different semiotic registers, and about the links they make between the relative positions of monomial curves and classic inequations such as  $x \leq x^2$  when  $x \geq 1$ . Moreover, if some students manage to apply properties on limits and zeros of polynomial functions, few succeed in recognizing graphs of polynomial functions of degree greater than 3.

## REFERENCES

- Bolondi, G., Ferretti, F., & Maffia, A. (2020). Monomials and polynomials: The long march towards a definition. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, 39(1), 1–12. <https://doi.org/10.1093/teamat/hry015>
- Buck, J. C. (1995). Fostering Connections between Classes of Polynomial Functions. In D. T. Owens (Ed.), *Proceedings of the 17th Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1–14).
- Chaachoua, H., Pilet, J., & Bessot, A. (2022). The Analysis of Dominant Praxeological Models with a Reference Praxeological Model: A Case Study on Quadratic Equations. In Y. Chevallard, B. Barquero, M. Bosch, I. Florensa, J. Gascón, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Advances in the Anthropological Theory of the Didactic* (pp. 229–238). Birkhäuser Cham. [https://doi.org/10.1007/978-3-030-76791-4\\_17](https://doi.org/10.1007/978-3-030-76791-4_17)
- Dede, Y., & Soybaş, D. (2011). Preservice mathematics teachers' concept images of polynomials. *Quality & Quantity*, 45(2), 391–402. <https://doi.org/10.1007/s11135-009-9303-2>
- Dorier, J.-L. (Ed.). (1997). *L'enseignement de l'algèbre linéaire en question*. La Pensée Sauvage.
- Doukhan, C. (2020). Mathematical modelling in probability at the secondary-tertiary transition, example of biological sciences students at university. In T. Hausberger, M. Bosch, & F. Chellougui (Eds.), *Proceedings of the Third conference of the International Network for Didactic Research in University Mathematics* (pp. 199–207).
- Duval, R. (2017). *Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations*. Springer Cham. <https://doi.org/10.1007/978-3-319-56910-9>

- Gueudet, G. (2008). Investigating the secondary–tertiary transition. *Educational Studies in Mathematics*, 67(3), 237–254. <https://doi.org/10.1007/s10649-007-9100-6>
- Ministère de l'Éducation nationale et de la Jeunesse. (2019). *Programme d'enseignement optionnel de mathématiques expertes de terminale générale*.
- Montoya Delgadillo, E., Páez Murillo, R. E., & Vivier, L. (2016). Conceptions spontanées et perspectives de la notion de tangente pour des étudiants de début d'université. In E. Nardi, C. Winsløw, & T. Hausberger (Eds.), *Proceedings of the First conference of the International Network for Didactic Research in University Mathematics* (pp. 103–112).
- Pleština, J. (2023). *Epistemološka i didaktička analiza pojma polinoma* [Doctoral thesis]. University of Zagreb, Faculty of Science.
- Pleština, J., & Milin-Sipus, Z. (2022). The notion of a polynomial in the secondary-tertiary transition. *Proceedings of the Fourth Conference of the International Network for Didactic Research in University Mathematics*, 100–109.
- Vandebrouck, F. (2011). Perspectives et domaines de travail pour l'étude des fonctions. *Annales de Didactique et de Sciences Cognitives*, 16, 149–185.
- Vergnaud, G. (2009). The Theory of Conceptual Fields. *Human Development*, 52, 83–94.
- Veuillez-Mainard, J. (2022). L'enseignement des polynômes lors de la transition secondaire-supérieur. *Repères IREM*, 128, 39–61.
- Veuillez--Mainard, J. (2023). *L'enseignement et l'apprentissage des polynômes à la transition secondaire-supérieur* [Master's thesis]. Université de Montpellier.

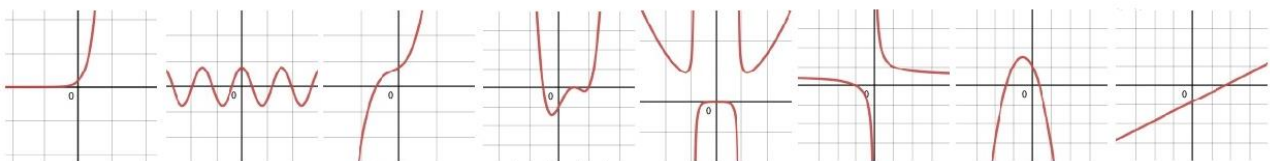
## APPENDIX 1: QUESTIONNAIRE (UNIVERSITY VERSION)

- A student in your class was absent during the lesson about polynomials. Write them a definition of a polynomial. Provide examples.
- How would you explain the use of polynomials in mathematics to a student in your class?
- Which of the following expressions are polynomials?
 

$\frac{x^2}{2}$       $x^2 + \pi$       $3x^{12} + \sqrt{x} - \sqrt{2}$       $\frac{x}{2} + ax + x^5$  ( $a \in \mathbb{R}$ )      $\sqrt{x} + 5$       $\frac{1}{x} - \frac{1}{x^2}$       $\frac{1}{x} + \frac{2}{x^4} - x^5$   
  $\frac{x + 5x^2}{x^3 - x^{23}}$       $x^n - x^{n-1}$  ( $n \in \mathbb{N}^*$ )      $x^5 - 3x^2 = 4$       $x + 1$       $x^2 - 2x + 1 = 0$       $(x - 5)(x + 12)(x^2 - x)$   
  $(x - 1)(x + \sqrt{2}) + 3x^4$       $(x - 1)^2 + 5$       $\sum_{i=0}^5 (-1)^i x^i$       $x^{10^6} + 1$
- Solve the following equations with unknown  $x \in \mathbb{R}$  providing the details of the solution.
 

A.  $(x - 4)(x + 5) = 0$     B.  $(x + 1)^2 = -12$     C.  $(x^2 + 2)^2 - 9 = 0$     D.  $9x^2 - 6x + 1 = 0$
- Do the following polynomials have (at least) one real root? Justify your answer.
 

A.  $P(x) = (x - \sqrt{2})(x^n + x - 1)$  ( $n \in \mathbb{N}$ )    B.  $Q(x) = x^n - x^{n-1}$  ( $n \in \mathbb{N}^*$ )    C.  $R(y) = y^5 - 3y^2 + 2$     D.  $S(x) = x^{1000} + 2$
- Which polynomials vanish at 5?
- Let  $P$  be a polynomial such that for all  $x \in \mathbb{R}$ ,  $P(e^x) = 0$ . What can be said about  $P$ ?
- Plot on the graph below the shapes of the functions  $x \mapsto x^n$ . Differentiate the cases based on the parity of  $n \in \mathbb{N}^*$ .
- Can the following graphs represent polynomial function? Provide a justification for each curve.



- Does the function  $f : x \mapsto (x + 3)^2 - 1$  have an optimum (maximum, minimum)? Justify. If so, determine it.