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Mathematics in mathematics courses and mathematics in physics courses: toward a comparison method using praxeologies

Pauline Hellio¹, Ghislaine Gueudet¹, and Aude Caussarieu²

¹Université Paris-Saclay, UR Études sur les Sciences et les Techniques (EST), France, pauline.hellio@universite-paris-saclay.fr; ²ENS de Lyon, UMR Interactions, Corpus, Apprentissages, Représentations (ICAR), France

Our aim in this paper is to develop a method to analyse the similarities and differences between mathematics in mathematics courses and mathematics in physics courses in the first year of university. Referring to the anthropological theory of the didactic, we propose an initial method using the concept of praxeology, and we test it by analysing solved exercises from a mathematics and a physics textbook. We identify types of tasks that are present in both; sometimes, these types of tasks intervene in physics as ingredients of techniques. We then compare the associated praxeologies in the two disciplines. We also uncover physics types of tasks containing elements where mathematics and physics are intertwined. Lastly, we discuss what we believe are necessary evolutions of the initial method.

Keywords: Teaching and learning of mathematics in other disciplines, Teaching and learning of analysis and calculus, Anthropological theory of the didactic, Mathematics in physics courses.

INTRODUCTION

The difficulties encountered by first-year ‘non-specialists’ students due to mathematics is an international issue that is increasingly being studied by research in mathematics education (González-Martín et al., 2021). Some of these difficulties come from differences between the mathematics in mathematics courses and the mathematics in other disciplines courses (Taylor & Loverude, 2023). Identifying these differences is thus a crucial first step towards a better understanding of the causes of students’ difficulties and the design of interventions. The study we present here aims to design a method for a systematic investigation of such differences.

Adopting an institutional perspective, we refer to the anthropological theory of the didactic (Chevallard, 1999). In the next section, we present this framework as well as background literature related to our work. We propose an initial method for the analysis of the differences between mathematics in the mathematics courses and mathematics in the physics courses. We then test this method and discuss its affordances and potential areas of improvement. Concerning mathematics, we focus on three fundamental concepts of calculus: derivation, integration, and differential equations. Concerning physics, we focus on mechanics as a domain where these concepts frequently intervene. This work is part of a broader study on the difficulties encountered by first-year physics students in mathematics and how to overcome them.

RELATED WORKS AND THEORETICAL FRAMEWORK

A first approach to related works

Mathematics and physics education research has evidenced that the mathematics in mathematics courses differ from the mathematics in physics courses. Karam et al. (2019) drew on the history of the two disciplines to demonstrate deep epistemological differences. They also showed that different conventions are used in the communities of physicists versus mathematicians. Redish and Kuo (2015) argue that there are ‘dramatic differences in how the disciplinary cultures of mathematics and physics use and interpret mathematical expressions’ (p. 562). Taylor and Loverude (2023) showed that students at the university level perceive differences between mathematics in mathematics and physics courses and that they cannot transfer to physics what they learned in mathematics. The authors gave the students a graph displaying a given object’s position relative to time, and the students were asked to determine its velocity. They were not able to reinvest their calculus knowledge in this physics task. White Brahmia (2023), focusing on modelling in physics, observes that the ‘physical world’ and the ‘mathematical world’ are not separate. The activity of modelling involves hybrid knowledge, situated at the intersection and not usually taught in either course.

The anthropological theory of the didactic (Chevallard, 1999) is a socio-cultural theory with a strong focus on epistemological aspects. It is thus relevant to identify differences between mathematics in mathematics and physics courses. Other authors have already made this choice; we present their works after introducing our theoretical framework.

Theoretical framework

The Anthropological Theory of the Didactic (ATD, Chevallard, 1999) posits that knowledge is shaped by the institutions where it lives. According to the ATD, an institution is any legitimate social group; hence, the physics courses and the mathematics courses for first-year students can be considered as two different institutions. How knowledge is shaped in the institutions is analysed by the ATD with the concept of praxeology. A praxeology comprises four elements: a *type of tasks* T ; a *technique* τ to perform this type of tasks; a *technology* θ , which is a discourse explaining and justifying the technique; and a *theory* Θ , which is a more general discourse supporting the technology. The pair $[T, \tau]$ constitutes the praxis block, while the pair $[\theta, \Theta]$ is called the logos block.

A type of tasks gathers all the tasks with a similar aim, e.g., ‘Solve a differential equation’. Following Chaachoua (2020), we consider a technique to be a ‘set of types of tasks called technique ingredients’ (p. 110). A technique for solving differential equations can be composed of the types of tasks ‘Find a particular solution of the differential equation’ and ‘Solve a homogeneous linear equation’, amongst others. While the concept of praxeology has been mostly used in mathematics education research so far, it can be applied to other disciplines (or even to any human activity). In physics courses, the knowledge is shaped as physical praxeologies, and we are interested in the mathematics present in these physical praxeologies.

Praxeological approaches to the gaps between mathematics and physics courses

Referring to the ATD, González-Martín (2021) studied how integrals were used in physics courses regarding bending moments and electric potentials. In both cases, integrals appeared in the logos block of the praxeologies in a very different way from mathematics courses. Indeed, elements of mathematics and engineering were mixed, and several properties of the integral were implicit. Hitier and González-Martín (2022) investigated the use of derivatives to study motion in five mechanics and five calculus textbooks. They compared the associated praxeologies and identified significant differences. For example, the definition of the derivative using a limit barely appeared in the techniques used in mechanics, whereas it was present in about 50 % of the tasks within a kinematics context in calculus. They also found that these tasks only dealt with velocity, whereas in mechanics, acceleration was also often present. The authors concluded from their textbook analysis that these inconsistencies were likely to ‘impact students’ ability to connect derivatives with the notions of velocity and acceleration’ (Hitier & González-Martín, 2022, p. 307).

In physics education, Caussarieu (2022) studied the differences between practices of mathematics in mathematics and physics exercises. Using the ATD in a non-systematic way, she found that these differences could be grouped into four categories: 1) *different notations*, for example, for the derivative: \square' in mathematics and $\frac{d\square}{dt}$ or $\dot{\square}$ in mechanics; 2) *differences in the objects manipulated*, for example, physicists manipulate physical quantities, whereas mathematicians manipulate functions; 3) *different techniques for a similar task*, for example in physics, when asked to find the minimum of a function, one is expected to find where a derivative is null whereas in mathematics one also has to study the sign of the derivative; 4) *different types of tasks performed using the same notion*, for example, the logarithm is often used in integration tasks in mathematics whereas, in physics, students often use it as the reciprocal of the power-of-ten function.

Hitier and González-Martín (2022) demonstrated the feasibility of a systematic analysis of physics and mathematics textbooks to identify differences and similarities between praxeologies, but they limited their study to tasks involving derivatives and one-dimensional motion. The work done by Caussarieu (2022) suggests that these differences might be grouped into categories. In this paper, we would like to extend these works to get a more comprehensive and systematic view of the differences between mathematics in physics and mathematics courses. Nevertheless, comparing praxeologies in mathematics and physics is complex. In particular, we need to find relevant criteria to decide what is similar or different. Thus, the first step in our broader study, which we present in this paper, is to establish a comparison method.

Our research question is: How can praxeologies in a physics course be compared to praxeologies in a mathematics course, with the aim of identifying similarities and differences between the mathematics present in both courses?

PRESENTATION OF A PRELIMINARY METHOD

In this section, we present a method that uses praxeologies to analyse the mathematics present in a mathematics course and a physics course proposed to the same students. In the next section, we carry out a test of this method to identify its affordances and limitations. Given textual resources we generically call ‘the mathematics course’ and ‘the physics course’, the principles of this method can be described as follows.

Establishing the list of praxeologies present in the physics course.

We first build a list of praxeologies present in the physics course. To do so, we identify tasks from the physics course, which we then gather into types of tasks. We define a type of tasks by a verb (e.g., ‘Determine’) followed by a direct object (e.g., ‘the velocity’). Then, we study the techniques associated with these types of tasks and, when needed, determine the ingredients composing these techniques. Finally, we describe the technological discourse justifying these techniques. We recall that the ingredients of the techniques are themselves types of tasks, and we determine their associated praxeologies. We obtain a list of praxeologies, which we call physical praxeologies to refer to the fact that they were found in the physics course. For each type of tasks, we specify whether it appears directly (in which case, we call it primary) and/or as an ingredient of techniques (in which case, we call it secondary).

Identifying which physics praxeologies incorporate mathematics.

We then identify, among these physical praxeologies, the ones that incorporate mathematical elements, whether in the type of tasks, technique, or technology. To discern whether an element is, in fact, mathematical, we refer to a corresponding mathematics course’s syllabus. This step provides us with a list of physical praxeologies incorporating mathematics and, for each praxeology, whether the type of tasks is primary or secondary.

Identifying common types of tasks and investigating the associated praxeologies.

In the third step, we determine the praxeologies present in the mathematics course using the same method as for the physical praxeologies. We compare the mathematical praxeologies with the physical praxeologies to identify which types of tasks are present in both. For these types of tasks, we determine the praxeologies appearing in the mathematics course (called mathematical praxeologies). We label each element of each praxeology with an M, P or MP corresponding to the course where we found the element (mathematics, physics, or both), and we call types of tasks that appear in both courses common types of tasks. This provides us with a second list of common types of tasks and their associated praxeologies. For a given common type of tasks, we analyse these praxeologies at the scale of the techniques and the technologies.

Analysing the types of tasks only present in the physics course.

In the final step, we go back to our first list and consider the remaining types of tasks. We investigate their features, trying to understand why they do not appear in the mathematics course.

TEST OF THE METHOD

We test our method on texts corresponding respectively to a mechanics course and a calculus course, and we limit our study to the following mathematical concepts: derivation, integration, and differential equations, as these are central concepts of calculus in the first year of university. We chose a first-year mathematics textbook (Boualem et al., 2013) and a first-year physics textbook (Brunel et al., 2015) from the same series. We analysed all exercises from the mechanics section of the physics textbook and the derivation, integration, and differential equations chapters of the mathematics textbook. There is no prescribed textbook at the national or university scale in France. Therefore, our choice of textbooks was motivated by our want to ensure both a coherent editorial line between the textbooks and a relatively high number of solved exercises and examples. Moreover, these books can be found in many French university libraries and, in the case of universit  Paris-Saclay, are relatively frequently borrowed. We analysed 101 calculus exercises and 52 physics exercises and worked examples, and our reference for mathematical content was the first-year calculus course summary provided at universit  Paris-Saclay for physics-chemistry-geoscience students.

Identifying types of tasks common to mathematics and physics

Our method identified multiple common types of tasks that students have to perform in both disciplines. Comparing the types of tasks identified at the level of the exercise questions and sub-questions leads to the identification of three primary types of tasks that are common to mathematics and physics. These are $T^{\text{MP}}_{\text{CompDerivative}}$: ‘compute the derivative’, $T^{\text{MP}}_{\text{StudyVariations}}$: ‘study the variations’, and $T^{\text{MP}}_{\text{ShowConstant}}$: ‘show that a quantity or function is constant’. We present task examples for these primary types of tasks in Table 1 below.

Type of tasks	Task example in physics	Task example in maths
$T^{\text{MP}}_{\text{CompDerivative}}$	Compute the derivative of the angular momentum at O with respect to time.	Compute the derivative of f defined on \mathbb{R} by $f(x) = 3x^2 + 7x - \sin x$.
$T^{\text{MP}}_{\text{StudyVariations}}$	Study the variations of the mechanical energy of a system.	Study the variations of f defined on \mathbb{R} by $f(x) = x^4 + 2x^3 - 2x + 1$.
$T^{\text{MP}}_{\text{ShowConstant}}$	Show that the angular momentum is constant.	Show that any function verifying, for all x and y , $ f(x) - f(y) \leq k x - y ^\alpha$ for a given k and $\alpha > 1$ is constant.

Table 1: Common types of tasks appearing as primary types of tasks in physics

Only a minority of the primary types of tasks we identify in physics incorporate mathematics. Most often, the formulation of the types of tasks derived from the physics textbook does not explicitly contain mathematical elements. We contend this does not mean the corresponding praxeologies do not incorporate mathematics. Indeed, further

analysis of the techniques associated with physical types of tasks leads to the identification of additional mathematical types of tasks appearing as ingredients of techniques. These are $T^{\text{MP}}_{\text{CompAntiderivative}}$: ‘Compute the antiderivative’, $T^{\text{MP}}_{\text{SolveDiffEq}}$: ‘Solve a differential equation’, and $T^{\text{MP}}_{\text{CompIntegral}}$: ‘Compute an integral’. Moreover, two of the primary types of tasks we already identified also appear as ingredients of techniques. Table 2 presents them with the primary type of tasks for which they are found as ingredients of techniques.

Mathematical T	Types of tasks they appear in as an ingredient of technique
$T^{\text{MP}}_{\text{CompAntiderivative}}$	$T^{\text{P}}_{\text{detPosition}}$: ‘Determine the position’ $T^{\text{P}}_{\text{detEnergy}}$: ‘Determine the energy of a system’
$T^{\text{MP}}_{\text{SolveDiffEq}}$	$T^{\text{P}}_{\text{detPosition}}$ $T^{\text{P}}_{\text{detVelocity}}$: ‘Determine the velocity’
$T^{\text{MP}}_{\text{CompDerivative}}$	$T^{\text{P}}_{\text{detVelocity}}$ $T^{\text{P}}_{\text{detAcceleration}}$: ‘Determine the acceleration’
$T^{\text{MP}}_{\text{ShowConstant}}$	$T^{\text{P}}_{\text{showMoveUniform}}$: ‘Show that the movement is uniform’ $T^{\text{P}}_{\text{studyMoveForm}}$: ‘Study the form of movement’
$T^{\text{MP}}_{\text{CompIntegral}}$	$T^{\text{P}}_{\text{detWork}}$: ‘Determine the work of a force’

Table 2: Mathematical types of tasks appearing as ingredients of techniques in physics

This shows that we would have missed several types of tasks common to physics and mathematics if we had not expanded the analysis to the ingredients of techniques.

Investigating the techniques and technologies associated with common types of tasks

Our method allows us to question whether tasks are performed in mathematics and physics using the same technique or not. We illustrate this through the example of $T^{\text{MP}}_{\text{CompDerivative}}$: ‘Compute the derivative’.

Three techniques are observed in mathematics. They are $\tau^{\text{M}}_{\text{rateOfChange}}$: ‘Compute the limit of the rate of change at that given point’, $\tau^{\text{MP}}_{\text{operations}}$: ‘Use the operations on derivatives to break the problem down to usual functions for which the derivative is known’, and $\tau^{\text{M}}_{\text{thmCalculus}}$: ‘Use the fundamental theorem of calculus’. In physics, we observe the technique $\tau^{\text{MP}}_{\text{operations}}$ as well as several more techniques. They are $\tau^{\text{P}}_{\text{coordinates}}$: ‘Derivate each coordinate (using $\tau^{\text{MP}}_{\text{operations}}$) and use the formula $\frac{d\vec{v}}{dt} = \dot{x}(t)\vec{e}_x + \dot{y}(t)\vec{e}_y$ for $v = x(t)\vec{e}_x + y(t)\vec{e}_y$ ’, $\tau^{\text{P}}_{\text{slope}}$: ‘Determine the slope of the function’s graph at each point’, and $\tau^{\text{P}}_{\text{phyEq}}$: ‘use a physical equation giving the derivative as a function of other quantities’. We note that $\tau^{\text{P}}_{\text{phyEq}}$ is the only technique appearing with $T^{\text{MP}}_{\text{CompDerivative}}$ as a primary type of tasks.

Varied technologies appear in physics, for example, with $\tau^{\text{P}}_{\text{phyEq}}$. One example is Newton’s second law of motion, $\theta^{\text{P}}_{\text{Newton}}$: ‘The acceleration multiplied by the mass of an object is equal to the sum of the forces applied to it’. Another, in the case of the angular momentum, is the technology $\theta^{\text{P}}_{\text{am}}$: ‘The derivative of the angular momentum

at a point O with respect to time is equal to the vectorial product of the position vector and the sum of the forces'. This allows us to point out that usage contexts are varied in physics in comparison to mathematics: praxeologies involving derivatives, integrals and differential equations are exclusively found over a real interval or an open subset of \mathbb{R} in mathematics. In physics, an analysis of the technologies shows the question of the interval and whether the subset is open is never discussed. However, derivation praxeologies appear with both real-valued and vector-valued functions.

Praxeologies incorporating mathematics but appearing only in physics

Our analysis unearthed the existence of praxeologies incorporating mathematics that were present only in physics. These praxeologies have a physical primary type of tasks, and neither the associated technique nor the associated technology could be found in the mathematics textbook. However, the ingredients of techniques comprising the technique of these praxeologies used both mathematics and physics. We illustrate this through the example of exercise 18.7 (Brunel et al., 2015, pp. 373–374), which we propose a translated version of below:

An object is moving along a straight line. Its acceleration is given by $a = -\omega^2 x$, where x represents the distance of the object to an origin point O.

Determine the expressions of $x(t)$ and $v(t)$ knowing that the object is at O when $t = 0$ and has initial velocity $v_0 = 4 \text{ m} \cdot \text{s}^{-1}$.

The types of tasks identified through the wording of the exercise are $T_{\text{detPosition}}$: 'Determine the position' and $T_{\text{detVelocity}}$: 'Determine the velocity'. These are physical types of tasks. The solution reads as follows:

The acceleration $a = -\omega^2 x$ is actually a second-order constant-coefficients differential equation, whose resolution is described by theorem 30.13: $\ddot{x} + \omega^2 x = 0$, where $\ddot{x} = \frac{d^2 x(t)}{dt^2}$. The solution of such an equation is $x(t) = x_0 \cos(\omega t + \varphi)$, where x_0 and φ are constants to be determined with the initial conditions. (Brunel et al., 2015, pp. 773–774)

In this solution, we identify the following technique: $\tau_{\text{idSolve}}^{\text{P}}$: 'Recognise that a given equation is a differential equation and solve the equation'. This technique is made up of two ingredients of techniques: $T_{\text{solveDiffEq}}^{\text{MP}}$: 'Solve a differential equation', which is found both in physics and mathematics and $T_{\text{recoDiffEq}}^{\text{P}}$: 'Recognise that a given equation is a differential equation'. $T_{\text{recoDiffEq}}^{\text{P}}$ and its associated technique only appear in the physics textbook, and the technique blends mathematical and physical concepts. Indeed, to recognise a differential equation one must both know what forms a differential equation may take, which is a mathematical concept, and know that $a = \ddot{x} = \frac{d^2 x(t)}{dt^2}$, which is a physical concept.

The fact that recognising a given equation as differential is not practised in the mathematics textbook may have an impact on students' ability to perform this type of tasks in physics.

DISCUSSION

A systematic comparison of the mathematics in the mathematics course and the physics course using praxeologies is a challenging project. So far, such comparisons have been limited to exercises in a kinematics context (Hitier & González-Martín, 2022) or made in a non-systematic way (Caussarieu, 2022). The test of our tentative method evidences that some of our choices are relevant, while others need to be questioned or revised.

We note the relevance of decomposing the techniques (in particular in the physics course) into ingredients of techniques. Indeed, if we had not decomposed the technique down to multiple ingredients of techniques, we would have missed several types of tasks incorporating mathematics that are present in physics.

Another significant choice in our method was to characterise a type of tasks by a verb followed by a complement, e.g., ‘Compute the derivative’. This led us to describe types of tasks with a high level of generality and allowed us to find similar types of tasks in the mathematics course and the physics course. If we had opted for a more precise definition of the type of tasks, ‘Compute the derivative of a polynomial function’ and ‘Compute the derivative of a position’ would have been two different types of tasks. In this case, we would have had to conclude that there are no types of tasks common to the physics and mathematics courses. As evoked above, one of the difficulties we face is to find relevant criteria to decide what is the same and what is different when we compare the two courses. Saying that everything is different would not be productive for our final aim of supporting students transitioning between courses.

This test also enabled the identification of some limitations in the tentative method and perspectives of improvement. First, we found it sometimes difficult to build complete praxeologies in the physics course, particularly regarding technologies. We think that this is a limitation coming from our use of exercises and worked examples, and this can be overcome by adding other material, e.g., lecture notes. Second, the test of our tentative method suggests that techniques in physics tend to apply to a more specific subset of the tasks of a type of tasks, whereas the techniques observed in mathematics tend to be more general. We could include what Castela (2008) describes as the efficiency domain of a technique in our framework to investigate this issue. Third, we observed, in physics, the presence of types of tasks intertwining mathematics and physics: this is a key output of the test of our tentative method. Nevertheless, we have probably missed other explanations for the presence of these types of tasks and how mathematics intervenes in the associated praxeologies. We could complement the initial method with an analysis whose starting point would be technologies in physics that incorporate mathematics (and using, e.g., lecture notes). Last, some important differences are not captured by our method, like differences in notations. This suggests the need for evolutions linked with concepts likely to complement the praxeologies, e.g., the comparison method could investigate ostensives (Bosch & Chevallard, 1999) appearing in mathematics courses versus physics courses.

CONCLUSION

Our research question was ‘How can praxeologies in a physics course be compared to praxeologies in a mathematics course, with the aim of identifying similarities and differences between the mathematics present in both courses?’ To answer this question, we proposed a tentative method and tested it on exercises and worked examples from a mathematics and a physics textbook.

This test evidenced that some aspects of the tentative method are relevant. Incorporating elements of techniques in the description of physics praxeologies allows us to identify mathematical types of tasks present in the physics course and then analyse the associated techniques and technologies in each course.

It also evidenced that other aspects need to be refined or modified. We chose to consider broad types of tasks; the consequences of choosing more precise types of tasks must be further investigated. Moreover, we plan to complement the initial method with an analysis taking technologies as a starting point to avoid overfocusing on the praxis block. This evolution should also be linked with the use of other material, e.g., more theoretical parts of textbooks or lecture notes. Indeed, the test presented here produced results concerning mainly the praxis block since the technological aspects were not often described in the solution of the physics exercises. Moreover, we also intend to complement the method by referring to other concepts: ostensives in particular.

We will continue to work on the design of the method and, at the same time, use it for the identification of differences between the mathematics in a mathematics course and a physics course. This should enable us in the next stages of our work to analyse student difficulties to design interventions taking these differences into account.

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