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# A Problem-Driven Approach to XR in Education: Teaching Non-Euclidean Geometries through Virtual Reality

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## ABSTRACT

Immersive technologies are increasingly used in education, offering unique benefits in fields like mathematics that require spatial understanding. These tools enable learners to interact with virtual environments, providing hands-on experiences that enhance comprehension and retention of abstract concepts. This is particularly valuable in areas like non-Euclidean geometries, which are difficult to visualize using traditional methods. This paper explores our research on using virtual reality to teach non-Euclidean geometries, highlighting both the benefits and challenges.

**Index Terms:** XR, Educational Technology, Mathematics

## 1 INTRODUCTION

Digital technologies have become central to everyday life, especially for children growing up in a digital world. Among these, immersive technologies are gaining traction, particularly in education and training. Their accessibility and ability to replace costly physical setups [10], reduce safety risks [5], and offer interactive, engaging learning experiences make them highly valuable. Unlike other digital tools, immersive technologies allow first-person virtual experiences, enabling learners to interact with and manipulate environments in ways that traditional methods cannot.

Embodied learning theory [9] emphasizes that effective learning involves not just cognitive processes but active interaction with the environment. Immersive technologies foster this by offering hands-on, experiential learning opportunities that deepen understanding. They help learners visualize and engage with abstract concepts in a dynamic way, improving both comprehension and retention.

The potential of immersive environments in education is widely acknowledged, particularly in fields that require high engagement and spatial awareness. These environments enable learners to perform tasks that would be difficult or risky in the real world, while also enhancing motivation and knowledge transfer. This is especially valuable in mathematics education, where complex, abstract concepts challenge traditional learning methods. Immersive technologies provide a more intuitive approach, helping students better visualize and manipulate mathematical principles.

This paper outlines our research on using virtual reality to teach non-Euclidean geometries, which are particularly difficult to grasp and represent. Their complexity makes them hard to intuit, as they do not align with everyday spatial experiences. In Section 2, we will discuss the challenges and opportunities of integrating immersive technologies into education, and in Section 3, we will explore our work on using XR technologies for teaching advanced mathematics to post-secondary students.

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## 2 DISCUSSING THE RESEARCH PROCESS

### 2.1 A Problem-Driven Research Approach

We believe it is essential that our research is motivated by a clear, tangible added value, rather than simply expanding the use of immersive technologies for their own sake. The mere adoption of new technologies does not automatically lead to meaningful progress or improvement in the educational landscape [8]. It is crucial that their application is purposeful, addressing real challenges and enhancing learning outcomes.

In the context of immersive technologies in education, the primary objective should be to conduct research that responds to a specific educational need identified by experts in the relevant field, rather than seeking to implement these technologies broadly without a clearly defined problem to solve. This targeted approach ensures that the adoption of immersive tools is aligned with the actual requirements of educators and learners, making the integration of such technologies both effective and meaningful.

Our research emerged from a precise and well-articulated need within the field of mathematics: the visualization of three-dimensional non-Euclidean geometries (see Figure 1, right). These are highly complex and counterintuitive spaces that challenge traditional ways of thinking and are notoriously difficult to conceptualize. Without the aid of advanced digital tools, such as immersive technologies, these geometries are nearly impossible to visualize in a way that is both accurate and comprehensible. While it is possible to create simplified, two-dimensional models to represent these concepts, such models — although they can be drawn — still fall short of providing a genuine, intuitive understanding of how a world governed by such non-Euclidean geometries would appear. For example, Figure 2 shows how different a geodesic (i.e. straight line) on the 2D model for hyperbolic geometry looks compared to a geodesic (modeled by a cylinder placed around a geodesic) in the actual simulation of 3D hyperbolic geometry. The ability to immerse oneself in these virtual spaces is essential for gaining a deeper, more meaningful insight into their properties and behaviors (see Figure 1, left). Thus, our research focuses on leveraging immersive technologies to meet this critical need for better visualization, transforming abstract mathematical concepts into tangible, interactive experiences.

### 2.2 A Risk-Aware Research Approach

It is crucial to carefully assess the potential impact of integrating immersive technologies into education.

Firstly, there are significant costs associated with these technologies. Economically, institutions must make substantial investments in the necessary hardware and software. Environmentally, immersive technologies require considerable resources for production and energy consumption, raising sustainability concerns. Health-wise, there are risks linked to the prolonged use of immersive devices, especially for children and adolescents, with concerns about physical and cognitive well-being [6].

In addition to these financial, environmental, and health-related costs, technical challenges arise when implementing immersive

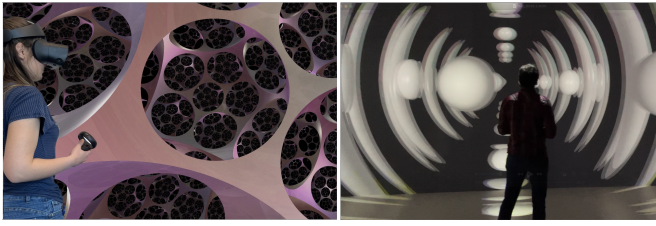


Figure 1: Left: The user is immersed through an HMD in a tiled hyperbolic space made up of sphere complements. Right: The user is immersed through a cave into the  $S^2 \times E$  geometry, within a scene featuring rows of spheres.

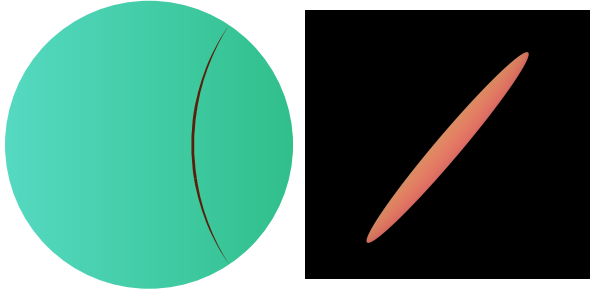


Figure 2: Left: 2D model of hyperbolic geometry with a geodesic in red. Right: Screenshot of an infinite cylinder placed around a geodesic in 3D hyperbolic geometry. Detailed explanations can be found in Section 3.4.

technologies in education. Institutions may face issues such as system glitches, high startup efforts, and unfamiliarity with the XR environment, all of which can slow adoption [2]. Moreover, there is a significant need for comprehensive training for educators, as teachers must adapt to new pedagogical methods and effectively use these tools [4]. These factors highlight the complexity of integrating immersive technologies and emphasize the importance of a thoughtful and cautious approach.

Our research, currently in the experimental phase and not yet deployed, focuses on an adult audience, which alleviates concerns related to the health of child users. Additionally, we continuously assess our experiments to account for the various limiting factors mentioned earlier, ensuring that we ultimately propose a solution that is both practical and usable for the target users.

### 2.3 A Humble Research Approach

As emphasized in the introduction, immersive technologies hold tremendous potential and, when applied thoughtfully, can be a powerful tool for enhancing education and training. These technologies open up new avenues for learning, providing unique opportunities to engage with complex concepts in ways that were previously unimaginable. However, it is crucial to maintain a sense of humility and recognize that while a technology may be revolutionary for teaching one subject, it may be completely unnecessary, or even counterproductive, for teaching another.

In the context of our research, the potential of immersive technologies is evident. They enable the visualization of non-Euclidean geometries in a manner that is entirely new, offering a novel perspective on these abstract, counterintuitive spaces. This capability provides a significant advantage, allowing learners to interact with and explore these complex concepts in ways that traditional methods cannot match. However, we also understand that digital technology’s ability to reveal the “invisible” or abstract does not automatically make it the best approach for every educational sce-

nario. While immersive technology can offer powerful insights into certain aspects of non-Euclidean geometry, it is not necessarily superior for all aspects of teaching this field. Some elements of the subject matter might be better suited to traditional methods or may not require the immersive experience at all.

For this reason, our approach has been to carefully consider where immersive technology provides a distinct and undeniable advantage in learning outcomes, and where traditional teaching methods may still be just as effective, if not more so. We have been deliberate in identifying the specific aspects of non-Euclidean geometry where technology enhances understanding and engagement, and conversely, where the use of immersive tools may not add significant value or could even be a hindrance. In doing so, we aim to ensure that the integration of immersive technologies is not just a trend, but a meaningful improvement to the learning experience that complements and enhances existing educational practices, rather than replacing them indiscriminately.

### 2.4 A Technology Aligned with the Problem

XR encompasses a wide range of technologies, each with its own strengths and limitations. It is crucial to base the choice of technology on the specific problem and its context. In fact, both the context and the subject matter play a key role in determining which technology is most suitable. Factors such as the target audience, the number of simultaneous users, the age group of the users, the financial resources available to the institution, and the specific topic to be addressed are all important considerations.

We carefully considered which type of technology to use. Given the educational goals of our research, we ruled out high-cost, complex setups (such as CAVE systems) and focused on more accessible devices, such as screens and virtual reality headsets, which are more readily available to educational institutions.

Our next challenge was to select the most appropriate technology for different contexts. To address this, we conducted a comparative user study involving 40 participants aged 19 to 60 ( $\bar{X} = 24.3$ ,  $SD = 6.4$ ), half of whom were math experts and half were math novices. Each participant engaged with one of three modalities: slides, screens, or VR headsets, focused on various geometric concepts and properties. We assessed their level of understanding before and after the experience to track progress and compare the effectiveness of each modality.

Although the experiment did not yield statistically significant results for several reasons, it nonetheless provided valuable insights. These included identifying which pedagogical modalities are best suited for different types of content, as well as understanding the entry costs associated with implementing virtual reality. Specifically, we observed a significant sense of disorientation among participants, which could be attributed to the combination of using the VR device and being immersed in an environment governed by rules that differ entirely from those of the physical world.

These insights help us refine our research focus, concentrating on areas where virtual reality offers a meaningful advancement, while also considering the time required for users to adapt to our simulations. Building on the results from the first experiment, we are currently conducting a second experiment focused solely on virtual reality. This new study leverages the strengths of VR technology to explore areas of non-Euclidean geometries identified as particularly suited to virtual reality.

### 2.5 Side benefits of XR in education

XR, as discussed, requires careful attention at every stage of research to ensure its application aligns with educational goals and enhances the learning experience. A systematic approach is essential to address the unique challenges and opportunities of immersive technologies, ensuring they are effectively integrated into teaching practices. When used thoughtfully, XR can make abstract concepts

more tangible and engaging. In addition to improving learning outcomes, XR can offer secondary benefits. For example, the enthusiasm some learners feel for immersive technologies can reduce their resistance to topics they might otherwise approach with reluctance. This shift in attitude can create a more open and receptive learning environment, encouraging students to engage with challenging material. Furthermore, the immersive experience allows learners to interact with content in ways that go beyond traditional methods. While immediate gains in knowledge or skills may not always be apparent, these experiences can broaden students' perspectives and deepen their connection to the subject matter. XR enables learners to explore new viewpoints, offering a more holistic understanding of complex concepts. Ultimately, when approached thoughtfully, immersive technologies can be powerful tools for both knowledge acquisition and cultural enrichment. By expanding how we interact with content, XR fosters critical thinking and creativity, encouraging learners to view problems from multiple angles and engage more dynamically with the material.

We observe these effects in our research on the use of virtual reality for visualizing and interacting with 3D non-Euclidean geometries. Immersion in these geometries can initially be quite disorienting, as users find themselves in a world where the fundamental rules of space have changed. It is not immediately apparent to them how to extract mathematical principles, models, and formulas — the key elements typically evaluated in mathematics courses — from this immersive experience. However, this very immersion allows users to see things differently, literally from a new perspective — that of an inhabitant of the space itself. This shift in viewpoint fosters an open-minded approach and facilitates a deeper understanding through experiential learning. Moreover, this immersive approach could enable individuals with no prior background in these mathematical concepts to become familiar with spaces they would otherwise never have been able to engage with.

### 3 AN EXPLORATION OF NON-EUCLIDEAN GEOMETRIES

#### 3.1 Contextualization

Non-Euclidean geometries are types of geometry that modify the classical principles of Euclidean geometry, the one typically taught in schools. In Euclidean geometry, one of the fundamental rules is that for a given line, there is exactly one parallel line passing through a point outside the line. Non-Euclidean geometries challenge this rule. In these geometries, the concept of parallel lines differs: either there are multiple parallels to a given line, or there are none at all. This leads to different properties for geometric shapes, such as triangles, where the sum of the angles may be different from what is observed in Euclidean geometry. These geometries are used to describe spaces where the rules of traditional geometry no longer apply, and they are crucial for understanding complex phenomena in mathematics and physics, such as the curvature of space or time. They show that space can be much more flexible and rich than what we perceive in our everyday experience.

The VR community has recently shown a strong interest in using XR for STEM education [13], as evidenced by the 3DUI Contest at IEEEVR 2024, which focused on the theme “Developing Next-Gen Calculators Using 3D User Interfaces.”. However, research on the use of XR for advanced mathematics is still limited, as is its application for students beyond the high school level in general. Highly advanced visualization solutions have already been developed, particularly in the medical field, but educational applications aimed at higher education are still rare.

#### 3.2 A Need Expressed by Mathematicians

These geometries, inherently different from the geometry learned in school and experienced in daily life, are therefore very challenging for students to grasp. This is why mathematicians have

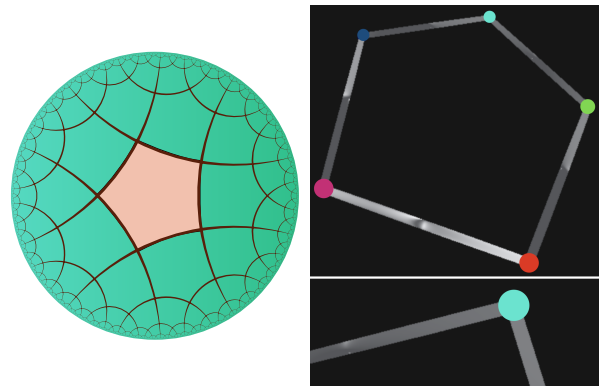


Figure 3: Left: 2D model of hyperbolic geometry with a right-angled pentagon in red. Right: Screenshot of the right-angled pentagon and a close-up of one angle in 3D hyperbolic geometry. Detailed explanations can be found in Section 3.4.

expressed the need for 3D environments to gain an intuitive understanding of these non-Euclidean geometries. Virtual reality provides a full immersion into these geometries, allowing students to experience these alternative geometries firsthand. For example, in non-Euclidean geometries, the properties of polygons change. While it is not immediately obvious, but still possible, to draw a right-angled pentagon on the 2D model of hyperbolic geometry, it is extremely difficult to truly perceive and experience these surprising five right angles. Virtual reality allows users to approach the angles and, through direct experience, observe these new geometric properties. Figure 3 illustrates this example.

#### 3.3 A Complex Problem on the Technical Level

We encountered three successive challenges in our research on using virtual reality for non-Euclidean geometries:

First, we needed to implement the non-Euclidean geometries and the mathematical laws that govern them. Overcoming this challenge requires close collaboration between computer scientists and mathematicians to select the appropriate models and gain access to the full mathematical description of the geometries to be implemented, before proceeding with the actual implementation.

Next, we had to develop a rendering system for these geometries. Indeed, standard rendering engines do not support non-Euclidean geometries, as they assume light rays travel in straight lines. Therefore, it was necessary to design a rendering engine capable of casting rays along the geodesics of the space being rendered. This work was carried out based on the approach developed by [3]. Other renderers of 3D non-Euclidean geometries include [12, 1, 7, 11].

Finally, once the environment is set up, the ongoing challenge is to determine the appropriate interfaces that will allow for control and interaction within these environments. There is a significant challenge in creating educational spaces based on these geometries. Users are often faced with highly disorienting spaces where it is not always clear whether they should interact according to Euclidean rules or those of the simulated geometry. Additionally, it is well-known that written text does not translate effectively into virtual reality, making it crucial to design environments that are intuitive and user-friendly for students, minimizing the need for extensive instructions or written explanations. Furthermore, as the environment is populated with basic mathematical objects, it is essential to work with simple building blocks to create an engaging and stimulating educational experience.

This work thus sits at the intersection of mathematics, XR, and pedagogy, aiming to produce a final product that addresses all of these challenges.

### 3.4 Illustrations

To illustrate this paper, we present screenshots taken in our simulation of 3D hyperbolic space, as well as drawings of the Poincaré disk.

The Poincaré disk is a geometric model used to represent two-dimensional hyperbolic space, which is an infinite, non-Euclidean space. In this model, points inside the disk correspond to points in hyperbolic space, while the boundary of the disk is not part of the space itself. Geodesics, or “straight lines” in hyperbolic space, are represented by arcs of circles that intersect the boundary of the disk at right angles. This model effectively captures the idea of an infinite, curved space, where distances and angles behave differently from those in flat, Euclidean geometry.

Figure 2 shows both the Poincaré disk and a geodesic, along with a screenshot of a geodesic in the 3D simulation. From this perspective, the geodesic in the simulation appears to be an oval and finite. This is because the perceived size of an object is related to the portion of the total view angle it occupies. In Euclidean geometry, a geodesic always spans the entire field of view, regardless of the observer’s position, extending from one end of the view to the other. In contrast, in hyperbolic geometry, the geodesic only occupies a fraction of the field of view due to the curvature of space. To illustrate this, imagine standing at the center of the disk, facing the geodesic with a 180-degree field of view. In this case, the geodesic would fit entirely within the center of your view, without crossing from one side to the other.

Figure 3 shows both the Poincaré disk and a right-angled pentagon, along with screenshots of the pentagon in the 3D simulation. The construction of a right-angled pentagon is made possible by the curvature of hyperbolic space. While the right angles are not immediately obvious when viewing the pentagon in the disk model, the simulation allows a closer examination of each angle, revealing that they are indeed right angles. The reason why angles appear not to be right angles from a distance, but seem correct as you get closer, is due to the curvature of the space. Objects or shapes near the observer appear to behave as if they were in Euclidean space, whereas objects further away are more strongly affected by the curvature, distorting their appearance.

### 3.5 Progress Report

We have been working on the project for a year now and have conducted a preliminary study to gather early insights into the potential benefits of using virtual reality to explore non-Euclidean geometries. Building on these initial findings and observations, we are currently developing a second experiment that will allow us to deepen our understanding of which aspects of non-Euclidean geometries truly benefit from the use of virtual reality, as well as the pedagogical methods best suited to our context.

## 4 CONCLUSION

In conclusion, immersive technologies, particularly virtual reality, hold significant promise for transforming the way complex concepts are taught and understood, especially in fields like mathematics that require a high degree of spatial reasoning. Our research into the use of VR for visualizing and interacting with non-Euclidean geometries highlights both the potential and the challenges of integrating immersive tools into educational settings. By enabling learners to experience abstract mathematical spaces firsthand, VR provides a unique opportunity to deepen understanding and engagement with concepts that are traditionally difficult to grasp.

However, as we have discussed, the application of immersive technologies in education must be approached with careful consideration of the context, the specific educational needs, and the potential challenges that may arise. It is essential to ensure that these technologies are used purposefully and that their integration into the curriculum adds tangible value to the learning experience, rather

than serving as a mere technological novelty. Moreover, while VR offers distinct advantages for certain aspects of learning, it is not a one-size-fits-all solution. The decision to use immersive technology should be driven by the specific requirements of the subject matter and the needs of the learners, with a clear understanding of when traditional methods may still be more effective.

Beyond the immediate educational benefits, our research also suggests that immersive technologies can generate valuable secondary benefits, such as increased engagement, a shift in attitude toward challenging content, and a broader, more open-minded perspective on learning. These benefits, while not always measurable in terms of immediate academic outcomes, contribute to the holistic development of learners and expand the possibilities for learning in the digital age.

Ultimately, our research underscores the importance of adopting a thoughtful, problem-driven, and context-aware approach when implementing immersive technologies in education. With careful planning and ongoing evaluation, immersive tools like VR can offer powerful new avenues for enriching the learning experience and fostering deeper, more meaningful connections with the material.

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