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REPAIRABLE SYSTEM RELIABILITY WITH HIDDEN FAILURE OF CONTROL DEVICES

VINCENT COUALLIER*

*Univ. Bordeaux, Institute of Mathematics of Bordeaux
34000 Talence, France
email: vincent.couallier@u-bordeaux.fr*

The aim of this paper is to describe how the reliability characteristics of a system living in a controlled environment with constant stress may be modified by taking into account of random failure of the control device. Such issue arise when reliability objectives have been demonstrated under strong assumption of constant environmental conditions, but some device that controls the environment (temperature, pressure, voltage, current intensity), may also be subject to failure. Standard results obtained by applying a cumulative damage model are introduced to provide updated predictive reliability characteristics (reliability function, mean time to failure) that mix reliability distributions of the principal component and the control device. An application on real data in the aeronautic field is proposed. The statistical analysis is also addressed and illustrated with simulated data under different observation and maintenance policies.

Keywords: Accelerated Failure Time models, time-dependant covariate, repairable systems, maintenance policy, dynamic stress

1. Introduction

In the framework of reliability analysis of repairable systems, the very well known standard AGAN model (Asher and Feingold [5]) assumes strong assumptions that may not be fulfilled in practice. First and foremost is the hypothesis of independent and identically distributed interarrival times of failure. Secondly is the testability of items, that induces the ability to monitor the state of the system, detect the failure immediately, and apply a perfect corrective maintenance.

In that paper, we maintain the assumption that the corrective maintenance is perfect (for instance by assuming that the replacement of a failed unit is done by a new one, whose reliability distribution does not change). We also assume that the system is sufficiently monitored to ensure that the

*Corresponding Author

failure is detected as soon as it occurs, and lead to a replacement of the failed unit.

These restrictions are well adapted to issues encountered in the aeronautic or automotive industry (the present work has been motivated by a real problem in aeronautics) : assume that electronic components are placed in a container where the temperature is maintained to a constant value by an air cooling system. The reliability demonstration test of the electronic system was planned and done with an objective fixed by engineers to a given temperature x_0 . The issue is to guarantee the same reliability objective by assuming that the air cooling system may fail, and thus loose the ability to provide the target temperature x_0 and deliver a higher temperature (say x_1). For that, it is assumed that the control device has a known reliability distribution, but its failure may be hidden or detected with a delay of repair. We thus face the problem of dormant failures of sub-system (control) that may precipitate the failure of the principal system due to the elevation of stress. Furthermore, we must consider maintenance aspects, such that inspection policies or testability of devices state, in order to fulfill a reliability objective, often given in terms of MTBF for repairable systems.

A lot of work has been done on the analysis of dormant or hidden failures of complex systems, most of them tackling the problem of optimizing the maintenance policy according to specific criteria, see ([1]) for the mathematical treatment with Markov processes and independent Poisson processes, [2] for an application in aircraft reliability, ([3]) in the medical field, or ([4]) in the electrical power systems industry. To our knowledge, this paper is the first attempt to provide a stochastic model that addresses the issue of the accelerated lifetime of a principal system precipitated by a hidden failure of a control sub-system.

In this paper, we provide closed form expressions for reliability functions and mean time to failure (MTTF) under various hypotheses of monitoring and maintenance policies. We then analyse the effect of the repair duration on the reliability of the principal system.

An exponential distribution for the cooling part and an Arrhenius-Weibull distribution for the principal component are assumed to illustrate the point.

We finally consider the statistical analysis of such data.

2. The model and the notations

Let us denote by T the time to failure of a main system, and $(T_i)_{i \geq 1}$ the sequence of i.i.d. successive duration times between failures of a renewal process. It is assumed that a replacement is immediately done and that the T_i 's are observed.

The environmental conditions (resumed in the covariate $x \in \mathbb{R}^p$) may affect the reliability of the system. Let us denote by R_0 the reliability function of T under the assumption of constant stress x_0 .

For the sake of clarity, we pose $p = 1$ and assume a parametric model $R(t) = P(T > t) = R(t, \theta)$ under the following accelerated failure time model: Denoting by θ_0 the parameter value under the stress $X = x_0$ and θ_1 the parameter value under the stress $X = x_1$, we assume that there exist a function AF such that

$$R_{x_1}(t) = R(t, \theta_1) = R(AF(x_0, x_1)t, \theta_0) = R_{x_0}(AF(x_0, x_1)t). \quad (1)$$

The reliability distribution usually lies in a "log-location-scale" family such that Weibull, Log-Normal, or Log-Logistic, see Meeker ([6]), Lawless ([7]), for further details.

The standard AFT model for constant stress is generalized to time-varying stress $x_{(\cdot)} = (x(t), t \geq 0)$ through the very natural assumption of cumulative damage (Nikulin et al. [8]):

$$R_{x_{(\cdot)}}(t) = R_{x_0} \left(\int_0^t AF[x_0, x(u)] du \right). \quad (2)$$

For the one-step stress $x(t) = x_0 \mathbf{1}_{0 \leq t \leq e} + x_1 \mathbf{1}_{t > e}$, with known change point e , (2) reduces to

$$R_{x_{(\cdot)}}(t) = R_{x_0}(t) \mathbf{1}_{t \leq e} + R_{x_1}(t - e + e^*) \mathbf{1}_{t > e} \quad (3)$$

where the time e^* is determined by $R_{x_0}(e) = R_{x_1}(e^*)$, i.e. $e^* = e/AF(x_0, x_1)$. We thus may rewrite the reliability under known step-stress:

$$R_{x_{(\cdot)}}(t) = R_{x_0}(t) \mathbf{1}_{t \leq e} + R_{x_0}(AF(x_0, x_1)t - e(AF(x_0, x_1) - 1)) \mathbf{1}_{t > e}. \quad (4)$$

The figure 1 represents the cumulative damage AFT model under one-step stress with known change point.

To conclude this section, let us define a model for the control sub-system and the monitoring and maintenance policies. Assume that a device is designed to maintain a constant stress to a nominal value x_0 . We consider

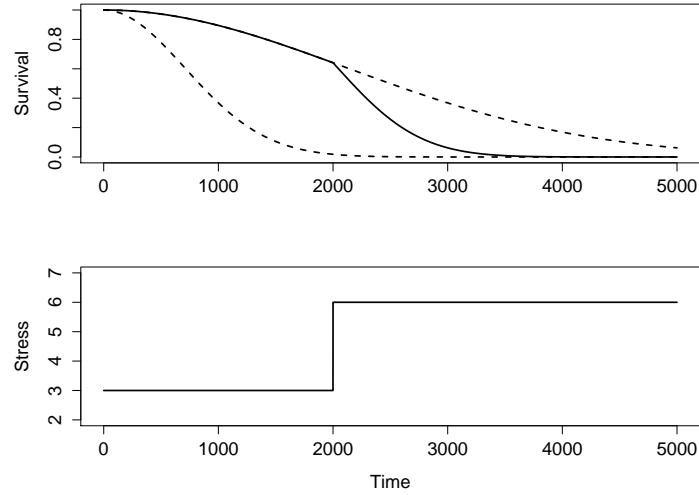


Figure 1. AFT model under one-step stress with known change point

that this control device may fail and denote by E the time to failure, with reliability function R_E and density function f_E . Hence, the principal system lives under a constant stress value x_0 until the failure of the control device, then switch to a non-controlled stress, which we shall assume to take a higher constant value x_1 . This time-varying stress model for T is evidently related to the previous one, except that the change-point is random.

The monitoring and maintenance policies that we consider are of two types :

- *Hidden non repaired control (HNRC)*: The principal system is continuously observed. At the time of failure, a perfect corrective maintenance of both principal and control system is carried out in a negligible duration. If a failure of the control device was hidden, then the stress level recovers the lower value immediately at the repair time. Note that in case of non failed control device, the replacement is nevertheless done resulting to a preventive replacement.
- *Observed repaired with delay control (ORDC)*: The monitoring allows the testability and the perfect observance of the control device's state as well as the principal system. The repair of the control device has a fixed known duration ΔI during which the principal

system, if alive, undergoes an overstress of value x_1 .

Obviously, the ORDC policy with $\Delta I = 0$ coincides with a perfect reliable control device providing a constant stress x_0 for the principal component. Both policies are still compatible with a AGAN renewal model for the principal system, the main difference being in the expression of the reliability function to be considered.

In the rest of the paper, we provide closed forms for reliability functions and mean time to failure, analyse the effect of the duration ΔI on the reliability - a major issue for practitioners, and conclude with a short remark on the statistical analysis of data.

3. Reliability analysis

3.1. Reliability of system with Hidden and non repaired control device HNRC

Denoting by E , the hidden random time of failure of the control device, the stress experienced by the principal system takes the form :

$$x(t) = x_1 \mathbf{1}_{t \leq E} + x_2 \mathbf{1}_{t > E}$$

where x_0 and x_1 are the lower and upper stress values. Assume that E has a density f_E then it can be shown that the reliability function of T is :

$$\begin{aligned} R_{x(\cdot)}(t) &= \int R_{x(\cdot)|E=e}(t) f_E(e) de \\ &= R_{x_0}(t) P(E > t) + \int_0^t R_{x_0}(AFt - e(AF - 1)) f_E(e) de, \quad (5) \end{aligned}$$

where AF is a short notation for $AF(x_0, x_1)$. It is worth noting that even if *log-location-scale* distribution families such that Exponential, Weibull or Log-Normal are stable with accelerated constant stress, the mixed distribution obtained in (5) does not lie in the original family of distributions. The figure 2 shows an example of $R_{x(\cdot)}$ for $R_{x_0} \sim W(3000, 2)$, $AF = 3$, $E \sim W(2000, 4)$.

We also get the following result for the mean time to failure (MTTF) under the HNRC-policy :

$$MTTF_{x(\cdot)} = MTTF_{x_0} - \left(1 - \frac{1}{AF}\right) \int_0^{+\infty} \left(\int_e^{+\infty} R_{x_0}(u) du \right) f_E(e) de \quad (6)$$

The equation (6) is the key point to analyse the loss of MTTF involved by a non reliable control device. The loss $MTTF_{x(\cdot)} - MTTF_{x_0}$ is clearly

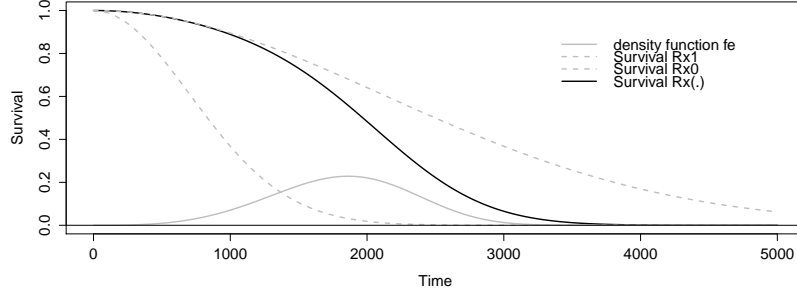


Figure 2. An example of th reliability function with hidden loss of control - HNRC

dependent of the acceleration factor and a complex relationship between the reliability function R_{x_0} and the density f_E .

3.2. Reliability of system with Observed and Repaired with Delay Control device ORDC

In this section, we assume that the control device is continuously monitored. The failure is thus immediately observed and triggers a repair action of fixed duration ΔI . Thus, for a fixed time period, the principal system lives under a higher stress x_1 , considering that the starting point of this overstress period is random. The stress profile takes the following expression:

$$x(t) = x_0 \mathbf{1}_{0 \leq t \leq E} + x_1 \mathbf{1}_{E < t \leq E + \Delta I} + x_0 \mathbf{1}_{t > E + \Delta I}. \quad (7)$$

The reliability function $R_{x(\cdot)}$ of the principal system under ORDC policy given in (7) takes the form:

$$R_{x(\cdot)}(t) = R_{x_0}(t)P(E > t) + R_{x_0}(t - \Delta I(1 - AF))P(E < t - \Delta I) + \int_{t - \Delta I}^t R_{x_0}(AFt - e(AF - 1))f_E(e)de, \quad (8)$$

The mean time to failure is also updated (compare with (6)):

$$MTTF_{x(\cdot)} = MTTF_{x_0} - \left(1 - \frac{1}{AF}\right) \int_0^{+\infty} \left(\int_e^{e + AF\Delta I} R_{x_0}(u)du \right) f_E(e)de$$

Note that theses equations generalize those given in the previous section by considering that $\Delta I \rightarrow +\infty$ could be interpreted as an absence of repair of the control device until the observed failure of the principal system.

Note also that the right-hand term of (8) is worth being minored which provides an easily handled expression:

$$R_{x(\cdot)}(t) \geq R_{x_0}(t) + P(E < t - \Delta I) [R_{x_0}(t - \Delta I(1 - AF)) - R_{x_0}(t)]. \quad (9)$$

The figure 3 shows an example of the ORDC policy that mimics the previous example in Figure 3.1, with $\Delta I = 300$.

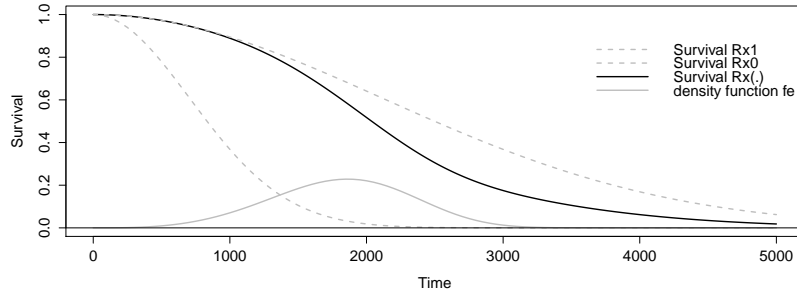


Figure 3. An example of reliability with observed and delayed repaired control- ORDC

4. Statistical Analysis of field data - Simulation study

As the above sections deal with predictive reliability, used in the design process of a system that requires reliability demonstration, we ask here the question of the statistical estimation of unknown parameters from various types of monitoring. The reliability function of a system undergoing a random stress due to failure of the control device has been provided in (5) and (8) according to the monitoring and maintenance policies HNRC and ORDC. Assume that $(D_i)_{i=1..n}$ are i.i.d. random variables with reliability function $R_{x(\cdot)}$. The likelihood of such data requires the derivation of the density function $f_x = -\frac{\partial}{\partial t} R_{x(\cdot)}$ where t appears both in the intergrand bounds and in the quantity to integrate. Such parametric integrals are derivated thanks to the well known Leibniz intergral rule for parametric integrals:

$$\frac{d}{dt} \left(\int_{a(t)}^{b(t)} f(x, t) dx \right) = f(b(t), t) \frac{db}{dt} - f(a(t), t) \frac{da}{dt} + \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t} dx$$

Heres, it reduces to (ORDC case):

$$f_x(t) = -\frac{\partial}{\partial t} R_{x(\cdot)}(t) = R_0(t)f_E(t) - R_0(t + \Delta I(AF - 1))f_E(t - \Delta I) + \frac{\partial}{\partial t} \int_{t-\Delta I}^t R_0(AFt - e(AF - 1))f_E(e)de.$$

We provide here the likelihood function for the ORDC data (the modification for the HNRC is straightforward):

$$\ln L(\eta_0, \beta, AF, \lambda) = \sum_{i=1}^n \ln \left[f_0(D_i)R_E(D_i) + f_0(D_i + \Delta I(AF - 1))F_E(D_i - \Delta I) + AF \int_{D_i - \Delta I}^{D_i} f_0(AF D_i - e(AF - 1))f_E(e)de \right]$$

This may easily be generalized to right censored samples.

Simulation studies show that the estimation of parameters is feasible, at least for standard parametric models. As an example, we have investigated the maximisation of the likelihood function by considering a Weibull distribution for the principal component under stress x_0 (with parameter η_0 and β), an accelerated failure time model with unknown acceleration factor AF between stress values x_0 and x_1 , an exponential distribution with unknown failure rate λ , and a fixed repair duration ΔI . The figure above shows for instance the contour profile of the log-likelihood $\ln L(\eta_0, \beta, \lambda, AF)$ in the plan (η_0, λ) , the other parameters being fixed to the estimated values.

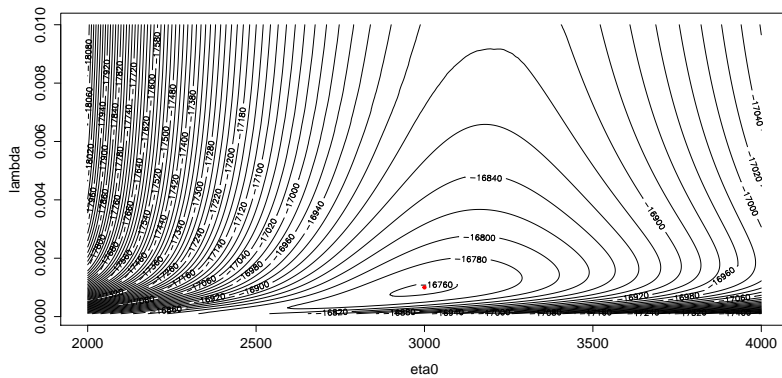


Figure 4. Log-Likelihood profile in (η_0, λ) for parametric ORDC simulated data

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