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ABSTRACT

This paper provides the first statistically validated experimental demonstration of a novel characterization procedure based on fast frequency sweeps. A MEMS structure with a time constant of several seconds is characterized in linear regime under mechanical actuation and optical detection of its motion. 450 series of fast sweeps are analyzed with different sweep durations to evaluate the precision and accuracy of the proposed approach compared to 450 traditional ring-down measurements. Our study shows that an estimation based on fast frequency sweeps in not only possible but even a competitive approach to estimate the parameters of a linear resonator with limited practical means. The precision and accuracy of this approach demonstrate its relevance.

1. Introduction

While MEMS (Micro-Electro-Mechanical Systems) technology is now becoming mature, new applications requiring unusual MEMS device geometries / dimensions, extreme values of system parameters, or novel operating principles present several interesting challenges and are still at relatively early development stages. Among these, resonant MEMS applications relying on the transduction of an inertial force, such as kinetic energy harvesters [1,2], micromachined gyroscopes [3,4] or resonant accelerometers, require either a very large quality factor Q and/or a relatively low resonant frequency f_0. During their development and production cycles, the values of f_0 and Q must be repeatedly assessed to ensure concordance with specifications. In practice, this can be very challenging because of the large response time $\tau = 2Q/\omega_0 = Q/\pi f_0$ of the resonators, which is typically on the order of 0.1 to 10 s.

The difficulties related to the characterization of a low-frequency resonant system get back to a very common issue in civil engineering, where the natural frequency of bridges can typically be in the range of 0.1 to 2 Hertz. Several procedures exist to characterize a bridge response, based either on ambient vibration, free vibrations or forced vibrations. Among them, forced sine-wave actuation with eccentric mass shakers installed into the bridge structure are still of interest and sometimes experimented to get the

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most accurate information about the response of the bridge at different actuation levels, each frequency sweep requiring thousands of seconds to perform [5]. In civil engineering, interesting alternative procedures exist to determine the frequency response of a bridge, for example based on passing vehicles which impose of forced (but uncontrolled) actuation [6]. However, the latter methods are specific to the context of civil engineering with relatively low quality factors and become quite impractical for high quality factor microscopic structures. When it comes to small-scale high-Q resonators, such as vibration energy harvesters, characterization is still generally performed with time-consuming quasi-static sine-wave frequency sweeps even in the most recent works [7].

In such contexts, steady-state characterization procedures relying on the measurement of a frequency response become impractical because of the sheer amount of time (typically a few hundred times τ) required to sweep slowly across the resonator bandwidth. Furthermore, precise control of the environment during the characterization process must be enforced, to avoid parameter drift, which makes steady-state procedures even more costly and difficult to set up.

In this respect, characterization relying on a transient regime, with a duration of the order of τ, are less demanding than steady state. The best-known method is based on "ring-down" measurements [4,8], which consist in measuring the freely decaying oscillations of the resonator. Several variants have been proposed over the years, extending its use from the case of a single-degree-of-freedom resonator to multiple-degrees-of-freedom, and/or nonlinear resonators [9–11]. Ring-down measurements are faster than steady-state ones, but they also have their own limitations. Any parasitic excitation during the free decay is a source of errors in the characterization results. For example, the actuator (e.g. a mechanical shaker) may not stop instantly when its supply voltage is turned off. Likewise, ambient vibrations impact the results of ring-down measurements, at least those performed on inertia-sensitive devices such as kinetic energy harvesters.

We have introduced in [12] an approach to estimate resonator parameters from fast frequency sweeps (with a duration of the order of τ) and illustrated it with simulations. This approach, which shares the same "Hilbert transform" formalism as the methods introduced by Feldman [13,14] in the field of vibration analysis, applies indifferently to linear or nonlinear resonators. Its practical use has been illustrated in the case of a piezoelectric gyroscope [12], operated in a well-controlled laboratory environment. In this paper, we demonstrate the potential of such fast frequency sweeps in a harsher environment, and compare, in this framework, its pros and cons with those of ring-down.

Performing a fast frequency sweep around the expected resonant frequency on a large interval (compared to the resonator bandwidth) is a common practical way to check if a resonator is working properly after fabrication and estimate its resonant frequency qualitatively. Fast frequency sweeps have several experimental advantages for qualitative observations. They are easy to set, require basic instrumentation and are quick to perform. However, no study has yet proven that these fast sweeps can be not only qualitatively but also quantitatively processed, to extract the natural frequency and the quality factor of the resonator with a precision that is comparable to other more classical methods. This paper aims at making this characterization procedure (including the experimental setup, the signal processing and the parameter estimation procedure) accepted by the scientific community as a valid option so that its adoption will grow as a convenient alternative to existing methods.

Our experimental validation is performed on a MEMS kinetic energy harvester with $f_0 \approx 40$Hz and $Q \approx 1000$. Its response time ($\tau \approx 7s$) is as large as more common MEMS resonators such as accelerometers and gyroscopes, which typically exhibit higher frequencies but larger quality factors. The setup includes a mechanical actuation by a shaker and an optical detection by a laser. After a statistical analysis of 450 fast sweeps of different durations compared to 450 ring-down measurements, we analyze how our experimental procedure compares with ring-down measurements.

After the theoretical background in section II and some simulations in a close-to-ideal scenario, we develop, in section III, the full experimental and statistical validation of the characterization procedure. Section IV aims at helping experimenters choose between ring-down or fast-sweep measurements for the characterization of future state-of-the-art resonators. Although there is no ideal procedure that would be better under all circumstances, we detail the most stringent constraints impacting the characterization results to help experimenters strike the best balance and choose between ring-down or fast-sweep measurements.

2. Theory and simulation

2.1. Description of the parameter estimation method

This study focusses on linear second-order mechanical resonators with mass M, stiffness K and damping coefficient c. The resonator displacement $x(t)$ is then governed by (1).

$$M\frac{d^2x}{dt^2} + c\frac{dx}{dt} + Kx = F(t) \tag{1}$$

We assume that the force F may be written (2) where $\psi(t)$ is the instantaneous phase and a is the acceleration provided by the external frame (e.g. a mechanical shaker).

$$F(t) = Ma\sin(\psi(t)) \tag{2}$$

$\psi(t)$ is assumed to be either measured by the experimental setup or known by other means (e.g. imposed by the experimenter). We also recall the expression of the instantaneous angular frequency, defined as $\omega(t) = \frac{d\psi}{dt}$. This description applies both to ring-down measurements and to fast frequency sweeps. In the former, measurements are performed by initiating a vibration close to resonance until steady state is reached, and then turning off the actuator, setting $a = 0$. In the latter, frequency sweeps are performed by sweeping $\omega(t)$

close to resonance starting from an arbitrary initial state and keeping $a \neq 0$ constant.

We write the solution x of (1) in the form (3) where $X(t)$ is the complex slowly varying amplitude of $x(t)$.

$$x(t) = \frac{X(t)}{2}\exp(j\psi(t)) + \frac{\overline{X}(t)}{2}\exp(-j\psi(t)) \tag{3}$$

We may then recast (1) as (4).

$$2j\omega(t)\dot{X}(t) = -\left(\omega_0^2 - \omega(t)^2 + j\omega\frac{\omega_0}{Q}\right)X(t) + a(t) \tag{4}$$

The derivation of (4) may not be straightforward for a someone who is not familiar with the "slowly-varying" envelope approximation. Previous works derive this same result along with the corresponding justifications (e.g. [15]). Other older works or textbooks derive similar expressions, in various domains for control theory, as in [16]. Some authors even add a supplementary relation (equation (5) in [17]) to their definition of the complex amplitude so that the higher-order terms disappear.

In characterization procedures based on ring-down measurements, the parameter estimation is performed in the absence of actuation so that $a(t) = 0$. In the two other scenarios, $a(t) = a$ is constant. Our objective is to estimate the 3 parameters $p_1 = \omega_0^2, p_2 = \omega_0/Q, p_3 = a$ appearing in (4) from a collection of measurements $X_m(t_1)$, …, $X_m(t_N)$, with $N \gg 3$. Typically, for a single frequency sweep, a very large number of samples can be measured. In our setup, N is always superior to 10^4.

To this end, we replace the time derivative $\dot{X}$ appearing in (4) with a finite-difference approximation $\widetilde{\dot{X}}$ derived from the measured samples with a central finite difference scheme, as in [11]. Doing this for all t_k ($k = 1$ to $k = N$) transforms (4) into a set of N equations $\boldsymbol{y} = \boldsymbol{Fp}$ written (5).

$$\begin{aligned}[y_k] &= \left[\omega(t)^2 X(t) - 2j\omega(t)\widetilde{\dot{X}}(t)\right]_{t_k} \\ [F_k] &= [\,X(t) \quad j\omega(t)X(t) \quad 1\,]_{t_k}\end{aligned} \tag{5}$$

$\boldsymbol{y}$ is a (complex) vector of observations, F is the regression matrix and

$\boldsymbol{p} = [p_1, p_2, p_3]^T$ is the real-valued vector of parameters. As for all techniques that require the numerical computation of derivative terms, experimenters should perform careful tests to check whether errors committed on these derivatives play a significant role in the estimation of the parameters they are looking for. In a completely noise-free (theoretical) scenario, the parameter estimation might be improved either by performing faster sweeps (smoother trajectories result from very fast sweeps, as illustrated in section III) or by using a higher-order finite-difference scheme (provided enough points are available) or by reducing the sampling time. However, in a noisy (realistic) context, there are clear trade-offs to these approaches. Faster sweeps result in less signal and thus poorer SNR and higher-order finite-difference schemes or smaller sampling time result in an increased sensitivity to noise.

The solution in the least-squares sense to this overdetermined set of equations is (6) where $\boldsymbol{F_r^T} = \left[\Re\left(\boldsymbol{F^T}\right)\Im\left(\boldsymbol{F^T}\right)\right]$ and $\boldsymbol{y_r^T} = \left[\Re\left(\boldsymbol{y}^T\right)\Im\left(\boldsymbol{y}^T\right)\right]$.

$$\widehat{\boldsymbol{p}} = \left(\boldsymbol{F_r^T F_r}\right)^{-1}\boldsymbol{F_r^T y_r} \tag{6}$$

Whether we apply this method to ring-down or frequency sweeps, determining $\widehat{\boldsymbol{p}}$ from (6) yields a unique solution. From $\widehat{\boldsymbol{p}}$, the optimal values of ω_0, Q and a are then derived.

In the case of sweeps, a is supposedly known since it is imposed by the shaker. However, estimating a along with the other parameters of the resonator mitigates uncertainties on the actual value of the acceleration amplitude when it is not measured and/or very noisy (which is especially true at very low acceleration levels). If the measurement of the acceleration is very noisy (for example because of a very low actuation level), or if acceleration is not measured at all, estimating a along with other parameters is required. If the actuation signal is precisely measured and if the acquisition chain does not introduce faulty synchronization (for example with a lock-in amplifier actuating a MEMS resonator electrically and measuring its motion via current detection), estimating a is not compulsory. In addition, the parameter estimation procedure remains unchanged if a is properly measured even if it not perfectly constant along the sweep. In our case, to account for various imperfections of the acquisition chain (e.g. phase delays, faulty synchronization) related to the use of a waveform generator, a shaker, optical detection and measurement with an oscilloscope, it is in fact preferable to estimate a as a complex value, i.e. $a = a_R + ja_I$. The corresponding regression matrix is then (7) instead of (5) and the last two entries of the corresponding vector of unknown parameters $\boldsymbol{p}$ are $p_3 = a_R$ and $p_4 = a_I$.

$$[F_k] = [\,X(t) \quad j\omega(t)X(t) \quad 1 \quad j\,]_{t_k} \tag{7}$$

In the case of ring-down measurements, the input acceleration is supposedly 0 and one should only keep the first two columns of $\boldsymbol{F}$ and $\boldsymbol{p} = [p_1, p_2]^T$.

The mathematical developments (1) to (7) could be applied to any stimulus F. However, in this paper, we focus on the comparison between the 3 following experimental setups:

1. the traditional ring-down where a sine-wave actuation at ω close to the resonance frequency is maintained for enough time to reach steady-state, before stopping the actuation (so that $a = 0$) to record the ring-down signal,
2. the unidirectional linear sweep from $\omega = \omega_{start}$ to $\omega = \omega_{end}$ between $t = T_{start}$ and $t = T_{end}$, where $\omega(t) = \omega_{start} + \frac{(\omega_{end}-\omega_{start})}{T_{end}-T_{start}}(t - T_{start})$,
3. the bidirectional linear sweep from $\omega = \omega_{start}$ at $t = T_{start}$ to $\omega = \omega_{end}$ at $t = \frac{T_{start}+T_{end}}{2}$ and then back to $\omega = \omega_{start}$ at $T = T_{end}$. In this case $\omega(t) = \omega_{end} - 2\frac{\omega_{end}-\omega_{start}}{T_{end}-T_{start}}\left|t - T_{start} - \frac{T_{end}-T_{start}}{2}\right|$.

The evolutions of $a(t)$ and $\omega(t)$ in the 3 experimental setups are plotted in Fig. 1.

Now that the theoretical background and assumptions has been presented, we provide simulations in sections II. B and II. C followed by experimental data in section III.

2.2. Simulation framework

To motivate our experimental study, simulations of (4) in the presence of acceleration noise (e.g. due to parasitic vibrations) or measurement noise are performed as follows:

1. the nominal resonator parameters are set to $f_0 = 40.52$Hz and $Q = 1000$ (comparable to the parameters of the actual device).
2. the ring-down is assumed to be initiated by a ring-up phase where the resonator is actuated at resonance with the same acceleration amplitude than the frequency sweeps.
3. a band-limited noise with normal distribution is generated and added to the acceleration. The root mean square (RMS) amplitude of this noise is typically a fraction of a_0.
4. the corresponding evolution of (4) is simulated thanks to a numerical solver for:
   - a 100s frequency sweep from 40.25Hz to 40.8Hz, followed by a 100s frequency sweep from 40.8Hz to 40.25Hz.
   - a 100s ring-up at $f = f_0$, followed by a 100s ring-down (a is set to 0, but noise is still present).

The number of (uniform) time steps for both simulations is 5×10^4. One sample out of ten is kept as measurement samples $X(t_1), \ldots, X(t_N)$ (with $N = 5 \times 10^3$).

5. a band-limited noise with normal distribution is generated and added to the samples to simulate measurement noise.

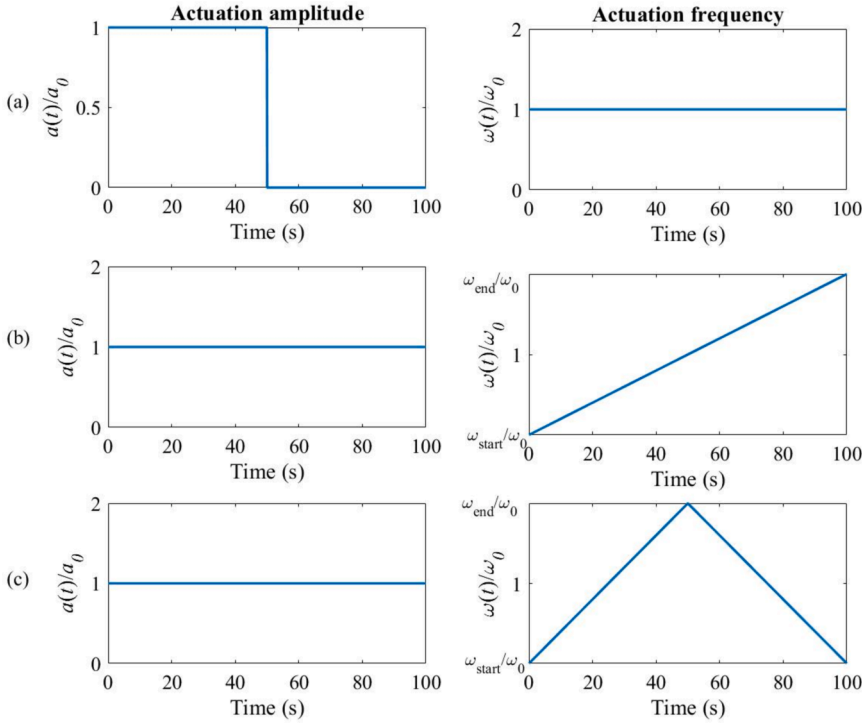

Fig. 1. Evolution of the amplitude (left) and the instantaneous frequency (right) of the actuation waveforms. (a) Ring-down measurement. (b) Unidirectional sweep. (c) Bidirectional sweep. Example for $T_{start} = 0$ and $T_{end} = 100s$ and an arbitrary actuation amplitude a_0.

6. the parameters estimated from these samples are calculated with (6). Four parameters are estimated from frequency sweeps, two from ring-downs, as discussed in section II. A.

Fig. 2 shows typical simulated sweeps and ring-down measurements obtained with the *nominal* parameters (ω_0, Q, and a) without and with acceleration noise and without measurement noise (outputs of step 4), superimposed to the simulated waveforms obtained with the parameters *estimated* at step 6 ($\widehat{\omega}_0$, $\widehat{Q}$, $\widehat{a}$). This qualitatively demonstrates the good fit that may be obtained with either sweeps or ring-downs. For a quantitative assessment of the two approaches, one may turn to the Monte-Carlo method to empirically determine the bias and variance of the estimated parameters, for different levels and types of noise.

Note that the accuracy of the parameter estimates obtained with ring-downs is largely dependent on which portion of the freely-decaying response is used – because the signal-to-noise ratio becomes increasingly worse over time. In this respect, simulations show there seems to be an optimum ring-down duration of the order of τ to 3τ, which yields the smallest bias for $\widehat{\omega_0}$ and $\widehat{Q}$. On the other hand, for sweeps, the smallest bias is always obtained when keeping all points from the dataset. Consequently, in section II. C below, we use our simulation framework to compare the quality of the estimates of ω_0 and Q (and a) obtained with either stimulus in a best-case scenario, by keeping only the first 20*s* of the ring-down response, and the whole 200s of the up and down sweeps.

2.3. Influence of actuation and measurement noise

2.3.1. Influence of actuation noise

Fig. 3 presents the results obtained when only *acceleration* noise is present (as in Fig. 2), based on 10^4 simulations for each noise level. It may seem surprising that the natural frequency of the resonator can be determined with such a good precision from noisy frequency sweeps like in Fig. 2. However, the reader should keep in mind that the provided estimations require the measurement of both the amplitude and phase responses. A quick look at the amplitude response alone is not sufficient to evaluate the chances of an accurate parameter estimation. This should be kept in mind especially by experimenters who usually analyze the steady-state frequency response obtained from a quasi-static step-by-step frequency sweep and identify a Lorentzian response. Additionally, since fast sweeps do not require to reach steady state at any time, the number of amplitude and phase samples measured on the frequency response can be extremely large (typically larger than 10^4 or 10^5) which is practically impossible for steady-state measurements, usually limited to a few hundred points at maximum because of the time required to reach steady-state at all frequencies.

The typical precision of the estimation of f_0 in this close-to-ideal scenario is in the range of $10^{-7}\frac{f_0}{2Q}$ to $10^{-2}\frac{f_0}{2Q}$ both for ring-down measurements and fast frequency sweeps, depending on the level of actuation noise. The typical precision of the estimation of Q in this scenario is in the range of $10^{-6}Q$ to $10^{-2}Q$. In practice, as we will see later in section III, the precision of the parameter estimation achieved by our setup is not as good as predicted by these simulations because of many reasons including real noise level, real noise spectrum, variability of this noise along the experiments, temperature drifts and possible other drifts of the resonator parameters.

The simulations provided in this section are given as an illustration that our characterization procedure works and is competitive with ring-down measurements in a close-to-ideal scenario. A comparison under real conditions is performed in section III.

For the comparison, one should remember that the bias quantifies the accuracy of the estimation whereas the standard deviation

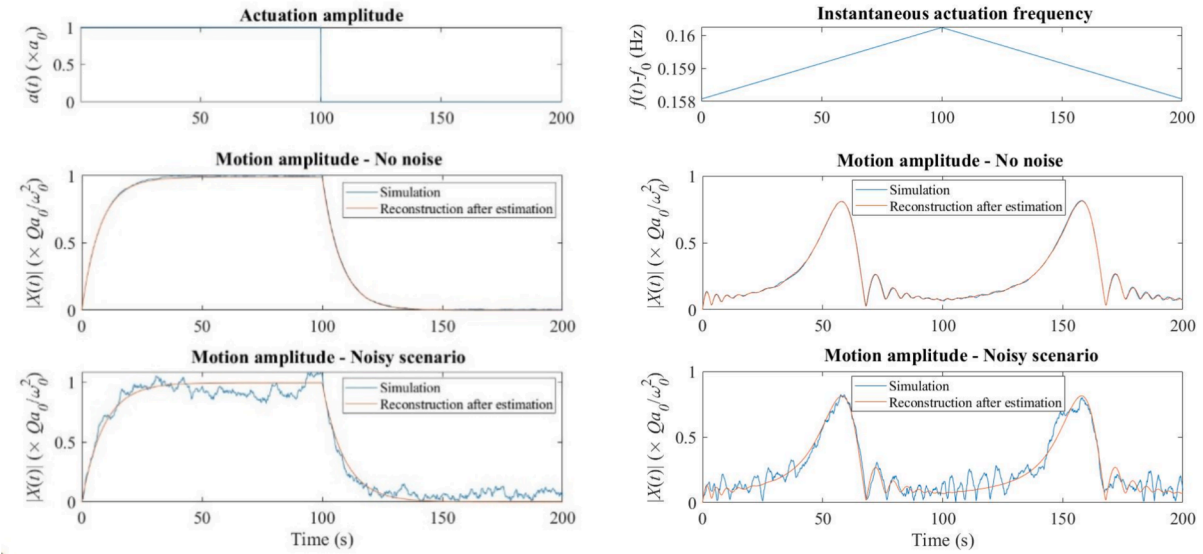

Fig. 2. Illustration of ring-down measurements (left) and fast frequency sweeps (right) in the presence of actuation noise. Blue: Simulated waveforms. Red: reconstructed after the parameter estimation procedure of section II. A. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

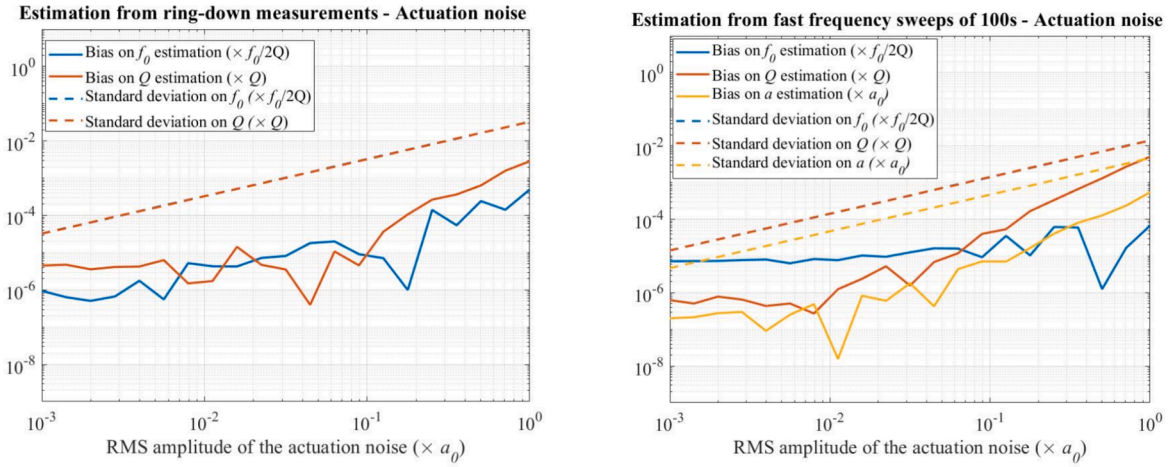

Fig. 3. Bias and standard deviation on each estimated parameter for ring-down measurements (left) and fast sweeps (right) in the presence of actuation noise. The standard deviation on f_0 is superimposed to the standard deviation on Q.

quantifies the precision of the estimation, related to the experimental repeatability. In the case of sweeps, the acceleration is estimated with both good accuracy and good precision: the standard deviation of $\widehat{a}$ (relative to a) is about three to ten times smaller than that of $\widehat{\omega}_0$ and $\widehat{Q}$.

Looking more closely, we find the following similar features for both approaches:

- When the acceleration noise increases, the precision of the estimations decreases. The standard deviation of all estimates increases proportionally with the RMS noise amplitude (the standard deviation of $\widehat{\omega}_0$, relative to $\omega_0/2Q$, is equal to the standard deviation of $\widehat{Q}$, relative to Q).
- Above a certain noise threshold (acceleration signal-to-noise ratio (SNR) around 40 dB), both characterization procedures become less accurate: the bias of all parameter estimates tends to increase proportionally to the *square* of the rms noise amplitude. Below this threshold, both methods have comparable accuracy, and the bias of the estimates is constant. Further simulations show that this threshold effect results from the discretization error of the finite difference scheme used for the approximation of $\dot{X}(t_k)$.

Distinguishing features are:

- Fast frequency sweeps lead to a slightly more precise estimation of ω_0 and Q than ring-down measurements: the standard deviation of $\widehat{\omega}_0$ or $\widehat{Q}$ is two times smaller when estimated from an up and down sweep than when estimated from a ring-down.

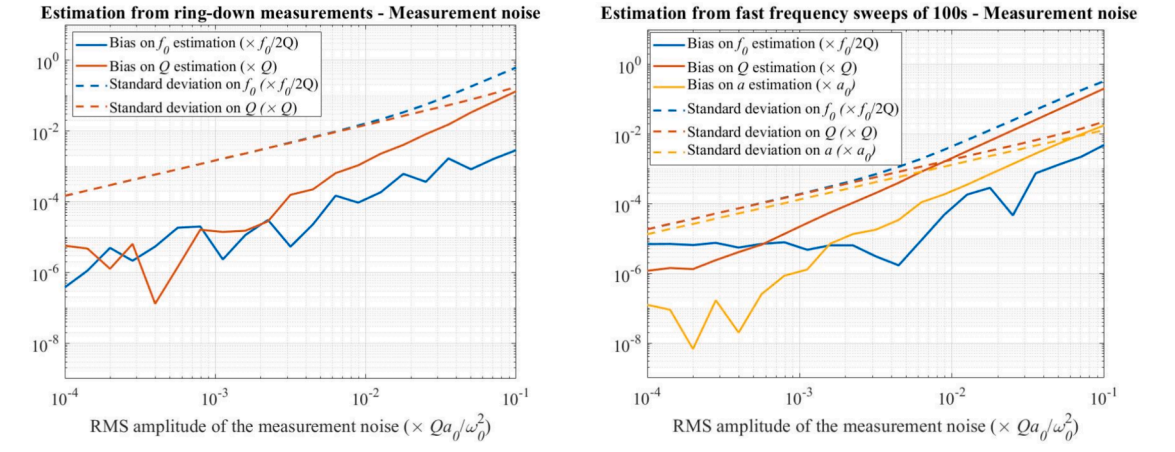

Fig. 4. Bias and standard deviation on each estimated parameter for ring-down measurements (left) and fast sweeps (right) in the presence of measurement noise.

- At low acceleration noise (acceleration SNR larger than 40 dB), the accuracy of the Q-factor estimation is better with fast sweeps than with ring-down measurements: below this noise threshold, the bias on the Q-factor estimate is smaller for sweeps than for ring-downs. The situation is reversed when acceleration becomes very noisy (acceleration SNR inferior to 20 dB).
- On the contrary, the accuracy of the natural frequency estimation is significantly better with ring-down measurements than with fast sweeps, at low acceleration noise. The situation is reversed when acceleration noise dominates.

Actuation noise is not the only significant noise that may affect the parameter estimation. In II.C.2, we analyze the effect of the presence of white measurement noise in the simulated data.

2.3.2. Influence of measurement noise

Fig. 4 shows the results obtained in simulations when only white *measurement* noise is present (based on 10^4 simulations for each noise level).

Again, the accuracy and precision of the estimation of the acceleration is very good compared to what one may expect from a direct acceleration measurement due to external vibration noise and sensor noise and bias. This illustrates why, unless it is very precisely measured and controlled, it may be better to consider the acceleration amplitude as an unknown (which is not measured) determined by the procedure than to collect the data from an accelerometer. About the natural frequency and the quality factor, the accuracy reached by the two methods in the presence of measurement noise is very similar, but the frequency sweeps tend to exhibit a better precision: the standard deviation of the estimates of f_0 and Q obtained from fast frequency sweeps is lower than those of ring-down measurements (up to 10 times). All these first conclusions prove that the two methods yield a comparable accuracy even for situations where the SNR related to measurement noise is relatively low.

The main findings of this statistical analysis are summarized in Table 1. This comparison shows that, in a close-to-ideal scenario, fast frequency sweeps can be a competitive alternative to ring-down measurements It highlights that the choice of the best method cannot be made a-priori without careful verification on a case-by-case basis. The choice of the method becomes even more complex when adding other imperfections of the setup such as noise spectrum, noise variability, drifts in the resonator parameters and possible instrument limitations. Ideally, fast frequency sweeps and ring-down measurements should both be performed each time a new resonator is characterized and/or for every modification of an experimental setups. Experimenters should remember that the results presented in Table 1 are an illustration and depend on several assumptions (e.g. finite difference step size, sweep duration, central frequency and bandwidth, ring-up excitation frequency) that have been introduced in this section. Hence, the latter features may not apply under all circumstances. A general comparison about the most accurate method would be interesting but is close to impossible due to the excessively large number of parameters to consider. Even if such a general comparison was available, there would be so many assumptions on the experimental setup that a case-by-case a-posteriori verification would anyway be required to guarantee that the best method has been chosen.

3. Experimental validation

The aim of this section is to validate our characterization procedure on a real experimental setup and illustrate how it competes with ring-down measurements.

3.1. Experimental setup

For the experimental demonstration, measurements are performed on a MEMS energy harvester [15] with a resonance frequency f_0 close to 40 Hz and a quality factor close to 1000, with a typical response time $\tau = \frac{Q}{\pi f_0} \simeq 8$s. A picture of the setup is reported in Fig. 5.

The MEMS is actuated by a shaker, driven by a Keysight 33600A waveform generator through a B&K 2718 power amplifier. The acceleration is set by applying a fixed actuation voltage to the shaker. The acceleration is measured at $2.75 mm/s^2 \pm 5\%$ with an accelerometer glued to the moving support of the MEMS. Considering the very low actuation level, the level of noise and the speed at which frequency varies during the sweeps, it is impossible to control the acceleration with an accuracy better than 5 %.

The motion of the MEMS is monitored by a differential displacement measurement setup with two LK-H050 laser heads connected to a LK-HD500 conditioner. The signal coming from the conditioner is recorded on an MSO-2 Tektronix oscilloscope set at a sampling

Table 1
Example of an ideal time-invariant linear second-order resonator under additive white noise. This qualitative comparison is obtained from simulations and does not claim to be general.

		White actuation noise		White measurement noise	
		Low	High	Low	High
Natural frequency	Best accuracy	Ring down	Fast frequency sweep	Ring down	Comparable
	Best precision	Fast frequency sweep	Fast frequency sweep	Fast frequency sweep	Fast frequency sweep
Quality factor	Best accuracy	Fast frequency sweep	Ring down	Comparable	Comparable
	Best precision	Fast frequency sweep	Fast frequency sweep	Fast frequency sweep	Fast frequency sweep

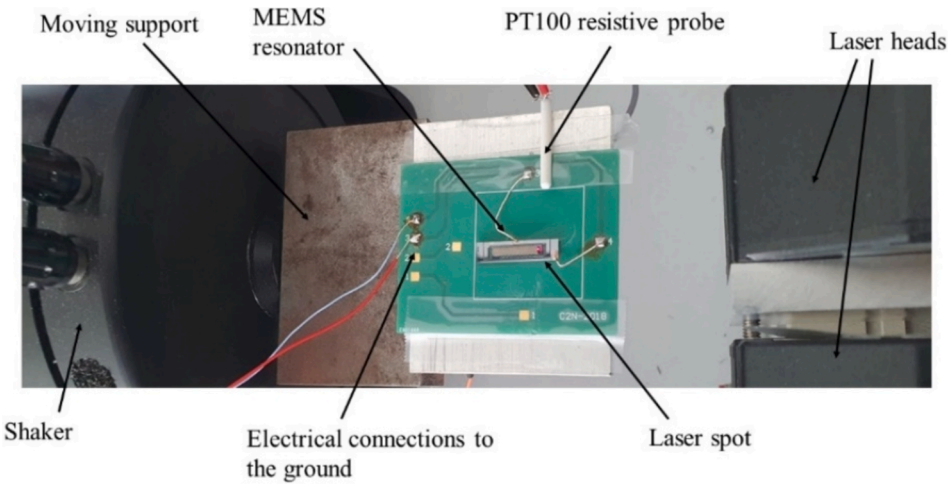

Fig. 5. Picture of the experimental setup.

frequency of 5kHz. Bidirectional frequency sweeps (upward followed by downward sweep) are performed from $f_{min} = 40.25Hz$ to $f_{max} = 40.8Hz$ on various durations from 10 s to 150 s by steps of 10 s. Hence, the full duration of each bidirectional frequency sweep ranges from 20 s to 300 s. After each bidirectional frequency sweep, a ring-down measurement is performed. Each ring-down measurement starts with a 50 s actuation at 40.521 Hz which is very close to the expected resonant frequency to reach steady-state. Then, actuation is stopped, and the exponential decrease of the free oscillations is recorded during 40 s.

For all the measurements (sweeps and ring-down), the vertical scale of the oscilloscope is kept constant to avoid a biased comparison. It is set so that the amplitude at resonance covers 95 % of the full vertical scale of the scope, for the best resolution. For each of the 15 different sweep durations, from 10 s to 150 s, the sequence performs a bidirectional fast frequency sweep followed by a ring-down measurement one after another. Each of them is repeated 30 times, for a total of $30 \times 15 \times 2 = 900$ series of measurements. The whole experiment is performed in a room with air conditioning and lasts around 2 days, during which temperature is monitored with a PT100 resistive probe attached close to the MEMS resonator (around 1 to 2 cm from the MEMS), connected to a Keysight 34470A multimeter.

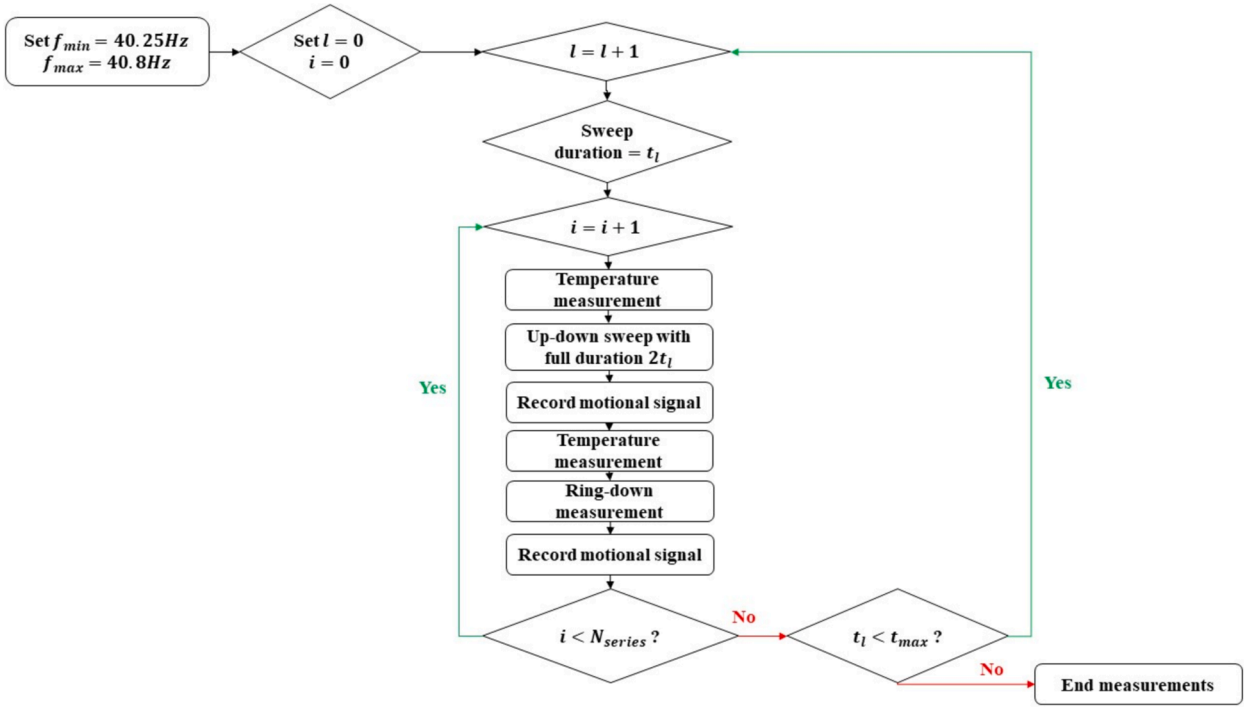

Fig. 6. Flow-chart of the full series of measurements.

The seismic mass of the harvester and its transduction electrodes are electrically grounded so that electrostatic forces are cancelled out. The flow chart of the whole series of measurements is presented in Fig. 6.

Typical waveforms are given in Fig. 7. They exhibit some white noise which appears to be independent of the level of signal (the SNR at large amplitudes being significantly better than the SNR at low amplitudes), which is consistent with an added white measurement noise of constant amplitude (assumption made in section II. C). Based on the previous simulations (see section II. C), fast frequency sweeps are expected to estimate the resonator parameters more precisely than ring-down measurements under these circumstances. This will be verified and discussed in section III. C.

3.2. Waveform processing

The complex amplitude of the displacement is extracted from the waveforms by demodulating them at a fixed frequency (40.521 Hz), with a 20Hz cutoff frequency. This choice is relevant to compare the two techniques on the same basis, but it is in fact not a critical parameter. Verifications have been performed and demodulation at the instantaneous frequency (like a lock-in amplifier would do) instead, or even at 40 Hz does not affect our conclusions and has a negligible impact on the parameter estimations. In the case of sweeps, the phase difference between the displacement and the driving force is recovered by assuming the evolution of the driving frequency is perfectly linear between f_{min} and f_{max} for the upward part (and between f_{max} and f_{min} for the downward part). Typical demodulated sweeps are shown in Fig. 8.

In Fig. 8, we can distinguish between the upward sweep (peak on the right, dark line) and the downward sweep (peak on the left, grey line). As the sweep time increases, the difference between the upward and the downward sweeps tends to fade, since measurements are getting closer to quasi-static regime. However, even with a total bidirectional sweep duration of 2×150s (5 min), transient effects appear clearly. Since both the measurement of amplitude and phase are required for the parameter estimation, we also provide, in Fig. 9 example of demodulated frequency sweeps in the complex plane for different sweep durations. All parameter estimations are based on measurements in the complex plane (like Fig. 9), knowing the instantaneous frequency at all times.

3.3. Parameter estimation

In this section, we focus on the best-case scenario corresponding to the comparison of bidirectional sweeps to 15 s ring-down measurements. The impact of this duration on the parameter estimation from a ring-down response will be discussed later, in Section IV. A. How to choose between a unidirectional sweep and a bidirectional sweep is related to practical synchronization issues and will be discussed in section IV. B.

The estimations of natural frequency and quality factor estimated from bidirectional sweeps are reported in Fig. 10 and compared with the estimations based on ring-down measurements. The estimations of acceleration are compared to the measurement coming from our accelerometer (see section III. A). The temperature profile is also reported.

In this section, we focus on the correlation between temperature and parameters. The quality of the estimation will be analyzed in sections III.C.1 and III.C.2.

The main impact of temperature can be seen in the estimation of the acceleration which is strongly correlated to temperature. The identified reason for this is a deviation in the gain of the laser heads, which is estimated at 2 μm/°C, as stated by the datasheet. Even with that limitation of the setup, the estimation of the acceleration based on frequency sweeps remains more reliable than a direct acceleration measurement at these low actuation levels, as shown by the size of the error bars.

The temperature has an impact not only on the setup but also on the MEMS itself. As a general trend, we notice that the natural frequency of the MEMS increases when the temperature decreases, with an order of magnitude of 1 mHz/°C to 2 mHz/°C. The most plausible explanation for this small deviation is a mismatch of dilatation between the glass layer and the silicon layer of the wafer. In terms of quality factor, we notice that the quality factor tends to increase when the temperature decreases. The order of magnitude is a 1 % Q-factor increase for a 1 °C decrease in temperature. We do not have a positive explanation for this evolution, but the Q-factor of the MEMS energy harvester is mainly determined by air damping. Hence, a possible explanation lies in the evolution of the viscosity of

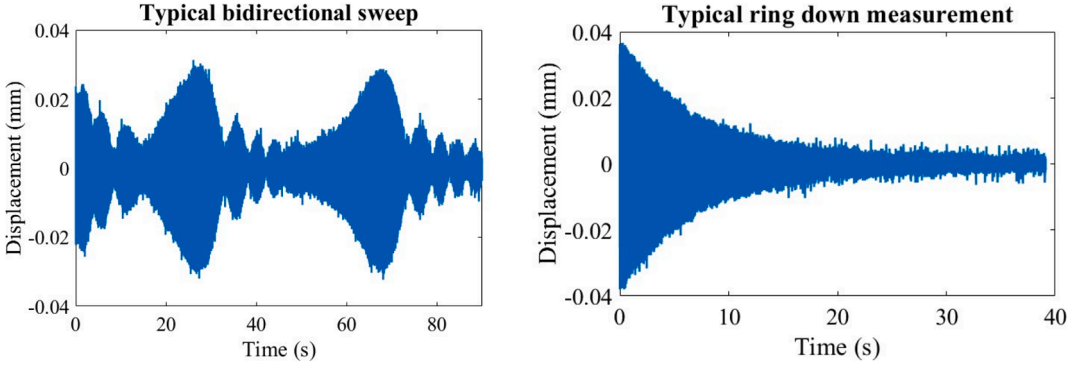

Fig. 7. Typical waveforms recorded for a bidirectional sweep (left) and a ring-down measurement (right).

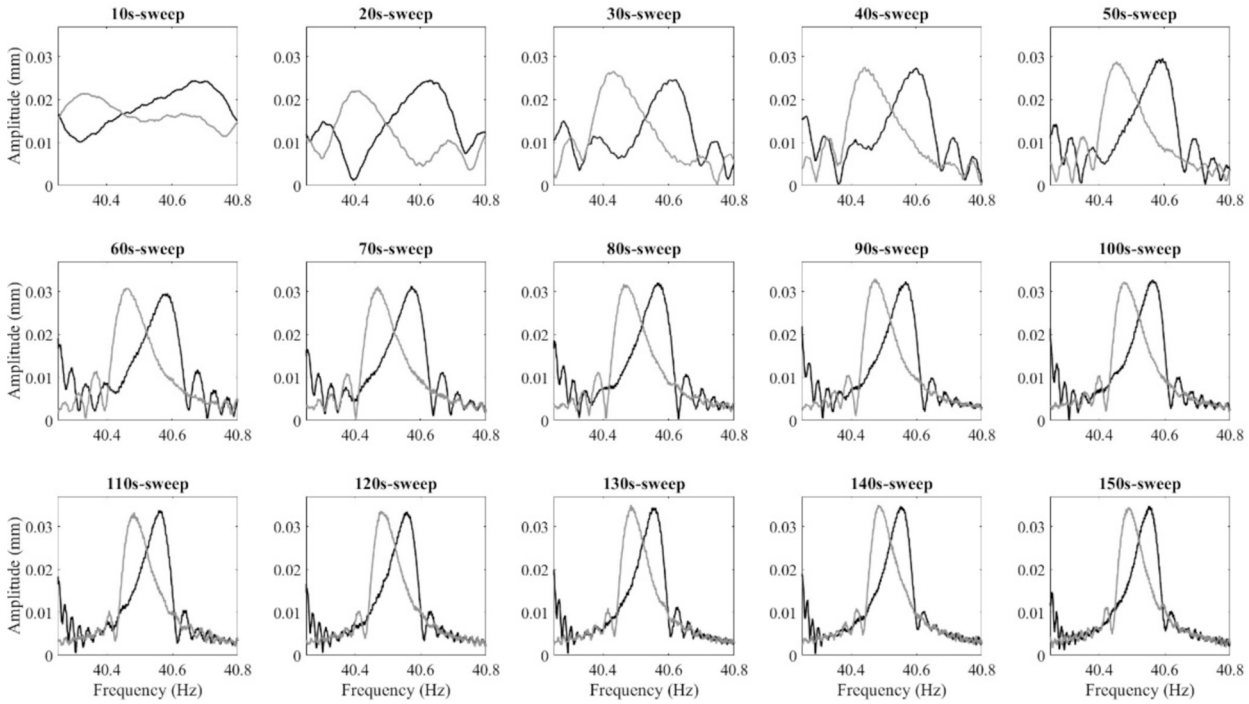

Fig. 8. Amplitude vs. frequency response of bidirectional sweeps after synchronous demodulation. Dark: upward sweep. Grey: downward sweep.

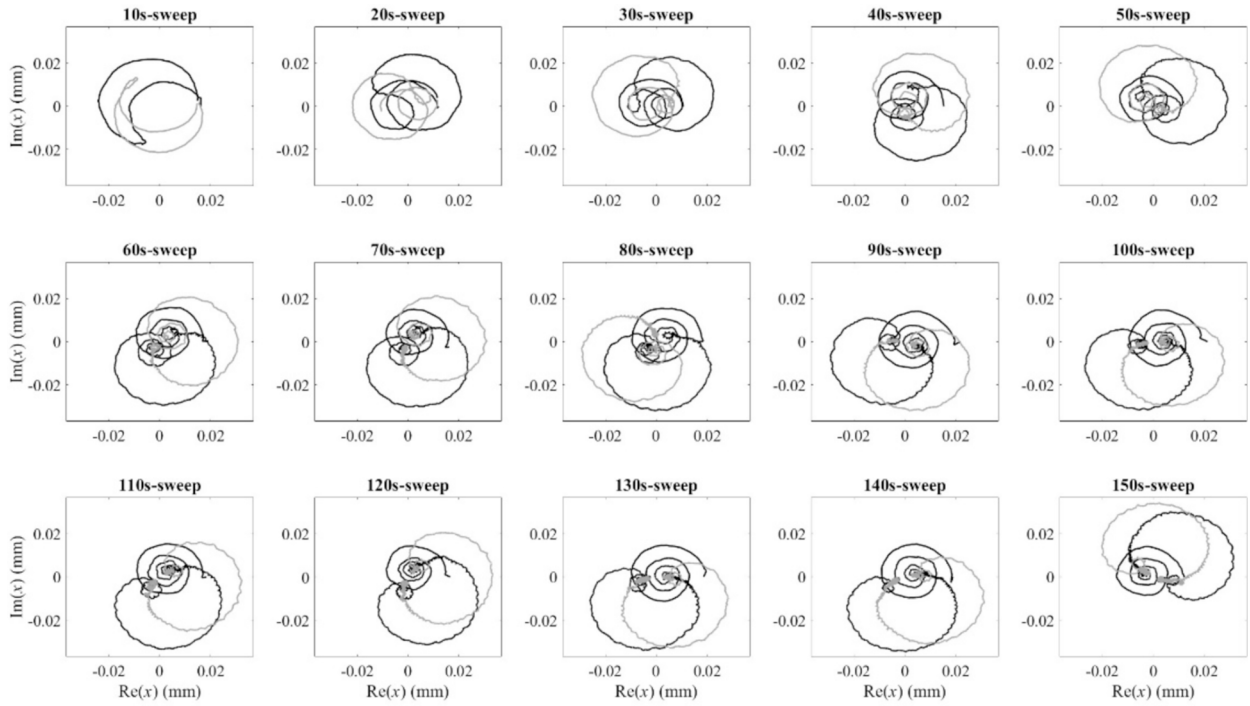

Fig. 9. Imaginary vs real part of the (complex) displacement of the resonator. Examples of experimental bidirectional sweeps for a unidirectional sweep duration of 10 s to 150 s, corresponding to a total duration of 20 s to 300 s. Dark: upward sweep. Grey: downward sweep.

air with temperature and/or humidity. At these temperatures, the viscosity of air depends on humidity, which is not controlled in our experiment, but an increase of viscosity between 0.5 %/°C and 1 %/°C is reasonable [16].

For the fast sweeps, the worst-case scenario (leading to the largest standard deviation) corresponds to sweeps whose duration is of

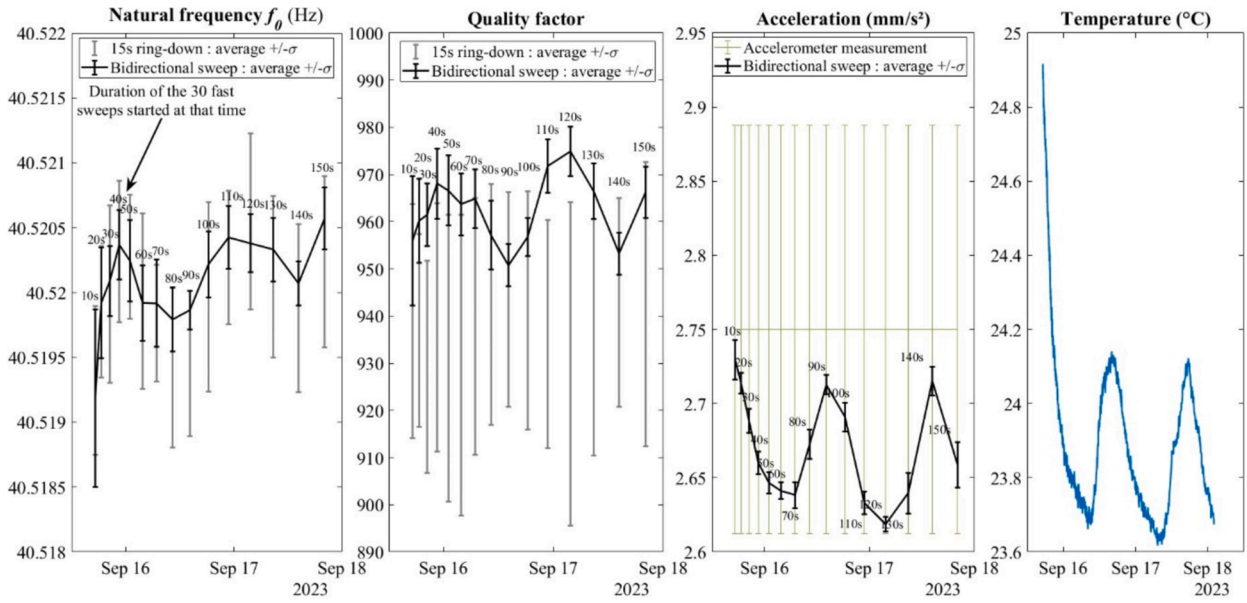

Fig. 10. Parameters estimated from the bidirectional sweeps, compared with the parameters estimated from the ring-down measurements. The temperature measured during the experiment is reported on the right. Each of the presented error bar corresponds to the average value estimated on a set of 30 measurements and its corresponding standard deviation.

the order of magnitude of the response time of the resonator τ (here sweep durations of 10 s). This is hardly surprising, when considering the relative featurelessness of the amplitude vs frequency response of the 10 s sweeps in Fig. 8. As a reasonable rule, the best scenario corresponds to a sweep duration equal to a few times τ, typically 3τ or 4τ, to ensure reliable data while the temperature remains as constant as possible. In our case, sweeps of 80 s to 90 s (corresponding to 11τ to 12τ) seem to be the best trade-off (see error bars both on frequency and Q-factor estimations). Longer sweeps do not yield a better estimation mostly because the temperature varies along each sweep when the sweep is too long.

In sections III.C.1 and III.C.2, we analyze further the reliability of the estimations provided by the fast sweeps for the MEMS natural frequency and its quality factor.

3.3.1. Natural frequency estimation

Like in the simulations of section II. C, the precision of the natural frequency estimation from the frequency sweeps of Fig. 9 may seem surprisingly good at first sight. However, this precision is significantly lower than the precision expected from the simulations of section II. C. This can be explained by several experimental limitations including – but not limited to:

- The noise levels, which may not be identical in all scenarios or even not constant along the experiment
- The actual noise spectrum
- Environmental fluctuations, including temperature drifts
- Other possible non-identified fluctuations of the resonator under used.

In addition, strong deviations in the sweep rate could generate discrepancies. However, even standard waveform generators have a frequency accuracy much better than the achieved precision ($1\mu Hz$ for our Keysight generator) and are able to generate a sinewave with a linearly varying frequency with a precision that is several orders of magnitude better than 1 mHz. Under all circumstances, the experiment must always ensure that the frequency resolution of the generator is much better than the expected precision of the estimation (which is not specific to our characterization procedure).

The estimations of the natural frequency based on the bidirectional sweeps are consistent with ring-down measurements. The standard deviation obtained from bidirectional sweeps is significantly smaller than the standard deviation of ring-down measurements, meaning that the frequency estimation based on ring-down measurements is less precise than the frequency estimation based on fast sweeps. This conclusion holds here mostly because of the very low actuation level of the MEMS which does not allow more than 15 s of the ring-down records to be exploited. Even at larger levels, the duration of the ringdown signal which can be properly used for characterization is limited by the vertical resolution of the ADC in the experimental setup (e.g. limited to 5 τ or 6 τ for an 8-bit ADC). Keeping this in mind, one must be very cautious when trying to estimate parameters based on noisy ring-down measurements, as we will analyze further in section IV. A.

3.3.2. Quality factor estimation

In terms of quality factor, we observe that the estimation provided by fast-sweeps is significantly more precise, at these levels, than the estimation provided by ring-down measurements (see error bars). However, the average Q-factor estimated from fast sweeps is consistently 2 % to 3 % larger than the average Q-factor estimated from ring-down measurements. Qualitatively at least, this is in accordance with the simulated results of section II. C, which predict that more accurate but less precise estimates of Q may be obtained from ring-downs in a noisy environment.

4. Practical constraints of ring-downs and fast sweeps

In this section, we discuss the practical limitations that must be kept in mind when opting for one or the other stimulus.

4.1. Ring-down measurements

The main limitations of ring-down measurements are:

- choosing a good prior value of f_0 for driving the resonator to a "high" resonant state during the ring-up phase.
- choosing only the most "significant" points of the ring-down phase to perform the parameter estimation.

Regarding the first item, for a given excitation amplitude, the largest signal-to-noise ratio (SNR) at the end of the ring-up phase (and consequently during the ring-down phase), is obtained when the excitation frequency is exactly equal to f_0. SNR drops sharply if there is any significant mismatch (say larger than $f_0/2Q$) between the ring-up frequency and f_0.

Regarding the second item, picking the best points to perform the parameter estimation is not an easy task. Indeed, the first data points of the ring-down response, with a good SNR, may be polluted by the free decay of the actuator response (for example if the shaker does not stop instantly), while the last points suffer from a poor SNR. This is illustrated in Fig. 11 which shows an example of signal recorded from a ring-down, with the corresponding amplitude and phase after demodulation in Fig. 12 (left). Amplitude decays to a non-zero value (due to acceleration or measurement noise). The instantaneous phase starts behaving erratically, instead of following a linear trend after demodulation. Fig. 12 shows how the natural frequency and quality factor estimated from the measurements of Fig. 11 depend on which portion of the signal (between T_{start} and T_{stop}) is used for the fitting procedure. The larger T_{start} is, the more dependent the estimated frequency becomes on the observation length $T_{stop} - T_{start}$, as can be expected from the outlook of the instantaneous phase. This effect is less noticeable in the case of Q, where it is shadowed by the strong dependency of the estimation on T_{start} rather than on T_{stop}.

4.2. Fast sweep measurements

As (fast) sweeps span a relatively large frequency range, compared to the resonator's bandwidth, prior knowledge of f_0 is not as stringent as in the case of ring-down measurements. Moreover, thanks to the continued excitation, less measurements are drowned in noise than in ring-down, so that, for reasonable sweep durations and spans, there seems to be no need to truncate or select samples, as opposed to the ring-down case.

On the other hand, a practical difficulty arises – which becomes more limiting as the duration of the sweep increases– for a proper demodulation of the measured signals. To this end, we require the exact knowledge of the instantaneous phase of the excitation, i.e. (8) for a linear sweep from frequency f_{start} at $t = T_{start}$ to $f_{end} = f_{start} + \Delta f$ at $t = T_{end} = T_{start} + \Delta T$.

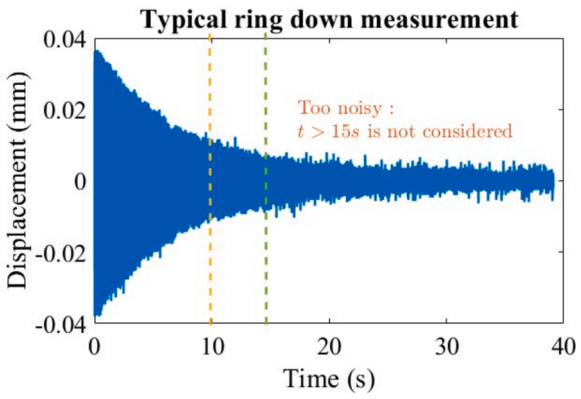

Fig. 11. Example of one of the 450 ring-down measurements performed during the whole experiment. The response after 15 s is not relevant and thus will not be considered due to the level of noise.

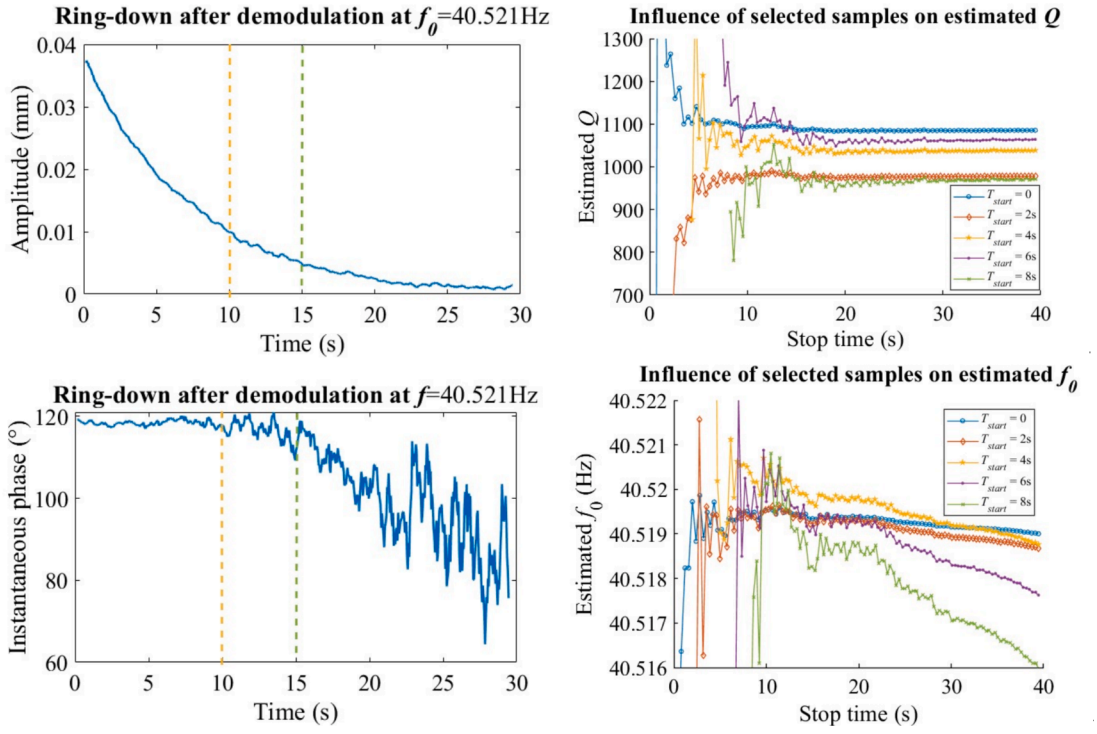

Fig. 12. Amplitude and instantaneous phase extracted from the ring-down measurement of Fig. 11. The estimated instantaneous phase illustrates why measurements after 15 s are not relevant to determine the resonance frequency.

$$\psi(t) = 2\pi\left(\frac{1}{2}\frac{f_{end} - f_{start}}{T_{end} - T_{start}} \times (t - T_{start}) + f_{start}\right)(t - T_{start}) \tag{8}$$

Any uncertainty on T_{start} (e.g. due to poor synchronization) or on the exact slope $\Delta f/\Delta T$ of the sweep results in a phase error that increases quadratically with time (and in an instantaneous frequency error that increases linearly with time). Thus, there is a setup- and resonator-dependent sweep duration above which frequency sweeps become impractical. To mitigate this issue, one should either:

- record and demodulate the excitation signal along with the motional signal. The phase errors on the excitation and detection will then compensate each other out.
- use bi-directional sweeps, as we did in this paper, so that the phase errors during the up- and down-sweeps compensate each other.

The benefit of the latter approach is illustrated in Fig. 13, in which the parameters estimated with bi-directional sweeps are compared to those estimated with the first (respectively second) half of the signal only (upward sweep, and respectively downward sweep). As the duration of the sweep increases, the mean values of the quality factor and resonance frequency estimated with either up- or down-sweeps tend to diverge. Of course, the use of well-synchronized equipment (e.g. a lock-in amplifier) would make this issue almost non-existent but it also comes with a price. Another possibility also lies in processing the data: one may seek to estimate the exact sweep parameters (e.g. T_{start}, $\Delta f/\Delta T$) along with the parameters of the resonator. However, the optimization problem then becomes nonlinear and is less well-conditioned. On the other hand, bidirectional sweeps provide consistent results, independently of the duration of the sweep. This is also confirmed by Monte-Carlo simulations. Furthermore, bidirectional sweeps are the best solution to verify the linearity of the resonator, compared to unidirectional ones.

5. Conclusion

We have presented a simple and efficient characterization procedure, based on fast frequency sweeps, for the quantitative estimation of the parameters of linear resonators. A simulation-driven statistical analysis was first led, in an idealized framework, demonstrating the validity of this technique and its potential benefits and shortcomings, in terms of precision and accuracy, compared to the reference (ring-down) approach.

The relevance of our approach was experimentally demonstrated on a MEMS resonator with a large time constant τ. Under the constraints of a standard experimental setup with low-level mechanical actuation and laser detection, we have shown that fast frequency sweeps of a few times τ on a frequency interval more than 10 times larger than the resonator bandwidth let us determine the natural frequency and the quality factor of the resonator with a better repeatability than ring-down measurements. Our approach thus

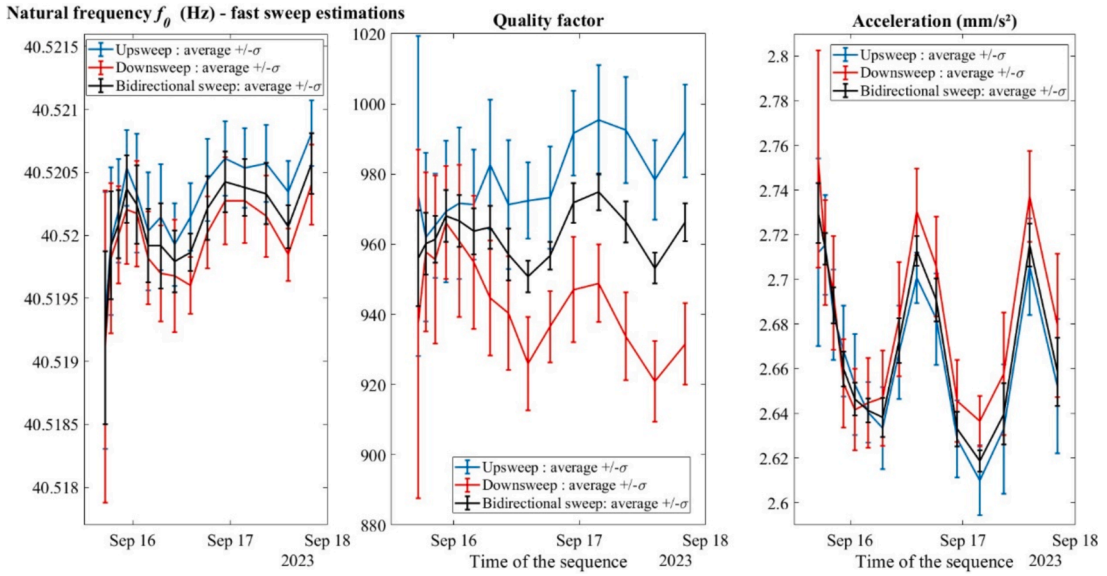

Fig. 13. Parameters estimated for unidirectional upward sweep (blue), unidirectional downward sweep (red) and bidirectional sweeps (black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

competes with existing state-of-the-art characterization procedures. We also showed that this procedure provides, as a by-product, a good estimation of the actuation applied to the resonator (here a shaker acceleration).

Finally, we discussed the practical difficulties associated with our approach, and with the ring-down approach. Regarding ring-down measurements, we highlighted why the precision and accuracy of the estimated parameters were dependent on the prior information on f_0 and on the selected portion of the free decay response. While our approach is less demanding in these respects, it does require a precise knowledge of the excitation's instantaneous phase during the whole frequency sweep, without which the estimation of the Q factor becomes less accurate. This effect, however, is mostly noticeable for very (exceedingly) long sweeps in noisy environments and can be minimized by using precision equipment – to the detriment of cost – or can be compensated, as we explained, thanks to bidirectional frequency sweeps.

Some theoretical, practical and application-related developments remain to be explored. As far as theory is concerned, analytical formulas for the bias and variance of the estimated parameters remain to be found. A general review of all possible techniques of characterization, providing a definitive answer and helping experimenters in opting for the most accurate one is still missing from the literature. Such a general study is extremely complex considering the number of scenarios to account for. It would have to include at least the effect of the noise level, the noise spectrum, the variability of the noise, both for the actuation noise and the measurement noise but also the number of samples recorded and the selection of samples for the estimation procedure. It would also need to account for the large diversity of possible signal processing techniques on the same datasets, the intrinsic drift of each resonator parameter (for example related to temperature) and other experimental limitations related to the instruments under use.

Hence, our study does not claim that fast sweeps are the most accurate and precise technique to estimate the parameters of a resonator. Such a general statement would be dubious towards any existing method. It only proves that fast frequency sweeps are as legitimate as ring-down measurements for the estimation of resonance parameters of a linear resonator. Our objective is to make this characterization procedure as widely accepted by the scientific community as other competing state-of-the-art methods, like ring-down, which is often considered as a reference.

The application of our approach to nonlinear resonators is also the subject of ongoing work. As far as applications are concerned: resonator characterization through fast frequency sweeps has little requirements in terms of prior information, hence it may be eligible as an industrial test method for wafer-level testing of a batch of MEMS devices in parallel. The embedded implementation of our approach for the *in situ* self-calibration of a single resonator is also a promising development –because of the relative simplicity of the processing it requires. Finally, resonant sensing applications may also benefit from our approach, which may be used to discriminate fluctuations of the quantity of interest (related to f_0) from fluctuations of influence quantities, such as temperature.

CRediT authorship contribution statement

Alexis Brenes: Writing – review & editing, Writing – original draft, Validation, Investigation, Data curation. **Jérôme Juillard:** Writing – review & editing, Resources, Methodology, Formal analysis, Conceptualization. **Jorge Cuevas Ayala:** Investigation, Formal analysis. **Lucca Reinehr:** Investigation, Conceptualization. **Erwan Libessart:** Formal analysis, Conceptualization. **Laurent Bourgois:** Writing – review & editing, Formal analysis. **Jean Guérard:** Supervision, Project administration. **Lucas Hudeley:** Resources, Conceptualization. **Puneet Gupta:** Investigation, Data curation. **Jose-Francisco Ambia Campos:** Visualization, Validation,

Investigation. **Elie Lefeuvre:** Validation, Supervision, Resources, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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