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Meije Gawinowski, Jérôme Enjalbert, Paul-Henry P.-H. Cournède, Timothée Flutre. Developing WHEAMM, a new FSPM for the simulation of wheat cultivar mixtures. Feedbacks on model conception, sensitivity analysis and automatic calibration in Julia. Presentation for the Wageningen FSPM group, Apr 2023, Online, France. hal-04936906

HAL Id: hal-04936906

<https://hal.science/hal-04936906v1>

Submitted on 9 Feb 2025

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Developing WHEAMM, a new FSPM for the simulation of wheat cultivar mixtures. Feedbacks on model conception, sensitivity analysis and automatic calibration in Julia.

Meije Gawinowski^{1,2,3}, Paul-Henry Cournède², Jérôme Enjalbert¹, Timothée Flutre¹

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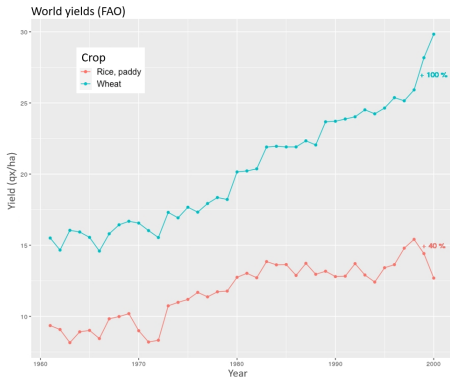
³ Université Paris Cité



The Green Revolution

Yield increase

- Breeding
- Chemical inputs
- Mecanization, irrigation



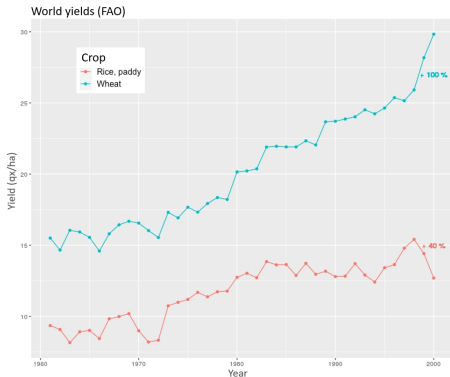
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Negative externalities

- Landscape homogenization
- Loss of agro-biodiversity
- Pollution → biodiversity, environment, human health



Climate change and yield stagnation

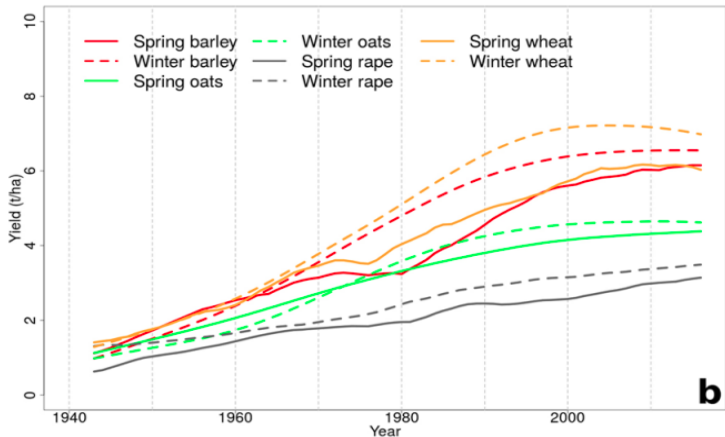


Figure from Schauburger et al. (2018)

The agroecological transition

- **Agroecology** : maximization of ecological processes to minimize the use of chemical inputs

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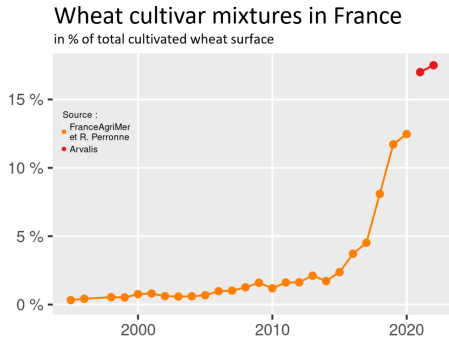
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- Importance of plant-plant interactions **within-plot diversification**
 - negative → competition
 - positive → complementarity and facilitation

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- Importance of plant-plant interactions **within-plot diversification**
 - negative → competition
 - positive → complementarity and facilitation
- Focus on **cultivar mixtures**
 - case study : winter wheat

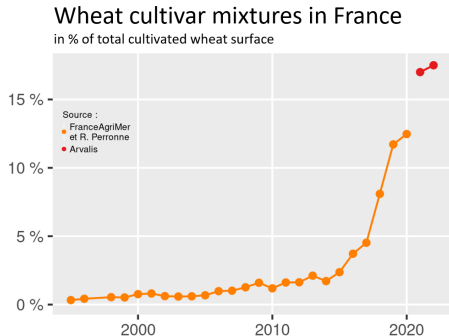
Wheat cultivar mixtures

- 17.5 % of wheat cultivated surface in France in 2022



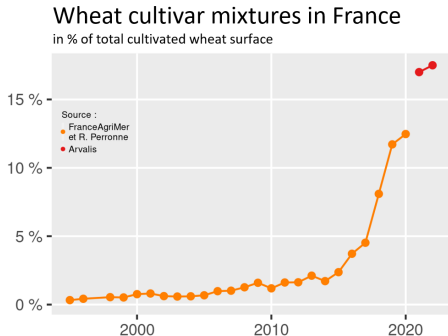
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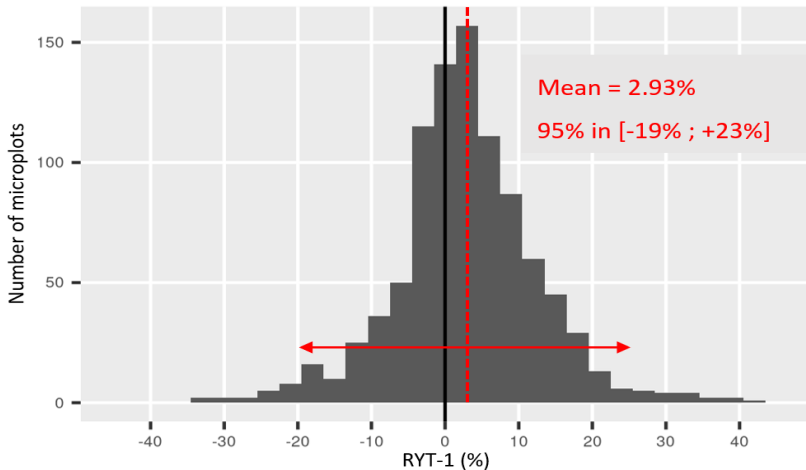
Wheat cultivar mixtures

- 17.5 % of wheat cultivated surface in France in 2022
- Few assembling rules
 - except for resistance to fungal diseases
- Difficulties to study cultivar mixtures
 - combinatorial explosion
 - difficult distinction between cultivars



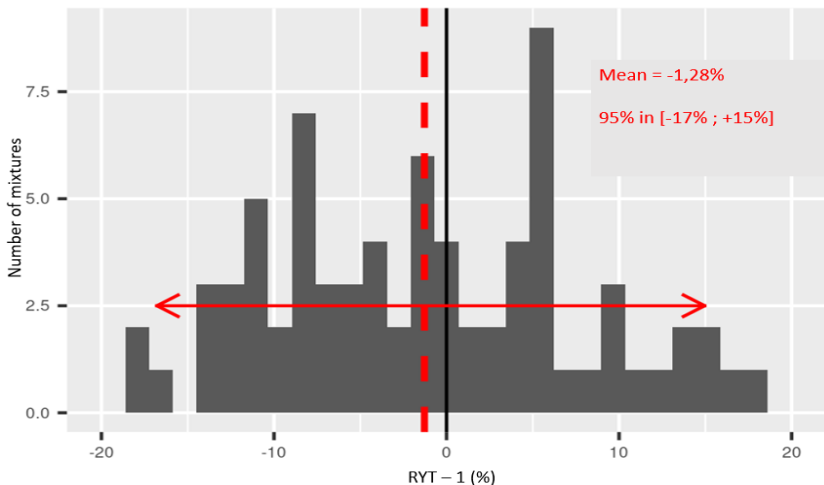
RYT distribution

Meta-analysis on wheat cultivar mixtures (Borg et al. 2018) on the distribution of the Relative Yield Total (RYT = Yield mixture / Average yield in monoculture) → large variance



RYT distribution

Large variance in the same environment: trial of 75 wheat cultivar mixtures at Le Moulon in France, 2014-2015 (Forst 2018)



Possible causes of RYT large variance

Observed yield
of mixture

versus
 \longleftrightarrow

Expected yield from
varietal proportions at sowing
and monoculture yields

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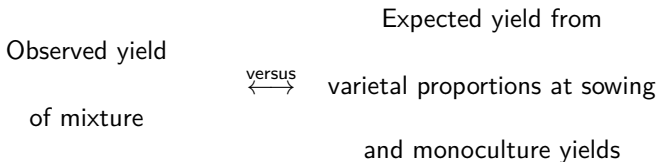
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- evolution of varietal proportions between sowing and harvest
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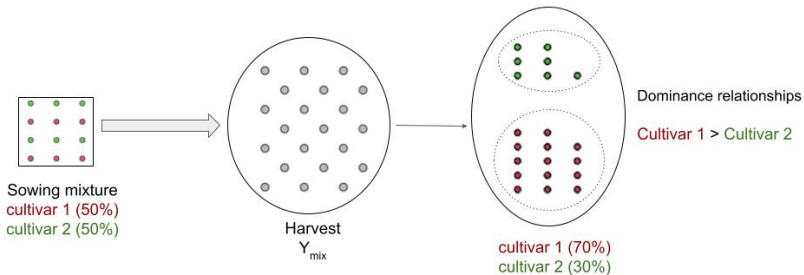


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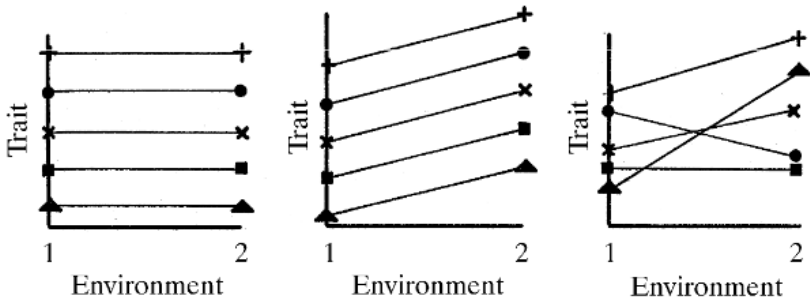
- evolution of varietal proportions between sowing and harvest
- inherent yield differences between monoculture and mixture for each cultivar (= phenotypic plasticity)

Hypothesis : these differences are caused by eco-physiological processes underlying plant-plant interactions

Differences in varietal proportions



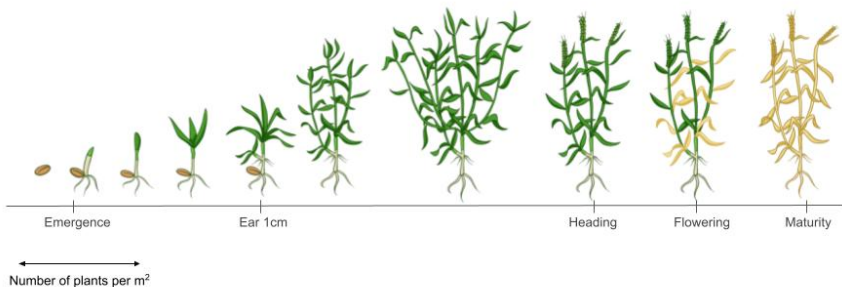
Phenotypic plasticity



Source : [Guntrip & Sibly \(1998\)](#)

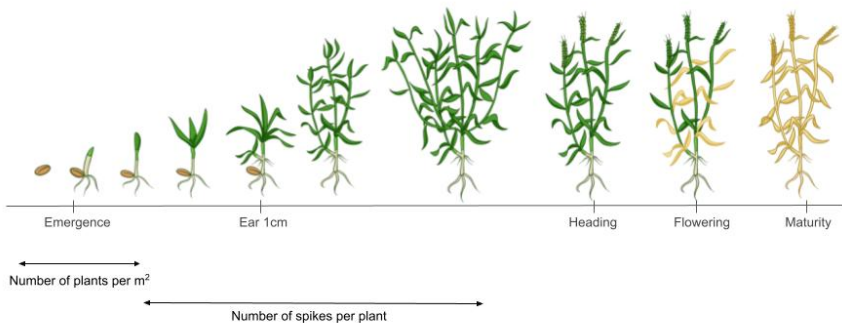
Yield components

Yield (g/m^2) = plant density \times #spikes/plant \times #grains/spike \times grain weight



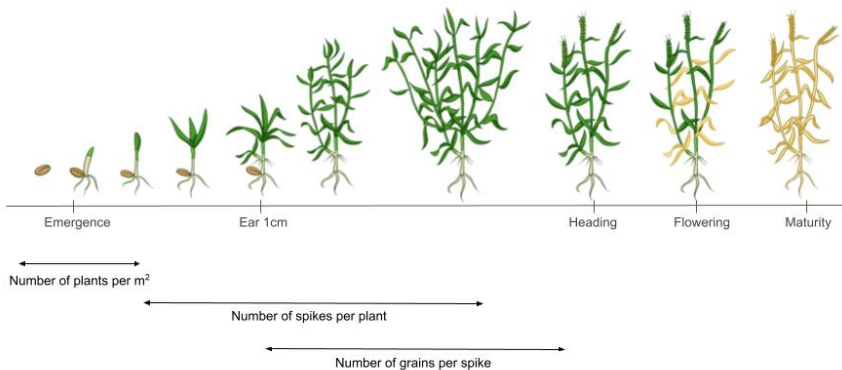
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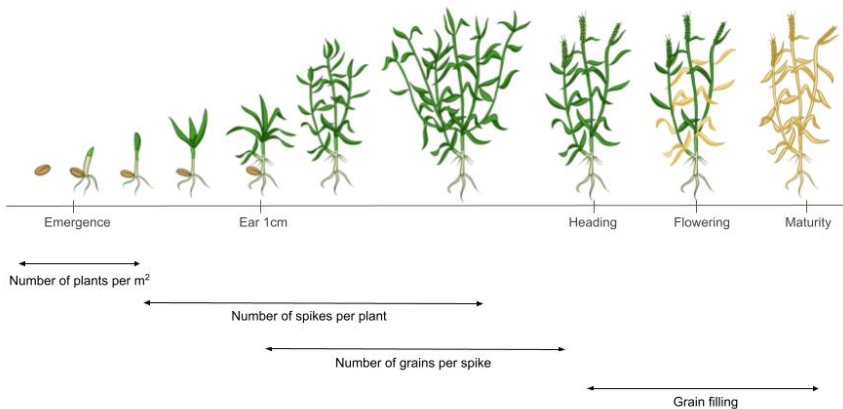
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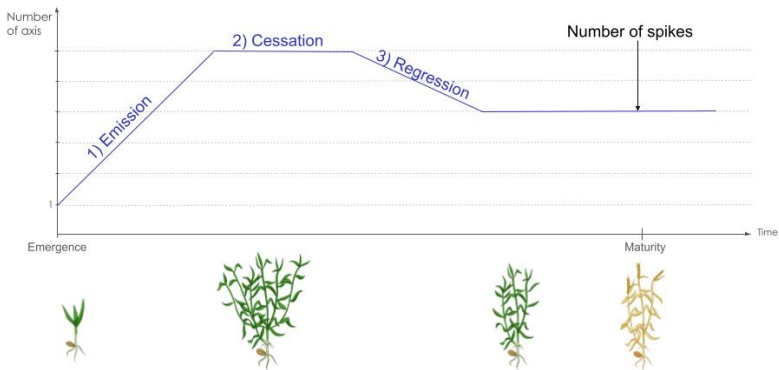


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Tillering dynamics



→ Focus on competition for light and its effect on tillering in conventional conditions

Research question

Understand the performance of cultivar mixtures from the evolution of varietal proportions between sowing and harvest by focusing on competition for light:

- Which traits are important for mixture performance?
- What is the importance of phenotypic plasticity for these traits and how is it shaped by plant-plant interactions?
- Are there differences in plastic responses among genotypes?

⇒ Interdisciplinary approach between field experiments and mechanistic modeling

Modeling to study cultivar mixtures

- Experiments are essential but onerous
 - Diversification complicates combinatory designs
 - Difficulties raised by cultivar distinction in cultivar mixtures → heavy and limited design (precision sowing)

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- FSPMs
 - Detailed 3D architecture + radiative model → long simulation times (ADEL-wheat, CN-wheat, WALTER)
 - GreenLab models: simplified architecture + source-sink formalism → short simulation times

Modeling strategy

- Objective of the model: simulate plants grown in cultivar mixtures

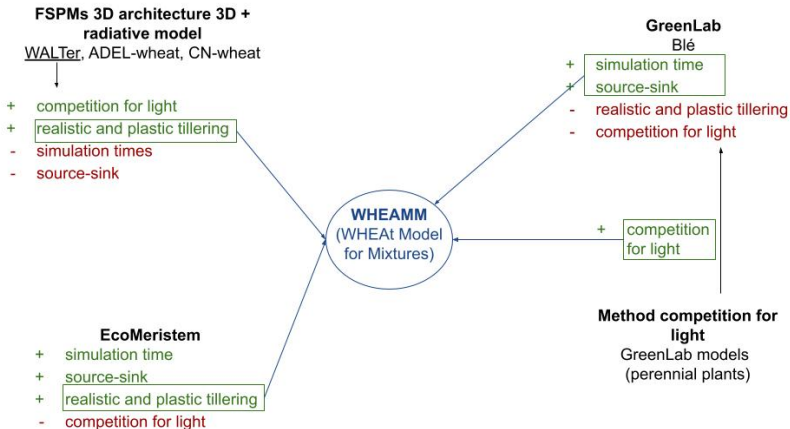
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- Future applications: tool to support breeding for mixtures
 - Generate large datasets in mixtures to predict mixing ability
 - Test assembling of cultivars in mixtures
 - Efficient calibration for new cultivars

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- Future applications: tool to support breeding for mixtures
 - Generate large datasets in mixtures to predict mixing ability
 - Test assembling of cultivars in mixtures
 - Efficient calibration for new cultivars
- Criteria
 - Short simulation time
 - Simulation of source-sink relations
 - Simulation of competition for light
 - Simulation of realistic and plastic tillering dynamics

Conception of WHEAMM



Conception of WHEAMM

- Organs: leaves, internodes, ears (no root system)
- Organogenesis → thermal time calendar
- Biomass production by photosynthesis with the Monteith law

$$q(n) = PAR_n \cdot S_{el}(n) \cdot RUE$$

n : Day

q : Produced biomass (g)

PAR : Photosynthetically Active Radiation ($MJ \cdot m^{-2}$)

S_{el} : Exposed leaf surface (m^2)

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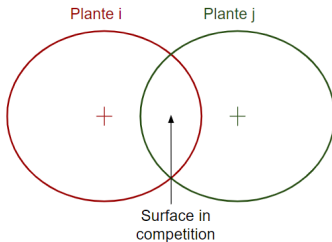
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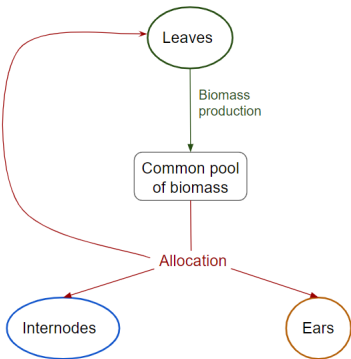
RUE : Radiation Use Efficiency ($g \cdot MJ^{-1}$)

- Approximation of S_{el} with a dedicated method for competition for light based on disks partition (Cournede et al. 2007)



Conception of WHEAMM

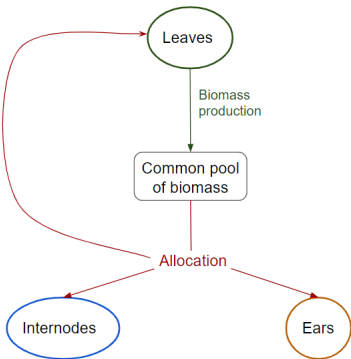
- Biomass allocation with organ-specific beta functions (Baey 2014)



Conception of WHEAMM

- Biomass allocation with organ-specific beta functions (Baey 2014)
- Height simulated as an independent process → final height of an internode of rank r (Lecarpentier et al. 2019) :

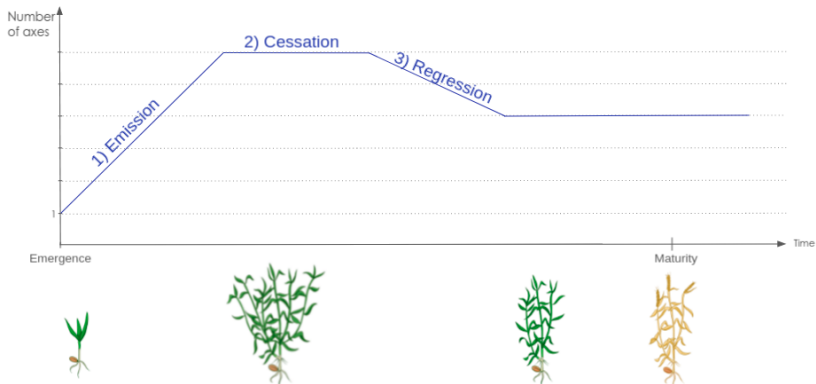
$$h(r) = (a \cdot r + b)^2$$



Conception of WHEAMM

■ Tillering dynamics

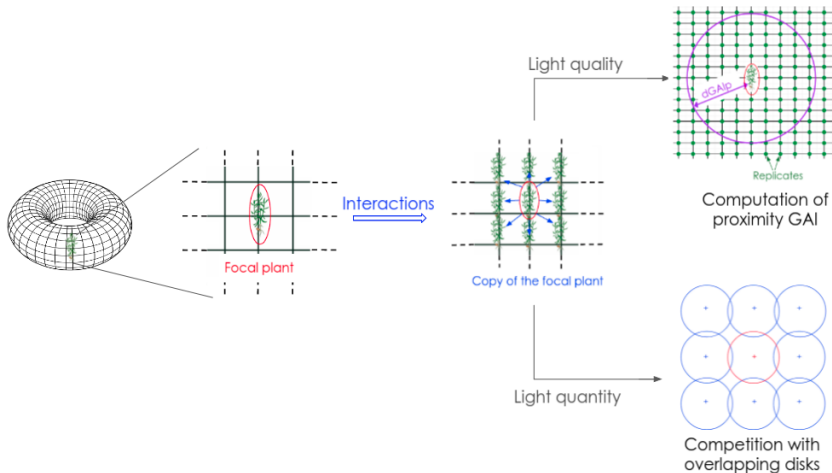
- Emission
- Cessation at R:FR ratio approximated by critical proximity Green Area Index GAI_c (Lecarpentier et al. 2019)
- Regression when allocated biomass to an axis below threshold T_k (Larue et al. 2019) with protection delay Δ_{prot} (Lecarpentier et al. 2019)



Conception of WHEAMM

Two model versions

- Individual plants (700 seconds)
- Average-plant (8 seconds) → homogeneous canopy



Experimental data (Gawinowski et al. 2022)

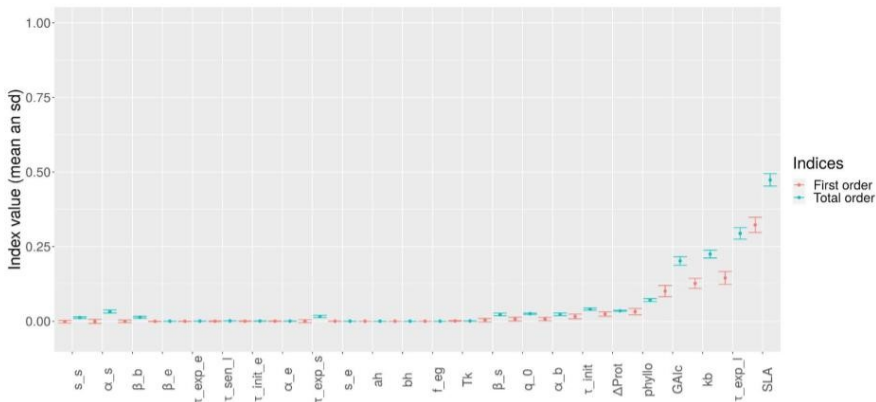
- Field experiment (Perfomix project) at Le Moulon, France in 2019-2020 and 2020-2021
- 8 winter wheat elite cultivars with different heading dates or final heights → two contrasted quaternary mixtures
- Precision sowing for mixed stands by respecting spatial distributions
- Phenotyping
 - Growth: height, number of tillers, plant biomass, leaves surface and weight
 - Harvest: yield components

B	Ber	Acc	Exp	Exp	Ber	Acc	Exp	Exp	Ber	Acc	Exp	15
C	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	14
D	Ber	Acc	Exp	Exp	Acc	Exp	Exp	Exp	Acc	Exp	Exp	13
E	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	12
F	Exp	Acc	Exp	Exp	Acc	Exp	Exp	Exp	Acc	Exp	Exp	11
G	Exp	Acc	Exp	Exp	Acc	Exp	Exp	Exp	Acc	Exp	Exp	10
H	Exp	Acc	Exp	Exp	Acc	Exp	Exp	Exp	Acc	Exp	Exp	9
A	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	8
B	Ber	Acc	Exp	Exp	Ber	Acc	Exp	Exp	Ber	Acc	Exp	7
C	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	6
D	Ber	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Ber	Exp	Exp	5
E	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	4
F	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	3
G	Exp	Acc	Exp	Exp	Acc	Exp	Exp	Exp	Acc	Exp	Exp	2
H	Acc	Exp	Exp	Acc	Exp	Acc	Exp	Exp	Acc	Exp	Exp	1
	1	2	3	4	5	6	7	8	9	10	11	12



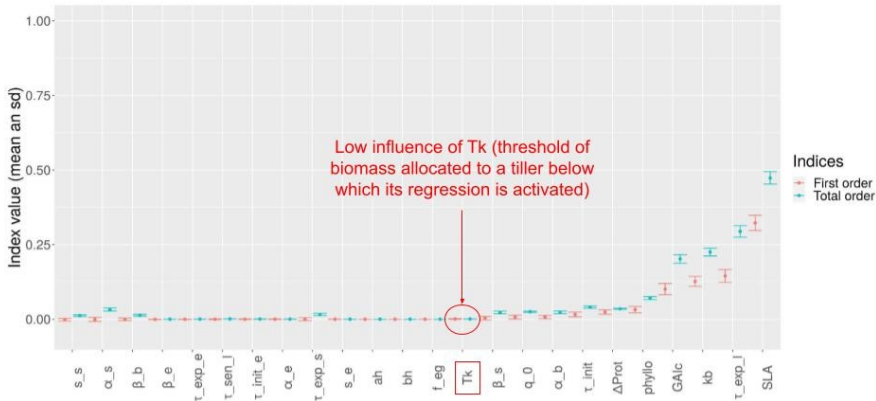
Global sensitivity of WHEAMM in pure stands

- GlobalSensitivityAnalysis Julia package ([Rennels & Anthoff 2022](#))
- Computation of first and total order Sobol indices ([Sobol 2001](#)) for 24 parameters on different outputs
- Number of tillers at harvest



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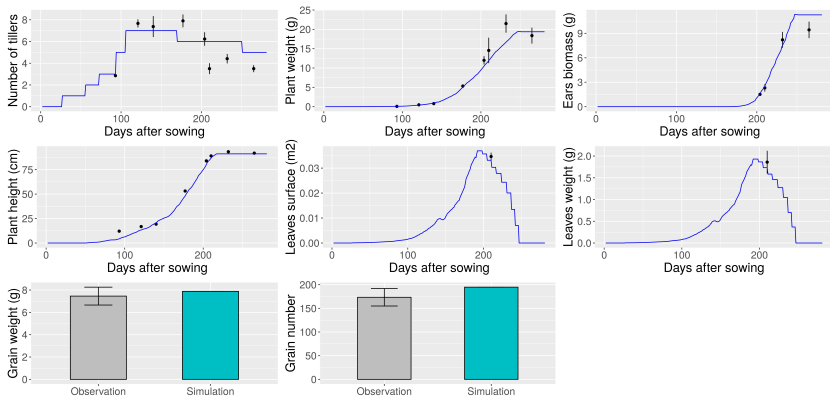
Automatic calibration of WHEAMM in pure stands

- Experimental data → Perfromix experiment for 8 elite cultivars grown in pure stands with variables throughout growth and at harvest
- Optim Julia package (Mogensen & Riseth 2018), simulated annealing algorithm with bounds (Goffe et al. 1994)
- 16 parameters jointly calibrated for each genotype and season separately
- Objective function (Seber & Wild 2003)

$$RMSE = \frac{1}{N_V} \sum_{v=1}^V \frac{1}{N_{T_v}} \sum_{t=1}^{T_v} \sqrt{\sum_{i=1}^{I_{vt}} \left(\frac{y_{vti,obs} - y_{vt,sim}}{\sigma_{vt}} \right)^2}$$

Automatic calibration of WHEAMM in pure stands

Example of simulations with calibrated parameters for genotype Expert in 2019-2020



Automatic calibration of WHEAMM in pure stands

- Better goodness-of-fit in 2019-2020 than 2020-2021

Automatic calibration of WHEAMM in pure stands

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- Calibrated parameters
 - Contrasted estimations between genotypes and/or seasons
 - 10% of parameters calibrated to their upper bounds
 - High values: extinction coefficient k_b , thermal time of emergence τ_{init} , biomass threshold activating regression T_k

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 - Solution: make regression irreversible and more progressive

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- Identification of misspecifications regarding tillering dynamics with parameter T_k
 - High values \rightarrow systematic activation, regression truly controlled by Δ_{prot}
 - Solution: make regression irreversible and more progressive
- Non conclusive validation
 - Another season \rightarrow joint calibration
 - Another density \rightarrow tillering misspecifications, calibration on several densities (Blanc et al. 2021)

Simulations in mixture

- "Individual plants" version
- Perspective to study differences in means and variances between pure and mixed stands on simulated data → implementation of stochasticity on the emergence date to simulate inter-individual variability between plants of the same genotype, even in pure stands

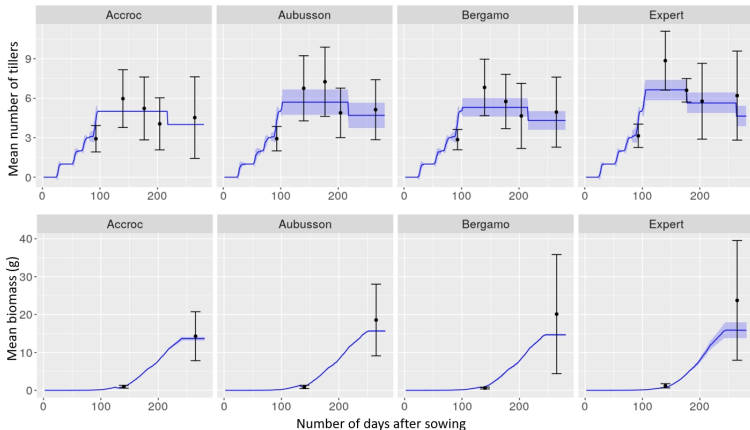
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- Sensitivity analysis in mixture
- Simulations in mixtures
 - Parameter values calibrated in pure stands in 2019-2020
 - Simulations of the two quaternary mixtures of the Perfomix experiment in 2019-2020
 - Mixture 1 (contrasted earliness)

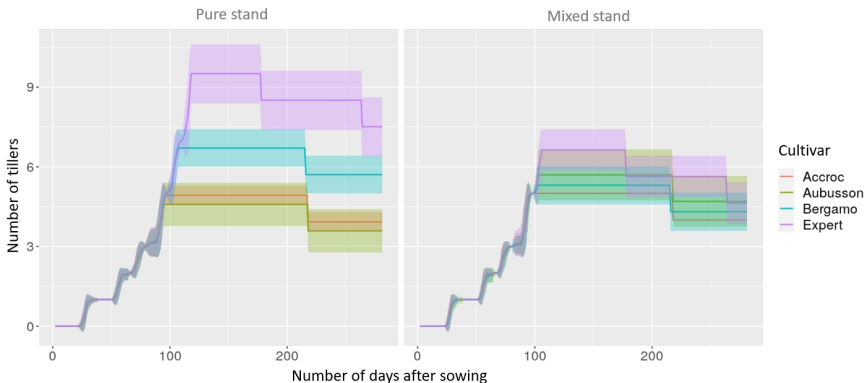
Simulations Mixture 1 in 2019



- Satisfying tillering (although peak too early and regression too weak for Expert) but biomass often underestimated
- Simulated variability largely underestimated

Convergence of tillering dynamics in mixture

Simulated data for mixture 1 in 2019-2020



→ In accordance with experimental data

Discussion

Development of WHEAMM

- All criteria

- Source-sink formalism + minimalist 3D architecture → fast simulations
- Realistic tillering dynamics in 3 phases with plastic responses to competition for light
- Implementation of competition for light without detailed 3D architecture

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Development of WHEAMM

■ All criteria

- Source-sink formalism + minimalist 3D architecture → fast simulations
- Realistic tillering dynamics in 3 phases with plastic responses to competition for light
- Implementation of competition for light without detailed 3D architecture

■ Confrontation to reality

- WHEAMM not fully validated on the PerfoMix experiment
- Satisfying calibration for many cultivars in pure stands
- Simulation of diverse plastic behaviors in mixtures
- Emergence of tillering convergence in mixtures

Discussion

Importance of sensitivity analysis and automatic calibration in a modeling cycle

- "The point of falsifying a model is not to learn that the model is false but rather to learn the ways in which a model is false" (Gelman & Shalizi 2013)

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- Essential to good modeling practices ([Cournède et al. 2013](#)) but too rare for FSPMs
- Manual calibration too subjective
- No consensus on calibration practices for crop models ([Wallach 2011](#)) but consideration of entire process

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Modeling challenges → computation time

- WHEAMM "average plant" 8 seconds vs WHEAMM "individual plants" 10 minutes

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- Parallel simulations → possible for sensitivity analyses but not always for automatic calibration (iterative process)
- Calibration → parallel computing within the one simulation (parallel computation of different plants at one timestep)
- Metamodeling: Gaussian processes (kriging) metamodels to approximate complex models coupled with a sequential EGO (expectation global optimization) method
 - Successful automatic calibration for tillering parameters in WALTer ([Blanc et al. 2021](#))
 - Further developments required for WHEAMM → plant scale, several variables

Conclusion

Conclusion

- Use of Julia for model development
 - Efficiency
 - Accessible
 - Important number of packages
 - Community
 - But lack of solutions for some very specific issues...
- Perspectives
 - Solve identified tillering misspecifications, add sources of stochasticity and a root sink
 - Many possible applications, notably coupling WHEAMM with population genetics models to study heterogeneous populations over sowing/resowing cycles

Thank you for your attention!

References I

- Baey, C. (2014), Modélisation de la variabilité inter-individuelle dans les modèles de croissance de plantes et sélection de modèles pour la prévision, PhD thesis.
- Blanc, E., Enjalbert, J. & Barbillon, P. (2021), Automatic calibration of a functional-structural wheat model using an adaptive design and a metamodelling approach, preprint, Bioinformatics.
URL: <http://biorxiv.org/lookup/doi/10.1101/2021.07.29.454328>
- Borg, J., Kiær, L., Lecarpentier, C., Goldringer, I., Gauffreteau, A., Saint-Jean, S., Barot, S. & Enjalbert, J. (2018), 'Unfolding the potential of wheat cultivar mixtures: A meta-analysis perspective and identification of knowledge gaps', *Field Crops Research* **221**, 298–313.
URL: <https://linkinghub.elsevier.com/retrieve/pii/S0378429017309498>
- Cournede, P.-H., Mathieu, A., Houllier, F., Barthelemy, D. & de Reffye, P. (2007), 'Computing Competition for Light in the GREENLAB Model of Plant Growth: A Contribution to the Study of the Effects of Density on Resource Acquisition and Architectural Development', *Annals of Botany* **101**(8), 1207–1219.
URL: <https://academic.oup.com/aob/article-lookup/doi/10.1093/aob/mcm272>
- Cournède, P.-H., Chen, Y., Wu, Q., Baey, C. & Bayol, B. (2013), 'Development and Evaluation of Plant Growth Models: Methodology and Implementation in the PYGMALION platform', *Mathematical Modelling of Natural Phenomena* **8**(4), 112–130.
URL: <http://www.mmnp-journal.org/10.1051/mmnp/20138407>
- Forst, E. (2018), Développement de méthodes d'estimation de l'aptitude au mélange pour la prédiction des performances et la sélection de mélanges variétaux chez le blé tendre et co-conception d'idéotypes de mélanges adaptés à l'agriculture biologique, PhD thesis, Paris-Saclay, Gif-sur-Yvette.

References II

- Gawinowski, M., Enjalbert, J., Cournede, P.-H. & Flutre, T. (2022), Contrasted phenotypic plasticities of life-history traits in interacting plants: case of pure vs mixed stands of wheat, preprint, *Plant Biology*.
URL: <http://biorxiv.org/lookup/doi/10.1101/2022.10.27.514050>
- Gelman, A. & Shalizi, C. R. (2013), 'Philosophy and the practice of Bayesian statistics: *Philosophy and the practice of Bayesian statistics*', *British Journal of Mathematical and Statistical Psychology* **66**(1), 8–38.
URL: <https://onlinelibrary.wiley.com/doi/10.1111/j.2044-8317.2011.02037.x>
- Goffe, W. L., Ferrier, G. D. & Rogers, J. (1994), 'Global optimization of statistical functions with simulated annealing', *Journal of Econometrics* **60**(1-2), 65–99.
URL: <https://linkinghub.elsevier.com/retrieve/pii/0304407694900388>
- Guntrip, J. & Sibly, R. M. (1998), 'Phenotypic plasticity, genotype-by-environment interaction and the analysis of generalism and specialization in *Callosobruchus maculatus*', *Heredity* **81**(2), 198–204.
URL: <http://link.springer.com/10.1046/j.1365-2540.1998.00354.x>
- Larue, F., Fumey, D., Rouan, L., Soulié, J.-C., Roques, S., Beurier, G. & Luquet, D. (2019), 'Modelling tiller growth and mortality as a sink-driven process using Ecomeristem: implications for biomass sorghum ideotyping', *Annals of Botany* **124**(4), 675–690.
URL: <https://academic.oup.com/aob/article/124/4/675/5429672>
- Lecarpentier, C., Barillot, R., Blanc, E., Abichou, M., Goldringer, I., Barbillon, P., Enjalbert, J. & Andrieu, B. (2019), 'WALTER: a three-dimensional wheat model to study competition for light through the prediction of tillering dynamics', *Annals of Botany* **123**(6), 961–975.
URL: <https://academic.oup.com/aob/article/123/6/961/5281415>

References III

- Mogensen, P. K. & Riseth, A. N. (2018), 'Optim: A mathematical optimization package for Julia', *Journal of Open Source Software* **3**(24), 615.
URL: <http://joss.theoj.org/papers/10.21105/joss.00615>
- Rennels, L. & Anthoff, D. (2022), 'GlobalSensitivityAnalysis'.
URL: <https://juliahub.com/ui/Packages/GlobalSensitivityAnalysis/dILX8/1.1.3>
- Schauberger, B., Ben-Ari, T., Makowski, D., Kato, T., Kato, H. & Ciais, P. (2018), 'Yield trends, variability and stagnation analysis of major crops in France over more than a century', *Scientific Reports* **8**(1), 16865.
URL: <http://www.nature.com/articles/s41598-018-35351-1>
- Seber, G. A. F. & Wild, C. J. (2003), *Nonlinear regression*, Wiley series in probability and statistics, Wiley-Interscience, Hoboken, N.J.
- Sobol, I. (2001), 'Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates', *Mathematics and Computers in Simulation* **55**(1-3), 271–280.
URL: <https://linkinghub.elsevier.com/retrieve/pii/S0378475400002706>
- Wallach, D. (2011), 'Crop Model Calibration: A Statistical Perspective', *Agronomy Journal* **103**(4), 1144.
URL: <https://www.agronomy.org/publications/aj/abstracts/103/4/1144>