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Insights of using Control Theory for minimizing Induced Seismicity in Underground Reservoirs

Diego Gutiérrez-Oribio^a, Ioannis Stefanou^{a,*}

^aNantes Université, Ecole Centrale Nantes, CNRS, GeM, UMR 6183, F-44000, Nantes, France

Abstract

Deep Geothermal Energy, Carbon Capture, and Storage and Hydrogen Storage have significant potential to meet the large-scale needs of the energy sector and reduce the CO₂ emissions. However, the injection of fluids into the earth's crust, upon which these activities rely, can lead to the formation of new seismogenic faults or the reactivation of existing ones, thereby causing earthquakes. In this study, we propose a novel approach based on control theory to address this issue. First, we obtain a simplified model of induced seismicity due to fluid injections in an underground reservoir using a diffusion equation in three dimensions. Then, we design a robust tracking control approach to force the seismicity rate to follow desired references. In this way, the induced seismicity is minimized while ensuring fluid circulation for the needs of renewable energy production and storage. The designed control guarantees the achievement of the control objectives even in the presence of system uncertainties and unknown dynamics. Finally, we present simulations of a simplified geothermal reservoir under different scenarios of energy demand to show the reliability and performance of the control approach, opening new perspectives for field experiments based on real-time regulators.

Keywords:

Energy geotechnics, Geothermal energy, Energy and energy product storage, Earthquake prevention, Induced seismicity, Robust control

1. INTRODUCTION

Deep Geothermal Energy, Carbon Capture and Storage, and Hydrogen Storage show promising potential in addressing the substantial energy sector demands while mitigating CO₂ emissions. However, their effectiveness relies on injecting fluids into the Earth's crust, a process that may potentially induce earthquakes [1, 2, 3]. The occurrence of induced seismicity poses a significant threat to the feasibility of projects employing these techniques. This concern has led to the closure of several geothermal plants worldwide, such as those in Alsace, France, in 2020 [4, 5], Pohang, South Korea, in 2019 [6, 7], and Basel, Switzerland, in 2009 [8, 9].

Earthquakes initiate when there is a sudden release of significant elastic energy stored within the Earth's crust due to abrupt sliding along fault lines [10, 11].

The injection of fluids can lead to the formation of new seismogenic faults or the reactivation of existing ones, which cause earthquakes [2, 3, 12]. More specifically, elevated fluid pressures at depth amplify the amount of elastic energy accumulated in the Earth's crust while reducing the friction along faults. As a result, the likelihood of earthquakes significantly increases, even in regions that are typically considered to have low seismic potential (see [2], [7], and [12], among others).

Therefore, earthquake prevention strategies are necessary to mitigate induced seismicity in the energy sector [2, 7, 12]. Traffic light systems, cyclic stimulation and fracture caging are the most widely used approaches for earthquake prevention [13, 14, 15, 16]. Nevertheless, to our knowledge, these methods rely on trial and error rather than a systematic control approach. They lack a mathematical foundation and cannot guarantee the avoidance of induced seismicity. Moreover, there is no proof that these methods cannot even trigger earthquakes of greater magnitudes than the ones they are supposed to mitigate [17]. The fundamental assumptions of

*Corresponding author.

Email addresses: diego.gutierrez-oribio@ec-nantes.fr (Diego Gutiérrez-Oribio), ioannis.stefanou@ec-nantes.fr (Ioannis Stefanou)

38 traffic light systems and its effectiveness were also ques- 90
39 tioned in [18, 19, 20].

40 More recently, significant progress has been made in 92
41 controlling the earthquake instability of specific, well- 93
42 defined, mature seismic faults [21, 22, 23, 24, 25, 26]. 94
43 These studies have employed various control algorithms 95
44 to stabilize the complex and uncertain nature of the un- 96
45 derlying underactuated physical system. The designed 97
46 controllers effectively stabilize the system and modify
47 its natural response time. As a result, the energy dissi-
48 pation within the system occurs at a significantly slower
49 rate, orders of magnitude lower than that of the natural
50 (uncontrolled) earthquake event. However, it is worth
51 noting that these investigations did not consider factors
52 such as the presence of multiple smaller faults, which
53 are typically found in deep geothermal reservoirs, as
54 well as fluid circulation constraints associated with en-
55 ergy production. Regarding the controllability, observ-
56 ability and parameter identification of geological reser-
57 voirs, we refer to [27, 28, 29].

58 In [30], the authors introduce a control strategy for 100
59 a deep geothermal system. However, it is important to 101
60 note that this study focused on a 1D diffusion system, 102
61 the control design was implemented based on a dis- 103
62 cretized model (not accounting for possible spillover), 104
63 and the output considered was the pressure over the 105
64 reservoir, rather than the seismicity rate (SR). There- 106
65 fore, this result presents many limitations to cope in- 107
66 duced seismicity over real applications. 108

67 This study accounts for water injections into an un- 109
68 derground reservoir using a simplified model of a 3D 110
69 diffusion equation. By utilizing this Partial Differen- 111
70 tial Equation (PDE), a robust tracking control strategy 112
71 is developed to regulate the SR, ensuring tracking to 113
72 a desired reference. The primary control objective is 114
73 to prevent induced seismicity while maintaining energy 115
74 production. The designed control scheme demonstrates 116
75 resilience against system uncertainties and unmodelled 117
76 dynamics. Simulations of the process are conducted to 118
77 validate the effectiveness of the control approach and 119
78 show the potential for optimizing underground fluid cir- 120
79 culation and thus energy production. Various simu- 121
80 lation scenarios, considering different energy demands 122
81 and constraints, are presented to provide a comprehen- 123
82 sive understanding of the system's behaviour and the re-
83 liability of the control strategy. This research opens new
84 perspectives for field applications at the kilometric scale
85 based on real-time regulators and control theory.

86 The structure of this paper can be outlined as follows. 124
87 In Section 2, a motivating example of a simplified un- 125
88 derground reservoir is presented showing how the SR 126
89 increases when we inject fluids. Section 3 introduces 127

the underlying 3D diffusion model and shows the feed- 90
back control used for minimizing induced seismicity. 91
To demonstrate the effectiveness of the proposed control 92
strategy, simulations are conducted in Section 4, consid- 93
ering different scenarios of intermittent energy demand 94
and production constraints. Finally, concluding remarks 95
are provided in Section 5, summarizing the key findings 96
of the study. 97

98 2. EXAMPLE OF INDUCED SEISMICITY IN A 99 100 RESERVOIR DUE TO FLUID INJECTIONS

101 Consider a simplified underground reservoir at ap- 102
proximately 4 [km] below the earth's surface, as de- 103
picted in Fig. 1. The reservoir is made of a porous rock 104
which allows the circulation of fluids through its pores 105
and cracks. In our example, the reservoir has a thickness 106
of ~ 100 [m] and covers horizontally a square surface 107
of dimensions ~ 5 [km] \times 5 [km]. Wells are injecting 108
and/or extracting fluids (*e.g.* water) at different in- 109
jection points in the reservoir, as shown in Fig. 1. For the 110
sake of simplicity, injection of fluids will refer to both 111
injection and fluid extraction from the reservoir. 112

113 Pumping fluids in-depth causes the circulation of flu- 114
ids in the reservoir, which, in turn, causes the host 115
porous rock to deform. The hydro-mechanical behav- 116
iour of the reservoir due to the injection of fluids at 117
depth can be described by Biot's theory [31]. According 118
to this theory, the diffusion of the fluid and the deforma- 119
tion of the host porous rock are coupled dynamic pro- 120
cesses. However, if the injection rates are slow enough, 121
with respect to the characteristic times of the system due 122
to inertia, and if the volumetric strain rate of the host 123
porous rock is negligible, then the diffusion of the fluid
in the host rock due to fluid injections can be described
by the following diffusion equation [32]

$$124 \quad u_t = -\frac{1}{\beta} \nabla q + s, \quad (1)$$

125 where $u = u(x, t)$ is the change of the fluid pressure in 126
the reservoir due to fluid injections, x is the spatial coor- 127
dinate, $t \geq 0$ is the time, u_t denotes the partial derivative 128
of u with respect to time, $q = -\frac{k}{\eta} \nabla u$ is the change of 129
the hydraulic flux and s is a source/sink term represent- 130
ing the fluid injections. Furthermore, k is the perme- 131
ability of the host rock, η is the dynamic viscosity of the 132
fluid, and β is the compressibility of the rock-fluid 133
mixture. All these parameters are assumed constant in 134
most of the following examples and, thus, they can de-
fine a simple expression for the hydraulic diffusivity of

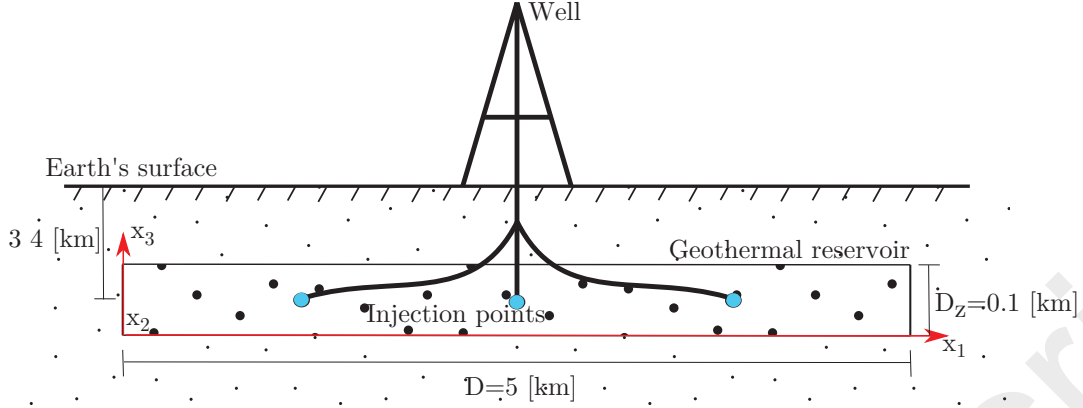


Figure 1: Underground reservoir diagram.

the system, $c_{hy} = k/\eta\beta$. A special case where heterogeneities are present in β and c_{hy} is covered in Section 4. Finally, the reservoir has volume V .

We consider drained boundary conditions at the boundary of the reservoir, *i.e.*, $u = 0$ at ∂V . Furthermore, we assume point source terms, as the diameter of the wells is negligible compared to the size of the reservoir. In particular we set $s = \frac{1}{\beta}\mathcal{B}(x)Q(t)$, where $Q(t) \in \mathcal{R}^m$, $Q(t) = [Q_1(t), \dots, Q_m(t)]^T$, are injection fluxes applied at the injection points, (x^1, \dots, x^m) , through the coefficient $\mathcal{B}(x) \in \mathcal{R}^{1 \times m}$, $\mathcal{B}(x) = [\delta(x - x^1), \dots, \delta(x - x^m)]$. The terms $\delta(x - x^i)$ are Dirac's distributions and m is the number of the wells in the reservoir. For the rigorous statement of the mathematical problem and its control we refer to Section 3 and Appendix A to Appendix E.

It is nowadays well established that the injection of fluids in the earth's crust causes the creation of new and the reactivation of existing seismic faults, which are responsible for notable earthquakes (see for instance [2], [12] and [7]). The physical mechanisms behind those human-induced seismic events is connected with the change of stresses in the host rock due to the injections, which intensify the loading and/or reduce the friction over existing or new discontinuities (faults). In other words, fluid injections increase the SR in a region, *i.e.*, the number of earthquakes in a given time window.

The seismicity rate, R , of a volume containing nucleation sources, *i.e.* a region, depends on the average stress rate change, $\dot{\tau}$, over the same region according to the following expression

$$\dot{R} = \frac{R}{t_a} \left(\frac{\dot{\tau}}{\dot{\tau}_0} - R \right), \quad (2)$$

where $\dot{}$ denotes the time derivative, t_a is a characteristic decay time and $\dot{\tau}_0$ is the background stress change rate in the region, *i.e.* the stress change rate due to var-

ious natural tectonic processes, and is considered to be constant. The above equation coincides with the one of Segall and Lu [33] (see also [34]), with the difference that here the SR is defined region-wise rather than point-wise. This choice results in a more convenient formulation as we mainly focus on averages over large volumes rather than point-wise measurements of the SR, which can be also singular due to point sources. Following Segall and Lu [33] we assume also that the stress change rate is a linear function of the pore fluid pressure change rate, *i.e.*, $\dot{\tau} = \dot{\tau}_0 + f\dot{u}$, where \dot{u} is the average fluid pressure change rate over a given region of the reservoir and f a (mobilized) friction coefficient. The latter linear hypothesis is justified on the basis of Coulomb friction over the fault planes and Terzaghi's effective stress principle [35].

It is worth emphasizing that, according to [34], the volume containing the nucleation sources must be small enough for all of them to experience uniform stress changes. Apparently, this is improbable during fluid injections in the Earth's crust, hence our simplifying assumption of averaging the Coulomb stress changes, $\dot{\tau}$, over a predefined region. The size of the averaging volumes (regions) could potentially be determined based on statistical arguments to accommodate the maximum expected induced earthquake magnitude (see for instance [36]). However, these considerations extend beyond the scope of the current work (see also Section 5 for more details about the limitations of the current approach). Finally, the point-wise SR model could be recovered at the limit of infinitesimal volumes. In this sense the region-wise SR approach used here is more general.

In the absence of fluid injections, $\dot{\tau} = \dot{\tau}_0$ and, therefore, $R \rightarrow 1$. In this case, the SR of the region reduces to the natural one. If, on the contrary, fluids are injected

204 into the reservoir, then $\dot{u} > 0$ and consequently, $\dot{\tau} > \dot{\tau}_0$,
 205 leading to an increase of the SR ($\dot{R} > 0$) over the region.
 206 To illustrate this mechanism, let us consider an injection
 207 of $Q = Q_{s_1} = 32$ [m³/hr] through a single injection
 208 well (see [37, 38] for enhanced geothermal systems
 209 with similar injection rates). In this numerical exam-
 210 ple, we consider the parameters of Table 1, we depth
 211 average Equation 1 as shown in Appendix C and we in-
 212 tegrate the resulting partial differential equation in time
 213 and space using a spectral decomposition method as ex-
 214 plained in Appendix D. We then calculate the SR over
 215 two distinct regions, one close to the injection point and
 216 one in the surroundings. Fig. 2 shows the location of
 217 the regions and of the injection point.

218 The SR in both regions as a function of time is shown
 219 in fig. 3. We observe that the maximum SR over V_1 is
 220 equal to $R_1 = 7975$, which means that 7975 more earth-
 221 quakes of a given magnitude in a given time window are
 222 expected over region V_1 , than without injections. The
 223 seismicity is even higher near the injection well (see re-
 224 gion V_2 in Fig. 3). Fig. 4 shows the evolution of the
 225 pressure over the reservoir through different times. The
 226 pressure experiences a gradual rise across extensive ar-
 227 eas near the injection point, eventually stabilizing at ap-
 228 proximately two years.

229 In the case of an Enhanced Geothermal System (EGS
 230 [39]), we would like to increase the permeability be-
 231 tween two wells by creating a small network of cracks
 232 that would facilitate the circulation of fluids between the
 233 wells [40]. The creation of those cracks would be as-
 234 sociated with a localized microseismicity in the region
 235 encircling the wells. This microseismicity is welcome,
 236 provided that the overall SR over the larger region of the
 237 reservoir remains close to one. Therefore, in the control
 238 problem addressed in this work, we will set as control
 239 objective the controlled increase of the SR in a small
 240 region surrounding some wells (*e.g.*, in region V_2 , see
 241 Fig. 2), while keeping constant and equal to one the SR
 242 over the larger area of the reservoir (*e.g.*, in region V_1 ,
 243 see Fig. 2). For this purpose, additional wells will be
 244 added in the reservoir, whose fluxes will be controlled
 245 by a specially designed controller. This controller will
 246 be robust to uncertainties of the system parameters and
 247 will achieve the aforementioned control objective under
 248 different production rates.

Table 1: Diffusion and Seismicity rate system parameters. Such pa-
 rameters are consistent with real applications like [41, 42].

Parameter	Description	Value and Units
c_{hy}	Hydraulic diffusivity	3.6×10^{-4} [km ² /hr]
D	Reservoir length	5 [km]
D_z	Reservoir depth	0.1 [km]
Q_s	Static flux	32 [m ³ /hr]
β	Mixture compressibility	1.2×10^{-4} [1/MPa]
f	Friction coefficient	0.5 [-]
τ_0	Background stressing rate	1×10^{-6} [MPa/hr]
t_a	Characteristic decay time	500100 [hr]

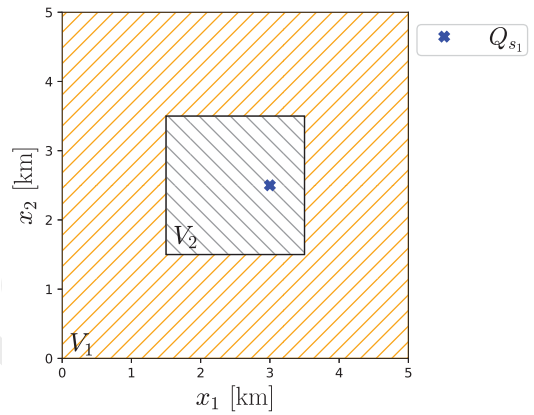


Figure 2: Regions V_1 and V_2 and location of the injection well with
 flux Q_{s_1} inside of region V_2 .

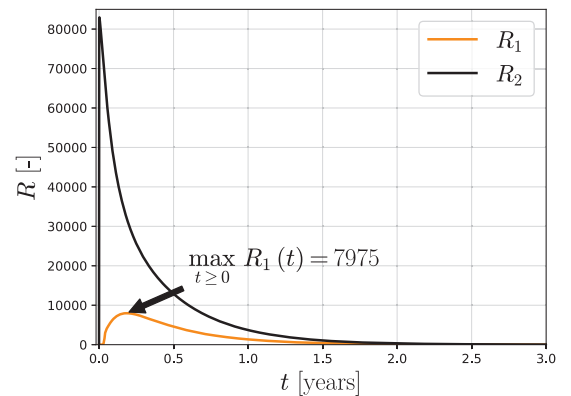


Figure 3: Seismicity rate in both regions, V_1, V_2 with constant injec-
 tion rate, Q_{s_1} . 7975 more earthquakes of a given
 time window are expected over the outer region of the reservoir due
 to the constant fluid injection.

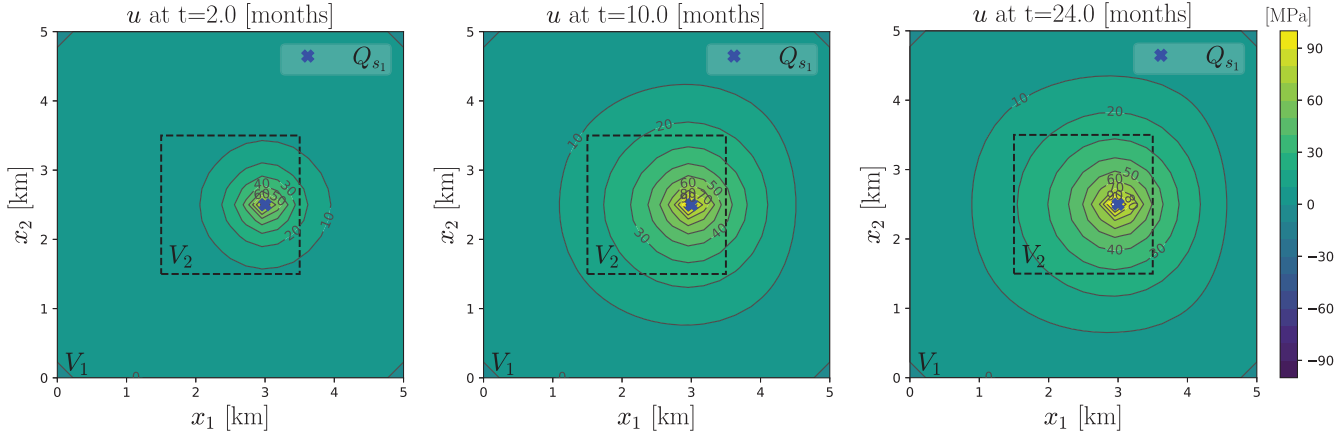


Figure 4: Solution, $u(x, t)$, of the pressure's reservoir at different times, with constant injection rate, Q_{s_1} . The solution presents high pressure profiles in wide areas next to the point of injection.

3. PROBLEM STATEMENT AND CONTROL DESIGN

The diffusion system (1) can be written as follows

$$\begin{aligned}
 u_t(x, t) &= c_{hy} \nabla^2 u(x, t) + \frac{1}{\beta} [\mathcal{B}_s(x) Q_s(t) + \mathcal{B}_c(x) Q_c(t)], \\
 u(x, t) &= 0 \quad \forall \quad x \in S,
 \end{aligned}
 \tag{3}$$

where $u(x, t)$ is the fluid pressure change evolving in the space $H^1(V)$, $x \in \mathfrak{R}^3$, $x = [x_1, x_2, x_3]^T$, is the space variable belonging to a bounded subset V of the space \mathfrak{R}^3 with boundary $S = \partial V$, and $t \geq 0$ is the time variable. As mentioned above, c_{hy} is the hydraulic diffusivity and β is the compressibility of the rock-fluid mixture. $Q_s(t) \in \mathfrak{R}^{m_s}$, $Q_s(t) = [Q_{s_1}(t), \dots, Q_{s_{m_s}}(t)]^T$, are fixed (not controlled) fluxes applied at the injection points, $(x_s^1, \dots, x_s^{m_s})$, trough the coefficient $\mathcal{B}_s(x) \in \mathfrak{R}^{1 \times m_s}$, $\mathcal{B}_s(x) = [\delta(x - x_s^1), \dots, \delta(x - x_s^{m_s})]$, and $Q_c(t) \in \mathfrak{R}^{m_c}$, $Q_c(t) = [Q_{c_1}(t), \dots, Q_{c_{m_c}}(t)]^T$, are the controlled fluxes applied at the injection points, $(x_c^1, \dots, x_c^{m_c})$, trough the coefficient $\mathcal{B}_c(x) \in \mathfrak{R}^{1 \times m_c}$, $\mathcal{B}_c(x) = [\delta(x - x_c^1), \dots, \delta(x - x_c^{m_c})]$. Note that the number of original inputs, m , in system (1) is equal to the sum of not controlled and controlled input of system (3), *i.e.*, $m = m_s + m_c$ ($Q(t) = \mathcal{B}_s(x) Q_s(t) + \mathcal{B}_c(x) Q_c(t)$). Since the right-hand side of (1) contains Dirac's distributions, the above boundary value problem is interpreted in the weak sense (see Appendix A for more details on the notation and Appendix B for the definition of weak solution).

As explained before, the SR in equation (2) is defined region-wise. In this study, we will define the SR over m_c regions, $V_i \subset V$, $i \in [1, m_c]$ of the underground reservoir

as follows

$$\dot{h}_i = \frac{f}{t_a \tau_0 V_i} \int_{V_i} u_t(x, t) dV - \frac{1}{t_a} (e^{h_i} - 1), \quad i \in [1, m_c],
 \tag{4}$$

where the change of variables

$$R_i = e^{h_i}, \quad i \in [1, m_c],
 \tag{5}$$

has been used for ease of calculation. The objective of this work is to design the control input Q_c driving the output $y \in \mathfrak{R}^{m_c}$ defined as

$$y = [h_1, \dots, h_{m_c}]^T,
 \tag{6}$$

of the underlying BVP (3)–(4) to desired references $r(t) \in \mathfrak{R}^{m_c}$, $r(t) = [r_1(t), \dots, r_{m_c}(t)]^T$. This is known as tracking. Note that solving such tracking problem results in solving the tracking for the SR system (2) due to the change of variables (5), *i.e.*, $R_i(t)$ will be forced to follow the desired reference $\bar{r}_i(t) = e^{r_i(t)}$ for $i \in [1, m_c]$.

For that purpose, let us define the error variable, $y_e \in \mathfrak{R}^{m_c}$, as follows

$$y_e = y - r,
 \tag{7}$$

and the control $Q_c(t)$ given by

$$\begin{aligned}
 Q_c(t) &= B_0^{-1} \left(-K_1 [y_e]^{1/l} + v + \dot{r} \right), \\
 \dot{v} &= -K_2 [y_e]^{1/l},
 \end{aligned}
 \tag{8}$$

where $K_1 \in \mathfrak{R}^{m_c \times m_c}$, $K_2 \in \mathfrak{R}^{m_c \times m_c}$ are matrices to be designed, and $l \in [-1, 0]$ is a freely chosen parameter [43, 44]. The matrix $B_0 \in \mathfrak{R}^{m_c \times m_c}$ is a nominal matrix that depends on the system parameters (see equation (E.6) in Appendix E for its definition). The function $[y_e]^l := |y_e|^l \text{sign}(y_e)$ is applied element-wise and

is determined for any $\gamma \in \mathbb{R}_{\geq 0}$ (see Appendix A for more details). Such control has different characteristics depending on the value of l . It has a discontinuous integral term when $l = -1$, *i.e.*, $[y_e]^0 = \text{sign}(y_e)$. When $l \in (-1, 0]$ the control function is continuous and degenerates to a linear integral control when $l = 0$. Note how the controller is designed with minimum information about the system (E.4), *i.e.* with only the measurement of $y(t)$ and the knowledge of the nominal matrix B_0 .

The tracking result for the output (4)–(6) is then in force:

Let system (3)–(4) be driven by the control (8) with $K_1 > 0$, $K_2 > 0$ and B_0 as in (E.6). Then, the error variable (7) will tend to zero in finite-time if $l = [-1, 0)$, or exponentially if $l = 0$.

In other words, we force the SR to follow desired references to avoid induced seismicity over the underground reservoir. (See Appendix E for the mathematical derivation of the proof and further details of the control algorithm.)

3.1. Energy demand and production constraints

We will consider a new scenario where an additional number of flux restrictions, m_r , to the fluid injection of the controlled injection points is needed. In other words, we will impose the weighted sum of the injection rates of some of the wells to be equal to a time-variant, possibly intermittent production rate.

For this purpose, we will augment the vector of controlled injection points as $\bar{Q}_c(t) \in \mathfrak{R}^{m_c+m_r}$, $\bar{Q}_c(t) = [\bar{Q}_{c_1}(t), \dots, \bar{Q}_{c_{m_c+m_r}}(t)]^T$. Notice that the number of controlled injection points, m_c , has to be increased to $m_r + m_c$. This does not change the previous theoretical results as shown below.

The condition imposed over the control input, $\bar{Q}_c(t)$, is

$$W\bar{Q}_c(t) = D(t), \quad (9)$$

where $W \in \mathfrak{R}^{m_r \times (m_c+m_r)}$ is a full rank matrix whose elements represent the weighted participation of the well's fluxes for ensuring the demand $D(t) \in \mathfrak{R}^{m_r}$. In order to follow this, the new control input will be designed as

$$\bar{Q}_c(t) = \bar{W}Q_c(t) + W^T(WW^T)^{-1}D(t), \quad (10)$$

where $Q_c(t)$ is the original control input designed as (8) and $\bar{W} \in \mathfrak{R}^{(m_c+m_r) \times m_c}$ is the null space of W . Note that if we replace (10) in (9), the demand over the controlled injection points will be strictly fulfilled at any time t .

Control (10) will ensure the linear combination of the new controlled fluxes $\bar{Q}_c(t)$ to be equal to a predetermined flux $D(t)$, which we called demand, according to

(9), while keeping the original output tracking result of the previous section. This can be of interest in geoenery applications to cope with different types of energy demand and production constraints (see Appendix E.1 for more details).

Note that by monitoring the SR in the regions of interest and using equations (8) and (10) it is theoretically possible to adjust the fluid flux of the wells in a reservoir and achieve the desired control objectives in terms of the SR, while achieving production constraints.

4. SIMULATIONS

In order to demonstrate our control approach, numerical simulations of (3) and (4) have been done in Python (scripts available upon request). Without loss of generality and for a simpler presentation of the results, we chose to depth average Equation 3 as shown in Appendix C and integrate the resulting two-dimensional partial differential equation in time and space using Runge-Kutta method RK23 [45] and spectral decomposition method as explained in Appendix D, respectively. The same parameters with the numerical simulations performed in section 2 were used (see Table 1).

Simulations were performed for three scenarios, *i.e.* one without any predetermined demand, one with constant demand and one with intermittent demand. In all scenarios we consider a fixed injection well with flux $Q_s(t) = Q_{s_1}(t) = 32$ [m³/hr] situated at the same location as the fixed injection well of the example presented in section 2. Moreover, following the same example, we consider two different regions, V_1, V_2 over which we calculate the SR, R_1, R_2 . Consequently, the number of outputs to be tracked is equal to two and, thus, at least two control inputs have to be designed ($Q_c(t) = [Q_{c_1}(t), Q_{c_2}(t)]^T$, $m_c = 2$). In Fig. 5 (top) we show the chosen location of the control wells. The initial conditions of the systems (3) and (4) were set as $u(x, 0) = 0$ and $h_1(0) = h_2(0) = 0$ (*i.e.*, $R_1(0) = R_2(0) = 1$).

To design the control (8), the matrices B_s, B_c of system (E.4) are needed (see Appendix E for more details), and they are defined as

$$B_s = \frac{f}{t_a \tau_0 \beta} \begin{bmatrix} 0 \\ \frac{1}{V_2} \end{bmatrix}, \quad B_c = \frac{f}{t_a \tau_0 \beta} \begin{bmatrix} \frac{1}{V_1} & 0 \\ 0 & \frac{1}{V_2} \end{bmatrix}. \quad (11)$$

For the scenarios with predetermined demand, we will apply a single flux restriction, *i.e.*, $m_r = 1$, with $W = [1, 1.01, 1]$ (see Section 3.1 for more details). As a result, an addition control will be needed (*i.e.* $m_c + m_r = 3$), whose location is depicted in Fig. 5 (bottom). Therefore, the matrices \bar{B}_c, B_c of system (E.10)

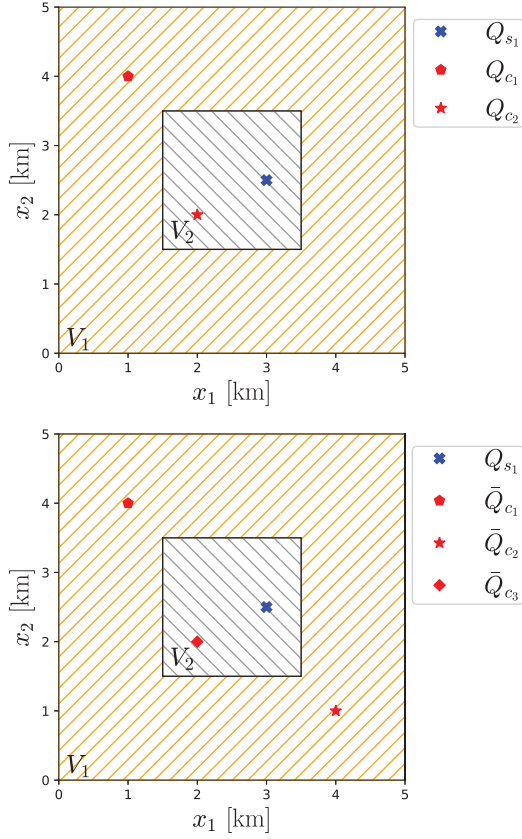


Figure 5: Regions V_1 and V_2 and location of the injection wells in the cases without demand (top) and with demand (bottom).

become

$$\bar{B}_c = \frac{f}{t_a \dot{\tau}_0 \beta} \begin{bmatrix} \frac{1}{V_1} & \frac{1}{V_1} & 0 \\ 0 & 0 & \frac{1}{V_2} \end{bmatrix}, \quad (12)$$

$$B_c = \bar{B}_c \bar{W} = \frac{f}{t_a \dot{\tau}_0 \beta} \begin{bmatrix} \frac{0.22}{V_1} & \frac{-0.78}{V_1} \\ \frac{-0.21}{V_2} & \frac{0.79}{V_2} \end{bmatrix}$$

and the matrix B_s is given in (11).

Finally, in all scenarios, the reference $r(t)$ was selected as $r(t) = [r_1(t), r_2(t)]^T$, where $r_1(t) = 0$ and $r_2(t)$ is a smooth function that reaches the final value of $\ln(5)$ in 1 [month] (see Figs 6, 9 and 13). This reference was chosen so the SR on every region, V_1, V_2 to converge to the final values 1 and 5, respectively. This selection aims at forcing the SR in the extended region V_1 to be the same as the natural one. Regarding, region V_2 we opt for an increase of the SR in order to facilitate the circulation of fluids and improve the production of energy.

4.1. Scenario 1: SR tracking without demand

In this scenario, the control (8) was implemented with a nominal matrix B_0 as

$$B_0 = \frac{f_0}{t_{a_0} \dot{\tau}_{0_0} \beta_0} \begin{bmatrix} \frac{1}{V_{1_0}} & 0 \\ 0 & \frac{1}{V_{2_0}} \end{bmatrix}, \quad (13)$$

where the subscript ‘0’ corresponds to the nominal values of the system’s parameters. In our case, we have chosen all the nominal values 10% higher than the real ones, e.g., $f_0 = 1.1f$.

The gain parameters of the control (8) were selected as

$$K_1 = \begin{bmatrix} 1.5 \times 10^{-2} & 0 \\ 0 & 6.7 \times 10^{-2} \end{bmatrix}, \quad (14)$$

$$K_2 = \begin{bmatrix} 1.1 \times 10^{-4} & 0 \\ 0 & 2.2 \times 10^{-3} \end{bmatrix}, \quad l = -0.6$$

The results are depicted in Figures 6 to 8. In both regions, seismicity rates align with the specified constant references after approximately one month, achieving a steady state more rapidly than the uncontrolled system, as illustrated in Figure 3, which took around two years to reach a steady state. This robust performance is attributed to the control strategy, which effectively addresses uncertainties in the system. Specifically, the control uses only the nominal matrix B_0 and compensates for the remaining error dynamics. The generated control signals, presented in Figure 8, exhibit continuous fluxes that can be used in practical pump actuators. Figure 7 displays the pressure profile, $u(x, t)$, at different time points. In contrast to the scenario without control (refer to Figure 4), the control strategy effectively prevents the propagation of high-pressure profiles around the static injection point.

4.2. Scenario 2: SR tracking with constant demand

In this scenario we consider the demand to be equal to $D(t) = -Q_{s_1} = -32$ [m³/hr] (see Fig. 11). This is interesting in applications where the extracted fluid is re-injected into the reservoir.

The control $\bar{Q}_c(t)$ was designed as (8),(10) with the nominal matrix B_0

$$B_0 = \frac{f_0}{t_{a_0} \dot{\tau}_{0_0} \beta_0} \begin{bmatrix} \frac{0.22}{V_{1_0}} & \frac{-0.78}{V_{1_0}} \\ \frac{-0.21}{V_{2_0}} & \frac{0.79}{V_{2_0}} \end{bmatrix}, \quad (15)$$

where the subscript ‘0’ corresponds to the nominal values of every system parameter. Again, we have chosen all the nominal values 10% larger than the real ones to test robustness. The control gains were selected as in

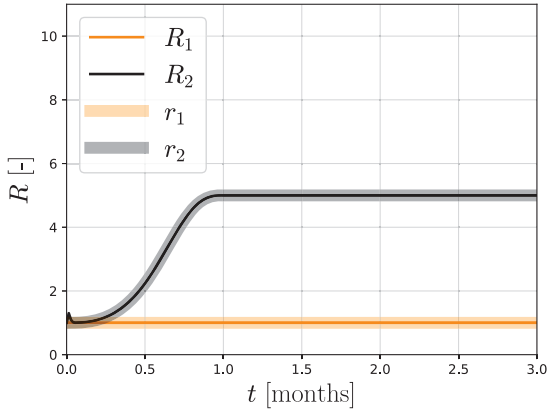


Figure 6: Seismicity rate in regions, V_1, V_2 . The control strategy forces the SR to follow the desired references preventing induced seismicity.

(14) and the results are shown in Figs. 9–11. Consistent with the earlier findings, the control effectively monitors the SR in two regions, but now under the influence of the imposed flux restriction (9) on the control wells (refer to Fig 11, bottom). The control strategy successfully regulates the seismicity rates across both regions, achieving a stable low pressure solution as in the previous case (see Figure 10).

Figure 11 (top) shows fast-frequency oscillations (chattering) in the control signals due to the discontinuous nature of the employed controller. One can reduce such oscillations by choosing the linear case of the control, *i.e.*, $l = 0$ instead of $l = -0.6$. Indeed, in Fig. 11 (center) we observe the linear control signal generated for this case, which is smoother, but its convergence is slower and less precise. The latter is demonstrated in Fig. 12, where the norm of the tracking error is shown for both cases. Therefore, the selection of the control parameter, l , offers a trade-off between smoothness and precision, resulting in a great flexibility depending on the requirements of specific practical applications.

4.3. Scenario 3: SR tracking with intermittent demand

In order to test a more plausible demand scenario, we will apply the same control strategy as in the previous case, but using an intermittent demand (*cf.* [41]), $D(t)$, as depicted in Fig. 15 (bottom). According to this injection plan, the demand varies abruptly between the flux of the fixed well and zero. The results are shown in Figs. 13–15. Despite the intermittent nature of the demand, leading to numerous transients in the SR as depicted in Figure 13, the generated control signals (Figure 15 (top)) swiftly restore the tracking performance for the SR. It is noteworthy that the control signals ad-

here strictly to the intermittent demand $D(t)$, as evident in Figure 15 (bottom). Additionally, the abrupt shifts in demand result in higher pressure profiles compared to previous scenarios, as illustrated in Figure 14. Nevertheless, the control strategy ensures the steady state of the pressure solution after approximately 24 months.

4.4. Scenario 4: SR tracking with heterogeneities

As a final test to validate the robustness of the presented control strategy, we will introduce a more realistic scenario where there is presence of heterogeneities in the reservoir, more specifically in the hydraulic diffusivity and the mixture compressibility parameters, *i.e.*, $c_{hy} = c_{hy}^{het}(x)$ and $\beta = \beta^{het}(x)$. Fig. 16 illustrates the spatial variation of these parameters normalized by their constant values, as considered in the previous paragraphs. This variation extends to three orders of magnitude.

Fig. 17 shows how the pressure evolves in the heterogeneous reservoir under the constant injection rate $Q_{s1}(t) = 32$ [m³/hr] (no control). We observe that the contours are not circular convex anymore as in Fig. 4 due to heterogeneity. Furthermore, now the steady state is reached in a longer time (around 30 years in this simulation) due to the new distribution of $c_{hy}^{het}(x)$ and $\beta^{het}(x)$ across the reservoir.

The results of the simulations for the control case (Scenario 1) under heterogeneous parameters are depicted in Figs. 18–20. Although there are some oscillations in the control signal of the injection well Q_{c2} (see Fig. 20) due to its location on the reservoir (see $\beta^{het}(x)$ value in Fig. 16), the control algorithm is still able to compensate such uncertainties and accomplish the tracking of the SR. The steady state of the pressure solution is achieved in a longer time compared to the case without heterogeneity (Fig. 7), but ten times faster than in the case without control (Fig. 17).

5. CONCLUSIONS AND LIMITATIONS

A new control strategy for minimizing induced seismicity while assuring fluid circulation for energy production in underground reservoirs is designed in this paper. Contrary to existing approaches for induced seismicity mitigation due to fluid injections in the earth's crust, the proposed control strategy ensures the robust tracking of desired seismicity rates over different regions in geological reservoirs. For this purpose, a robust controller uses the measurement of averages of SR over different regions to generate continuous control signals for stabilizing the error dynamics, despite the presence

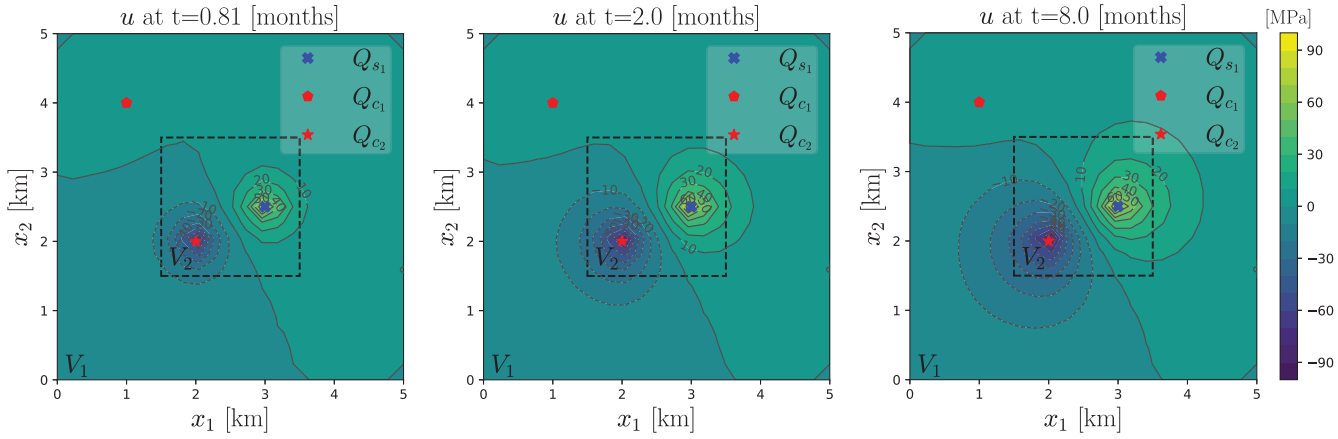


Figure 7: Fluid pressure distribution, $u(x, t)$, in the reservoir at different times. The control strategy prevents the propagation of high-pressure profiles around the static injection point.

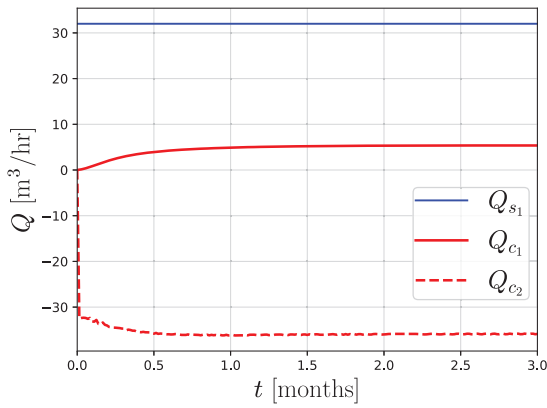


Figure 8: Static flux input Q_{s_1} and controlled flux inputs Q_{c_1}, Q_{c_2} .

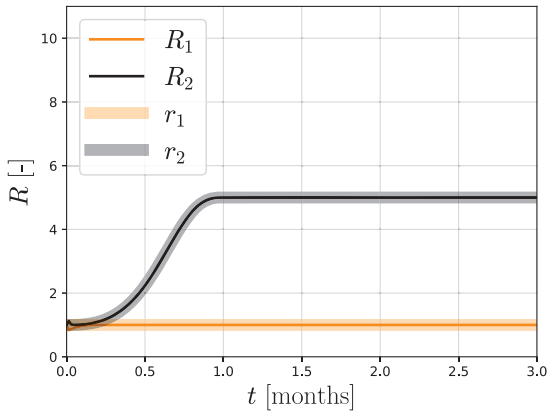


Figure 9: Seismicity rate in regions, V_1, V_2 under the constraint of constant demand. The control strategy forces the SR to follow the desired references while respecting the imposed energy demand.

522 tem where it is always difficult to measure the real sys-
 523 tem parameters (*e.g.*, diffusivity and compressibility),
 524 or there are errors in the sensing of physical quantities
 525 (*e.g.*, SR and accelerometers).

526 A series of numerical simulations confirm the effec-
 527 tiveness of the presented theory over a simplified model
 528 of an underground reservoir under different scenarios.
 529 This provides a new direction for using robust control
 530 theory for this challenging application that involves an
 531 uncertain, underactuated, non-linear system of infinite
 532 dimensionality for mitigating induced seismicity while
 533 maximizing renewable energy production and storage.

534 However, assessing earthquake risk solely based on
 535 the seismicity rate may pose limitations, as earthquake
 536 magnitude holds greater significance than seismicity
 537 rate (*c.f.*, Pohang EGS project [38, 46] vs. Basel EGS
 538 project [37] earthquakes). The magnitude of expected
 539 earthquakes can be related to their frequency accord-
 540 ing to modified Richter-Gutenberg distributions as pre-
 541 sented in [36] and related works. The same holds for
 542 the maximum magnitude of the expected earthquakes,
 543 which may be connected to the size of the activated re-
 544 gions, V_i . However, the incorporation of this kind of
 545 statistical analysis exceeds the scope of this work.

546 Furthermore, the inclusion of fault discontinuities
 547 should be studied in real scenarios. Controlling multiple
 548 faults within a reservoir (*cf.*, [47]) remains a challenge
 549 due to complexity and faster spatio-temporal scales as-
 550 sociated with poroelastodynamic phenomena activated
 551 by intermittent injections. Controlling this dynamics to-
 552 gether with point-wise SR, instead of region-wise, non-
 553 linear constraints for the fluxes and error in measure-
 554 ments, among others, is under investigation.

It is worth mentioning that the current approach has

520 of system uncertainties and unknown error dynamics.
 521 This is of great importance on this complicated sys-

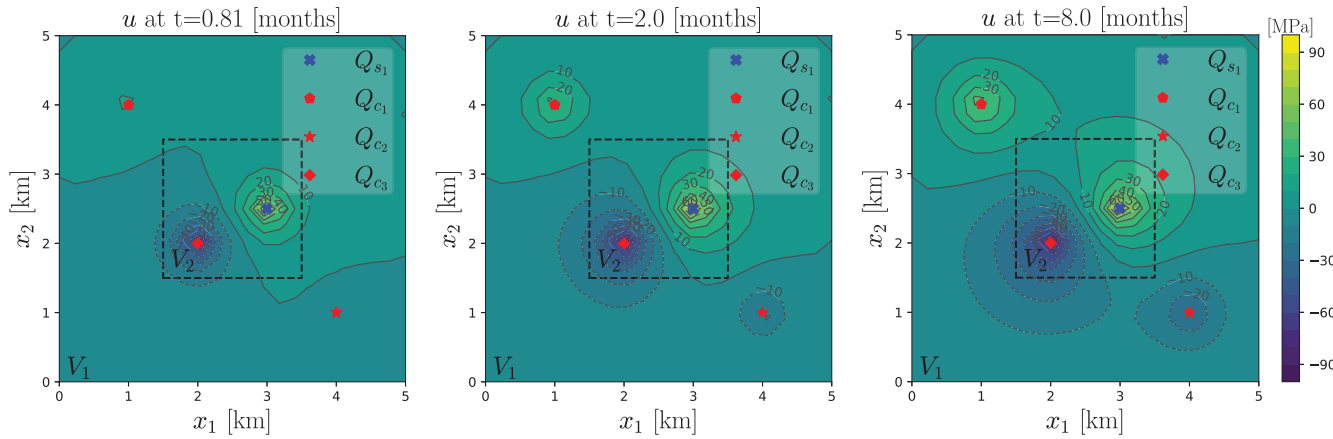


Figure 10: Pressure distribution, $u(x, t)$, in the reservoir at different times, under the constraint of constant demand. The control strategy regulates the SR across both regions, achieving stable low pressure solution profiles.

not yet been experimentally tested. A first step towards the experimental testing of the control of the slip of a single fault has been successful in the laboratory (see [23]), but this test seems rather adequate for intermittent injections and induced seismicity at the level of reservoirs. Moreover, field-scale testing is with no doubt necessary despite the methodological challenges related to repeatability and other methodological problems. Hence, this work is part of a more general effort for providing rigorous mathematical proofs and numerical simulations for exploring the extent up to which induced seismicity can be prevented in practice, while maximizing renewable energy production and storage.

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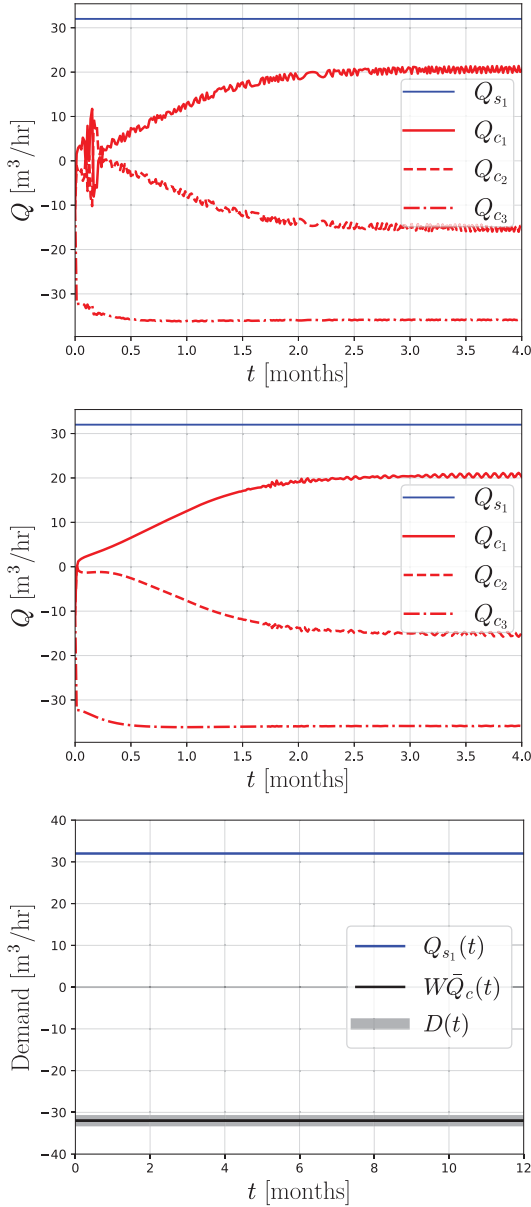


Figure 11: Injection flux of fixed well Q_{s_1} and injection fluxes of the controlled wells Q_{c_1} , Q_{c_2} , Q_{c_3} , under the constraint, $D(t) = W\bar{Q}_c(t) = -Q_{s_1}(t)$. Discontinuous case (top) and continuous case (center). The demand (bottom plot) is always ensured.

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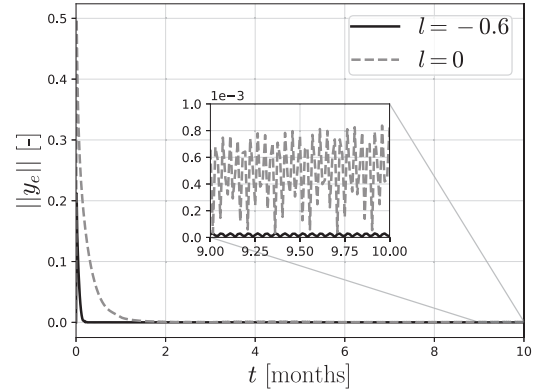


Figure 12: Norm of the tracking error, y_e , with constant demand using discontinuous control (continuous black line) and continuous control (dotted gray line).

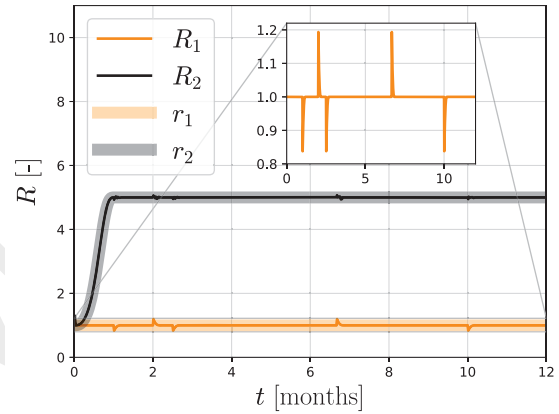


Figure 13: Seismicity rate in regions, V_1, V_2 under the constraint of intermittent demand. The control strategy forces the SR to follow the desired references while respecting the intermittent energy demand.

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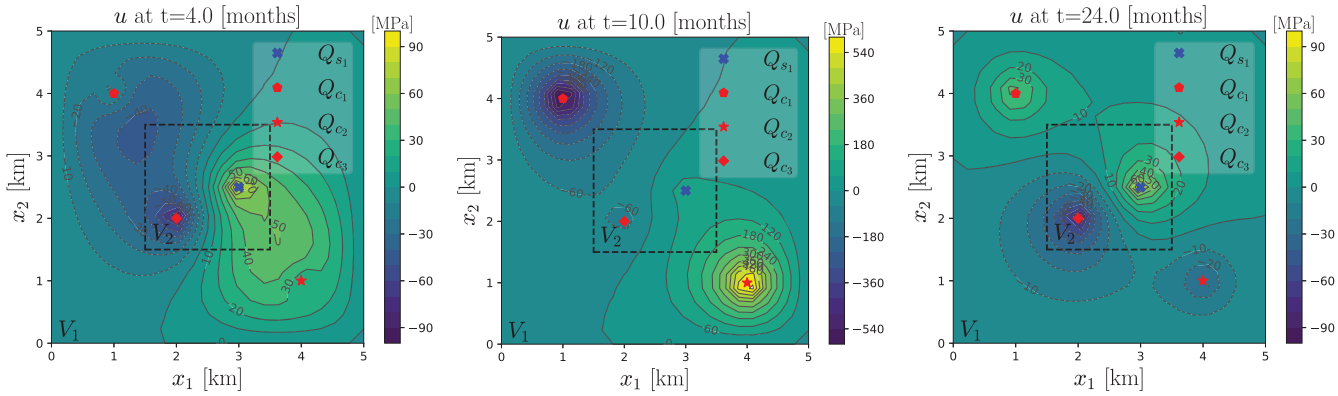


Figure 14: Fluid pressure distribution, $u(x, t)$, in the reservoir at different times, under the constraint of intermittent demand. The abrupt shifts in demand result in higher pressure profiles compared to previous scenarios. Nevertheless, the control strategy ensures the steady state of the pressure solution after approximately 24 months.

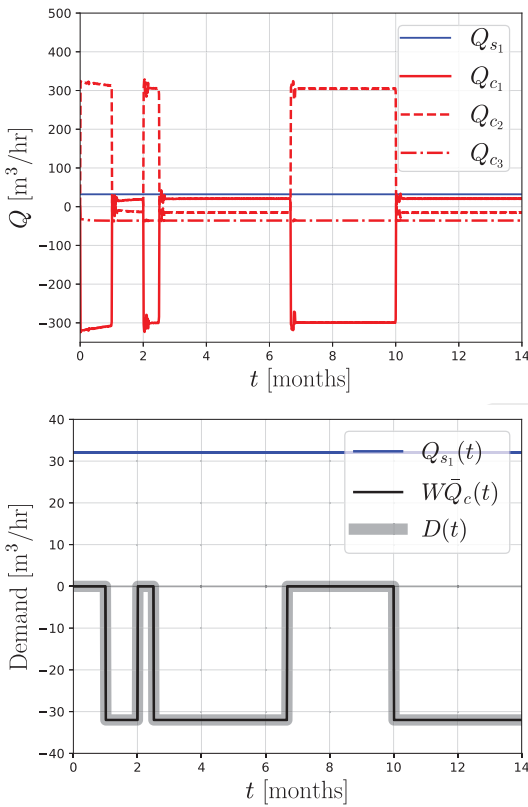


Figure 15: Injection flux Q_{s_1} of the fixed well and injection fluxes $Q_{c_1}, Q_{c_2}, Q_{c_3}$ of the control well under the constraint of intermittent demand. The demand (bottom) is strictly ensured again.

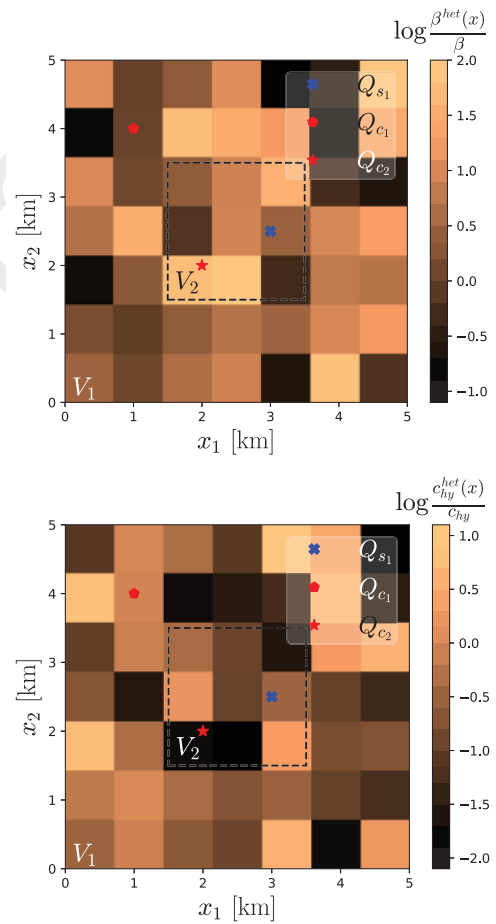


Figure 16: Heterogeneity of $\beta^{het}(x)$ and $c_{hy}^{het}(x)$ in the underground reservoir. The average weight of hydraulic diffusivity in the reservoir is $c_{hy}^{het}(x)/c_{hy} = 1.08$.

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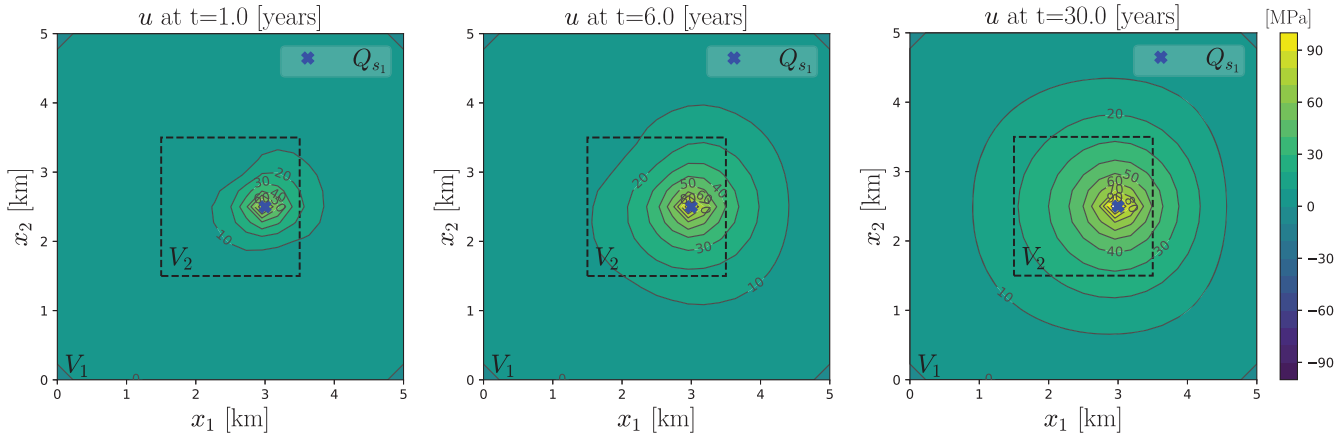


Figure 17: Pressure distribution, $u(x, t)$, at different times, under heterogeneities in the reservoir.

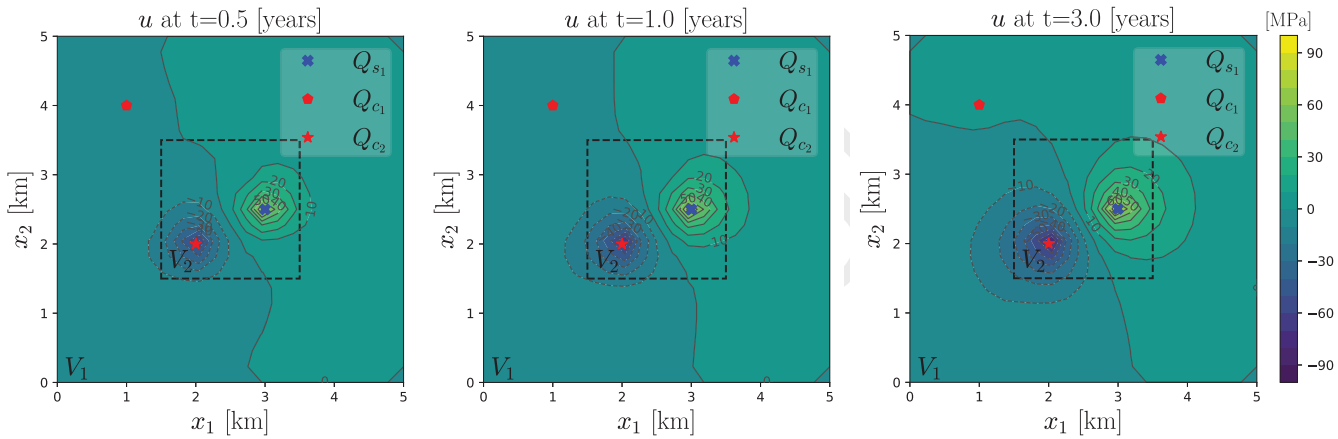


Figure 18: Pressure distribution, $u(x, t)$, at different times, under heterogeneities in the reservoir. The control strategy regulates the SR across both regions, achieving a stable low pressure solution after approximately 3 years.

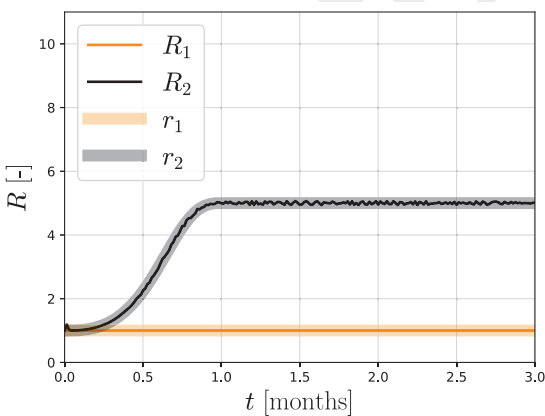


Figure 19: Seismicity rate in regions, V_1, V_2 . The control strategy forces the SR to follow the desired references preventing induced seismicity.

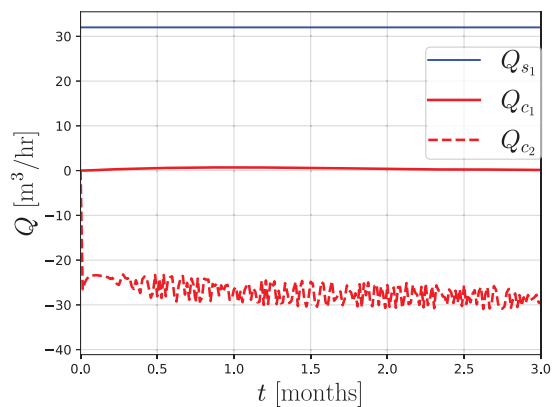


Figure 20: Static flux input Q_{s1} and controlled flux inputs Q_{c1}, Q_{c2} .

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Appendix A. Notation

We denote $\|\cdot\|$ as the euclidean norm of the n -dimensional Euclidean space, \mathfrak{R}^n . If $y_e \in \mathfrak{R}$, the function $[y_e]^\gamma := |y_e|^\gamma \text{sign}(y_e)$ is determined for any $\gamma \in \mathbb{R}_{\geq 0}$.

810 If $y_e \in \mathfrak{K}^m$, the functions $[y_e]^\gamma$ and $|y_e|^\gamma$ will be applied 840
811 element-wise.

812 Consider the Sobolev space, $H^1(V)$, of abso-
813 lutely continuous scalar functions $u(x)$, $x \in \mathfrak{K}^3$,
814 $x = [x_1, x_2, x_3]^T$, defined on a bounded subset
815 V of the space \mathfrak{K}^3 with boundary $S = \partial V$ as
816 $H^1(V) = \{u \mid u, \nabla u \in \mathcal{L}^2(V)\}$. Its H^0 -norm is defined
817 as $\|u(\cdot)\|_{H^0(V)} = \sqrt{\int_V [u(\cdot)]^2 dV}$. The time deriva-
818 tive is denoted by $u_t = \partial u / \partial t$, the gradient as $\nabla u =$
819 $[\partial u / \partial x_1, \partial u / \partial x_2, \partial u / \partial x_3]$, and the Laplacian as $\nabla^2 u = \partial^2 u / \partial x_1^2 +$
820 $\partial^2 u / \partial x_2^2 + \partial^2 u / \partial x_3^2$. We define the Dirac's distribution, $\delta(x)$,
821 as $\int_{V^*} \phi(x) \delta(x - x^*) dV = \phi(x^*)$, $\forall x^* \in V$, $V^* \subset V$, on
822 an arbitrary test function $\phi(x) \in H^1(V)$.

823 For their use, important inequalities are recalled:

824 **Poincaré's Inequality:** For $u(x) \in H^1(V)$ on a bounded
825 subset V of the space \mathfrak{K}^3 of zero-trace (i.e., $u(x, t) = 0$
826 for all $x \in \partial V$), the inequality

$$\|u(x)\|_{H^0(V)}^2 \leq \gamma \|\nabla u(x)\|_{H^0(V)}^2 \quad (\text{A.1})$$

827 with $\gamma > 0$ depending on V , is fulfilled.

828 **Cauchy-Schwarz Inequality:**

$$\int_V f(x)g(x) dV \leq \|f(x)\|_{H^0(V)} \|g(x)\|_{H^0(V)}, \quad (\text{A.2})$$

829 for any $f, g \in \mathcal{L}_2(V)$.

830 Appendix B. Weak solution of the 3D diffusion 831 equation

832 **Definition 1.** [48, 49] A continuous function $u(x, t) \in$
833 $H^1(V)$ is said to be a weak solution of the BVP (3) on
834 $t \geq 0$, if for every $\phi(x) \in H^1(V)$ under BCs $\phi(x) = 0$
835 $\forall x \in \partial V$, the function $\int_V u(x, t)\phi(x) dV$ is absolutely
836 continuous on $t \geq 0$ and the relation

$$\begin{aligned} \int_V u_t(x, t)\phi(x) dV &= -c_{hy} \int_V \nabla u(x, t) [\nabla \phi(x)]^T dV \\ &+ \frac{1}{\beta} [\phi(x_s^1), \dots, \phi(x_s^{m_s})] Q_s(t) \\ &+ \frac{1}{\beta} [\phi(x_c^1), \dots, \phi(x_c^{m_c})] Q_c(t), \end{aligned} \quad (\text{B.1})$$

837 holds for almost all $t \geq 0$.

838 The weak solution (B.1) is obtained by multiplying
839 (3) by the test function $\phi(x)$ and integrating with respect

to the space variable:

$$\begin{aligned} \int_V u_t(x, t)\phi(x) dV &= c_{hy} \int_V \nabla^2 u(x, t)\phi(x) dV \\ &+ \frac{1}{\beta} \int_V [\mathcal{B}_s(x)Q_s(t)] \phi(x) dV \\ &+ \frac{1}{\beta} \int_V [\mathcal{B}_c(x)Q_c(t)] \phi(x) dV. \end{aligned}$$

841 Using integration by parts and the definition of the
842 Dirac's distribution, the latter expression can be rewritten
843 as

$$\begin{aligned} \int_V u_t(x, t)\phi(x) dV &= c_{hy} \int_V \nabla \cdot [\nabla u(x, t)\phi(x)] dV \\ &- c_{hy} \int_V \nabla u(x, t) [\nabla \phi(x)]^T dV \\ &+ \frac{1}{\beta} [\phi(x_s^1), \dots, \phi(x_s^{m_s})] Q_s(t) \\ &+ \frac{1}{\beta} [\phi(x_c^1), \dots, \phi(x_c^{m_c})] Q_c(t). \end{aligned}$$

844 Finally, to retrieve expression (B.1) from the latter ex-
845 pression, the divergence theorem and the BCs were used
846 in the first term of the RHS.

847 Appendix C. Depth average of the 3D diffusion 848 equation

849 The system described by (3) is a three-dimensional
850 system, whose solution would be difficult to plot in a
851 simplified manner. For this purpose and without loss of
852 generality of the theoretical results presented in this
853 study, we chose to limit our numerical simulations to
854 a two-dimensional boundary value problem, which was
855 derived by depth averaging the full, three-dimensional
856 problem given in (3) (see [50] for another example of
857 depth averaging). The depth averaging was performed
858 as follows

$$\begin{aligned} \frac{1}{D_z} \int_0^H u_t(x, t) dx_3 &= \frac{c_{hy}}{D_z} \int_0^H \nabla^2 u(x, t) dx_3 \\ &+ \frac{1}{\beta D_z} \int_0^H [\mathcal{B}_s(x)Q_s(t)] dx_3 \\ &+ \frac{1}{\beta D_z} \int_0^H [\mathcal{B}_c(x)Q_c(t)] dx_3 \\ &= \frac{c_{hy}}{D_z} \int_0^H \frac{\partial^2 u(x, t)}{\partial x_1^2} dx_3 \\ &+ \frac{c_{hy}}{D_z} \int_0^H \frac{\partial^2 u(x, t)}{\partial x_2^2} dx_3 \\ &+ \frac{c_{hy}}{D_z} \int_0^H \frac{\partial^2 u(x, t)}{\partial x_3^2} dx_3 \\ &+ \frac{1}{\beta D_z} [\bar{\mathcal{B}}_s(\bar{x})Q_s(t) + \bar{\mathcal{B}}_c(\bar{x})Q_c(t)], \end{aligned}$$

859 where H is the height of the reservoir, the new space
 860 variable $\bar{x} \in \mathfrak{X}^2$, $\bar{x} = [x_1, x_2]^T$. $\bar{\mathcal{B}}_s(\bar{x}) = [\delta(\bar{x} -$
 861 $\bar{x}_s^1), \dots, \delta(\bar{x} - \bar{x}_s^{m_s})]$ and $\bar{\mathcal{B}}_c(\bar{x}) = [\delta(\bar{x} - \bar{x}_c^1), \dots, \delta(\bar{x} - \bar{x}_c^{m_c})]$.
 862 We note that $\int_0^H \frac{\partial^2 u(x,t)}{\partial x_3^2} dx_3 = 0$ due to the BC, $u(x, t) =$
 863 $0 \quad \forall x \in S$. Defining the depth average pressure as
 864 $\bar{u}(\bar{x}, t) = \frac{1}{D_z} \int_0^H u(x, t) dx_3$, the last expression becomes

$$\begin{aligned} \bar{u}_t(\bar{x}, t) &= c_{hy} \nabla^2 \bar{u}(\bar{x}, t) + \frac{1}{\beta D_z} \left[\bar{\mathcal{B}}_s(\bar{x}) Q_s(t) + \bar{\mathcal{B}}_c(\bar{x}) Q_c(t) \right], \\ \bar{u}(\bar{x}, t) &= 0 \quad \forall \bar{x} \in \partial S. \end{aligned} \quad (\text{C.1})$$

865 Note how the systems (3) and (C.1) obtain finally
 866 the same form, allowing the theoretical developments of
 867 section Appendix E to be applied without any change.
 868 This 2D diffusion equation is numerically solved in Sec-
 869 tions 2 and 4 using the spectral decomposition presented
 870 in the following appendix.

871 Appendix D. Spectral Decomposition of the 2D dif- 872 fusion equation

873 We decompose the function $\bar{u}_t(\bar{x}, t)$ of the BVP (C.1)
 874 according to

$$\bar{u}_t(\bar{x}, t) = \sum_{\substack{n=1 \\ m=1}}^{\infty} z_{nm}(t) \phi_{nm}(\bar{x}), \quad (\text{D.1})$$

875 where $z_{nm}(t) = \langle \bar{u}_t(\bar{x}, t), \phi_{nm}(\bar{x}) \rangle$ is the nm -th Fourier
 876 coefficient of $\bar{u}_t(\bar{x}, t)$ and $\phi_{nm}(\bar{x})$ is the nm -th orthonor-
 877 mal eigenfunction satisfying the BC. The expression
 878 $\langle \cdot, \cdot \rangle$ denotes the inner product, *i.e.*, $\langle f(\cdot), g(\cdot) \rangle =$
 879 $\int_S f(\cdot) g(\cdot) dS$. For the case of the BVP (C.1), the eigen-
 880 function, $\phi_{nm}(\bar{x})$, and the corresponding eigenvalues λ_{nm}
 881 are

$$\begin{aligned} \phi_{nm}(\bar{x}) &= 2 \sin\left(\frac{n\pi x_1}{D}\right) \sin\left(\frac{m\pi x_2}{D}\right), \\ \lambda_{nm} &= 2 \frac{\pi^2}{D^2} (n^2 + m^2). \end{aligned} \quad (\text{D.2})$$

882 In order to simplify the notation, we adopt the map-
 883 ping $k = h(n, m)$, which leads to the more compact form

$$\bar{u}_t(\bar{x}, t) = \sum_{k=1}^{\infty} z_k(t) \phi_k(\bar{x}). \quad (\text{D.3})$$

884 Substituting expression (D.3) in (C.1) results in

$$\begin{aligned} \dot{z}_k(t) &= -c_{hy} \lambda_k z_k(t) + \frac{1}{\beta D_z} [\phi_k(\bar{x}_s^1), \dots, \phi_k(\bar{x}_s^{m_s})] Q_s(t) \\ &\quad + \frac{1}{\beta D_z} [\phi_k(\bar{x}_c^1), \dots, \phi_k(\bar{x}_c^{m_c})] Q_c(t), \\ z_k(0) &= \langle \bar{u}_t(\bar{x}, 0), \phi_k(\bar{x}) \rangle, \quad \forall k \in [1, \infty). \end{aligned} \quad (\text{D.4})$$

885 Systems (C.1) and (D.3)–(D.4) are equivalent when
 886 $k \rightarrow \infty$, but the significant difference is that system
 887 (D.4) is an ODE that can be easily implemented numer-
 888 ically with k finite. In our numerical simulations, we
 889 were limited to 160 eigenmodes, which was more than
 890 enough according to convergence analyses. These con-
 891 vergence analyses are standard and were omitted from
 892 the manuscript.

893 Appendix E. Output Feedback Tracking Control 894 Design

895 The control design will be performed under the fol-
 896 lowing assumptions for system (3)–(4):

897 **Assumption 1.** *The diffusion system (3) fulfils*

$$\|\nabla^2 u_t(x, t)\|_{H^0(V)} \leq L_u, \quad (\text{E.1})$$

898 *with known constant $L_u \geq 0$.*

899 **Assumption 2.** *The fixed, not controlled flux input,*
 900 *$Q_s(t)$, in system (3) fulfils*

$$\|\dot{Q}_s(t)\| \leq L_Q, \quad (\text{E.2})$$

901 *with known constant $L_Q \geq 0$.*

902 **Assumption 3.** *The reference to be followed, $r(t)$, is de-*
 903 *signed to fulfil*

$$\|\dot{r}(t)\| \leq L_r, \quad \|\ddot{r}(t)\| \leq L_{\ddot{r}}, \quad (\text{E.3})$$

904 *with known constants $L_r \geq 0$, $L_{\ddot{r}} \geq 0$.*

905 **Assumption 4.** *All the parameters of the system (3)–(4)*
 906 *are uncertain and only nominal values are known (e.g.,*
 907 *a nominal value, f_0 , is known for the parameter f).*

908 **Remark 1.** Assumption 1 is feasible due to energy con-
 909 servation on the realistic system (3). Furthermore, as-
 910 sumptions 2-4 are easily met in control applications.

911 The first step on the design, will be to obtain the error
 912 dynamics of (7) as

$$\dot{y}_{e_i} = \frac{f}{t_a \tau_0 V_i} \int_{V_i} u_t(x, t) dV - \frac{1}{t_a} (e^{y_{e_i} + r_i} - 1) - \dot{r}_i,$$

913 for $i \in [1, m_c]$. Using the 3D diffusion equation (3) and
914 the divergence theorem, the error dynamics becomes

$$\begin{aligned} \dot{y}_{e_i} &= \frac{c_{hy}f}{t_a \dot{\tau}_0 V_i} \int_{V_i} \nabla^2 u(x, t) dV \\ &+ \frac{f}{t_a \dot{\tau}_0 \beta V_i} \int_{V_i} [\mathcal{B}_s(x) Q_s(t) + \mathcal{B}_c(x) Q_c(t)] dV \\ &- \frac{1}{t_a} (e^{y_{e_i} + r_i} - 1) - \dot{r}_i \\ &= \frac{c_{hy}f}{t_a \dot{\tau}_0 V_i} \int_{V_i} \nabla^2 u(x, t) dV \\ &+ \frac{f}{t_a \dot{\tau}_0 \beta V_i} \sum_{j=1}^{m_s} \int_{V_i} \delta(x - x_s^j) Q_{s_j}(t) dV \\ &+ \frac{f}{t_a \dot{\tau}_0 \beta V_i} \sum_{j=1}^{m_c} \int_{V_i} \delta(x - x_c^j) Q_{c_j}(t) dV \\ &- \frac{1}{t_a} (e^{y_{e_i} + r_i} - 1) - \dot{r}_i, \end{aligned}$$

915 for $i \in [1, m_c]$.

916 The error dynamics can be represented in matrix form
917 as follows

$$\dot{y}_e = \Psi(t) + B_s Q_s(t) + B_c Q_c(t) - \Phi(t) - \dot{r}(t), \quad (\text{E.4})$$

918 where $B_s = [b_{ij}^s] \in \mathfrak{R}^{m_c \times m_s}$, $B_c = [b_{ij}^c] \in \mathfrak{R}^{m_c \times m_c}$, $\Psi(t) \in$
919 \mathfrak{R}^{m_c} and $\Phi(t) \in \mathfrak{R}^{m_c}$ are defined as

$$\begin{aligned} b_{ij}^s &= \begin{cases} \frac{f}{t_a \dot{\tau}_0 \beta V_i} & \text{if } x_s^j \in V_i \\ 0 & \text{if } x_s^j \notin V_i \end{cases}, \quad i \in [1, m_c], j \in [1, m_s], \\ b_{ij}^c &= \begin{cases} \frac{f}{t_a \dot{\tau}_0 \beta V_i} & \text{if } x_c^j \in V_i \\ 0 & \text{if } x_c^j \notin V_i \end{cases}, \quad i \in [1, m_c], j \in [1, m_c], \\ \Psi(t) &= \begin{bmatrix} \frac{c_{hy}f}{t_a \dot{\tau}_0 V_1} \int_{V_1} \nabla^2 u(x, t) dV \\ \vdots \\ \frac{c_{hy}f}{t_a \dot{\tau}_0 V_{m_c}} \int_{V_{m_c}} \nabla^2 u(x, t) dV \end{bmatrix}, \\ \Phi(t) &= \begin{bmatrix} \frac{1}{t_a} (e^{y_{e_1} + r_1} - 1) \\ \vdots \\ \frac{1}{t_a} (e^{y_{e_{m_c}} + r_{m_c}} - 1) \end{bmatrix}, \end{aligned} \quad (\text{E.5})$$

920 where the definition of Delta's distribution has been
921 used.

922 The matrices B_c , $\Psi(t)$ and $\Phi(t)$ are assumed to fulfil

$$B_c = \Gamma B_0, \quad \|\dot{\Psi}(t)\| \leq L_1, \quad \|\dot{\Phi}(t)\| \leq L_2, \quad (\text{E.6})$$

923 where $B_0 \in \mathfrak{R}^{m_c \times m_c}$ is a known regular matrix (con-
924 sequently, B_c is assumed to be regular matrix as well),
925 $\Gamma \in \mathfrak{R}^{m_c \times m_c}$ is an uncertain matrix with positive diago-
926 nal entries, and $L_1 \geq 0$, $L_2 \geq 0$ are known constants.

Remark 2. The assumption over the term $\Phi(t)$ in (E.6)
928 requires Assumption 3 to be fulfilled and the bounded-
929 ness of the error vector derivative, \dot{y}_e , as $\|\dot{y}_e(t)\| \leq L_{y_e}$,
930 $L_{y_e} > 0$. Therefore, only local results on system (E.4)
931 are considered in this paper. Furthermore, the condition
932 over the term $\Psi(t)$ requires further analysis, which will
933 be performed in the next Lemma.

934 **Lemma 1.** The term $\Psi(t)$ in system (E.4),(E.5) fulfils
935 the condition (E.6) if Assumption 1 is fulfilled.

936 **PROOF.** Calculating the norm of the term $\Psi(t)$ defined in
937 (E.5) results in

$$\begin{aligned} \|\Psi(t)\| &= \frac{c_{hy}f}{t_a \dot{\tau}_0} \sqrt{\sum_{i=1}^{m_c} \frac{1}{V_i^2} \left[\int_{V_i} \nabla^2 u(x, t) dV \right]^2} \\ &\leq \frac{c_{hy}f}{t_a \dot{\tau}_0} \sqrt{\sum_{i=1}^{m_c} \frac{1}{V_i^2} \int_{V_i} [\nabla^2 u(x, t)]^2 dV}. \end{aligned}$$

938 Taking the time derivative of the last expression reads
939 as

$$\begin{aligned} \|\dot{\Psi}(t)\| &\leq \frac{c_{hy}f}{t_a \dot{\tau}_0} \sqrt{\sum_{i=1}^{m_c} \frac{1}{V_i^2} \int_{V_i} [\nabla^2 u_t(x, t)]^2 dV} \\ &\leq \frac{c_{hy}f}{t_a \dot{\tau}_0} \sqrt{\sum_{i=1}^{m_c} \frac{1}{V_i^2} \|\nabla^2 u_t(x, t)\|_{H^0(V)}^2} \end{aligned}$$

940 which is bounded as in (E.6) if the assumption (E.1) is
941 fulfilled. ■

942 The closed-loop system of (E.4) with control (8)
943 reads as

$$\begin{aligned} \dot{y}_e &= \Gamma (-K_1 [y_e]^{\frac{1}{l-1}} + x_I), \\ \dot{x}_I &= -K_2 [y_e]^{\frac{1+l}{l-1}} + \dot{\Delta}(t), \end{aligned} \quad (\text{E.7})$$

944 where

$$\begin{aligned} x_I &= v + \Delta(t), \\ \Delta(t) &= \Gamma^{-1} \Psi(t) + \Gamma^{-1} B_s Q_s(t) - \Gamma^{-1} \Phi(t) + (I - \Gamma^{-1}) \dot{r}(t), \\ \dot{\Delta}(t) &= \Gamma^{-1} \dot{\Psi}(t) + \Gamma^{-1} B_s \dot{Q}_s(t) - \Gamma^{-1} \dot{\Phi}(t) + (I - \Gamma^{-1}) \dot{r}(t). \end{aligned} \quad (\text{E.8})$$

945 The system of equations (E.7)–(E.8) has a discontinu-
946 ous right-hand side when $l = -1$ due to the definition
947 of function $[\cdot]^{\gamma}$ in Appendix A. In this special case, the
948 solutions are understood in the sense of Filippov [51].
949 The term $\Delta(t)$ is assumed to fulfil

$$\|\dot{\Delta}(t)\| \leq L_3, \quad (\text{E.9})$$

950 with *a priori* known constant $L_s \geq 0$. This is always the
 951 case due to Assumptions 1–3 and (E.6).

952 The tracking result for the output (4)–(6) is then in
 953 force.

954 **Theorem 1.** *Let system (E.4) assumed to fulfil (E.3),*
 955 *(E.6), and (E.9), be driven by the control (8) with $K_1 >$*
 956 *0 , $K_2 > 0$. Then, the origin of the error closed-loop*
 957 *system (E.7)–(E.8), is locally:*

- 958 1. *Finite-time stable for any $L_s \geq 0$, if $l = -1$.*
- 959 2. *Finite-time stable for $L_s = 0$, if $l = (-1, 0)$.*
- 960 3. *Exponentially stable for $L_s = 0$, if $l = 0$.*
- 961 4. *Exponentially ISS w.r.t. $\Delta(t)$ for $L_s > 0$, if $l =$*
 962 *$(-1, 0]$.*

963 **PROOF.** Following [43, 44], the trajectories (y_e, x_l) of
 964 system (E.7)–(E.8) are ensured to reach the origin if K_1
 965 is a matrix with positive diagonal elements and K_2 is a
 966 positive quasi-definite matrix. ■

967 **Remark 3.** As a consequence of the stability of the
 968 closed-loop system trajectories, (y_e, x_l) , and due to the
 969 definition of the perturbation term $\Delta(t)$ in (E.8), the in-
 970 tegral term, v , of the control (8) is able to provide an
 971 estimate of such term, *i.e.*, $v(t) \rightarrow -\Delta(t)$ as $t \rightarrow \infty$.

972 *Appendix E.1. Energy demand and production con-* 973 *straints*

974 For the case where additional number of flux restric-
 975 tions to the fluid injection of the controlled injection
 976 points are considered, the vector of controlled injection
 977 points of system (E.4) is defined as follows

$$\dot{y}_e = \Psi(t) + B_s Q_s(t) + \bar{B}_c \bar{Q}_c(t) - \Phi(t) - \dot{r}(t), \quad (\text{E.10})$$

978 where $\bar{Q}_c(t) \in \mathfrak{R}^{m_c+m_r}$, $\bar{Q}_c(t) = [\bar{Q}_{c_1}(t), \dots, \bar{Q}_{c_{m_c+m_r}}(t)]^T$
 979 are the new controlled fluxes and $\bar{B}_c = [\bar{b}_{ij}^c] \in$
 980 $\mathfrak{R}^{m_c \times (m_c+m_r)}$ is the new input matrix defined as

$$\bar{b}_{ij}^c = \begin{cases} \frac{f}{i_a \bar{\tau}_0 \beta V_i} & \text{if } x_c^j \in V_i \quad i \in [1, m_c] \\ 0 & \text{if } x_c^j \notin V_i \quad j \in [1, m_c + m_r] \end{cases}, \quad (\text{E.11})$$

981 where x_c^j , $j \in [1, m_c + m_r]$, are the injection points over
 982 the total region V . If we replace (10) in (E.10), the link
 983 between the new input matrix, \bar{B}_c , and the original input
 984 matrix, B_c , defined in (E.5) is stated as $\bar{B}_c \bar{W} = B_c$.