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Erratum: Deconvolution with unknown noise distribution is possible for multivariate signals

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The erratum offers another way to obtain an inequality similar to that given in Proposition A.2 in [Gassiat et al., 2022], since an error has been found in its proof.

In this note we use the setting and the notations introduced in [Gassiat et al., 2022]. We first show how their proof works, and where the issue is.

For any ϕ and Φ_{R^*} in $\Upsilon_{\rho,S}$, define $\Delta\phi = \phi - \Phi_{R^*}$, and $N : \mathbb{R}^{d_1} \times \mathbb{R}^{d_2} \rightarrow \mathbb{C}$ such that for all $(t_1, t_2) \in \mathbb{R}^{d_1} \times \mathbb{R}^{d_2}$,

$$N(t_1, t_2; \Delta\phi, \Phi_{R^*}) = \Delta\phi(t_1, t_2)\Phi_{R^*}(t_1, 0)\Phi_{R^*}(0, t_2) - \Phi_{R^*}(t_1, t_2)\Delta\phi(t_1, 0)\Phi_{R^*}(0, t_2) - \Phi_{R^*}(t_1, t_2)\Phi_{R^*}(t_1, 0)\Delta\phi(0, t_2),$$

so that

$$M^{\text{lin}}(\Delta\phi, \Phi_{R^*}; \nu) = \int_{[-\nu, \nu]^{d_1} \times [-\nu, \nu]^{d_2}} |N(t_1, t_2; \Delta\phi, \Phi_{R^*})|^2 dt_1 dt_2.$$

If it is possible to prove that there exists a constant c depending only on ρ_0 , ν , and S such that for all $\rho \leq \rho_0$, for all ϕ and Φ_{R^*} in $\Upsilon_{\rho,S} \cap \mathcal{H}$, for all integer m ,

$$\int_{[-\nu, \nu]^{d_1} \times [-\nu, \nu]^{d_2}} |N(t_1, t_2; T_m \Delta\phi, T_m \Phi_{R^*})|^2 dt_1 dt_2 \geq c^m \|T_m \Delta\phi\|_{2,\nu}^2, \quad (1)$$

then, following the proof of [Gassiat et al., 2022], it holds that for all $\delta > 0$, there exists $\eta > 0$ and a constant \tilde{c} depending only on δ , ρ_0 , ν , and S such that for all $\rho \leq \rho_0$, for all ϕ and Φ_{R^*} in $\Upsilon_{\rho,S} \cap \mathcal{H}$, as soon as $\|\phi - \Phi_{R^*}\|_{2,\nu} \leq \eta$, then

$$M_\star(\phi; \nu) \geq \tilde{c} \|\phi - \Phi_{R^*}\|_{2,\nu}^{2(1+\delta)}. \quad (2)$$

The error in [Gassiat et al., 2022] lies in the proof of Lemma B.1, which aims to show (1). It relates N with a matrix A with properties described in Lemma I.1 of Appendix I in the supplementary material of [Gassiat et al., 2022]. In particular, this Lemma entails that A restricted to the polynomials of degree at most m is injective (lower triangular with diagonal coefficients equal to -1), with explicit inverse, thus

allowing the control of its lowest singular value. The problem lies in point iv) of this Lemma: A is actually not injective. Its diagonal entries with coordinates $((i_1, 0), (i_1, 0))$ and $((0, i_2), (0, i_2))$ for $i_1, i_2 \geq 1$ are zero, and its entry with coordinate $((0, 0), (0, 0))$ is +1.

For some structured submodels, (1) may be proved directly. To deal with more general cases, we introduce an alternative assumption which allows to prove (2) directly.

1 Structured submodels

Equation (1) may be proved by lower bounding

$$\int_{[-\nu, \nu]^{d_1} \times [-\nu, \nu]^{d_2}} |N(t_1, t_2; T_m \Delta \phi, T_m \Phi_{R^*})|^2 dt_1 dt_2$$

with something satisfying (1). This is what is done in [Capitao-Miniconi et al., 2024] for the repeated measurements model, in which $X^{(1)} = X^{(2)} = X$, so that \mathcal{H} is the set of functions ϕ such that $\phi(t_1, t_2) = \Phi(t_1 + t_2)$. It is proved in [Capitao-Miniconi et al., 2024] that there exists a constant c_1 depending only on ρ_0, ν , and S such that for all $\rho \leq \rho_0$, for all ϕ and Φ_{R^*} in $\Upsilon_{\rho, S} \cap \mathcal{H}$, for all integer m ,

$$\int_{[-\nu, \nu]^{2d}} |N(t_1, t_2; T_m \Delta \phi, T_m \Phi_{R^*})|^2 dt_1 dt_2 \geq c_1^m \int_{[-\nu, \nu]^d} |N(t, t; T_m \Delta \phi, T_m \Phi_{R^*})|^2 dt,$$

see Lemma 8.1, and then that there exists a constant c_2 depending only on ρ_0, ν , and S such that for all $\rho \leq \rho_0$, for all ϕ and Φ_{R^*} in $\Upsilon_{\rho, S}$, for all integer m ,

$$\int_{[-\nu, \nu]^d} |N(t, t; T_m \Delta \phi, T_m \Phi_{R^*})|^2 dt \geq c_2^m \|T_m \Delta \phi\|_{2, \nu}^2,$$

see Lemma 8.2, so that (1) holds.

2 General case

Consider now the following assumption. Let $\delta \in (0, 1)$.

H(δ): There exists $\nu_0 > 0$ such that for all $\nu \in (0, \nu_0]$, there exists $c(\nu, \delta) > 0$ such that for all R^* such that $\Phi_{R^*} \in \mathcal{H} \cap \Upsilon_{\kappa, S}$,

$$\forall \phi \in \mathcal{H} \cap \Upsilon_{\kappa, S} \quad \text{s.t.} \quad \phi \neq \Phi_{R^*}, \quad \frac{\left\| \frac{\phi}{\Phi_{R^*}} - \frac{\phi^{(1)}}{\Phi_{R^*}^{(1)}} - \frac{\phi^{(2)}}{\Phi_{R^*}^{(2)}} + 1 \right\|_{2, \nu}}{\max \left(\left\| \frac{\phi^{(1)}}{\Phi_{R^*}^{(1)}} - 1 \right\|_{2, \nu}, \left\| \frac{\phi^{(2)}}{\Phi_{R^*}^{(2)}} - 1 \right\|_{2, \nu} \right)}^{1+\delta} \geq c(\nu, \delta). \quad (3)$$

Proposition 1. Assume $\mathbf{H}(\delta)$ holds. There exists $\bar{\nu} > 0$ depending only on S and ρ such that for all $\nu \leq \bar{\nu}$, there exists a constant $\tilde{c} > 0$ depending only on c_ν and $c(\nu, \delta)$ such that for all R^* such that $\Phi_{R^*} \in \mathcal{H} \cap \Upsilon_{\kappa, S}$, all $Q^* \in \mathbf{Q}(\nu, c_\nu, c_Q)$ for some $c_Q \in (0, \infty]$, for all $\phi \in \mathcal{H} \cap \Upsilon_{\kappa, S}$, as soon as

$$\left\| \frac{\phi}{\Phi_{R^*}} - \frac{\phi^{(1)}}{\Phi_{R^*}^{(1)}} - \frac{\phi^{(2)}}{\Phi_{R^*}^{(2)}} + 1 \right\|_{2, \nu} \leq \min \left\{ \left(c(\nu, \delta)^2 2^{-(1+\delta)} (2\nu)^{d(1+\delta)} \right)^{\frac{1}{1-\delta}} ; 1 \right\}, \quad (4)$$

it holds

$$M_\star(\phi; \nu) \geq \tilde{c} \|\phi - \Phi_{R^*}\|_{2, \nu}^{2(1+\delta)}. \quad (5)$$

In [Gassiat et al., 2022], \mathcal{H} is chosen as a closed subset of $\mathbf{L}^2(B_{\nu_{\text{est}}}^d)$ such that all elements of \mathcal{H} satisfy **H2**. In the choice of \mathcal{H} we may require that $\mathbf{H}(\delta)$ holds, since this choice comes from the prior modeling that allows to fix **H2**, see examples in Section 2 of [Gassiat et al., 2022]. If we add $\mathbf{H}(\delta)$ in the choice of \mathcal{H} , then Theorem 3.2 and Theorem 3.3 follow from the arguments developed in [Gassiat et al., 2022] with no modification.

Notice that the uniform consistency of the estimator (Section A.1 in [Gassiat et al., 2022]) holds without any change, that is without assuming $\mathbf{H}(\delta)$ which is only used to get rates.

Let us now prove Proposition 1. First, notice that for all $\phi \in \Upsilon_{\rho, S}$, $|\phi(t) - 1| \leq \sup_{u: \|u\| \leq \|t\|} \|\phi'(u)\| \|t\|$, where $\phi'(u)$ denotes the gradient of ϕ at u (recall that ϕ is multivariate analytic), so that it is possible to choose $\bar{\nu}$ depending only on S and d such that for all ρ , for all $\phi \in \Upsilon_{\rho, S}$, for all $\nu \leq \bar{\nu}$, all $t \in B_\nu^d$, $|\phi(t)| \geq 1/2$. For instance, one can choose $\bar{\nu} = 1/2Sd^2$. Let now R^* be such that $\Phi_{R^*} \in \mathcal{H} \cap \Upsilon_{\kappa, S}$, and let $Q^* \in \mathbf{Q}(\nu, c_\nu, c_Q)$ for some $c_Q \in (0, \infty]$. Let ϕ be any function in $\mathcal{H} \cap \Upsilon_{\kappa, S}$ such that (4) holds. Denote

$$g_1 = \frac{\phi^{(1)}}{\Phi_{R^*}^{(1)}} - 1, \quad g_2 = \frac{\phi^{(2)}}{\Phi_{R^*}^{(2)}} - 1 \quad \text{and} \quad G = \frac{\phi}{\Phi_{R^*}} - 1 - g_1 - g_2.$$

Then, M_\star rewrites

$$M_\star(\phi; \nu) = \|\Phi_{R^*} \Phi_{R^*}^{(1)} \Phi_{R^*}^{(2)} (G - g_1 g_2) \Phi_{Q^*}^{(1)} \Phi_{Q^*}^{(2)}\|_{2, \nu}^2, \quad (6)$$

and we get $M_\star(\phi; \nu) \geq \frac{c_\nu^4}{2^6} \|G - g_1 g_2\|_{2, \nu}^2$. Using $\mathbf{H}(\delta)$ and (4), we get

$$\|g_1\|_{2, \nu} \|g_2\|_{2, \nu} \leq \left(\frac{\|G\|_{2, \nu}}{c(\nu, \delta)} \right)^{2/(1+\delta)} \leq \frac{\|G\|_{2, \nu}}{2} (2\nu)^d$$

(recall $0 < \delta < 1$). We then get

$$\begin{aligned} M_\star(\phi; \nu) &\geq \frac{c_\nu^4}{2^6} (\|G\|_{2, \nu} - \|g_1 g_2\|_{2, \nu})^2 \\ &= \frac{c_\nu^4}{2^6} (\|G\|_{2, \nu} - (2\nu)^{-d} \|g_1\|_{2, \nu} \|g_2\|_{2, \nu})^2 \\ &\geq \frac{c_\nu^4}{2^8} \|G\|_{2, \nu}^2. \end{aligned}$$

On the other hand,

$$\begin{aligned}
\|\phi - \Phi_{R^*}\|_{2,\nu} &= \|G + g_1 + g_2\|_{2,\nu} \\
&\leq \|G\|_{2,\nu} + \|g_1\|_{2,\nu} + \|g_2\|_{2,\nu} \\
&\leq \|G\|_{2,\nu} + 2(c(\nu, \delta)^{-1} \|G\|_{2,\nu})^{1/(1+\delta)} \\
&\leq (1 + 2c(\nu, \delta)^{-1/(1+\delta)}) \|G\|_{2,\nu}^{1/(1+\delta)}
\end{aligned}$$

since $\|G\|_{2,\nu} \leq 1$. Hence, we may conclude

$$M_*(\phi; \nu) \geq \frac{c_\nu^4}{2^8(1 + 2c(\nu, \delta)^{-1/(1+\delta)})^{2(1+\delta)}} \|\phi - \Phi_{R^*}\|_{2,\nu}^{2(1+\delta)}.$$

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