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## Some Thoughts Concerning Spacetime Models in Cosmology : Role of Gravitation and Electromagnetism

L. R. Rakotomanana\*

*Institut de Recherche Mathématique de Rennes, Université de Rennes,  
Rennes, 35402, France*

*\*E-mail: lalaonirina.rakotomanana-ravelonarivo@univ-rennes.fr  
<https://math.univ-rennes.fr>*

Considering an Einstein-Cartan vacuum spacetime, this work focusses on the influence of electromagnetic field on the accelerated expansion of the universe. In addition to metric and torsion, we study the influence of the compatibility of both of them with the volume-form, needed to correctly define any Lagrangian of the vacuum. The action including Einstein-Palatini and Yang-Mills Lagrangians is considered. Application of Variation Procedure allows us to derive fields equations : (1) Maxwell, (2) Einstein-Maxwell and (3) a new linking relation giving what we called distortion of the spacetime. This (third-order) distortion tensor is expressed explicitly and is generated by a particular component of the electromagnetic field : the Chern-Simmons current which is constituted by Magnetic Helicity and Spin Angular Momentum of the wave. It is deduced that norm of the torsion-vector acts as either dark energy (spacelike torsion) or dark matter (timelike torsion) depending its sign. The integral curves of the torsion-vector field might be suggested as the tangent field of gigantic filaments, and confirming that the two major forces remain the gravitation and magnetic forces.

*Keywords:* Einstein-Cartan spacetime; Gravitation; Electromagnetism; Dark energy.

### 1. Introduction

Cosmology is the branch of astrophysics which is interested in the origin, nature, structure and evolution of the universe taken as a whole<sup>1</sup>. Briefly, the term Cosmology in its modern version goes back to the physicist Alexander Friedmann who was the first to express it in its actual form in 1922<sup>2</sup>. During the development of his Relative Gravitation theory, Einstein proposed in 1917 a first cosmological solution to his theory. This solution, although idealistic, is obtained from the Einstein field equations and can be considered as the first truly scientific cosmological model. On the one hand, Relative Gravitation can be nowadays considered as a plausible theory, at least for large scale length, tests are much better namely the precession of the perihelion of Mercury, the bending of light in gravitational fields, and the gravitational redshift, to name but a few. On the other hand, cosmology theories still lean mostly on the Relative Gravitation, together with Quantum Physics for small scales, and most of time remain as an extrapolations. It should be pointed out that the length scale characteristic of the tests accuracy in Relative Gravitation is lower than  $10^{11}[m]$  while for Cosmology Theories, the scale length characteristic is nearly  $10^{26}[m]$ . By the way, quantum effects are relevant at dimension lower than Planck length  $\ell_P \simeq 10^{-35}[m]$ . Currently, a parameter more or less accepted

as a hypothesis of the different theories of cosmology is that the universe consists essentially of vacuum but is filled with matter, called baryonic, whose mean density is very low  $\rho \simeq 10^{-28} [kg/m^3]$ . Baryonic matter is made up of protons and neutrons. However, most of the matter, called dark matter to maintain the universe, remains unknown to this day to respect the physical observations on accelerated expansion. The hypothesis of the existence of this dark matter as well as dark energy remains a major hypothesis of modern cosmology. This hypothesis is essential to respect the major experimental observations of the universe evolution which is undergoing accelerated expansion<sup>3,4</sup>. There are numerous publications and journals which report on the different theories underlying the various major international projects whose main objective is, for example, to observe and thus clarify the notion of dark energy<sup>5</sup>.

Like most of any scientific theory, Cosmology is based on mathematical modeling, and on experimental observations and measurements. These two aspects are of essential and comparable importance. They constitute the two crutches that allow us to advance in the detailed understanding of the universe. The object of our study is limited to those portions of the universe between galaxies or aggregates of galaxies which we call vacuum spacetime in which we note that physical action can be transmitted even in the absence of matter. The concept of action-at-a-distance is intimately linked to the concept of spacetime, the latter being the playing medium of the various actions that are present in physics<sup>6</sup>. From different points of view, the notion of space and time has always been the object of interest of scientists for millennia<sup>1</sup>. On the one hand, on a mathematical level, the consistency of the theory is to respect the initial hypotheses inspired by physical observations while avoiding as much as possible the addition during development of other hypotheses which could contradict those initially admitted. On the other hand, the physical consistency of the theory is to be able to describe real physical phenomena as best as possible using mathematical models. The two "consistencies" are essential to make it an acceptable theory, which of course will not be immune from questioning following new observations or more detailed experimental measurements which could lead to new concepts. This is the scientific dialectic. This work is written not to claim to build models of Cosmology and not even of Relative Gravitation. Far from it, it remains with a view to analyzing all the possible consequences that can be drawn from an initially assumed geometric structure of spacetime, as simple as possible. The present thought therefore remains limited to a study of theoretical consistency and does not claim an experimental confrontation, other than those which already exist in the literature. In this paper, we would like to respond to two questions concerning the accelerated expansion of the universe: (a) Why should we use an Einstein-Cartan spacetime, and (b) Why introduce electromagnetism as an external source coupled with gravity? And incidentally, what would be the consequences of these choices on the derivation of cosmological equations, and by the way their relation with the cosmological constant ?

## 2. Standard model of spacetime in cosmology

### 2.1. Principles of Cosmology

It is worth to remind some basics on the cosmology principles. Historically, the Copernicus Principle is based on the hypothesis that any observer studying the evolution of the universe can position himself in any location in this universe, and that the direction of observations has no influence on the results. In other words, if one assumes the Principle of Copernicus, then the universe appears homogeneous (same properties everywhere) and isotropic (same properties for all directions). In reality, experimental measurements by astronomers assess that the universe has non-uniform structures up to the scale of galaxies, galactic clusters, gigantic filaments and maybe intersidereal vacuum. However, the spacetime underlying the universe is assumed to be more and more homogeneous and isotropic on larger and larger scales, indicating that that homogeneity as well as the isotropy might be violated but nevertheless always assumed. The present paper only wanted to propose a simple theoretical model which takes up Friedman's equations by considering relative gravitation and electromagnetism. Derivation of cosmology theory is essentially based on the assumptions of spacetime homogeneity and isotropy at very large scale. From the outset, the construction of Friedman-Lemaître-Roberston-Walker (FLRW) spacetime was originally based on the derivation of metrics assuming these two hypotheses together with the Birkhoff theorem.

### 2.2. Derivation of Friedman equations

Friedman developed equations governing the expansion of the universe and presented solutions to the Einstein field equations, with a time dependent scale factor  $R(x^0)$ , assuming perfect isotropic fluid as matter, with an energy density  $\rho$  and pressure  $p$ . Nearly one hundred years ago, with these equations, Friedman discovered that the Relative Gravitation of Einstein predicts a non-stationary expansion of the universe. This was a key step for the cosmology since then this revolutionary change about the vision of human and scientific constitutes a landmark for the space science. <sup>a</sup> To formulate the Friedman equations, the source term is due to a perfect fluid with energy density  $\rho$  and pressure  $p$ . Derivation starts from the Hilbert-Einstein action with an *ad hoc* cosmological constant  $\Lambda$ :

$$\mathcal{S}_{HE} := \frac{1}{2\chi} \int_{\mathcal{B}} (\overline{\mathcal{R}} - 2\Lambda) \overline{\omega}_n$$

where the Levi-Civita connection  $\overline{\nabla}$  is compatible with metric  $g_{\alpha\beta}$  and with the Riemann volume-form  $\overline{\omega}_n := \sqrt{-g} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$ .  $\overline{\mathcal{R}}$  denotes the scalar

<sup>a</sup>At the time of the discovery of Relative Gravitation, the acceptance that the universe is dynamic was not considered. Einstein himself had refuted this idea when he read Friedman's mathematical results of May 1922 which showed that Einstein's field equations necessarily implied the non-stationarity of spacetime. This did not facilitate the acceptance of Friedman's discovery concerning these equations, but which Einstein had finally recognized as exact in May 1923.

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curvature calculated with  $\bar{\nabla}$ . The variation of the action gives:

$$\delta\mathcal{S}_{HE} = \frac{1}{2\chi} \int_{\mathcal{B}} [\delta(\bar{\mathcal{R}} - 2\Lambda)\bar{\omega}_n + (\bar{\mathcal{R}} - 2\Lambda)\delta\bar{\omega}_n] \quad (1)$$

where  $\delta\bar{\omega}_n = -\frac{1}{2} g_{\mu\nu} \bar{\omega}_n \delta g^{\mu\nu}$ , and where the variation of the curvature may be written as follows:

$$\delta\bar{\mathcal{R}} = \delta g^{\mu\nu} \bar{\mathcal{R}}_{\mu\nu} + \delta^\alpha_\gamma g^{\beta\lambda} \left[ \bar{\nabla}_\alpha (\delta\Gamma^\gamma_{\beta\lambda}) - \bar{\nabla}_\beta (\delta\Gamma^\gamma_{\alpha\lambda}) \right]$$

Arbitrariness of the variation  $\delta g^{\mu\nu}$  induces the Einstein field equations in presence of perfect fluid,  $\bar{G}_{\mu\nu}$  is the Einstein tensor,:

$$\bar{G}_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}^{\text{fl}}, \quad \bar{G}_{\mu\nu} := \bar{\mathcal{R}}_{\mu\nu} - \frac{\bar{\mathcal{R}}}{2} g_{\mu\nu} \quad (2)$$

by accounting for a fluid energy-momentum  $T_{\mu\nu}^{\text{fl}} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$ . The curvature is formulated with the well-known FLRW metric. The generic form of the standard metric of the Friedmann-Lemaître-Robertson-Walker metric in cosmology:

$$ds^2 = (dx^0)^2 - R^2(x^0) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right), \quad k = -1, 0, +1 \quad (3)$$

$k$  characterizes the universe curvature (spatially open, flat, or closed), is constant throughout a particular solution, but may vary from one solution to another.

**Friedmann equations within Einstein spacetime (1929).** Deduced from (2) and (2), the resulting Friedman equations constitute the basic model of the expanding universe<sup>2</sup>,  $G$  being the Newton constant of gravitation,:

$$\begin{cases} \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho - \frac{\Lambda}{3} - \frac{k}{R^2} \\ \frac{\ddot{R}}{R} = -\frac{4\pi G}{3} (\rho + 3p) - \frac{\Lambda}{3} \end{cases} \quad (4)$$

Equations (4) which represent Einstein field equations highlight that the universe is not static in general. For a FLRW spacetime model, there are only two independent equations. Most solutions in various situations can be found in the book<sup>7</sup>. Owing the discovery of the acceleration of the universe expansion by<sup>3,4</sup>, in the framework of Einstein Relative Gravitation, the way to give explanation was to consider the existence of exotic matter for which the quantity  $\rho + 3p < 0$  source of the so-called dark energy. The sign of the guessed cosmological constant  $\Lambda$  plays a key role in (4). Since then, the determination of the dark energy remains a crucial challenge for physics and mathematics of cosmology. These equations justify the role of the cosmological constant<sup>8</sup>:

- (1)  $\Lambda < 0$  contributes to accelerated expansion;
- (2)  $\Lambda = 0$  plays no role;
- (3)  $\Lambda > 0$  contributes to contraction.

**Remark 2.1.** Relative Gravitation is based on continuum physics e.g.<sup>9</sup>, leaving aside the aspect of quantum particle physics of micro-particles e.g.<sup>10</sup>, and from historical point of view e.g.<sup>11</sup>. More precisely, our effort is dedicated to the phenomenological cosmological constant  $\Lambda$  and its possible origins in the field of electromagnetism and especially the magnetic component of this field e.g.<sup>12, 13, 14</sup>. Although this remind is limited to Einstein Relative Gravitation without torsion, introduction of weak torsion field into the Friedman's equations has been already done e.g.<sup>15, 16</sup> where, in the first reference, the two equations are considered as a dynamical system and where the focus was on their stability analysis, which allowed these authors to study the influence of the torsion-vector on the system behavior. Hereafter, we will consider a systematic approach by considering a Riemann-Cartan manifold.

### 3. EC spacetime with connection-compatible volume-form $\omega_n$

Let consider an Einstein-Cartan spacetime with a metric and an independent connection. Several reasons have been already given to justify the use of Einstein-Cartan spacetime to describe theoretical models of cosmology e.g.<sup>17</sup>. One of them is linked to the universe evolution itself, which is supposed to move from an extremely condensed hot state to lower temperature expanded situations, passing through several phases of inflation, expansion and cooling. The torsion as well as the electromagnetic field constitute a dynamical component of the spacetime geometry and field of physical fields in spacetime.

#### 3.1. Defects and evolution of the universe

By analogy with the behavior of condensed matter processes, it is then possible that during some phase of universe coolings, there were mismatches which led to the formation of defect field at higher density. Thermomechanical process of cooling of the universe which accompanied its expansion from nearly  $100 - 200 [MeV]$  at the origin of the Bing Bang ( $10^{-5} [s]$ ) towards a temperature of  $10^{-4} [eV]$  could be a source of defects. In the hypothesis of spacetime continuum, main defects found in condensed matter are point, line, and surface defects corresponding in geometry to non-metricity, torsion, and curvature fields respectively<sup>18</sup> which may linked to Magnetic Monopole e.g.<sup>19</sup>. The initial moments of the universe formation are essentially governed by the behavior of particle physics in the infinitely small length scale, namely the quantum physics. Conversely cosmology is instead turning towards very large astronomical distances and even galactic dimensions. What is highly interesting is therefore the introduction of variables modeling the defect fields in cosmological models since all the defects in question (such as torsion and curvature<sup>20</sup>) may be on the scale of particle physics. As for condensed matter, the hypothetic proliferation of defects is accompanied by time and space redistribution in the long term to form cellular structures and filaments patterns. *In fine, we consider a generalized continuum model for the spacetime embodied by the Einstein-Cartan spacetime*<sup>21</sup>.

### 3.2. Motivation for introducing electromagnetism

It is well-known that the only two actions-at-distance of fundamental physics are gravitation and electromagnetism, with an infinite radius of action. Both of them may act in a vacuum spacetime. This is a reason to account for electric and magnetic fields within vacuum spacetime model. Furthermore, numerous studies have stated that magnetic fields exist in vacuum and are closely related to filaments structure connecting galaxies<sup>22</sup>. On the one hand, recent works have also shown the role of electromagnetic field in the acceleration of universe expansion, using usual physics of gravitation and electromagnetism e.g.<sup>23, 24</sup>. On the other hand, numerous studies have focused on the influence of spacetime defects, via the torsion field, on this acceleration e.g.<sup>25, 16, 26</sup>. Given these gravito-electromagnetic findings, clearly and mathematically showing the coupling between torsion and electromagnetism<sup>27</sup>, we consider Einstein-Cartan spacetime framework. Nevertheless, most past studies already developed the equations that govern the coupling of Einstein's relative gravitation and Maxwell's electromagnetism by using a connection  $\nabla$  compatible with  $\mathbf{g}$  but not with the Riemann- volume-form  $\bar{\omega}_n$ . The equations are explicitly completed but there remains one point which is not worth concerning the Principle of Minimal Coupling because the equation of electromagnetic wave (resulting from Maxwell's equation) included an additional term which does not allow to have exactly the same form of equation as for the case without torsion. Although this drawback could be considered only cosmetic, we worthily must develop a model where connection is compatible with both metric and volume form.

### 3.3. Motivation for using of compatible volume-form $\omega_n$

If we use the volume-form  $\bar{\omega}_n$  to define the action  $\mathcal{S}$  that encompasses relative gravitation and electromagnetism, variational procedure mathematically induces the system of Maxwell's equation, Einstein-Maxwell gravitational equation, and a third linking equation which allows us to calculate a distortion  $\mathcal{D}_\gamma^{\mu\nu}$  of spacetime as function of the electromagnetic field among others. However, there remains a concern regarding Maxwell's equation which does not respect the Minimal Coupling Principle since we do not find the generic form of this equation as in Special Relativity and in Einstein's General Relativity (without torsion). In relativity and electromagnetism, the rule of thumb to render covariant the fields equations is to change the partial derivatives  $\partial_\alpha$  to the covariant derivatives  $\bar{\nabla}_\alpha$ , and to change the Minkowski metric  $\eta_{\alpha\beta}$  into Riemann metric  $g_{\alpha\beta}$ . A major problem may arise when dealing with Riemann-Cartan spacetime due to the non-vanishing torsion<sup>28</sup>. Indeed, the incompatibility of the connection  $\nabla$  with the Riemann volume-form does not allow us to use the divergence theorem, in particular to derive the conservation laws. This is the main motivation.

### 3.4. Expression of compatible volume-form $\omega_n$

Compatibility of the volume-form  $\omega_n$  with the connection was studied in the framework of Riemann–Cartan spacetime e.g.<sup>29</sup> which made it possible to define a volume-form of any manifold with torsion and curvature<sup>30</sup>, and related with the gauge invariance of electromagnetic fields in e.g.<sup>31, 29</sup> and also recently in e.g.<sup>32</sup>. Compatibility of the connection with the volume-form is expressed with the Lie derivative along an arbitrary vector field  $\xi$ :

$$\mathcal{L}_\xi \omega_n = (\nabla_\mu \xi^\nu) \omega_n$$

The expression of such volume-form was already given<sup>29</sup>, we re-derive the proof with a direct calculus. For this purpose, let express the Lie derivative of the volume-form components:

$$\begin{aligned} \mathcal{L}_\xi \omega_{\alpha_1, \dots, \alpha_n} &= \xi^\mu \nabla_\mu \omega_{\alpha_1, \dots, \alpha_n} + (\nabla_{\alpha_1} \xi^\mu) \omega_{\mu, \alpha_2, \dots, \alpha_n} \\ &+ (\nabla_{\alpha_2} \xi^\mu) \omega_{\alpha_1, \mu, \alpha_3, \dots, \alpha_n} + \dots + (\nabla_{\alpha_n} \xi^\mu) \omega_{\alpha_1, \dots, \alpha_{n-1}, \mu} \\ &+ \xi^\mu \aleph_{\mu\alpha_1}^\nu \omega_{\nu, \alpha_2, \dots, \alpha_n} + \dots + \xi^\mu \aleph_{\mu\alpha_n}^\nu \omega_{\alpha_1, \dots, \alpha_{n-1}, \nu} \end{aligned}$$

Since there is only one component  $\Omega := \omega_{\alpha_1 < \alpha_2 < \dots < \alpha_n}$ , the Lie derivative reduces to:

$$\mathcal{L}_\xi \Omega = \xi^\mu \nabla_\mu \Omega + (\nabla_\mu \xi^\mu) \Omega + \tau_\mu \xi^\mu \Omega, \quad \text{with} \quad \tau_\mu := \aleph_{\mu\nu}^\nu = \sum_{\nu=0}^{\nu=3} \aleph_{\mu\nu}^\nu \quad (5)$$

Imposing the compatibility of  $\nabla$  with  $\omega_n$  induces the relation:

$$\xi^\mu \nabla_\mu \Omega + \tau_\mu \xi^\mu \Omega = 0 \quad (6)$$

By definition  $\Omega$  is always positive, it is then possible to make a change of variable  $\Omega := e^\vartheta$  to obtain,  $\xi^\mu$  being arbitrary:

$$\nabla_\mu \vartheta = -\tau_\mu \quad (7)$$

Coming back to the projection of the volume-form onto an arbitrary tensor base  $dx^1 \wedge \dots \wedge dx^n$ , we may write:

$$\omega_n = e^\vartheta \omega_{12\dots n} dx^1 \wedge \dots \wedge dx^n = e^\vartheta \text{Det}(\partial_1, \partial_2, \dots, \partial_n) dx^1 \wedge \dots \wedge dx^n \quad (8)$$

Particularly, we can suggest a compatible volume-form with the connection:

$$\omega_n = e^\vartheta \bar{\omega}_n, \quad \vartheta = \vartheta(\aleph) \quad (9)$$

The introduction of the determinant is necessary to have an invariant component during a base transformation. For completeness, it is also possible to assume  $\vartheta = \vartheta(\mathbf{g}, \aleph)$ , but it will add other terms but not change the methodology. To avoid lengthy equations, we can remain with  $\vartheta = \vartheta(\aleph)$  if necessary.

### 3.5. Fields equations within EC spacetime

Extending Friedman equations to Einstein–Cartan spacetime is not as simple as expected. Most authors have paid attention to metric compatibility of connection, but not that of connection with volume-form *a priori*. Let start from action (10).



### 3.5.1. Action for Gravitation and Electromagnetism

In the following, we therefore consider a connection  $\nabla$  compatible with both  $\mathbf{g}$  and  $\omega_n$ . The vacuum spacetime action includes Einstein-Palatini and Yang-Mills Lagrangians:

$$\mathcal{S}_{GEM} := \frac{1}{2\chi} \int_{\mathcal{M}} \mathcal{R}(\mathbf{g}, \nabla) \omega_n - \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \omega_n, \quad \omega_n := e^\vartheta \bar{\omega}_n \quad (10)$$

In the relation  $\omega_n := e^\vartheta \bar{\omega}_n$ , the trace of the torsion tensor is a gradient  $\aleph_{\mu\nu}^\mu = -\nabla_\nu \vartheta$  with  $\vartheta(\aleph)$  e.g.<sup>29</sup>. The relevance of this choice was reinforced<sup>30</sup> by the will to respect the Minimal Coupling Principle for the action<sup>32</sup>.

### 3.5.2. System of fields equations

We introduce the Faraday strength tensor  $\mathcal{F}_{\mu\nu} := \nabla_\mu A_\nu - \nabla_\nu A_\mu$  where  $A_\mu$  is the electromagnetic potential. The variation of the action holds:

$$\delta \mathcal{S}_{GEM} = \frac{1}{2\chi} \int_{\mathcal{M}} (\delta g^{\beta\lambda} \mathcal{R}_{\beta\lambda} + g^{\beta\lambda} \delta \mathcal{R}_{\beta\lambda}) \omega_n + g^{\beta\lambda} \mathcal{R}_{\beta\lambda} \delta \omega_n \quad (11)$$

$$- \frac{1}{4} \int_{\mathcal{M}} \delta (\mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu}) \omega_n + \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \delta \omega_n \quad (12)$$

in which the primal variables are respectively the metric, the connection (or the torsion), and the electromagnetic potential. The choice of this form of Faraday tensor, with connection  $\nabla$ , is not only a matter of the Minimal Coupling Principle. Indeed, this choice seems to violate gauge invariance at first glance, but such is not the case for several reasons by taking the action (10), one of the most relevant being the introduction of an *ad hoc* weighting function  $\phi$  of the Riemann volume-form<sup>33</sup>, that we find deductively in the present paper as  $\phi := e^{\vartheta \mathfrak{b}}$ . This is again a consequence of the choice of a  $\nabla$  compatible with  $\omega_n$ . By the way, we give a rational explanation of the torsion potential with relation (7) which was problematic. The variation of the volume-form may be explicitly derived as, since  $e^{\vartheta(\aleph)} > 0$ :

$$\omega_n := e^{\vartheta(\mathbf{g}, \aleph)} \bar{\omega}_n \implies \delta \omega_n = \left( \frac{\partial \vartheta}{\partial g^{\beta\lambda}} \delta g^{\beta\lambda} + \frac{\partial \vartheta(\aleph)}{\partial \aleph_{\alpha\beta}^\gamma} \delta \aleph_{\alpha\beta}^\gamma - \frac{1}{2} g_{\beta\lambda} \delta g^{\beta\lambda} \right) \omega_n \quad (13)$$

For the curvature, we remind the formula<sup>35</sup>:

$$\delta \aleph_{\alpha\beta\lambda}^\gamma = \nabla_\alpha (\delta \Gamma_{\beta\lambda}^\gamma) - \nabla_\beta (\delta \Gamma_{\alpha\lambda}^\gamma) + \aleph_{\alpha\beta}^\mu \delta \Gamma_{\mu\lambda}^\gamma \quad (14)$$

and for the Faraday strength tensor<sup>9</sup>:

$$\delta \mathcal{F}_{\mu\nu} = \nabla_\mu (\delta \Gamma_\nu) - \nabla_\nu (\delta \Gamma_\mu) - A_\rho \delta \aleph_{\mu\nu}^\rho \quad (15)$$

<sup>b</sup>The present approach does not postulate the absence of magnetic monopoles which would avoid certain problems concerning gauge invariance. But even in their absence, similar method based on Yang-Mills action with  $\nabla \neq \bar{\nabla}$  generalizes situation in electromagnetism where analogous of Gauss' equations shows that Lie-Jacobi brackets of fields  $\mathbf{A}$  with  $\mathbf{E}$ , and  $\mathbf{A}$  with  $\mathbf{B}$  play role of "electric charge density" and "magnetic monopole density" respectively<sup>34</sup>.

with the connection  $\nabla$  compatible with  $\omega_n$ . In addition, we have the variation:

$$\delta(\mathcal{H}_{\mu\nu}\mathcal{F}_{\mu\nu}) = -2\epsilon_0\mathcal{F}_{\mu\alpha}g^{\alpha\beta}\mathcal{F}_{\beta\nu} + 2\mathcal{H}^{\mu\nu}\delta\mathcal{F}_{\mu\nu} \quad (16)$$

As previously, the variation of the curvature may be written as follows:

$$\delta\mathcal{R} = \delta g^{\mu\nu}\mathcal{R}_{\mu\nu} + \delta_\gamma^\alpha g^{\beta\lambda} \left[ \nabla_\alpha (\delta\Gamma_{\beta\lambda}^\gamma) - \nabla_\beta (\delta\Gamma_{\alpha\lambda}^\gamma) + \mathbb{N}_{\alpha\beta}^\mu \delta\Gamma_{\mu\lambda}^\gamma \right]$$

A point deserves to be highlighted. Taking this function  $\vartheta$  into account seems to complicate the model since it introduces additional terms, but at the same time it allows taking into account coupling terms, which is worth, and using the divergence theorem. Putting together all of these variations, we obtain:

$$\begin{aligned} \delta\mathcal{S}_{GEM} &= \frac{1}{2\chi} \int_{\mathcal{M}} \left( \mathcal{R}_{\mu\nu} - \frac{\mathcal{R}}{2}g_{\mu\nu} + \mathcal{R} \frac{\partial\vartheta}{\partial g^{\mu\nu}} \right) \delta g^{\mu\nu} \omega_n + \mathcal{R} \frac{\partial\vartheta}{\partial \mathbb{N}_{\mu\nu}^\gamma} \delta \mathbb{N}_{\mu\nu}^\gamma \omega_n \\ &+ \frac{1}{2\chi} \int_{\mathcal{M}} \delta_\gamma^\alpha g^{\beta\lambda} \left[ \nabla_\alpha (\delta\Gamma_{\beta\lambda}^\gamma) - \nabla_\beta (\delta\Gamma_{\alpha\lambda}^\gamma) + \mathbb{N}_{\alpha\beta}^\mu \delta\Gamma_{\mu\lambda}^\gamma \right] \omega_n \\ &- \frac{1}{4} \int_{\mathcal{M}} -2\epsilon_0\mathcal{F}_{\mu\alpha}g^{\alpha\beta}\mathcal{F}_{\beta\nu} \delta g^{\mu\nu} \omega_n \\ &- \frac{1}{4} \int_{\mathcal{M}} 2\mathcal{H}^{\mu\nu} (\nabla_\mu(\delta A_\nu) - \nabla_\nu(\delta A_\mu) - A_\rho \delta \mathbb{N}_{\mu\nu}^\rho) \omega_n \\ &- \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu}\mathcal{F}_{\mu\nu} \left( \frac{\partial\vartheta}{\partial \mathbb{N}_{\alpha\beta}^\gamma} \delta \mathbb{N}_{\alpha\beta}^\gamma - \frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu} \right) \omega_n \end{aligned}$$

The resulting terms in divergence vanish by imposing the boundary conditions  $\delta\Gamma_{\mu\nu}^\gamma \equiv 0$  and  $\delta A_\nu \equiv 0$ . The integral term, by using the metric compatibility,:

$$\int_{\mathcal{M}} \delta_\gamma^\alpha g^{\beta\lambda} \left[ \nabla_\alpha (\delta\Gamma_{\beta\lambda}^\gamma) - \nabla_\beta (\delta\Gamma_{\alpha\lambda}^\gamma) \right] \omega_n \rightarrow \int_{\mathcal{M}} \nabla_\alpha \xi^\alpha \omega_n$$

becomes a 3-surface integral, and so is the case for the two integrals, after applying integration by parts the two integrands of the 4<sup>th</sup> row,:

$$\int_{\mathcal{M}} \nabla_\mu (\mathcal{H}^{\mu\nu}\delta A_\nu) \omega_n \quad \text{and} \quad \int_{\mathcal{M}} \nabla_\nu (\mathcal{H}^{\mu\nu}\delta A_\mu) \omega_n \rightarrow \int_{\mathcal{M}} \nabla_\alpha \xi^\alpha \omega_n$$

### 3.5.3. Fields equations

Due to arbitrariness of  $\delta A_\nu$ ,  $\delta g^{\mu\nu}$  and  $\delta\Gamma_{\mu\nu}^\gamma$ , we deduce the following.

**Theorem 3.1.** (*Gravitation with electromagnetism in vacuum*). *The system of fields equations within Einstein-Cartan where the connection  $\nabla$  is compatible with both the metric  $\mathbf{g}$  and the volume-form  $\omega_n$  holds:*

$$\left\{ \begin{array}{l} \nabla_\mu \mathcal{H}^{\mu\nu} = 0 \\ \frac{1}{2\chi} G_{\mu\nu} + T_{\mu\nu}^{\text{em}} = 0 \\ \frac{1}{2\chi} \mathcal{D}_\gamma^{\mu\nu} + \mathcal{H}^{\mu\nu} A_\gamma - \frac{1}{4}(\mathcal{H}^{\alpha\beta}\mathcal{F}_{\alpha\beta}) \left( \frac{\partial\vartheta}{\partial \mathbb{N}_{\mu\nu}^\gamma} - \frac{\partial\vartheta}{\partial \mathbb{N}_{\nu\mu}^\gamma} \right) = 0 \end{array} \right. \quad (17)$$

with the Einstein tensor and the spacetime distortion tensor:

$$G_{\mu\nu} := \mathfrak{R}_{\mu\nu} - \frac{\mathcal{R}}{2} g_{\mu\nu} + \mathcal{R} \frac{\partial\vartheta}{\partial g^{\mu\nu}}, \quad \mathcal{D}_{\gamma}^{\mu\nu} := g^{\beta\nu} \aleph_{\gamma\beta}^{\mu} + \mathcal{R} \left( \frac{\partial\vartheta}{\partial \aleph_{\mu\nu}^{\gamma}} - \frac{\partial\vartheta}{\partial \aleph_{\nu\mu}^{\gamma}} \right) \quad (18)$$

where we have the relationship  $\aleph_{\nu\mu}^{\nu} \equiv \nabla_{\mu}\vartheta$ .

The first equation of (17) is the covariant Maxwell electromagnetic equation, the second the extended Einstein-Cartan-Maxwell equation, and the third a new linkage equation coupling the distortion tensor  $\mathcal{D}_{\gamma}^{\mu\nu}$  with physics. This model does not introduce new physical phenomena other than those contained in these two actions, and eventually their coupling. By the way, introducing the expression of the Faraday tensor in Maxwell's equations (17) implies the covariant extended electromagnetic wave equations within an Einstein-Cartan spacetime<sup>9</sup>:

$$-g^{\alpha\beta} \nabla_{\alpha} \nabla_{\beta} A^{\mu} + g^{\mu\alpha} \nabla_{\alpha} \nabla_{\beta} A^{\beta} - g^{\mu\alpha} \aleph_{\beta\alpha}^{\gamma} \nabla_{\gamma} A^{\beta} + g^{\mu\alpha} \mathfrak{R}_{\alpha\beta} A^{\beta} = 0 \quad (19)$$

In an Einstein curved spacetime, the electromagnetic wave equation reduces to:

$$-g^{\alpha\beta} \bar{\nabla}_{\alpha} \bar{\nabla}_{\beta} A^{\mu} + g^{\mu\alpha} \bar{\nabla}_{\alpha} \bar{\nabla}_{\beta} A^{\beta} + g^{\mu\alpha} \bar{\mathfrak{R}}_{\alpha\beta} A^{\beta} = 0 \quad (20)$$

which models the usual theory on the light-bending phenomenon. In the case of special relativity, in a flat Minkowski spacetime, assuming the Lorenz gauge, (20) projected onto an inertial frame gives the classical wave equation:

$$-\eta^{\alpha\beta} \partial_{\alpha} \partial_{\beta} A^{\mu} = 0 \quad (21)$$

with  $\eta^{\alpha\beta} := \text{diag}[+1, -1, -1, -1]$ .

**Chern-Simons current and spin angular momentum** At this stage, we can observe in the third equation of (17) the quantity and its trace:

$$\mathcal{H}^{\mu\nu} A_{\lambda} \implies \mathcal{H}^{\mu\nu} A_{\nu} \quad (22)$$

where we recognize as the Chern-Simons current<sup>37</sup>. The vectorial form gives:

$$\mathcal{H}^{\mu\nu} A_{\nu} \rightarrow \begin{pmatrix} \mathbf{B} \cdot \mathbf{A} \\ \mathbf{E} \times \mathbf{A} \end{pmatrix} \quad (23)$$

The first component  $\mathbf{B} \cdot \mathbf{A}$  of the four-vector is the magnetic helicity density and the three-dimensional vector  $\mathbf{E} \times \mathbf{A}$  is the spin angular momentum<sup>38</sup>. The extremely coupled system of equations aims to show that one cannot use parts of these equations carelessly without impunity. Maxwell's equation (first line) has the same shape as usually, the second equation (Einstein-Maxwell) also keeps almost exactly the same form. However the third equation couples electromagnetism with torsion and is new. The Chern-Simons current, on the other hand, appears explicitly. Considering this model (17), we see that the electromagnetic field acts not only on the curvature which bends spacetime but also on the distortion which "tears" this spacetime to obtain a set infinite number small grains of spacetime (named microcosms<sup>39</sup>) of very small length scales (18). As in condensed materials, the history

of the thermal evolution of the universe would not be without consequences for its structure and its contents. By analogy, huge temperature changes starting from the estimated temperature at  $T_0 \simeq 10^{31}$  [K] at the initial time  $t \simeq 10^{-43}$  [s] and which drops to the currently measured temperature of  $T_{\text{CMB}} \simeq 2.72528 \pm 0.000057$  [K] e.g.<sup>40</sup> (CMB means Cosmic Microwave Background) can be sources of nucleation and proliferation of defects in the vacuum spacetime of the universe. These defects are governed by the third equation of (17).

**Remark 3.1.** Tensor  $\mathcal{D}_\gamma^{\mu\nu}$  is the counterpart of the Einstein tensor but for the torsion. Without electromagnetism, the field equations of an Einstein-Cartan vacuum reduce to  $G_{\mu\nu} = 0$  and  $\mathcal{D}_\gamma^{\mu\nu} = 0$ . More detailed research about this question constitutes new outlook. The non dependence of function  $\vartheta$  on torsion implies that the torsion necessarily vanishes, and by the way, the model reduces to the Einstein Relative Gravitation.

#### 4. Particular case of Torsion-vector and Field Equations

The development of Relative Gravitation models including torsion and particularly the torsion vector has already been the subject of advanced studies<sup>41</sup>. We take up this aspect with the model derived in this study.

##### 4.1. Variation of compatible $\omega_n$

The torsion tensor may be represented by the relation, where we have introduced the 1-form  $\aleph_\mu$  as the trace of the torsion tensor, :

$$\aleph_{\mu\nu}^\gamma = \aleph_\mu \delta_\nu^\gamma - \aleph_\nu \delta_\mu^\gamma \quad \implies \quad \aleph_\mu = \frac{1}{3} \aleph_{\mu\nu}^\nu \quad (24)$$

Let consider the particular case where we have the dependence:

$$\omega_n := e^{\vartheta(\tau)} \bar{\omega}_n, \quad \tau := g^{\mu\nu} \aleph_\mu \aleph_\nu \in \mathbb{R} \quad (25)$$

This particular expression  $\tau$  is dictated by the seek of formulation invariance of the scalar function  $\vartheta(\aleph)$ , by virtue of the Cauchy theorem on scalar-valued function<sup>42</sup>. The Euler variation of the volume-form takes the form of:

$$\delta\omega_n = -\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu} \omega_n + \vartheta'(\tau) \delta\tau \omega_n$$

where  $\delta\tau = \delta(g^{\mu\nu} \aleph_\mu \aleph_\nu) = \aleph_\mu \aleph_\nu \delta g^{\mu\nu} + 2 g^{\mu\nu} \aleph_\nu \delta \aleph_\mu$ . Variation is taken as the directional derivative along the geometric variables  $\delta g^{\mu\nu}$  and  $\delta \aleph_\mu$ . The variation may be expressed either in terms of 1- form  $\aleph_\mu, \mu = 0, 1, 2, 3$ :

$$\delta\omega_n = \left( -\frac{1}{2} g_{\mu\nu} + \vartheta'(\tau) \aleph_\mu \aleph_\nu \right) \delta g^{\mu\nu} \omega_n + 2 \vartheta'(\tau) g^{\mu\nu} \aleph_\nu \delta \aleph_\mu \omega_n \quad (26)$$

This expression is used when we directly utilize the function  $\vartheta(\tau)$  and the variations are formulated taking  $\delta g^{\mu\nu}$  and  $\delta \aleph_\mu$  as independent variables.

#### 4.2. Variation of Einstein-Palatini action

Say a vacuum spacetime with gravitation and electromagnetic fields, and let consider the particular case of torsion-vector. Let remind the gravitational action of vacuum spacetime:

$$\mathcal{S}_{EP} := \frac{1}{2\chi} \int_{\mathcal{M}} \mathcal{R}(\mathbf{g}, \nabla) \omega_n \quad (27)$$

where the curvature is function of the metric and the torsion, as independent variables. Since  $\mathcal{R} := g^{\beta\lambda} \mathcal{R}_{\beta\lambda}$ , the variation of the action is given by:

$$\delta \mathcal{S}_{EP} = \frac{1}{2\chi} \left( \int_{\mathcal{M}} \delta g^{\mu\nu} \mathcal{R}_{\mu\nu} \omega_n + \int_{\mathcal{M}} g^{\beta\lambda} \delta \mathcal{R}_{\beta\lambda} \omega_n + \int_{\mathcal{M}} \mathcal{R} \delta \omega_n \right)$$

where we have:

$$g^{\beta\lambda} \delta \mathcal{R}_{\beta\lambda} = \delta_{\gamma}^{\alpha} g^{\beta\lambda} \left[ \nabla_{\alpha} (\delta \Gamma_{\beta\lambda}^{\gamma}) - \nabla_{\beta} (\delta \Gamma_{\alpha\lambda}^{\gamma}) + \aleph_{\alpha\beta}^{\mu} \delta \aleph_{\mu\lambda}^{\gamma} \right] \quad (28)$$

in which the term  $\delta \Gamma_{\mu\lambda}^{\gamma}$  may replaced by  $\delta \aleph_{\mu\lambda}^{\gamma}$ . The only term that could be non zero after integration<sup>c</sup> is:

$$\delta_{\gamma}^{\alpha} g^{\beta\lambda} \aleph_{\alpha\beta}^{\mu} \delta \aleph_{\mu\lambda}^{\gamma} = g^{\beta\lambda} \aleph_{\gamma\beta}^{\mu} \delta \aleph_{\mu\lambda}^{\gamma} = 3 g^{\beta\lambda} \aleph_{\beta} \delta \aleph_{\lambda}$$

Application of the metric compatibility and the divergence theorem implies:

$$\begin{aligned} \delta \mathcal{S}_{EP} &= \frac{1}{2\chi} \int_{\mathcal{M}} \left( \mathcal{R}_{\mu\nu} - \frac{\mathcal{R}}{2} g_{\mu\nu} + \mathcal{R} \vartheta'(\tau) \aleph_{\mu} \aleph_{\nu} \right) \delta g^{\mu\nu} \omega_n \\ &+ \frac{1}{2\chi} \int_{\mathcal{M}} 3 g^{\mu\nu} \aleph_{\nu} \delta \aleph_{\mu} \omega_n + \frac{1}{2\chi} \int_{\mathcal{M}} 2 \mathcal{R} \vartheta'(\tau) g^{\mu\nu} \aleph_{\nu} \delta \aleph_{\mu} \omega_n \end{aligned}$$

By introducing the expression of the Einstein-like tensor, we obtain:

$$\begin{aligned} G_{\mu\nu} &= \overline{G}_{\mu\nu} + g^{\sigma\epsilon} [\aleph_{\sigma} \aleph_{\epsilon} - (\overline{\nabla}_{\sigma} \aleph_{\epsilon} + \overline{\nabla}_{\epsilon} \aleph_{\sigma})] g_{\mu\nu} \\ &+ (\overline{\nabla}_{\mu} \aleph_{\nu} + \overline{\nabla}_{\nu} \aleph_{\mu}) + 2 \aleph_{\mu} \aleph_{\nu} \end{aligned} \quad (29)$$

where we recognize the Einstein tensor  $\overline{G}_{\mu\nu}$  of the classical Relative Gravitation. Finally, we have the variation:

$$\begin{aligned} \delta \mathcal{S}_{EP} &= \frac{1}{2\chi} \int_{\mathcal{M}} [G_{\mu\nu} + \mathcal{R} \vartheta'(\tau) \aleph_{\mu} \aleph_{\nu} + \Lambda_{\aleph} g_{\mu\nu}] \delta g^{\mu\nu} \omega_n \\ &+ \frac{1}{2\chi} \int_{\mathcal{M}} [3 + 2 \mathcal{R} \vartheta'(\tau)] g^{\mu\nu} \aleph_{\nu} \delta \aleph_{\mu} \omega_n \end{aligned}$$

where for convenience, we define the quantity  $\Lambda_{\aleph}$  similar as cosmological constant:

$$\Lambda_{\aleph} := g^{\alpha\beta} \aleph_{\alpha} \aleph_{\beta} \quad (30)$$

**Remark 4.1.** We notice that without physical sources the second integrand vanishes  $\aleph_{\beta} = 0$ , therefore the second line implies a zero torsion-vector. In such a case, the variation reduces to the classical Einstein field equation  $\overline{G}_{\mu\nu} = 0$ .

<sup>c</sup>The metric compatibility together with the divergence theorem is applied for that purpose.

### 4.3. Variation of Yang-Mills action

Let again consider the electromagnetic action of the vacuum spacetime action where the torsion is reduced to the torsion-vector:

$$\mathcal{S}_{YM} := -\frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \omega_n \quad (31)$$

with  $\mathcal{F}_{\mu\nu}$  is the Faraday strength tensor and  $\mathcal{H}^{\mu\nu}$  the electromagnetic excitation tensor. The variation of this action writes:

$$\delta\mathcal{S}_{YM} := -\frac{1}{4} \int_{\mathcal{M}} (\delta\mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} + \mathcal{H}^{\mu\nu} \delta\mathcal{F}_{\mu\nu}) \omega_n - \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \mathcal{F}_{\mu\nu} \delta\omega_n$$

Each term of this variation can be formulated as:

$$\begin{aligned} \delta\mathcal{S}_{YM} = & -\frac{1}{4} \int_{\mathcal{M}} (-2\epsilon_0 \mathcal{F}_{\mu\alpha} g^{\alpha\beta} \mathcal{F}_{\beta\nu} \delta g^{\mu\nu} + \mathcal{H}^{\mu\nu} \delta\mathcal{F}_{\mu\nu}) \omega_n - \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \delta\mathcal{F}_{\mu\nu} \omega_n \\ & - \frac{1}{4} \int_{\mathcal{M}} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} \left[ \left( -\frac{1}{2} g_{\mu\nu} + \vartheta'(\tau) \aleph_\mu \aleph_\nu \right) \delta g^{\mu\nu} + 2 \vartheta'(\tau) g^{\mu\nu} \aleph_\nu \delta \aleph_\mu \right] \omega_n \end{aligned}$$

with:  $\delta\mathcal{F}_{\mu\nu} = \nabla_\mu(\delta A_\nu) - \nabla_\nu(\delta A_\mu) - A_\gamma \delta \aleph_{\mu\nu}^\gamma$ , and for the case of torsion vector:

$$\delta\mathcal{F}_{\mu\nu} = \nabla_\mu(\delta A_\nu) - \nabla_\nu(\delta A_\mu) - (\delta \aleph_\mu A_\nu - \delta \aleph_\nu A_\mu) \quad (32)$$

After some re-arrangements, we obtain the following expression:

$$\begin{aligned} \delta\mathcal{S}_{YM} = & \int_{\mathcal{M}} \left[ \frac{1}{8} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} g_{\mu\nu} + \frac{1}{4} \mathcal{H}^{\lambda\rho} (g_{\mu\lambda} \mathcal{F}_{\rho\nu} + \mathcal{F}_{\mu\lambda} g_{\rho\nu}) \right] \delta g^{\mu\nu} \omega_n \\ & - \int_{\mathcal{M}} \frac{1}{4} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} \vartheta'(\tau) \aleph_\mu \aleph_\nu \delta g^{\mu\nu} \omega_n - \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} \nabla_\mu(\delta A_\nu) \\ & + \int_{\mathcal{M}} \mathcal{H}^{\mu\nu} A_\nu \delta \aleph_\mu \omega_n - \int_{\mathcal{M}} \frac{1}{2} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} \vartheta'(\tau) g^{\mu\nu} \aleph_\nu \delta \aleph_\mu \omega_n \end{aligned}$$

To generalize the classical case, we suggest the definition of the extended electromagnetic energy-momentum:

$$\tilde{T}_{\mu\nu}^{\text{em}} := \frac{1}{8} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} g_{\mu\nu} + \frac{1}{4} \mathcal{H}^{\lambda\rho} (g_{\mu\lambda} \mathcal{F}_{\rho\nu} + \mathcal{F}_{\mu\lambda} g_{\rho\nu}) - \frac{1}{4} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} \vartheta'(\tau) \aleph_\mu \aleph_\nu \quad (33)$$

**Remark 4.2.** The first two terms constitute the usual contribution for electromagnetic energy-momentum e.g.<sup>43</sup>. The function  $\vartheta(\tau)$  is related to the so-called Dilation for Brans-Dicke like theory of gravitation e.g.<sup>44</sup>.

### 4.4. Fields Equations for Gravito-Electromagnetism

The variation of Yang-Mills within an Einstein-Cartan spacetime holds:

$$\delta\mathcal{S}_{YM} = \int_{\mathcal{M}} \tilde{T}_{\mu\nu}^{\text{em}} \delta g^{\mu\nu} \omega_n + \int_{\mathcal{M}} \nabla_\mu \mathcal{H}^{\mu\nu} \delta A_\nu \omega_n \quad (34)$$

$$+ \int_{\mathcal{M}} \left[ \mathcal{H}^{\mu\nu} A_\nu - \frac{1}{2} \mathcal{H}^{\sigma\epsilon} \mathcal{F}_{\sigma\epsilon} \vartheta'(\tau) g^{\mu\nu} \aleph_\nu \right] \delta \aleph_\mu \omega_n \quad (35)$$

and for the Einstein-Palatini action:

$$\begin{aligned} \delta \mathcal{S}_{EP} &= \frac{1}{2\chi} \int_{\mathcal{M}} [G_{\mu\nu} + \mathcal{R} \vartheta'(\tau) \aleph_{\mu} \aleph_{\nu} + \Lambda_{\aleph} g_{\mu\nu}] \delta g^{\mu\nu} \omega_n \\ &+ \frac{1}{2\chi} \int_{\mathcal{M}} (3 + 2 \mathcal{R} \vartheta'(\tau)) g^{\mu\nu} \aleph_{\nu} \delta \aleph_{\mu} \omega_n \end{aligned}$$

Let us define an extension of the Einstein tensor:

$$\begin{aligned} \tilde{G}_{\mu\nu} &= \bar{G}_{\mu\nu} + \Lambda_{\aleph} g_{\mu\nu} - (\bar{\nabla}_{\sigma} \aleph_{\epsilon} + \bar{\nabla}_{\epsilon} \aleph_{\sigma}) g_{\mu\nu} \\ &+ [2 + \mathcal{R} \vartheta'(\Lambda_{\aleph})] \aleph_{\mu} \aleph_{\nu} + (\bar{\nabla}_{\mu} \aleph_{\nu} + \bar{\nabla}_{\nu} \aleph_{\mu}) \end{aligned} \quad (36)$$

**Main results on the accelerated expansion.** Now, we observe that the term  $\Lambda_{\aleph} := g^{\alpha\beta} \aleph_{\alpha} \aleph_{\beta}$  in (36) may be negative or positive since the metric is pseudo-Riemannian. *It therefore plays the role of a cosmological constant. For time-like torsion  $\Lambda_{\aleph} > 0$ , it acts as dark matter, whereas for space-like torsion  $\Lambda_{\aleph} < 0$ , it rather acts as dark energy.*

For the localization of the variation equation, arbitrariness of  $\delta A_{\nu}$ ,  $\delta g^{\mu\nu}$ ,  $\delta \aleph_{\mu}$ , implies the theorem:

**Theorem 4.1.** *Let consider an Einstein-Cartan spacetime endowed with a metric and a connection, compatible with both the metric and the volume-form, with the action (10). The fields equations for relative gravitation coupled with electromagnetism are the following:*

$$\left\{ \begin{array}{l} \nabla_{\mu} \mathcal{H}^{\mu\nu} = 0 \\ \frac{1}{2\chi} \tilde{G}_{\mu\nu} + \tilde{T}_{\mu\nu}^{\text{em}} = 0 \\ \frac{1}{2\chi} \mathcal{D}_{\nu}^{\mu\nu} + \mathcal{H}^{\mu\nu} A_{\nu} - \frac{\vartheta'(\tau)}{2} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} g^{\mu\nu} \aleph_{\nu} = 0 \end{array} \right. \quad (37)$$

where  $\tilde{G}_{\mu\nu}$  is defined by (36) and the distortion tensor by:

$$\mathcal{D}_{\nu}^{\mu\nu} := (3 + 2\vartheta'(\tau)\mathcal{R}) g^{\mu\nu} \aleph_{\nu} \quad (38)$$

Tensors  $\tilde{G}_{\mu\nu}$  and  $\mathcal{D}_{\nu}^{\mu\nu}$  are purely deduced from geometric structure of the spacetime whereas the other terms includes both geometry and physics.

**Approximation .** If we can neglect the term  $\vartheta'(\tau) \simeq 0$ , meaning that  $\vartheta$  varies slowly in terms of torsion, and assume that the torsion field is quasi-homogeneous  $\bar{\nabla} \aleph \simeq 0$ , we obtain a significantly simpler fields equations:

$$\left\{ \begin{array}{l} \nabla_{\mu} \mathcal{H}^{\mu\nu} = 0 \\ \frac{1}{2\chi} (\bar{G}_{\mu\nu} + \Lambda_{\aleph} g_{\mu\nu} + 2 \aleph_{\mu} \aleph_{\nu}) + \tilde{T}_{\mu\nu}^{\text{em}} = 0 \\ \frac{3}{2\chi} g^{\mu\nu} \aleph_{\nu} + \mathcal{H}^{\mu\nu} A_{\nu} = 0 \end{array} \right. \quad (39)$$

We recover the usual form of the electromagnetic energy-momentum:

$$\tilde{T}_{\mu\nu}^{\text{em}} := \frac{1}{8} \mathcal{H}^{\alpha\beta} \mathcal{F}_{\alpha\beta} g_{\mu\nu} + \frac{1}{4} \mathcal{H}^{\lambda\rho} (g_{\mu\lambda} \mathcal{F}_{\rho\nu} + \mathcal{F}_{\mu\lambda} g_{\rho\nu}) \quad (40)$$

From the third row of (39), we deduce the torsion-vector in term of the magnetic helicity and the spin angular momentum<sup>38</sup> (Chern-Simons current):

$$\aleph_\mu = -\frac{2}{3}\chi g_{\mu\alpha} \mathcal{H}^{\alpha\beta} A_\beta \quad (41)$$

This approximated case is apparently simpler because some coupling terms vanish. It nevertheless clearly highlights that the magnetic field is a major physical contributor to shaping the large structure of the universe<sup>22</sup> and to the acceleration of the expansion. In a general manner, it should be always stressed that the electromagnetic field breaks the spacetime isotropy due to either directly the energy-momentum tensor  $T_{\mu\nu}^{\text{em}}$ , or the isotropic breaking from the tensor product  $\aleph \otimes \aleph$ , except in very rare cases.

## 5. Discussion and concluding remarks

The seminal paper of Hubble showed the uniform expansion of universe and definitely led to a turning point in the understanding of the universe evolution<sup>45</sup>. Nevertheless in 1998, two teams of astronomers reported that the universe was pulling itself apart<sup>3,4</sup>. This modified completely the way of thinking, the common and maybe the simplest path was to postulate in addition to the existence of dark matter generating attractive force to maintain universe, the existence of a dark energy for a repulsive force source of accelerated expansion. Both of them might be approached phenomenologically by the cosmological constant. We find that the coupling of gravitation and electromagnetic fields may be alternative candidate for explain in part this accelerated expansion, say  $\Lambda_\aleph < 0$  for dark energy, and  $\Lambda_\aleph > 0$  for the dark matter. The developed model implicitly considers two hierarchies since electromagnetism has several facets: classical macroscopic approach and also first quantized form of the photon equation. We have established a formal link between residual electromagnetic field, primordial or due to plasma flows, and torsion without any need to consider matter-spin relationship. This result may provide new insights for latter model of cosmology e.g.<sup>17</sup>. Dark energy is then reasonably more than simply a cosmological constant  $\Lambda$  used to fit the observed data resulting from astronomical and astrophysical measurements of universe elements and galaxies: it remains a supreme enigma for physics, thousands of papers and ideas were published and suggested, but at the present this enigma remains unsolved.

The present work shows that the tensor  $\mathcal{D}_\gamma^{\mu\nu}$  homogeneously function the contortion tensor, quantifying the spacetime distortion, is explicitly related to the Chern-Simons current and gives new interesting insights for analyzing the magnetic helicity density and spin angular momentum density<sup>47,48</sup>. Integral of the magnetic helicity gives the Chern-Simons number which is clearly shown to be related to topological defects of the spacetime. The cosmological magnetic helicity is nearly conserved quantity for the universe is a high conductive medium e.g.<sup>49</sup>. On the one hand, torsion is a variable measuring the spacetime defects, on the other hand it is also closely linked to electromagnetic field, in particular what this is called a magnetic



monopole<sup>50</sup>. In a way this geometrization of the electromagnetic field by the torsion and the link which unites the torsion with the cosmological constant is in conformity with recent studies which place the torsion as a very probable element, at least in part, to quantify the dark mass and energy of cosmology. Accounting for the torsion induces spacetime anisotropy.

The standard model of cosmology is based on the hypotheses of isotropy and homogeneity of the universe on very large scales. However, in reality, spacetime locally presents anisotropy. One of the questions is whether this anisotropy is pronounced so as not to be negligible or not. The guiding principle for carrying out the present work was to base ourselves on the geometry of spacetime, in particular on the geometry of Riemann-Cartan, linked to the coupling of gravitation-electromagnetism<sup>51, 52</sup>. It is a modest but reasonable generalization because it constitutes a certain continuity of the method with the approach specific to Relative Gravitation based on spacetime of Einstein-(anti)-de Sitter with the additional role of the torsion field. The attribution of electromagnetic field and *in fine* torsion field within the vacuum spacetime therefore remains a plausible approach to further study the notion of dark matter and dark energy<sup>53</sup>.

The present work deliberately omitted particle microphysics, which is beyond of its scope, and instead it focused on the continuum geometry of spacetime and particularly on defects. Torsion is indeed associated to translational defects while curvature is due to rotational defects of the spacetime. Non-compatibility of quantum physics and general relativity arises in part from the fact that the metric tensor and the connection are physical and dynamic variables. Spacetime is curved and maybe twisted. Which shows that spacetime geometry stands apart from other aspects of micro-physics, notably quantum physics, whose definitions of variables are still based on the existence of classical non-relativistic gravitational spacetime. The unification of all physical theories is thus dependent on the possibility of making the geometry of spacetime and physical particles consistent down to the smallest scales. A critical question is to know up to what length scale we can consider that spacetime is a continuum (compact and connected), at the Planck length scale ?<sup>54</sup>. And consequently, what is the geometric structure of possibly discrete spacetime for smaller scales which allows a continuous and valid transition to be made with the continuum assumption. The present work shows that taking into account curvature and torsion seems to provide some (fragmentary) answers. But this question still presents gray areas despite recent quantum theories of microphysics and especially with the recent evolution of cosmology with accelerated expansion of the universe.

The mutual interweaving of dark energy, dark matter, the diffuse electromagnetic field in space, and the plasmas creation due to charged fluids in the universe remain very complicated. In this regard, some works also suggested the role of plasmas regarding proof of the existence of Magnetic Monopoles<sup>55, 56</sup>. Beyond the classical approaches, quantum phenomena have received particular attention regarding their coupling with macroscopic electromagnetic fields in a vacuum. In this

extremely complicated and dense set and whose concepts are often at the stage of hypotheses like dark energy and dark matter (unknown but accounting for nearly 95% of the contributions in the evolution of the universe in its expansion<sup>57</sup>), despite the spectacular advances in experimental observations in cosmology, the development of a global mathematical model including all these phenomena is more than desirable but still remains at the stage of wishes and perspectives, at least in the current state of cosmology and astrophysics. In a modest level, taking into account the torsion field and the curvature field amounts to performing an increasingly fine modeling of continuous media. If the curvature is associated with gravitation, the torsion seems to be associated with electromagnetism<sup>48</sup>. Therefore, since Maxwell's equations are the continuous versions of Schrödinger's equations<sup>58</sup>, the question is whether ultimately taking torsion into account would not constitute a path towards quantization? Much if not everything remains to be done, but the model of spacetime as generalized continuum, with torsion and therefore taking into account the electromagnetic fields of the universe, could hopefully provide new insights into current problems of cosmology, in particular its accelerated expansion<sup>59, 60</sup> and the formation of galaxies and gigantic filaments.

To conclude, the two basic action-at-distance interactions as gravitational mechanics and electromagnetism are investigated in vacuum spacetime. The common thread for the writing of this work remains the analysis and mathematical consequences of the choice of geometric variables which structure spacetime: metric, connection and volume-form. This choice aims to satisfy their mutual compatibilities which must be rigorously respected during the theoretical development. Beyond the mutual influences between physics and mathematics in the search for a model that we hope is more or less reliable to predict the genesis and evolution of the universe, it is clear that cosmology itself remains closely linked to philosophical concept<sup>61</sup> and metaphysical approach<sup>62</sup>, as shown by numerous works for thousands of years and in view of multitude of publications.

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