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Logic and grounding

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Abstract

The word “grounding” is nowadays the object of a large and flourishing philosophical debate. However, without further specification, this expression is ambiguous between (at least) two different perspectives: a metaphysical perspective according to which existent items are ordered in a hierarchy where some items ground other; a scientific perspective, according to which the truths of a scientific theory are ordered in a hierarchy where some truths are the reasons or grounds of others. These two different perspectives have given rise to two different logical analyses. The aim of this paper is to present a critical overview of the major results obtained by means of these two logical analyses, comparing their differences and similarities.

1. Introduction

This paper belongs to the part of the Handbook dedicated to “Logic and Metaphysics;” given that it focuses on the links between logic and grounding, and that grounding is one of the central topics in contemporary metaphysics,¹ it seems reasonable that it has been assigned to this part of the Handbook. However, and probably surprisingly for most of the readers, the word “grounding” has not always had a purely metaphysical connotation; quite on the contrary, in the history of philosophy, the expression “grounding” has been used in relation to a particular conception of science, where mathematics was a paradigm: from Aristotle to Bolzano,² from Leibniz to Frege, there has been a wide consensus in considering

¹ E.g., see Bliss and Trogon (2014), Rosen (2010) and Schaffer (2009).

² Aristotle and Bolzano are often mentioned as precursors of the notion of metaphysical grounding, e.g., see Correia and Schnieder, (2012). However, there also exists a tradition that link them to a different conception of grounding mainly relevant for science and mathematics, e.g., see Detlefsen (1988). Without entering in a historical debate that although interesting would lead us far away from the main topic of this paper, we are sympathetic and thus follow this second line of thought.

the truths of mathematics as ordered in a grounding hierarchy. Given the tight relations between logic and science,³ and in particular between logic and mathematics,⁴ it is not at all surprising that even this second conception of grounding has given rise to several studies concerning the links between “logic and grounding.” However, as far as we know, no systematic comparison between the logics coming from these two different, yet kindred, traditions have been dealt with in the literature. The main aim of the present paper is precisely to fill this gap. This will be done by keeping the discussion to a not too formal level: the technical details of the different logical analysis of grounding have already been provided in other articles like Mc Sweeney (2020a), Poggiolesi (2020a, 2016b).

The paper will be organised as follows. In *Section 2*, we will introduce the notion of metaphysical grounding in its generality together with its link to a logical study. In *Section 3*, we will do the same operation for the other notion of grounding, coming from the science, that we might call “conceptual grounding.” *Section 4* will be used to draw a systematic comparison between the logic of metaphysical grounding and the logic of conceptual grounding. In *Section 5*, we will end up the paper with some conclusion and directions for future research.

2. Metaphysical Grounding

One of the central philosophical tenets used to analyse reality is the relation of causation, namely a binary relation amongst two relata: the cause(s) and the effect. More recently, philosophers have become sensitive to the idea that there might be another relation that link facts but not via some causal mechanism, rather via some form of determination. Such a relation is called metaphysical grounding.⁵ This relation is often conveyed by the expression ‘because’ and typical examples are the following:

³ See the chapter *Logic and the natural sciences* of this Handbook by G. Schurz.

⁴ See the chapter *Logic and the mathematical sciences* of this Handbook by R. Zach.

⁵ E.g., see Raven (2020).

- The glass is fragile because of its molecular structure,
- ‘Socrates is pale’ is true because there is a trope of paleness in Socrates,
- John is in pain because he is in brain state P .

Whether these statements are true or false, and no matter what they concern, they all seem to convey a relation that is non-causal and explanatory in nature: in each of these cases, the fact denoted by what is at the right of the ‘because’ is the ground or the reason why the fact that corresponds to what is at the left of the because holds. Metaphysical grounding has been said to accommodate an intuitive idea of metaphysical apriority, which is spelled out in terms of fundamentality. For example, it seems intuitive to say that the structure of the glass is more fundamental than its fragility, and indeed there is a grounding relation that runs from the former to the latter. Another way of conveying the same issue is obtained by saying that grounding is essentially an asymmetric relation, as any explanatory relation is, and that this asymmetry is provided by a notion of fundamentality: more fundamental facts ground less fundamental ones, see Correia (2021a, 2021b, 2021c).

Metaphysical grounding is like causation naturally linked to a notion of explanation, in particular with a notion of metaphysical explanation: whether metaphysical grounding is the same as metaphysical explanation (*unionist* view), or it backs such a relation (*separatist* view) is a subject of great debate, e.g., see Maurin (2019).

Let us come to the links between metaphysical grounding and logic. In the literature it is taken as a given that the metaphysical grounding relation is formalized either as a sentential operator⁶ or as a predicate,⁷ and that certain axioms (or rules) describe the relations between such operator, or predicate, and logical connectives. Although there exist several papers dedicated to this formalization,⁸ there is a common consensus around the principles regulating these links (we will see them in detail in

⁶ E.g., see Correia (2017, 2014), Fine (2012a, 2012b), Schnieder (2011).

⁷ See Korbmacher (2018a, 2018b).

⁸ E.g., see Batchelor (2010), Litland (2018, 2016).

the next sections). Other two features receive general agreement: the first is that grounding principles respect a logical complexity increase from the grounds to the conclusion.⁹ In other word, logical complexity seems to play for formulas the same role that fundamentality plays for facts.¹⁰ The second feature that is common to the several logical grounding principles so far proposed is the following: since metaphysicians assume grounding to be a primitive notion, these principles are motivated by intuitions¹¹ and they are nowhere further justified.

The most important problems that metaphysicians discuss concerning the logic of grounding are about the formal features that this relation satisfies. In particular, each of the following properties has been giving rise to great debates¹² (addressing the issue of whether the grounding relation enjoys it or not):¹³

- Irreflexivity: if A grounds B , then A and B are distinct.
- Asymmetry: if A grounds B , then B does not ground A .
- Transitivity: if A grounds B and B grounds C , then A grounds C .
- Non-monotonicity: from A grounds B , it does not follow that for any C , A and C ground B .
- Factivity: from A grounds B , it does follow that A and B exist/are true.
- Hyperintentionality: if A grounds B and B is necessarily equivalent to C , it does not follow that A grounds C ; if A grounds B and A is necessarily equivalent to C , it does not follow that C grounds B .
- Necessitation: from A grounds B , it follows that it is necessary that if A exists/is true, then B exists/is true.

⁹ E.g., see Correia (2017), Kramer (2021).

¹⁰ Despite their strict connection, we think important to underline that the issue of the links between fundamentality and complexity is, as far as we know, not treated in the literature.

¹¹ See Mc Sweeney (2020).

¹² E.g., see De Rosset (2015), Jenkins (2011), Raven (2013) and Schnieder (2011).

¹³ We convey the following properties by focusing on the grounding relation with a unique ground, as we believe that this is the most common way used by metaphysicians. However, they can be easily generalized to grounding relations where the grounds form a set of formulas.

The two most widespread logics of metaphysical grounding so far introduced are Fine (2012b) and Correia (2017). Whilst Fine assumes the relation of grounding he aims at formalizing to be transitive, asymmetric, non-monotonic and factive,¹⁴ he remains open to the possibility that it might be reflexive – this is *weak* grounding - or irreflexive – this is *strict* grounding. As for the issues of hyperintentionality and necessitation, these properties are not treated by Fine’s logic. As for Correia’s approach, it follows the path drawn by Fine with two notable exceptions: the formalized grounding relation is non-factive and the issue of hyperintentionality is adequately treated thank to the introduction of an operator for propositional equivalence.

Finally, it is worth noticing that in the metaphysical literature there is a widespread distinction between a worldly and a representation conception of grounding. We use Fine’s passage to describe it:

A statement represents the world as being a certain way. We may therefore distinguish between the way it represents the world as being and how it represents the world as being that way. The worldly content of the statement is just a matter of the way it represents the world, while the conceptual content is also a matter of how it represents that content. The worldly conception of ground is one that is blind to anything other than factual content [i.e., the factual content of the statements which flank the linguistic item that expresses grounding], while the conceptual conception of ground is one that also takes into account the representation of the factual content. Fine (2010).

The distinction worldly versus representational becomes particularly important in cases involving logically equivalent formulas. Assuming the grounding operator or predicate to be irreflexive, whilst on a representational conception any formula *A*

¹⁴ Although Fine (2012b) does not explicitly address the issue of factivity, Correia (2017) reformulates Fine’s calculus by taking into account this feature.

grounds the formulas $A \wedge A$, $A \vee A$, and $\neg\neg A$, A cannot possibly ground either of these since they have the same factual content as A . Both Fine (2012b)'s and Correia (2017)'s logics of grounding adopt a representational conception of grounding.

3. Conceptual Grounding

In present-days metaphysics, more than one scholar has mentioned the possibility of the grounding relation, not to be just of metaphysical character, but also of a conceptual nature, namely such that it links truths according to the concepts these truths contain.¹⁵ Typical examples of the so-called conceptual grounding are the following:

- That animal is a vixen because it is a female and it is a fox,
- John is a bachelor because he is a man and he is unmarried,
- The wall is red because it is scarlet.

Each of these statements seems to convey a grounding relation, namely a relation between truths such that one or more are the reasons why another truth is so; however, in these cases, differently from the examples of the previous section, the grounding relation holds in virtue of the concepts involved in the examples.

In the contemporary literature, the idea of conceptual grounding has been treated in relation to the identity criteria (e.g., see Carrara and de Florio 2020), or in relation to metaphysical grounding, namely the question has been raised on whether conceptual grounding is part of metaphysical grounding or rather it is a different notion, e. g. see Smithson (2020). Apart from a few exceptions (e.g., see Pincock 2015, Poggiolesi and Genco 2023, Sebestik 1992), scholars tend to neglect what we deem to be the most salient features of conceptual grounding, namely its tight relation to the notion of theory and proof, its relevance in a mathematical context, as well as its illustrious historical roots. Indeed, way before our contemporary notion

¹⁵ E.g. see Berker (2018), Betti (2010), Schnieder (2006), Smithson (2020).

of conceptual grounding, there exists a long tradition of philosophers starting from Aristotle and passing through Leibniz, Bolzano and Frege (see for further details Deftelsen (1988)) who shape their theory of science – and in particular mathematics – around the idea of grounding. A scientific conception based on the idea of grounding consists in the idea that not all truths of a science are on a par, but they are structured: some truths are so because of others. In other words, the truths of a science are well-ordered in a hierarchy of more or less complex truths and the grounding relation reveals this order in that simpler truths are the grounds or the reasons why more complex truths are such.

Consider, for example, mathematics and the two mathematical truths: (i) the sum of the angles of any triangle is 180° degrees; (ii) the sum of the angles of any quadrangle is 360° . According to the scientific conception exposed above, these two truths do not stand on a par, but the former grounds the latter. In other words, if we wonder why the sum of the angles of any quadrangle is 360° , the answer is provided by (ii), namely because the sum of any triangle is 180° degrees. Note that the links between (i) and (ii) obtain in virtue of the concepts that these truths contain and (i) grounds (ii) rather than vice-versa because (i) is less complex than (ii).

In this scientific perspective, proofs play a fundamental role. First of all, not all proofs are the same: some proofs only prove that the theorems are true; others, on the contrary, show the reasons why theorems are true. These latter are explanatory proofs, or (mathematical) explanations *tout court*, in that they reveal the grounding relation that order the mathematical truths of the theory where they are developed.¹⁶ In other words, from this perspective, a scientific explanation, which takes the form of a proof, amounts to uncovering the grounding relation amongst the truths of a science, namely their objective connexion.

Let us come to the relationship between conceptual grounding and logic. Because of the tight links between conceptual grounding and proofs and the pivotal role of the notion of complexity, the first steps in this direction have been moved by looking

¹⁶ E.g., see Betti (2010).

at normal derivations in natural deduction calculi:¹⁷ more than one scholar could not help at underlying that normal derivations in natural deduction calculi enjoy many of the properties that proofs backed by a grounding relation should satisfy.¹⁸ One for all consists in the fact that normal derivations enjoys a peculiar relation with logical complexity which strongly reminds the fact that in a conceptual grounding relation grounds are supposed to be less complex than their conclusion. However, the analysis has never gone beyond these loose analogies. A further step onward has been taken in the work of Poggiolesi (2018, 2020d). Poggiolesi introduces a novel metalinguistic relation, called *formal explanation* such that, whenever one has that a multiset¹⁹ of formulas M formally explains a formula A , in symbol $M \mid\sim A$, then M and A stands in a grounding relation. The relation of formal explanation is to be thought of as a refinement of the derivability relation: it is indeed a special kind of derivability, which also possesses an explanatory power.²⁰ In structural terms, it is a type of derivability which is irreflexive, asymmetric, non-monotonic, non-factive, hyperintentional and transitive.

Note that in Poggiolesi's account the relation of grounding, which backs the metalinguistic relation of formal explanation, is not taken as primitive but modelled by three necessary and sufficient conditions.²¹ The first two conditions are expressed in terms of logical consequence and establish that in a grounding relation, not only the conclusion logically follows from its grounds, but it is also the case that the negation of the conclusion follows from the negation of each of its grounds. These two conditions are meant to ensure that grounding is a dependence relation. The third condition is meant to provide the directionality or asymmetry of the

¹⁷ A normal derivation in natural deduction calculus is an analytic derivation, namely a derivation where any formula occurring in it is less complex than the formula which ends the derivation. Normal derivations are central in proof theory and there exists an immense literature underlying their value and interest, e.g. see Gentzen (1969).

¹⁸ E.g. see Casari (1987), or Rumberg (2013), Stovall (2020).

¹⁹ A multiset is a set where the number of occurrences of the same element counts.

²⁰ As recently shown by Genco (2021), it is possible to construct a natural deduction calculus for classical logic where one has both standard derivability plus the new metalinguistic relation of formal explanations. Thus, in this calculus, not only can one prove that certain formulas are valid, but moreover one can display the explanatory steps of the proof. Moreover one can show that this calculus normalizes.

²¹ We will not describe in detail the three conditions. The interested reader is referred to Poggiolesi (2016, 2020b, 2020c) and Poggiolesi and Francez (2021).

grounding relation. It comes in the form of a measure of complexity adequate for a grounding framework - indeed called *g-complexity* – which can be easily defined in the following way:

Definition of g-complexity. The *g-complexity* of a formula of the classical propositional language A , $gcm(A)$, is defined in the following way:

- $gcm(p) = gcm(\neg p) = 0$,
- $gcm(A \vee B) = gcm(A \wedge B) = gcm(\neg(A \vee B)) = gcm(\neg(A \wedge B)) = gcm(A) + gcm(B) + 1$,
- $gcm(\neg\neg A) = gcm(A) + 1$.

The main insight behind the notion of *g-complexity* is that it is a notion for the grounding framework, and grounding - conceptual grounding in particular - is concerned entirely with truths. Accordingly, *g-complexity* tracks relationships among the truths expressed by the formulas if they were true. Consider first conjunction and disjunction: in these cases, *g-complexity* coincides with the standard notion of logical complexity. If, for example, A and B express truths, then the truth expressed by $A \wedge B$ is obtained from the previous truths using a single operation. However, this is not so for the case of negation. Since (at most) one of p and $\neg p$ will express a truth, then only one of the formulas will ever be an object of the grounding framework. Thus, there seems to be no reason to count $\neg p$ as more complex than p : in other terms, $\neg p$ can no longer be seen as constructed from p , since if the former is true, the latter is false. We should rather look at them as two formulas on the same level and this is precisely what the *g-complexity* does. Analogous reasoning can be applied to the *g-complexity* of more complex formulas like $\neg(A \wedge B)$. We can no longer count the complexity of $\neg(A \wedge B)$ as the complexity of $A \wedge B$ plus one, as standard logical complexity does, since if $\neg(A \wedge B)$ is true, then $A \wedge B$ is false and thus it cannot be constructed from it. We should rather think of $A \wedge B$ and $\neg(A \wedge B)$ as the two faces of the same medal, two formulas at the same level and thus having the same *g-complexity*.

Let us finally move to the case of double negation. In this case, the negation counts since $\text{gcm}(\neg\neg A) = \text{gcm}(A) + 1$. This is in harmony with what has been said up to now as $\neg\neg A$ and A may both express a truth, and thus the former can be seen as constructed from the latter.

Once the notion of g-complexity introduced, it plays a cornerstone role in the model of conceptual grounding proposed by Poggiolesi: indeed, the grounds should always be less g-complex than their conclusion. In other words, and as we have already pointed out, it is the notion of g-complexity that provides the directionality or asymmetry of the conceptual grounding relation.

Finally note that the distinction between worldly and representational grounding is not present in Poggiolesi's account: since it is based on concepts, the approach is quite naturally spelled out in what metaphysicians would call a representational approach.

4. Systematic comparison between two logical approaches

We have introduced two approaches to the notion of grounding, one metaphysical and one conceptual, and we have attempted to clarify their peculiarities, underlying that each of them has given rise to a different logical framework. In this section, we will compare these approaches along the following three perspectives. The distinction between full and complete grounding, the former typical of the metaphysical approach, whilst the latter of the conceptual approach (Section 4.1). The two different ways each approach deals with the negation connective (Section 4.2), as well as the two different ways each approach deals with the existence of several different logics (Section 4.3).

4.1 Complete-partial versus full-partial

As Fine (2012b) has put it, one can say that the notion of grounding comes in different flavours, namely there are different ways of conceiving the notion independently from the context it is considered in. Here we focus on a flavour which is central for the metaphysical approach, namely the distinction between *full* and *partial* ground. According to Fine, A is a partial ground of C if A on its own or together with some other truths is a ground of C . The notion of full ground is never explicitly defined but it is suggested that A is a full ground of C if A is sufficient to guarantee that C holds. In the conceptual framework, an analogous, yet not identical, distinction arises, and it is the one between *complete*²² and *partial* grounding. The multiset of all, and only, those truths each of which contributes to ground the truth C is a complete ground of C . On the other hand, each proper multiset of the complete ground of C is a partial ground of C .

Different grounding principles for the logical connectives emerge depending on whether we deal with the notion of full or complete grounding. Hence, in order to appreciate the consequences that each of these notions involve, we consider the paradigmatic cases of conjunction and disjunction. As far as conjunction is concerned, the grounding principles linked to the notions of full and complete grounds coincide since A and B together are the full, but also the complete, grounds of $A \wedge B$. In the case of disjunction, on the contrary, the two notions diverge. A is sufficient for $A \vee B$ to hold, the same goes for B , hence A and B , separately, are the full ground of $A \vee B$. Because of the *amalgamation principle*,²³ which is a principle that naturally follows from the structural features attached to the formalization of grounding both in Fine's and Correia's logics, also A and B together are the full ground of $A \vee B$, even if A and B together clearly represent the complete, rather than the full, ground of $A \vee B$. Hence, not only does the notion of full ground straightforwardly involve the notion of complete grounding, but also, because of the coexistence of A , or B , or A and B , as three available options for the full ground of

²² Note that Shaffer (2016) uses the word “total” for what we call “complete.”

²³ The amalgamation principle establishes that there always be a maximum ground for a grounded truth, namely if M fully grounds A , then there exists a M^* such that (i) M^* fully grounds A and (ii) $N \subseteq M^*$, whenever N fully grounds A .

$A \vee B$, it gives rise to the well-known problem of the overdetermination of the disjunction, a feature that many have criticized.²⁴

As for the complete grounds of $A \vee B$ things are in this case quite different. If both A and B are true, then together they are the complete grounds of $A \vee B$. However, this is not the only possible scenario. Consider indeed the case where A is true. A is then a ground for $A \vee B$. But is A the complete ground for $A \vee B$? The answer would seem to depend on the truth value of B . If B is true, then A and B together, as we have seen above, are the complete grounds of $A \vee B$. And if B is false? In this case, A would seem to constitute the complete ground of $A \vee B$: but this is only because B is false. Indeed, as just noted, if B were true, A would no longer constitute the complete ground of $A \vee B$ (it would merely be a partial ground). Hence, in the case where B is false, it still has a role to play in determining the grounds of $A \vee B$: its falsity ensures that, or is a condition for A to be the complete ground for $A \vee B$. To capture this role, we shall say that A is the complete ground for $A \vee B$ under the *robust condition* that the negation of B is true.²⁵ Thus, according to this analysis of disjunctive truths, in order to give the complete grounds of $A \vee B$, a distinction between grounds and robust conditions is required: indeed, either both disjuncts are the complete grounds of the disjunctive truth, or if only one of them is, this can happen under the condition that the negation of the other is true.²⁶ We thus have grounding relations for disjunctive formulas which fit with the notion of complete grounding and do not create the overdetermination problem.

As it has been shown by Poggiolesi (2020c), there exists a translation function according to which the notion of full ground can be defined in terms of the notion of complete ground. Because of the presence of robust conditions in a conceptual and complete perspective, one can easily conjecture that it is not possible to find a

²⁴ E.g., see Mc Sweeney (2020).

²⁵ The precise formulation is the following. A is the complete ground for $A \vee B$ under the robust condition that the converse of B is true. However, the notion of “converse of a formula” will only be introduced in Section 4.2. Hence in this section, we limit ourselves to the more imprecise, although more intuitive, formulation in terms of negation.

²⁶ Here again the correct formulation would be: “under the condition that the converse of the other is true.”

translation from the notion of complete grounding back to the notion of full ground. So that the former is strictly stronger than the latter.

Let us now move the duo full-complete to the first-order level. At this level, the metaphysical approach is notoriously affected by some paradoxes which seems to stem from the notion of full ground, which is linked to it.²⁷ We briefly illustrate an example of this kind of paradoxes and then show why they cannot arise in the conceptual framework where the notion of complete grounding dominates.

Consider the fact that snow is white is true. This fact is fully grounded by snow's being white. It is the fact that snow is white which explains why snow is white is true. Broadly speaking, we can claim that for any fact A , A fully grounds $[A]$ is true (let us call this principle (TG)²⁸). On the other hand, existential generalizations are fully grounded in their instances, given that the notion of full ground only requires being sufficient for a fact to hold. Hence, we can claim that for any fact A , A fully grounds $\exists x A$ (let us call this principle (EG)). Suppose to have a ball of white snow, then this ball's being white grounds the fact that something is white. But here is where the paradox lies. For consider the fact that something is true. Call this fact S . Since truth is grounded in being - (TG), S fully grounds $[S]$ is true. But $[S]$ is true is an instance of S . So, since existential generalizations are grounded in their instances – (EG) –, $[S]$ is true grounds S . Hence, S grounds $[S]$ is true and $[S]$ is true grounds S and we thus violate the asymmetry of ground and, assuming grounding is transitive, we violate the irreflexivity of grounding as well.

Several solutions have been proposed for this kind of paradox, e.g., Lovett (2019). To discuss them in detail would lead us too far away from the main topic of the paper. We rather observe that in the conceptual framework, where the main notion is that of complete grounding, the paradox does not emerge since an existential

²⁷ E.g., see Fine (2010), Krämer, S. (2013).

²⁸ Principle (TG) stands as a first remarkable connection between the notions of grounding and truth. This connection, which is occasionally mentioned in the literature, e.g., Korbmacher (2018a, b), would deserve further attention. For the more general connection between *Truth and logic*, the reader is referred to the related chapter of this Handbook by S. Moruzzi and N. Pedersen.

quantifier is not completely grounded by at least one of its instances.²⁹ But blocking the principle (EG) seems to block the paradox itself.

Note however that in the metaphysical setting, the paradox can also be reformulated in terms of partial ground, without any of the principles mentioned above to be changed: (EG), as well as (TG), hold for the notion of full as well as the notion of partial ground. So, what happens in the conceptual setting when complete grounding is substituted by partial grounding in principles (EG) and (TG)? Does the paradox arise then? Contrary to the duo full-partial, where the two notions may overlap, the notions of complete and partial grounding are disjoint, i.e., they never coincide. This is so because a ground is partial in the conceptual framework only when it is a *proper* (non-empty) subset of a multiset of complete grounds. Hence, given that in (TG) the complete ground amounts to an unique formula, the principle cannot be reformulated in terms of partial ground.³⁰ As a result, the paradox is blocked again given that we no longer have one of the two principles that generate it.

4.2. The key role of negation.

A set of formulas which is interesting to analyse from a grounding perspective are negative formulas, namely formulas of the form $\neg A$. Depending on the form of A , depending on whether A is itself a negated formula, or rather a conjunction or a disjunction, different grounds emerge. The metaphysical and conceptual perspectives diverge on how to account for these grounds. In order to explain their difference, we need to first introduce the notion of *converse of a formula*,³¹ which is a pivotal notion for the conceptual perspective.

²⁹ As shown in Genco et al. (2021), the complete grounding principle for the existential quantifier does not involve the use of instances at all, but rather exploits the use of the symbol *epsilon*, introduced by Hilbert, and which indicates an indeterminate object.

³⁰ This case is analogous to that of A and $\neg\neg A$: the former is only the complete ground of the latter and not the partial ground.

³¹ Note that we have already mentioned the notion of “converse of a formula” in the previous section.

Definition. The converse of a formula A , written A^* , is defined as follows:

$$A^* = \neg^{(n-1)} E, \text{ if } A = \neg^n E, \text{ and } n \text{ is odd, or}$$

$$\neg^{(n+1)} E, \text{ if } A = \neg^n E, \text{ and } n \text{ is even.}$$

where the main connective in E is not a negation, $0 \leq n$ and 0 is taken to be an even number.

Given a formula A , its converse A^* is such that $A \wedge A^*$ is a contradiction and they have the same g-complexity (see Section 3). To give an example of the notion of converse, suppose A is of the form $\neg p$, where p is an atomic formula, then the converse of A is p . On the other hand, if A is of the form p , its converse is of the form $\neg p$.

Equipped with the notion of converse of a formula, we can compare the metaphysical and the conceptual perspectives on negative formulas. As for formulas of the form $\neg\neg A$, the two perspectives coincide in that they both take the full or complete ground of $\neg\neg A$ to be A -- let us call this principle ($\neg\neg$ G). Let us then compare them on formulas of the form $\neg(A \vee B)$.³² Whilst for the metaphysical conception, the full grounds of formulas of the form $\neg(A \vee B)$ correspond to the pair $\neg A$ and $\neg B$ -- let us call this principle ($\neg\vee$ GM) --, for the conceptual conception, the complete grounds of the formulas of the form $\neg(A \vee B)$ correspond to the pair A^* and B^* -- let us call this principle ($\neg\vee$ GC). The difference between the two become evident considering the following two examples. Suppose that A and B are the two atomic formulas p and q . Then the metaphysical and conceptual approaches coincide in considering the (full or complete) grounds of $\neg(p \vee q)$ the pair $\neg p$ and $\neg q$. In this case, they coincide because the converse of an atomic

³² A case analogous to the negation of disjunction is the negation of conjunction. However, in this latter case, and contrary to the former, considerations concerning the difference between full and complete ground naturally emerge. For this reason, to keep the distinction full/complete and the treatment of negation as two separated issues, we focus on the negation of disjunction.

formula is the same as the negation of an atomic formula. However, in the case where A and B are, say, each the negation of an atom, namely $\neg p$ and $\neg q$, then in those cases, whilst the full grounds of $\neg (p \vee q)$ are the formulas $\neg\neg p$ and $\neg\neg q$, the complete grounds of $\neg (p \vee q)$ are the formulas p and q .³³ Hence, the two approaches clearly diverge.

First, we discuss their difference at the intuitive level and then at the formal level. Consider the sentence “it is not the case that Paul is not tall and not thin.” Suppose one is asked what kind of grounds explain this sentence. The simplest answer – it seems to us – would be “Paul is tall” and “Paul is thin.” Indeed, if Paul is both tall and thin, then these are the reasons, complete (and immediate³⁴), why “it is not the case that Paul is not tall and not thin.” The answer “it is not the case that Paul is not tall” and “it is not the case that Paul is not thin,” although logically equivalent to the former (at least in classical logic), would sound rather complicated, and unnecessary tortuous. However, they represent the answer provided by the metaphysical conception. Therefore, the conceptual perspective seems preferable.

Let us now turn to the technical side. Indeed, as Wilhelm (2021) has recently showed, if we add to the metaphysical grounding principles (for negated formulas), two apparently innocent identities, namely De Morgan’s laws:

(DM1) $A \vee B$ is identical to $\neg(\neg A \wedge \neg B)$

(DM2) $A \wedge B$ is identical to $\neg(\neg A \vee \neg B)$

and we assume that the grounds of two identical formulas are themselves identical, then we can easily derive several contradictions.

Let us illustrate informally how to generate one of the inconsistencies identified by Wilhelm. It follows from ($\neg\vee$ GM) that $\neg\neg p$ is a (partial) ground of $\neg(\neg p \vee \neg p)$

³³ Indeed, whilst p and q are less g-complex than $\neg(p \vee q)$, $\neg\neg p$ and $\neg\neg q$ are not.

³⁴ Another notable distinction concerning grounding is that between *immediate* and *mediate* grounding. A grounding relation is immediate when it does not seem to be any further reducible; mediate grounding is taken to be the transitive closure of immediate grounding.

and from $(\wedge G)^{35}$ that p is a (partial) ground of $p \wedge p$. However, since, by (DM2), $p \wedge p$ is identical to $\neg(\neg p \vee \neg p)$, it follows that p is identical to $\neg\neg p$. However, $(\neg\neg G)$ states that p is a ground of $\neg\neg p$. By substitution, we get that p is a ground of p , which is inconsistent with the assumption that grounding is irreflexive (namely with one of the structural features that metaphysical grounding enjoys, see Section 2).

One way out from these inconsistencies is to reject (DM1) and (DM2). This is precisely one of the routes taken by Litland (2022) who argues against these principles. Despite his motivations, it remains artificial to reject in the context of metaphysical grounding two identities which are implied by its own grounding principles, for one can easily show that $A \wedge B$ and $\neg(\neg A \vee \neg B)$ eventually have the same metaphysical grounds and thus they should at least be compatible with an idea of identity in a grounding framework.

The inconsistency rather seems to highlight an equivocation about which 'grounding level' $\neg\neg p$ is on, in comparison with p . And any route out of it will have to clearly take a position on this question: is $\neg\neg p$ on a level higher than p , as implied by $(\neg\neg G)$, or on the same level, as implied by De Morgan identities and the grounding principles for conjunction and negation of disjunctions? Whilst Litland (2022) also propose to reject $(\neg\neg G)$ which is a viable solution, the account of grounding that we found in the conceptual framework naturally offers a solution in an opposite direction, namely it keeps the $(\neg\neg G)$ principle and changes, as we have seen before, the principle concerning the negation of disjunction with $(\neg\vee GC)^{36}$. As shown by Poggiolesi (2023), thanks to $(\neg\vee GC)$, no inconsistency arises. Note that in the conceptual framework the grounding principles for negative truths are prior to the discovery of the inconsistencies and independently motivated by the rigorous measure of g-complexity (see Section 3).

³⁵ Let us call $(\wedge G)$ the grounding principle stating that A and B are the (full or complete) grounds of $A \wedge B$.

³⁶ The same holds for the principles concerning the negation of conjunction.

4.3 Intuition versus modelling: how to apply grounding to other logics?

We use this section to compare the metaphysical and the conceptual conception of grounding on one last perspective. When scholars do research in the logic of grounding, whatever notion of grounding they might embrace, they seem to be guided by one central question, which is: what are the reasons why a certain (logical) connective is true? For example, why is a conjunction true? Why is a disjunction true? However, a more careful reflection leads one to see that these questions, as they stand, are not properly addressed. Indeed, in the logic realm, connectives like conjunction and disjunction are not some absolute and independent object; rather they are elements of formal languages on which logics are based. Moreover, each logic describes its own connectives in a different way. As a result, the meaning of logical connectives change from one logic to another: for example, a conjunction in classical logic is not the same as a conjunction in linear logic. Therefore, the questions that motivate research in the logic of grounding should rather be spelled out in the following more accurate terms: what are the reasons why a certain connective of a certain logic L is true? Why is a L -conjunction true? Why is a L -disjunction true? Now, it seems to us that most logics of grounding so far developed investigate the grounds of classical connectives: this is explicitly stated in papers like Korbmacher (2018), or Genco (2021), whilst it can (probably) be assumed in papers like Correia (2017, 2014) or Fine (2012a).³⁷

Hence, so far only classical logic has been treated within a grounding analysis. However, it could be interesting to also treat other logics. And by this, not only do we mean to extend the grounding analysis to some extensions of classical logic, like modal logic, but also to explore substructural logics. We indeed firmly believe that going to weaker logics could be a revealing exercise. To see this important point, it is enough to reflect on the paradigmatic case of the implication connective. The

³⁷ Note that more recent work on the grounding system introduced by Fine (2012a) can be found in de Rosset and Fine (2022).

formal grounds of classical implication, i.e., material implication,³⁸ are analogous to the grounds of a disjunction, namely given a material implication of the form $A \rightarrow B$, then its full grounds will amount the pair $\neg A$ or B , or just to $\neg A$, or to just to B . However, these formal grounds do not correspond at all at any intuition one might have with ordinary-language conditionals. Let us for example consider the following two sentences:

- if Anne pushes the ball, then it rolls,
- if it is sunny, then John will take the car and go to the sea.

When asked why the first conditional is true, nobody – we guess – would ever answer that this is because Anne did not throw the ball, or because the ball rolls, as the formal analysis of material implication would suggest us to do. One would rather say that the conditional is true because the ball is a sphere: it is because the ball is a sphere that if it is pushed, then it rolls. As for the second conditional, the answer is analogous; nobody would ever say that the conditional is true because it is not sunny, or because John takes the car and goes to the sea. The reasons of the truth of this conditional seem to be that if it is sunny, John will go to the sea and that if it is sunny, John will take the car: it is because if it is sunny, John does both the above-mentioned actions that then it is true that if it is sunny, then John takes the car and goes to the sea.³⁹

These examples suggest not only that the grounding analysis of material implication has little, if not anything, to do with the grounds of ordinary-language conditionals, but also that if we want to properly formalize these intuitions, we need to do that in a logic different from classical logic, a logic where the implication connective formalizes conditionals that have a connection between antecedent and consequent. So, for example relevance logic. In other words, if classical logic is a good and natural start for any new logical analysis, then in the specific case of the grounding

³⁸ On the links between natural language conditionals and material implication, see Egré and Rott (2021).

³⁹ See also Poggiolesi (2022).

framework, logics different and weaker than classical logics might provide results that better fit with our insights.

But what happens if we try to adapt the metaphysical as well as the conceptual conception of grounding to logics other than classical logic? As for the conceptual conception, there already are results in this direction,⁴⁰ therefore it is easier to make some remarks. In the conceptual approach, grounding is not a primitive notion, but rather a notion modeled via the concepts of logical consequence and g-complexity (see Section 3). If we want to adapt the conceptual approach to logics different from classical logic, both these ingredients need to be modified in accordance with the new setting. Whilst logical consequence is easily changeable, i.e., instead of taking logical consequence in classical logic, one takes logical consequence in the new logic under consideration, g-complexity is a more difficult notion to adapt; in other words, there is no meta-instruction that tells us which element one needs to modify to pile up formulas of a new logic in a hierarchy of g-complexity which is adequate for the notion of grounding in that logic. Hence, this is a (open) problem for the conceptual conception.

We now move to the metaphysical conception, where, as far as we know, there is no attempt to explore logics other than classical logics. Note that in the metaphysical conception, grounding is taken as a primitive notion and the grounding principles, as we have already pointed out, are chosen and formulated on a mere account of intuitions. Therefore, in the metaphysical approach, there is not recipe to move from one logic to another: one simply needs to re-start from scratch. This, of course, does not involve that one cannot adapt the metaphysical approach to logics different from classical logic, just that it will be a quite demanding task to do so.

5. Conclusion

In this paper we have broached the subject of the interactions between logic and grounding from a novel perspective, namely by starting from two different although

⁴⁰ See Poggiolesi (2020b, 2020d).

related conceptions of grounding: one metaphysical, the other conceptual. Each conception comes with certain flavours, like that between full-partial, or that between complete-partial, and each flavour leads to different logical principles. However, and at least as far as we can see, these flavours are not entangled in a particular conception, they are simply associated with it. We could very well think of the distinction between full-partial in a conceptual framework, and the distinction complete-partial in a metaphysical one. As a result, there is no essential feature of a conception that pushes us to choose certain grounding principles over others. After a careful reflection and comparison with notions similar to that of grounding, this actually comes with no surprise. Consider for example the concept of necessity: there are several conceptions of necessity, e.g., metaphysical, physical, epistemic, nevertheless there is a unique framework that capture the formal features of them all, and it is the framework usually called *modal logic*. Now, in modal logic one can find different axioms and thus different systems; each of them is supposed to capture a different flavour of the notion of necessity, even if these flavours are not entangled with a particular conception rather than another. The strength and power of modal logic precisely lies in its generality and the capacity of unification.

In the light of these remarks, one could adopt a more optimistic look at the differences between divergent conceptions that have been outlined in this chapter. At least some of these differences could indeed be seen as begging for a wider grounding framework that could encompass them all: a framework that reveals its strength and power in its capacity of generality and unification. This indeed sounds as a promising line of future research.

References

Batchelor, R. (2010), 'Grounds and consequences.' *Grazer Philosophische Studien*, 80(1): 65–77.

Berker, S. (2018), 'The unity of grounding,' *Mind*, 127: 442-456.

Bliss, R. and Trogdon, K. (2014), 'Metaphysical grounding.' In Zalta, E., *Stanford Encyclopedia of Philosophy*, p. 1-46, Stanford.

Betti, A. (2010), 'Explanation in metaphysics and Bolzano's theory of ground and consequence.' *Logique et analyse*, 211:281–316.

Bolzano, B. (2015), *Theory of Science.* P. Rusnok and R. George (eds.), Oxford, Oxford University Press.

Carrara, M. and de Florio, C. (2020), 'Identity criteria: an epistemic path to conceptual grounding,' *Synthese*, 197:3151-3169.

Casari, E. (1987), 'Matematica e verità.' *Rivista di Filosofia*, 78:329-350.

Corriea, F. (2021a), 'A Kind Route from Grounding to Fundamentality.' *Synthese*, 199: 8299-8315.

Corriea, F. (2021b), 'Fundamentality from Grounding Trees.' *Synthese*, 199, 5965-5994.

Corriea, F. (2021c), 'The Logic of Relative Fundamentality.' *Synthese*, 198, 1279-1301.

Correia, F. (2017), ‘An Impure Logic of Representational Grounding.’ *Journal of Philosophical Logic*, 46: 507-538.

Correia, F. (2014), ‘Logical Grounds.’ *Review of Symbolic Logic*, 7: 31-59.

Correia, F. and Schnieder, B. (2012), ‘Grounding: An opinionated introduction’. In F. Correia, and B. Schnieder (eds.), *Metaphysical Grounding: Understanding the Structure of Reality*, pp. 6-32, Cambridge, Cambridge University Press.

Detlefsen, M. (1988), ‘Fregean hierarchies and mathematical explanation.’ *International Studies in the Philosophy of Science*, 3:97–116.

De Rosset, L. and Fine, K. (2022), “A semantics for the impure logics of grounding.” *Journal of Philosophical Logic*, 1-57, forthcoming.

De Rosset, L. (2015), ‘Better semantics for the pure logic of ground.’ *Analytic Philosophy*, 56: 229-252.

Egré, P. and Hans, R. (2021), "The Logic of Conditionals", *The Stanford Encyclopedia of Philosophy* (Winter 2021 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/win2021/entries/logic-conditionals/>>.

Fine, K. (2012a), ‘The Pure Logic of Ground.’ *Review of Symbolic Logic*, 5: 1-25.

Fine, K. (2012b), ‘Guide to ground.’ In F. Correia, F. and B. Schnieder (eds.), *Metaphysical Grounding: Understanding the Structure of Reality*, pp. 37–80, Cambridge, Cambridge University Press.

Fine, K. (2010), ‘Some Puzzles of Ground.’ *Notre Dame Journal of Formal Logic*, 51: 97-118.

Jenkins, C. (2011), ‘Is metaphysical dependence irreflexive?’ *The Monist*, 94: 267-276.

Genco, F. (2021), ‘Formal explanations as logical derivations.’ *Journal of Applied Non-Classical Logics*, 31: 279-342.

Genco, F., and Poggiolesi, F. and Rossi, L. (2021), ‘Grounding, quantifiers and paradoxes.’ *Journal of Philosophical Logic*, 50:1417-1448.

Gentzen, G. (1969), ‘Investigations concerning logical deduction.’ In M. Szabo, (ed.), *The Collected Papers of Gerhard Gentzen*, pp. 68–131, the Netherlands: North-Holland.

Krämer, S. (2021), ‘Ground-theoretic equivalence’. *Synthese*, 198: 1643-1683.

Krämer, S. (2013), ‘A simpler puzzle of ground.’ *Thought*, 2: 85-89.

Korbmacher J. (2018a), ‘Axiomatic theories of partial ground I. The base theory.’ *Journal of Philosophical Logic*, 47: 161-91.

Korbmacher J. (2018b), ‘Axiomatic theories of partial ground II. Partial ground and typed truth.’ *Journal of Philosophical Logic*, 47: 193-226.

Litland, J. (2022), ‘A note on the Wilhelmine inconsistency.’ *Analysis*, 81: 639-647.

Litland, J. (2018), 'Pure logic of iterated full ground.' *Review of Symbolic Logic*, forthcoming.

Litland, J. (2016), 'Pure logic of many grounds.' *Journal of Philosophical Logic*, 45(5): 531-577.

Lovett, A. (2019), 'The puzzles of ground.' *Philosophical Studies*, 45:61-78.

Maurin, A.-S. (2019), 'Grounding and metaphysical explanation: it's complicated.' *Philosophical Studies*, 176:1573-1594.

Mc Sweeney, M. (2020), 'Logic.' In Raven, M. (ed), *Routledge Handbook on Metaphysical Grounding*, pp. 420-432, London, Routledge.

Pincock, C. (2015), 'The unsolvability of the quintic: A case study in abstract mathematical explanation.' *Philosophers' Imprint*, 17:1–19.

Poggiolesi, F. and Genco, F. (2023), 'Conceptual (and hence mathematical) explanation, conceptual grounding, and proof.' *Erkenntnis*, 88: 1481-1507.

Poggiolesi, F. (2023), 'Grounding and propositional identity: a solution to Wilhelm's inconsistencies.' *Logic and Logical Philosophy*, 1-3, forthcoming.

Poggiolesi, F. (2022), 'Bolzano, the appropriate relevant logic and grounding rules for implication', in *Bolzano and Grounding*, S. Roski and B. Schnieder (eds.), Oxford University Press.

Poggiolesi, F. and Francez, N. (2021), 'Towards a generalization of the logic of grounding.' *Theoria*, 36: 5-24.

Poggiolesi, F. (2020a), 'Logics.' In Raven, M. (ed), *Routledge Handbook on Metaphysical Grounding*, pp. 213-227, London, Routledge.

Poggiolesi, F. (2020b), 'Grounding principles for (relevant) implication.' *Synthese*, 198:1-26.

Poggiolesi, F. (2020c), 'A proof-theoretical framework for several types of grounding.' *Logique et Analyse*, 252: 387-414.

Poggiolesi, F. (2020d), 'Grounding rules for relevant implication.' *Journal of Applied Non-classical Logics*, 8:1-30.

Poggiolesi, F. (2018), 'On constructing a logic for the notion of complete and immediate formal grounding.' *Synthese*, 195: 1231-1254.

Poggiolesi, F. (2016a), 'On defining the notion of complete and immediate formal grounding.' *Synthese*, 193: 3147-3167.

Poggiolesi, F. (2016b), 'A critical overview of the most recent logics of grounding.' In *Objectivity, Realism and Proof*, F. Boccuni and A. Sereni (eds.), pp. 291-309, Boston Studies in the Philosophy and History of Science, Berlin, Springer.

Raven, M. (ed.) (2020), *The Routledge Handbook of Metaphysical Grounding*, Routledge.

Raven, M. (2013), 'Is ground a strict partial order?' *American Philosophical Quarterly*, 50(2): 193-201.

Rosen, G. (2010), 'Metaphysical Dependence: Grounding and Reduction.' In B. Hale and A. Hoffman, (eds.), *Modality: Metaphysics, Logic, and Epistemology*, pp.109-136. Oxford, Oxford University Press.

Roski, S. (2017), '*Bolzano's conception of grounding.*' Frankfurt am Main: Klostermann.

Rumberg, A. (2013), 'Bolzano's concept of grounding (Abfolge) against the background of normal proofs.' *Review of Symbolic Logic*, 6:424–459.

Schaffer, J. (2016), 'Grounding in the image of causation,' *Philosophical studies* 173: 49-100.

Schaffer, J. (2009), On what grounds what. In D. Chalmers and D. Manley (eds.), *Metametaphysics: new essays on the foundations of ontology*, pp. 347–383. Oxford, Oxford University Press.

Schnieder, B. (2011), 'A logic for 'because.' *Review of Symbolic Logic*, 4(3): 445-465.

Schnieder, B. (2006), 'A certain kind of trinity: Dependence, substance, and explanation.' *Philosophical Studies*, 129: 393-419.

Sebestik, J. (1992), '*Logique et mathématique chez Bernard Bolzano.*' Paris, Vrin.

Smithson, R. (2020), 'Metaphysical and conceptual grounding.' *Erkenntnis*, 86:1501-1525.

Stovall, P. (2021), 'Essence as modality: a proof-theoretic and nominalist analysis.' *Philosophers' Imprint*, 21:1-28.

Tatzel, A. (2002) 'Bolzano's theory of ground and consequence'. *Notre Dame Journal of Formal Logic*, 43(1):1-25.

Wilhelm, I. (2021), 'Grounding and propositional identity.' *Analysis*, 81:80-81.