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Un modèle moyenné en épaisseur pour le problème inverse de l'imagerie électrocardiographique

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A depth-averaged heart model for the inverse problem of cardiac electrophysiology

CANUM 2024

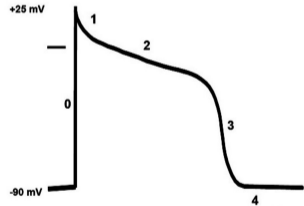
Ile de Ré, 29/05/2024

Yves Bourgault, Yves Coudière, Emma Lagracie, Lisl Weynans



- ▶ **Context**
- ▶ Forward averaged model
- ▶ Inverse problem
- ▶ Numerical results

- Cardiac rhythm disorders: pathologies of heart's electrical activity
- Electrical activation: gives the signal that precedes the contraction



- Action potential
- Activation maps
- Goal: detect pathologies from torso potentials, ECG imaging. Very ill posed inverse problem 🐱

Potential $u \in H^1(\Omega_T \cup \Omega_H)$, voltage $v \in H^1(\Omega_H)$.

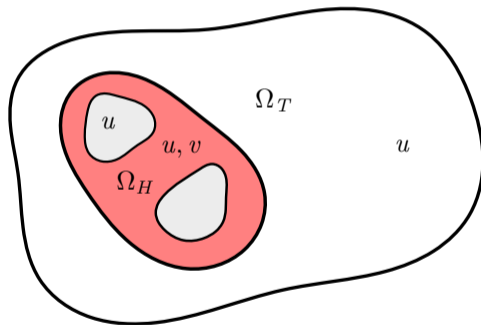
$$\operatorname{div}(\sigma_i \nabla(u + v)) = \partial_t v + I_{ion}(v, t, h) \quad \Omega_H$$

$$\operatorname{div}((\sigma_e + \sigma_i) \nabla u) = -\operatorname{div}(\sigma_i \nabla v) \quad \Omega_H$$

$$\operatorname{div}(\sigma_T \nabla u) = 0 \quad \Omega_T$$

$$\sigma_i \nabla(u + v) \cdot n = 0 \quad \partial\Omega_H$$

$$\sigma_T \nabla u \cdot n = 0 \quad \partial\Omega_T$$



Find u and/or v that satisfy (part of) the bidomain equations, matching torso data z_T .

Find the activation map.

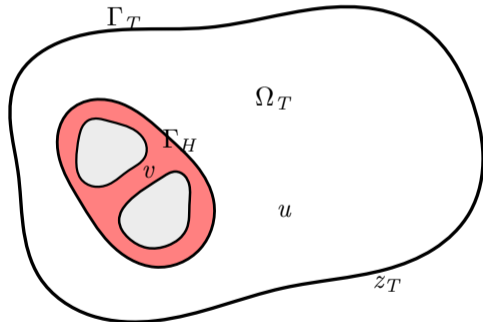
$$\operatorname{div}((\sigma_e + \sigma_i)\nabla u) = -\operatorname{div}(\sigma_i\nabla v) \quad \Omega_H$$

$$\operatorname{div}(\sigma_T\nabla u) = 0 \quad \Omega_T$$

$$\sigma_i\nabla(u + v) \cdot n = 0 \quad \partial\Omega_H$$

$$\sigma_T\nabla u \cdot n = 0 \quad \partial\Omega_T$$

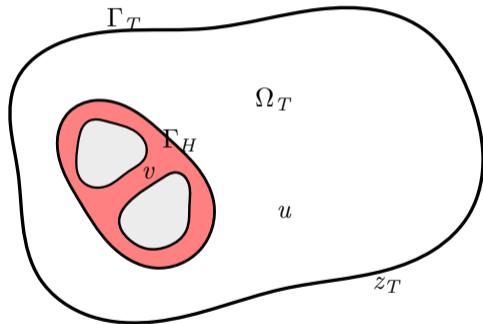
- In the heart surface: u : uniqueness (up to a constant) but non existence, v can be found up to a constant
- In the heart volume: infinite-dimensional kernel for v



Cauchy problem:

$$\begin{array}{ll} -\Delta u = 0 & \Omega_T \\ u = z_T & \Gamma_T \\ \partial_n u = 0 & \Gamma_T \end{array}$$

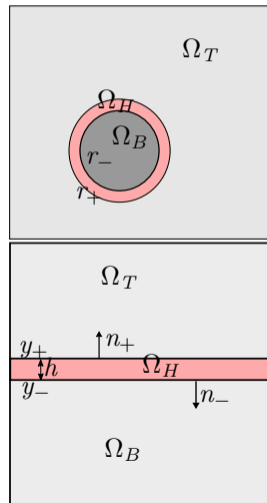
- No interactions with the heart
- Does not allow to reconstruct v
- No information on the heart volume



- ▶ Context
- ▶ **Forward averaged model**
- ▶ Inverse problem
- ▶ Numerical results

- 2D + smooth
- Ω_H separates blood cavity from torso
- Depth $h \ll L$, $h'(x) \ll 1$
- Γ_H smooth curve embedded in Ω_H
- Parallel fibers:

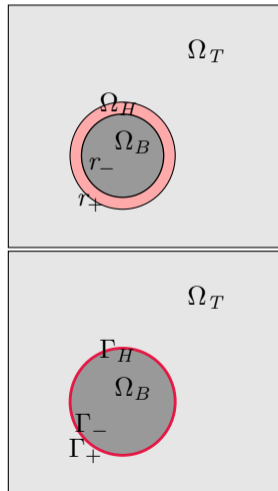
$$\sigma_{i/e} = \begin{pmatrix} \sigma_{i/e,l} & 0 \\ 0 & \sigma_{i/e,p} \end{pmatrix}$$



$$\begin{aligned}
 \sigma_\ell h \frac{\partial^2}{\partial s^2} \bar{u} + \sigma_T \frac{\partial u_T}{\partial n} \Big|_{\Gamma_H} + \sigma_B \frac{\partial u_B}{\partial n} \Big|_{\Gamma_H} + \sigma_{il} h \frac{\partial^2}{\partial s^2} \bar{v} &= 0 & \Gamma_H \\
 \operatorname{div}(\sigma_T \nabla u_T) &= 0 & \Omega_T \\
 \operatorname{div}(\sigma_B \nabla u_B) &= 0 & \Omega_B
 \end{aligned}$$

+BC:

$$\begin{aligned}
 \sigma_T \partial_n u &= 0 & \Gamma_T \\
 \sigma_{ep} \frac{u_T - \bar{u}}{h} &= (1 - \alpha) \sigma_T \partial_n u_T & \Gamma_H \\
 \sigma_{ep} \frac{u_B - \bar{u}}{h} &= \alpha \sigma_B \partial_n u_B & \Gamma_H
 \end{aligned}$$



Find (\bar{u}, u_T, u_B) in $V = H^1(\Gamma_H) \times H^1(\Omega_T)/\mathbb{R} \times H^1(\Omega_B)$ such that

$$\begin{aligned} & \sigma_\ell h \int_{\Gamma_H} \frac{\partial}{\partial s} \bar{u} \frac{\partial}{\partial s} \bar{\varphi} + \sigma_T \int_{\Omega_T} \nabla u_T \cdot \nabla \varphi_T + \sigma_B \int_{\Omega_B} \nabla u_B \cdot \nabla \varphi_B \\ & + \frac{\sigma_{e,p}}{(1-\alpha)h} \int_{\Gamma_H} (\bar{u} - u_T)(\bar{\varphi} - \varphi_T) + \frac{\sigma_{e,p}}{\alpha h} \int_{\Gamma_H} (\bar{u} - u_B)(\bar{\varphi} - \varphi_B) = \int_{\Gamma_H} F \bar{\varphi}, \quad \forall (\bar{\varphi}, \varphi_T, \varphi_B) \in V \end{aligned}$$

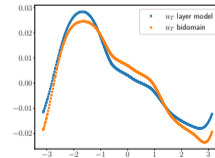
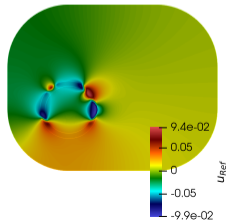
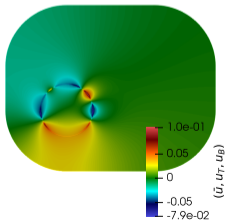
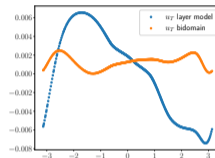
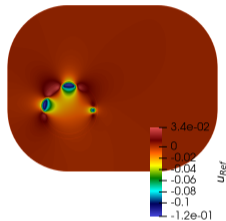
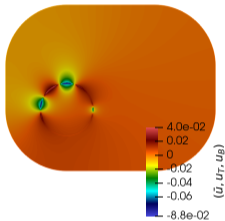
$$F = h\sigma_{i,\ell} \partial_{ss} \bar{v}$$

Well posed, all Lax-Milgram hypotheses are satisfied !

Numerical validation

Comparison with the bidomain

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- ▶ Context
- ▶ Forward averaged model
- ▶ **Inverse problem**
- ▶ Numerical results

$$\min_{\mathcal{E}} J(U[F], F) = \frac{1}{2} \int_{\Gamma_T} |u_T - z_T|^2 + \frac{\varepsilon_{\text{regul}}}{2} |F|_{H^1(\Gamma_H)}^2$$

Over space $\mathcal{E} = \{(U, F) \mid U = (u_T, u_B, \bar{u}) = AF + \mathbb{R}, F \in H^1(\Gamma_H)/\mathbb{R}\}$ with operator A :

$$\begin{aligned} & \sigma_\ell h \int_{\Gamma_H} \frac{\partial}{\partial s} \bar{u} \frac{\partial}{\partial s} \bar{\varphi} + \sigma_T \int_{\Omega_T} \nabla u_T \cdot \nabla \varphi_T + \sigma_B \int_{\Omega_T} \nabla u_B \cdot \nabla \varphi_B \\ & + \frac{\sigma_{e,p}}{(1-\alpha)h} \int_{\Gamma_H} (\bar{u} - u_T)(\bar{\varphi} - \varphi_T) + \frac{\sigma_{e,p}}{\alpha h} \int_{\Gamma_H} (\bar{u} - u_B)(\bar{\varphi} - \varphi_B) = \int_{\Gamma_H} F \bar{\varphi} \end{aligned}$$

Existence & uniqueness of the optimal control

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With regularization

Steps of the proof:

■ Existence:

1. **Minimizing sequence**
2. **Weakly convergent subsequences**
3. **Use of lower semi continuity**

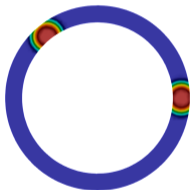
■ Uniqueness: Strict convexity

$$J(U, F) = \frac{1}{2} \int_{\Gamma_T} |u_T - z_T|^2 + \frac{\varepsilon_{\text{regul}}}{2} |F|_{H^1}^2$$

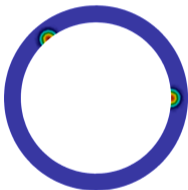
$$\begin{aligned}\mathcal{L}(\bar{u}, u_T, u_B, \bar{\lambda}, \lambda_T, \lambda_B, F) &= \frac{1}{2} \int_{\Gamma_T} |u_T - z_T|^2 + \frac{\varepsilon_{\text{regul}}}{2} \int_{\Gamma_H} |\partial_s F|^2 \\ &+ \sigma_\ell h \int_{\Gamma_H} \partial_s \bar{u} \partial_s \bar{\lambda} + \frac{\sigma_{e,p}}{(1-\alpha)h} \int_{\Gamma_H} (\bar{u} - u_T)(\bar{\lambda} - \lambda_T) \\ &+ \frac{\sigma_{e,p}}{\alpha h} \int_{\Gamma_H} (\bar{u} - u_B)(\bar{\lambda} - \lambda_B) \\ &+ \sigma_T \int_{\Omega_T} \nabla u_T \cdot \nabla \lambda_T + \sigma_B \int_{\Omega_B} \nabla u_B \cdot \nabla \lambda_B - \int_{\Gamma_H} F \bar{\lambda}\end{aligned}$$

1st order optimality conditions \rightarrow *linear* system to solve 😊

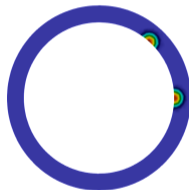
- ▶ Context
- ▶ Forward averaged model
- ▶ Inverse problem
- ▶ **Numerical results**



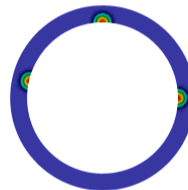
(a) Pacing protocol 1



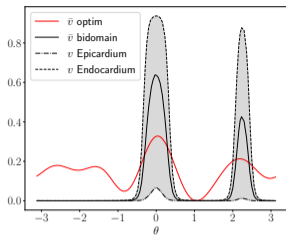
(b) Pacing protocol 2



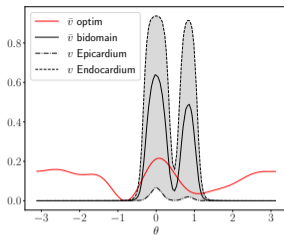
(c) Pacing protocol 3



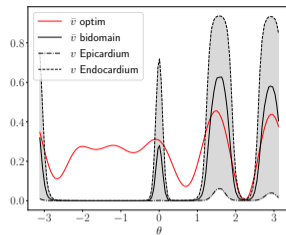
(d) Pacing protocol 4



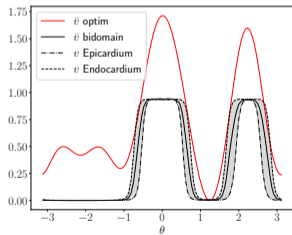
(a) Protocol 2. $t=6$ ms



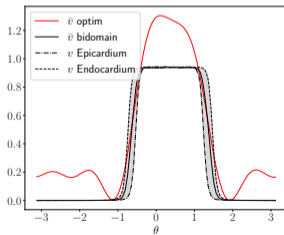
(b) Protocol 3. $t=6$ ms



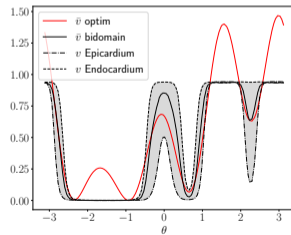
(c) Protocol 4. $t=6$ ms



(d) Protocol 2. $t=13$ ms



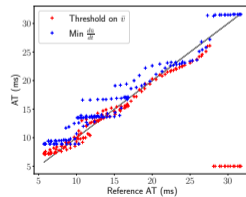
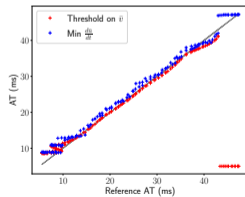
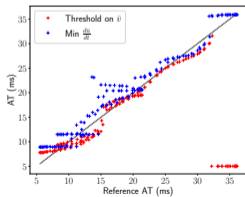
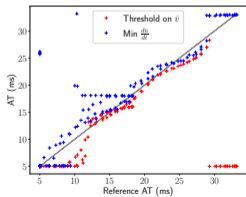
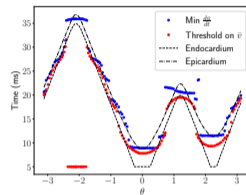
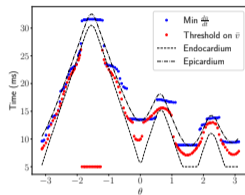
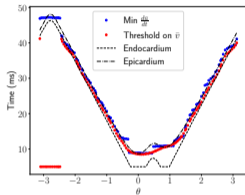
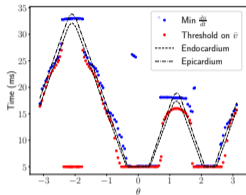
(e) Protocol 3. $t=13$ ms



(f) Protocol 4. $t=13$ ms

Activation maps

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(a) Protocol 1

(b) Protocol 2

(c) Protocol 3

(d) Protocol 4

- Accurate activation maps, bounded by endocardial & epicardial ATs
- We recover an average v , some information on the volume
- Comparison with the usual epicardial reconstruction
- Perspective of 3D + more realistic geometry

Thank you for your attention !

 Emma Lagracie, Yves Bourgault, Yves Coudière and Lisl Weynans, *A depth-averaged heart model for the inverse problem of cardiac electrophysiology*, Submitted

Operator $T: AF|_{\Gamma_T} = ([\text{Trace}] \circ A)F = TF$ continuous Banach to Banach

- **Minimizing sequence:** $m = \inf_{\mathcal{E}} J(U, F) \geq 0$, minimizing sequence $(U_n, F_n) \in \mathcal{E}_2$ s.t.
 $J(U_n, F_n) \xrightarrow{n \rightarrow \infty} m$.
- **Weakly convergent subsequences**

$$\frac{1}{2} \int_{\Gamma_T} |u_{n_T} - z_T|^2 + \frac{\varepsilon_{\text{regul}}}{2} |F_n|_{H^1}^2 \leq J(0, 0) = Cte. \quad (1)$$

Subsequences

- | | |
|---|---|
| <ul style="list-style-type: none"> — (F_n) bounded — $(u_{n_T} _{\Gamma_T}) = (TF_n + C_n)$ bounded + (TF_n) bounded (continuity) $\Rightarrow (C_n)$ bounded | <ul style="list-style-type: none"> — $(F_n) \xrightarrow{n \rightarrow \infty} \bar{F}$ — $(C_n) \xrightarrow{n \rightarrow \infty} \bar{C}$ — $(T \text{ weakly continuous}) TF_n \xrightarrow{n \rightarrow \infty} T\bar{F}$ |
|---|---|

- use of lower semi continuity $F \mapsto \|F\|^2$ weakly l.s.c.

$$\frac{\varepsilon_{\text{regul}}}{2} \|\bar{F}\|^2 \leq \liminf_{n \rightarrow \infty} \frac{\varepsilon_{\text{regul}}}{2} \|F_n\|^2,$$

$$\frac{1}{2} \int_{\Gamma_T} |T\bar{F} + \bar{C} - z_T|^2 \leq \liminf_{n \rightarrow \infty} \frac{1}{2} \int_{\Gamma_T} |u_{nT} - z_T|^2$$

Finally,

$$J(\bar{U}, \bar{F}) \leq \liminf_{n \rightarrow \infty} J(U_n, F_n) = m.$$

Uniqueness: J strictly convex.