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The role of FDI along transitional dynamics of the host country in an endogenous growth model

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Abstract

We investigate the role of foreign direct investment (FDI) in the transitional dynamics of host countries by using an optimal growth model. FDI may be beneficial for the host country because local people can work for multinational firms to get a favorable salary. However, if the host country only focuses on FDI, it may face a middle-income trap. We show that if the host country invests in research and development, its economy may have sustained growth. Moreover, in this case, FDI helps the host country only at the first stages of its development process.

Keywords: Optimal growth, FDI, R&D, fixed cost.

JEL Classifications: D15, F23, F4, O3, O4.

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1 Introduction

Over the past few decades, opening up to the global economy and attracting foreign direct investment (FDI) have been significant policy priorities in developing countries to promote their economic development. The main argument is that multinational enterprises (MNEs) would boost investment, bring new technologies and (management) skills, and generate FDI spillovers on domestic firms. However, the effects of FDI on the host country's development are far from clear.

Overall, empirical literature finds that the effect of FDI on the host country's economic growth is relatively weak (Carkovic and Levine, 2005; Gunby et al., 2017) and the link between FDI and growth varies over time (Bénétrix et al., 2023). More precisely, whether this effect is significant or not depends on local conditions such as the host country's income levels, institutional strength (Baiashvili and Gattini, 2020), the level of human capital (Li and Liu, 2005), or the development of local financial markets (Alfaro et al., 2004, 2010). For instance, the impact is positive in countries with high development level of human capital or financial markets (Borensztein et al., 1998; Alfaro et al., 2004). Besides, there is an inverted-U shaped connection between the extent of FDI impact and the host country's income-levels. The effect is larger for low- to middle-income countries, before declining from transition to high-income ones (Baiashvili and Gattini, 2020).¹

Despite substantial empirical research on the nexus FDI-growth,² there is still a lack of theoretical analysis. Hence, our paper aims to understand the role of FDI along the transitional dynamics of the host economy by investigating fundamental questions:

- (i) What is the optimal strategy of a country receiving FDI?
- (ii) How can FDI help the host country escape the middle-income trap and potentially achieve economic growth in the long run?

We address these questions by using an optimal growth model with FDI and endogenous growth. Let us briefly describe the ingredients of our model.

¹See, for example, Almfraji and Almsafir (2014) for a literature review on the FDI-growth's relationship.

²Over the five decades of research on FDI, the link FDI-GDP (economic growth) is the most often investigated. Indeed, 107 of 500 published articles considered in Paul and Feliciano-Cestero (2021) study the impacts of FDI on the host country's economic growth.

The host country is assumed to be a small open economy with three goods: consumption, physical capital, and new goods. These commodities are freely tradable with the rest of the world. There are two agents: a representative agent of the host country and an MNE. The representative agent has three choices for investment: (1) buy physical capital to produce consumption good, (2) invest in training to improve her/his skills and then work for MNE in order to get a salary, and (3) invest in research and development (R&D) to get innovations. If innovations are good enough, they will improve the productivity of domestic firms.

First, we show that if the host country has a low initial resource and a weak research process efficiency, it should never invest in R&D but focus on FDI. In this case, its economy converges to a steady state, which is higher than that of the economy without FDI.

Second, consider a low-income country so that the country cannot immediately invest in R&D and new technology (because the fixed cost in R&D is so high). In this situation, we prove that if the leverage of new technology is high enough or the country has good potential in R&D, the optimal strategy of the country should be as follows:

- Stage 1: the country should train specific workers.
- Stage 2: specific workers will work for the MNE to get a favorable salary and improve the country's income and capital accumulation.
- Stage 3: once the country's income is high enough, it should focus on R&D to create new technology that increases the domestic firms' total factor productivity (TFP). Thanks to this, its economy may have economic growth in the long run.

Our model also shows that a country may get economic growth in the long run without FDI. Our analyses suggest that FDI only plays a role as a catalyst for the host country's economic growth, especially in the first stages of the host country's development process.

This research has two significant contributions to the literature. First, we theoretically advance the understanding of how FDI affects economic growth. The existing literature provides some theoretical models to study the effect of FDI on growth. Looking back to history, [Findlay \(1978\)](#) examines the role of FDI in a dynamic framework by assuming that the sequences of domestic and foreign firms' capital stocks are determined by a continuous time dynamical

system.³ A key insight in [Findlay \(1978\)](#) is his assumption of the ‘contagion’ effect: the level of efficiency of domestic firms depends on (but is lower than) that of the advanced part of the world. [Wang \(1990\)](#) develops this idea by assuming that there is technology diffusion: the host country’s human capital stock is an increasing function of the ratio of foreign investment to domestically owned capital. By using this modeling of FDI and a two-country model with free capital mobility and exogenous propensities to save, [Wang \(1990\)](#) shows that opening to FDI has beneficial implications for the host country.

Notice, nevertheless, that in [Wang \(1990\)](#), the propensity to save is exogenous. Some other research considers models with endogenous saving rates. In a continuous time model with a continuum of varieties of capital goods,⁴ [Borensztein et al. \(1998\)](#) model FDI as the fraction of varieties produced by foreign firms in the total varieties of products. Under specific setups (Cobb-Douglas production and CRRA utility functions), they compute the rate of growth in the steady state equilibrium, which is an increasing function of the fraction of varieties produced by foreign firms in the total varieties of products. [Berthélemy and Démurger \(2000\)](#) extend [Borensztein et al. \(1998\)](#)’s model by endogenizing the numbers of varieties produced by domestic and foreign firms. As in [Borensztein et al. \(1998\)](#), [Berthélemy and Démurger \(2000\)](#) focus on the steady state equilibrium and compute the growth rate of the host country in the case of Cobb-Douglas production and CRRA utility functions. Using a continuous time product variety-based endogenous growth, [Alfaro et al. \(2010\)](#) study the role of local financial markets in enabling FDI to promote growth through backward linkages.⁵ They focus on the balanced growth path and their calibration shows that an increase in FDI leads to higher growth rates in financially developed countries compared to those observed in financially poorly developed ones.

Unlike the above studies, we focus neither on the steady state nor on the balanced growth path with specific functions. Instead, we explore the global and transitional dynamics of the optimal paths in endogenous growth models with FDI and without restrictions on the utility function.⁶ To be best of our

³This system’s parameters include domestic and foreign firms’ technological efficiency that are exogenous.

⁴For this kind of growth models, see [Romer \(1990\)](#), [Grossman and Helpman \(1991\)](#).

⁵In [Alfaro et al. \(2010\)](#), the development level of the local financial market is modeled by the difference between the instantaneous borrowing rate and the lending rate.

⁶[Nguyen-Huu and Pham \(2018, 2024\)](#) study the nexus between FDI spillovers and

knowledge, we are the first to do so. Consequently, our results seem to be more robust. Moreover, our analyses of transitional dynamics allow us to better understand the optimal strategy for the host countries (as we have explained), while the existing literature does not.

Second, our paper also contributes to the literature on optimal growth with thresholds (see [Azariadis and Drazen, 1990](#); [Bruno et al., 2009](#); [Le Van et al., 2010, 2016](#) among others) and increasing returns (see [Romer, 1986](#); [Jones and Manuelli, 1990](#); [Kamihigashi and Roy, 2007](#) among others). Our added value is to show the role of FDI. We point out that FDI may partially contribute to the capital accumulation of the host country and hence enable the country to overcome the threshold at the first stage of its development process. However, whether a host country can obtain growth in the long run does not depend on FDI but on the local conditions (mainly its innovation capacity and the efficiency of its investment in R&D). From a technical point of view, our analysis is far from trivial because of the presence of both domestic and foreign firms. For instance, the method used in [Bruno et al. \(2009\)](#), [Le Van et al. \(2010\)](#) cannot be directly applied in our model.

The paper is structured as follows. Section 2 introduces an endogenous growth model with FDI. Section 3 investigates the interplay between FDI, R&D, and economic growth of the host country. Section 4 concludes. Formal proofs are presented in the Appendix.

2 An endogenous growth model with FDI

Let us start with a benchmark model in which there is a small open economy with three kinds of goods: consumption, physical capital, and so-called new goods. The consumption good is taken as numéraire. The price (in terms of consumption good) of physical capital is exogenous and denoted by p .

In each period, there is an MNE in the host country. It produces the new good by using two inputs: physical capital and specific labor. We assume that there is no domestic firm in this sector.

At each date t , the foreign firm (without market power) chooses $K_{e,t}$ units of physical capital and $L_{e,t}^D$ units of specific labor in order to maximize its

industrial policy of a host country in a two-period and an exogenous growth models respectively. However, they do not consider endogenous growth.

profit:

$$(F_t) : \quad \pi_{e,t} = \max_{K_{e,t}, L_{e,t}^D \geq 0} \left[p_n F_t^e(K_{e,t}, L_{e,t}^D) - p K_{e,t} - w_t L_{e,t}^D \right] \quad (1)$$

where p_n is the exogenous price (in terms of consumption good) of the new good.

Assumption 1. *We assume that $F_t^e(K_{e,t}, L_{e,t}^D) = A_e K_{e,t}^{\alpha_e} (L_{e,t}^D)^{1-\alpha_e}$, where $\alpha_e \in (0, 1)$.*

There is a representative agent in the host country. Taking prices and wages as given, the agent chooses the allocation of resources to maximize the intertemporal welfare of the whole population.

The host country has three choices for investment at each date t . First, it can buy $K_{c,t+1}$ units of physical capital to produce $A_c K_{c,t+1}^\alpha$ units of the consumption good at period $t+1$, where $\alpha \in (0, 1)$.

Second, it can use H_{t+1} units of the consumption good for training to generate $A_h H_{t+1}^{\alpha_h}$ units of specific labor, where $\alpha_h \in (0, 1)$. The latter works for the MNE to get a total wage of $w_{t+1} A_h H_{t+1}^{\alpha_h}$ (units of the consumption good).

The last choice is to invest in R&D to create new technology: If the host country invests N_{t+1} units of the consumption good in R&D at period t , it will obtain bN_{t+1}^σ units of new technology in period $t+1$, where b represents the efficiency of the research process. We assume that $\sigma \in (0, 1)$. The new technologies can improve the old sector's productivity, but only if the amount of investment in R&D exceeds a critical threshold such that $bN_{t+1}^\sigma > \bar{x}$, where $\bar{x} > 0$ represents a fixed cost. In this case, the productivity goes up to $A_c + a(bN_{t+1}^\sigma - \bar{x})$ where the parameter a indicates the efficiency or the leverage of the new technology.⁷

To sum up, the representative agent solves the dynamic growth problem below:

$$(P) : \quad \max_{(c_t, K_{c,t}, H_t, N_t, L_{e,t})_{t=0}^{+\infty}} \left[\sum_{t=0}^{+\infty} \beta^t u(c_t) \right] \quad (2)$$

⁷To introduce R&D, we can also write, for example, $A_c + \gamma((N_{t+1} - N^*)^+)^{\sigma}$ instead of $A_c + a(bN_{t+1}^\sigma - \bar{x})^+$. However, the main results have similar insights.

subject to

$$0 \leq c_t, K_{c,t}, H_t, L_{e,t}, N_t \quad (3a)$$

$$c_t + pK_{c,t+1} + N_{t+1} + H_{t+1} \leq \left(A_c + a(bN_t^\sigma - \bar{x})^+ \right) K_{c,t}^\alpha + w_t L_{e,t} \quad (3b)$$

$$L_{e,t} \leq A_h H_t^{\alpha_h}. \quad (3c)$$

for every $t \geq 1$. Here, $\beta \in (0, 1)$ is a rate of time preference while u is the instantaneous utility function.

We require the following assumption.

Assumption 2. *The utility function u is in C^1 , strictly increasing, concave, and $u'(0) = \infty$. Assume that $A_c > 0, A_h > 0, \alpha \in (0, 1), \alpha_h \in (0, 1)$.*

We assume that $a\bar{x} > A_c$, implying the fixed cost \bar{x} is not too low.

At initial date, assume that $N_0 = 0$ while $K_{c,0}, L_{e,0} > 0$ are given.

We provide a formal definition of equilibrium.

Definition 1. *An intertemporal equilibrium is a list $(c_t, K_t, H_t, N_t, L_{e,t}, L_{e,t}^D, K_{e,t}^D, w_t)_{t=0}^\infty$ satisfying 3 conditions:*

- (i) *Given $(w_t)_{t=0}^\infty, (c_t, K_t, H_t, N_t, L_{e,t})_{t=0}^\infty$ is a solution of the problem (P),*
- (ii) *Given $w_t, (L_{e,t}^D, K_{e,t}^D)$ is a solution of the problem (F_t)*
- (iii) *Labor market clears: $L_{e,t}^D = L_{e,t}$.*

At equilibrium, we have $L_{e,t}^D = L_{e,t} > 0$. Hence, the first order conditions of the problem (F_t) imply that, for every t :

$$w_t = w := \left(\alpha_e^{\alpha_e} (1 - \alpha_e)^{1 - \alpha_e} \frac{p_n A_e}{p^{\alpha_e}} \right)^{\frac{1}{1 - \alpha_e}}. \quad (4)$$

Wage w depends not only on the foreign firm TFP but also on the prices of physical capital and new goods.

Denote $S_{t+1} = pK_{c,t+1} + N_{t+1} + H_{t+1}$ the total savings of the host country. In our framework, it is equal to the aggregate investment. By using Equation (4), the problem (P) can be rewritten as follows:

$$(P') : \quad \max_{(c_t, S_{t+1})_{t=0}^{+\infty}} \left[\sum_{t=0}^{+\infty} \beta^t u(c_t) \right] \text{ subject to: } c_t, S_t \geq 0, \quad c_t + S_{t+1} \leq G(S_t) \quad (5)$$

for any $t \geq 1$, and $c_0 + S_1 \leq X_0$, where $X_0 \equiv A_c K_{c,0}^\alpha + w_0 L_{e,0}$ and $G(S)$ is defined by

$$(G_S) : G(S) \equiv \max_{K_c, N, H} \left\{ g(K_c, N, H) : pK_c + N + H \leq S; K_c, N, H \geq 0 \right\} \quad (6a)$$

$$\text{where } g(K_c, N, H) \equiv \left(A_c + a(bN^\sigma - \bar{x})^+ \right) K_c^\alpha + wA_h H^{\alpha_h}. \quad (6b)$$

Notice that the function $G(\cdot)$ is continuous, strictly increasing and $G(0) = 0$. However, it may be non-concave and non-smooth.

Remark 1. *In the absence of the MNE and R&D, we recover an economy without FDI. In this case, the problem (P) becomes the standard Ramsey optimal growth model with the budget constraint: $c_t + pK_{c,t+1} \leq A_c K_{c,t}^\alpha \forall t$. We can prove that $\lim_{t \rightarrow \infty} S_t = S_a$, where S_a is defined by $S_a^{1-\alpha} = \alpha \beta A_c / p^\alpha$.*

Let us now consider a case where there is the MNE but no R&D. In this case, the problem (P) becomes

$$(P'_1) : \max_{(c_t, S_{t+1})_{t=0}^{+\infty}} \left[\sum_{t=0}^{+\infty} \beta^t u(c_t) \right] \text{ subject to } c_t, S_t \geq 0, \quad c_t + S_{t+1} \leq F(S_t)$$

where the function $F(S)$ is defined by

$$F(S) \equiv \max_{pK_c + H \leq S, K_c \geq 0, H \geq 0} \{ A_c K_c^\alpha + wA_h H^{\alpha_h} \}. \quad (8)$$

We can check that $F(S)$ is strictly increasing, strictly concave, smooth and satisfies Inada condition $F'(0) = \infty$. By using the standard argument in the dynamic programming, we obtain the following result.

Proposition 1. *Assume that there is the MNE, but the country does not invest in R&D. Then, S_t converges to S_b defined by*

$$\beta F'(S_b) = 1. \quad (9)$$

Moreover, S_b increases in A_c, w, A_h , and $S_b > S_a$.

In a particular case where $\alpha = \alpha_h$, the value S_b can be explicitly computed by:

$$S_b^{1-\alpha} = \alpha \beta A \text{ where } A \equiv \left(\left(\frac{A_c}{p^\alpha} \right)^{\frac{1}{1-\alpha}} + (wA_h)^{\frac{1}{1-\alpha}} \right)^{1-\alpha}. \quad (10)$$

The property $S_b > S_a$ means that with the presence of the MNE, the economy's investment stock converges to a steady state which is higher than that of the economy without FDI. Moreover, the steady state level S_b is increasing in the TFP of the old sector, wage, as well as the TFP of the MNE. It implies that the effect of FDI on the steady state output depends on both FDI and the host country's circumstances. This is consistent with several studies mentioned in the Introduction section.

3 Roles of FDI on the economy's dynamics

We now investigate the global dynamics of the allocation to explore the role of FDI. We first provide some static analysis (Subsection 3.1) and then global dynamic analysis (Subsection 3.2).

3.1 Static analysis

In this subsection, given the savings S , we study the optimal allocation of the host country. Formally, we look at the optimization problem (G_S). First, it is easy to see that this problem has a solution. However, since the objective function is not concave, the uniqueness of solutions may not be ensured.

We start our exposition by the following result.

Proposition 2.

- (i) If $bS^\sigma \leq \bar{x}$ then at optimum, we have $N = 0$ for any a .
- (ii) If $bS^\sigma > \bar{x}$ and $\left[A_c + a \left(\left(b^{\frac{1}{\sigma}} \frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2} \right)^\sigma - \bar{x} \right) \right] \frac{1}{p^\alpha} \left(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}} \right)^\alpha > F(S)$, then $N > 0$ at optimum.

Proof. See Appendix A.1. □

Point (i) of Proposition 2 indicates that if either the efficiency of the research process or the initial resource is low or the fixed cost is high, the host country may not invest in R&D. Besides, point (ii) implies that the country invests in R&D when a and b are high enough (because $F(S)$ depends neither on a nor b).

If we have increasing return to scale (i.e., $\sigma + \alpha \geq 1$), then condition in point (ii) in Proposition 2 is satisfied for any S high enough. It suggests

that the host country should invest in R&D once it is rich enough. We complement Proposition 2 by the following result.

Proposition 3. *Assume that $\alpha + \sigma \geq 1$.*

(1) *There exists a unique S^* such that: (i) $G(S) - F(S) = 0$ for any $S \leq S^*$, and (ii) $G(S) > F(S)$ and $N > 0$ at optimum for any $S > S^*$.*

(2) *We also have $b(S^*)^\sigma - \bar{x} > 0$. Moreover, we have $G'(S) = F'(S)$ if $S < S^*$, and $G'(S) = G'_0(S) > F'(S)$ if $S > S^*$. At $S = S^*$, the left derivative is $F'(S^*)$ and the right derivative is $G'_0(S^*)$.*

Proof. See Appendix A.1. □

Proposition 3 plays a crucial role in our analysis. It is in line with Lemma 3 in Bruno et al. (2009). However, notice that the method used in Bruno et al. (2009), Le Van et al. (2010) cannot be directly applied in our model.⁸

Let us provide a sketch of our proof. First, we introduce functions g_0 and G_0

$$g_0(K_c, N, H) \equiv (A_c + a(bN^\sigma - \bar{x}))K_c^\alpha + wA_hH^{\alpha_h}$$

$$G_0(S) = \max\{g_0(K_c, N, H) : pK_c + N + H \leq S; K_c, N, H \geq 0; bN^\sigma \geq \bar{x}\}$$

Observe that $G_0(S) \leq G(S)$. More importantly, we have that

$$G(S) - F(S) = \max\{F(S), G_0(S)\} - F(S) = \max\{0, G_0(S) - F(S)\} \quad (11)$$

Second, we prove that $G_0(S) - F(S)$ is strictly increasing in S . The value S^* is in fact the unique solution of the equation $G_0(S^*) = F(S^*)$.

3.2 Global dynamic analysis

In this subsection, we explore the global dynamics of equilibrium. First, we have the monotonicity of the savings path (S_t) .

Proposition 4. *The optimal path $(S_t)_t$ is monotonic. Moreover, S_t does not converge to zero.*

Proof. See Appendix A.2. □

⁸Indeed, their method relies on the set B defined on page 291 of Bruno et al. (2009). In our model with FDI and $\alpha \neq \alpha_h$, this trick no longer works.

Second, we study the boundedness of the allocation. Let us define the sequence (x_t) as $x_0 = X_0, x_{t+1} = F(x_t)$, where the function F is given in (8). Denote x^* and \bar{S} be uniquely defined by:

$$F(x^*) = x^* \text{ and } \bar{S} := \max\{X_0, x^*\}. \quad (12)$$

Notice that x^* and \bar{S} depend on (i) the productivity A_c and capital elasticity α of the consumption good sector, (ii) the efficiency of specific labor training A_h, α_h , and (iii) wage w ,⁹ but not on the TFP A_d of the potential domestic firm in the new sector.

It is important to mention some properties of the function F and the threshold \bar{S} .

Lemma 1. (1) $F(x) \leq F(x^*) = x^*$ for every $x \leq x^*$ and $F(x) \leq x$ for every $x \geq x^*$.

(2) In equilibrium, we have $S_t \leq x_t \leq \bar{S} \forall t$

Proof. See Appendix A.2. □

By consequence, we obtain the following result.

Proposition 5 (middle income trap). *If $X_0 \equiv A_c K_{c,0}^\alpha + w_0 L_{e,0} \leq x^*$ and $b(x^*)^\sigma \leq \bar{x}$, where x^* is defined by (12), then $N_t = 0$ for any t . In this case, we have $\lim_{t \rightarrow \infty} S_t = S_b$ (S_b is defined in Proposition 1).*

Proof. See Appendix A.2. □

Proposition 5 indicates that when the host country has both a low initial resource (in the sense that $X_0 \equiv A_c K_{c,0}^\alpha + w_0 L_{e,0} \leq x^*$) and a weak research process efficiency (in the sense that $b(x^*)^\sigma \leq \bar{x}$), it never invests in R&D ($N_t = 0$ for $\forall t$). In this case, both savings S_t and the output are bounded from above (this can be viewed as a middle income trap). More precisely, S_t converges to the same value S_b , defined by (9), as in the economy with FDI but without investment in R&D.

We now study the case under which the economy may grow without bound.

⁹If $\alpha_h = \alpha$, we can explicitly compute that $x^* = (\frac{A_c}{p^\alpha})^{\frac{1}{1-\alpha}} + (wA_h)^{\frac{1}{1-\alpha}}$.

Proposition 6 (convergence and growth with increasing return to scale).
Assume that $\alpha + \sigma \geq 1$, $\alpha_h + \frac{1}{\alpha} \geq 2$, and

$$\beta \min \left(F'(S^*), \Gamma(a, b, \bar{x}) \right) > 1 \quad (13)$$

$$\text{where } \Gamma(a, b, \bar{x}) \equiv \frac{\left(\frac{\alpha A_c}{p\sigma} \right)^\alpha \bar{x}^{-\frac{(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma (\alpha A_c)^\alpha} \right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h - \alpha}{\sigma(1-\alpha_h)}}} \right)^\alpha} \quad (14)$$

Then, for any level of initial resource, we have $\lim_{t \rightarrow \infty} S_t = \infty$. Moreover,

$$\lim_{t \rightarrow \infty} \frac{N_t}{S_t} = \frac{\sigma}{\alpha + \sigma}, \quad \lim_{t \rightarrow \infty} \frac{pK_{c,t}}{S_t} = \frac{\alpha}{\alpha + \sigma}, \quad \lim_{t \rightarrow \infty} \frac{H_t}{S_t} = 0. \quad (15)$$

Proof. See Appendix A.2. □

Condition (13) ensures that the marginal productivity of function G is high enough in the sense that $\beta D^+ G(S) > 1 \forall S > 0$, where $D^+ G(S)$ is the Dini derivative of function G .¹⁰ This happens if a and b are high enough because the function $\Gamma(a, b, \bar{x})$ is strictly increasing in a and b .

Notice that the conditions given in Proposition 6 do not depend on the initial resource $X_0 \equiv A_c K_{c,0}^\alpha + w_0 L_{e,0}$ which is less than x^* . So, our theoretical results lead to an interesting implication: Consider a low-income country characterized by condition $bX_0^\alpha < \bar{x}$. According to Proposition 2, we have $N_1 = 0$, i.e., the country cannot immediately improve the local firm TFP. Now, suppose that the leverage of new technology a is high enough and conditions in Proposition 6 hold. In this case, the country obtains a sustained growth (in the sense that $\lim_{t \rightarrow \infty} S_t = \infty$). Moreover, the sequence S_t is increasing in time. According to point (ii) of Proposition 2, there is a date t_0 along the optimal path such that the country should focus on R&D from date t_0 on (i.e., $N_t = 0 \forall t \leq t_0$ and $N_t > 0$ for any $t > t_0$). Therefore, the optimal strategy of the country should be as follows.

- First, the country should train specific workers.
- Second, specific workers will work for the MNE to improve the country's income and capital accumulation.

¹⁰The Dini derivatives of a function f are defined by $D^+ f(x) = \limsup_{\epsilon \downarrow 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$ and $D^- f(x) = \liminf_{\epsilon \downarrow 0} \frac{f(x) - f(x-\epsilon)}{\epsilon}$.

- Third, once the country's resource is high enough, it should focus on R&D to create new technology that increases the country's TFP. Hence, its economy may grow faster and converge to a high-income country.

Proposition 6 is related to the economic growth literature with increasing return to scale (Romer, 1986; Jones and Manuelli, 1990; Bruno et al., 2009; Le Van et al., 2010). Our main contribution is to introduce and study the role of FDI in an optimal growth model. In our model, FDI is beneficial to the host country but only at the first stages of its development process. Moreover, the property $\lim_{t \rightarrow \infty} S_t = \infty$ and condition (15) indicate that in the long run, when the host country's resource is high enough, it should focus on domestic investment in physical capital and R&D but not on FDI.

Remark 2 (growth without FDI). *It is interesting to note that conditions in Proposition 6 can be satisfied even if $A_e = w = 0$. In other words, a host country may get economic growth in the long run even in the absence of FDI. The key factors for such growth are the efficiency of investment in R&D (parameter b), the new technology's leverage on the firm TFP (parameter a), and increasing return to scale.*

In the case of decreasing return to scale, the capital stock may converge to a finite steady state, which is higher than that of an economy described in Proposition 1. Formally, we have the following result.

Proposition 7 (decreasing return to scale). *Let X_0 be such that $X_0 < S_b$. Assume that $\alpha + \sigma < 1$. The optimal path (S_t) increasingly converges to a finite value S_t and $S_d \geq S_b$. Moreover, $S_d > S_b$ if a and b are high enough.*

Proof. See Appendix A.2. □

So far, we have provided several theoretical results to show the role of FDI on the host country. In general, the host country benefits from FDI. However, the effect of FDI on economic growth depends not only on the nature of FDI but, more importantly, on the circumstances of the host country (initial resources, domestic firms' TFP, education system, efficiency of R&D process, ...). Indeed, look back at Proposition 1, if the host country only focuses on FDI, the steady state S_b , that is higher than the steady state of the economy without FDI, is increasing in the local conditions (the domestic TFP A_c , the efficiency of the training process A_h) and the TFP of the MNE. Moreover, according to Propositions 6 and 7, if the country invests in R&D and the local

conditions are good enough, the host country may get a sustained growth in the long run. This may happen even the country does not receive FDI.

Our point concerning the conditional impact of FDI on economic growth is supported by several empirical studies ([Borensztein et al., 1998](#); [Berthélemy and Démurger, 2000](#); [Li and Liu, 2005](#); [Alfaro et al., 2004, 2010](#)).

4 Conclusion

We have investigated the nexus between FDI, R&D and growth in a host country by using infinite-horizon optimal growth models. According to our results, the very question does not rely on whether or not developing countries should attract inward FDI, but instead on how they implement policies to benefit from FDI spillovers. We have proved that FDI can act as a catalyst, helping a host developing country to avoid a middle income trap and potentially attain a higher income. However, to reach sustained economic growth in the long run, the host country should focus on domestic investment and R&D.

A Formal proofs

A.1 Static analysis

Proof of Proposition 2. Let $x := bS^\sigma - \bar{x}$. Since $x > 0$, there exists $\alpha_n \in (0, 1)$ such that $bS^\sigma \alpha_n^\sigma = \bar{x}$. Define K_c, N, H by

$$N = (\alpha_n + \epsilon)S, \quad pK_c = \epsilon S, \quad H = 0 \quad (\text{A.1})$$

where $\epsilon > 0$ such that $\alpha_n + 2\epsilon = 1$ (so that $N + pK_c = S$). Precisely, $\epsilon = \frac{1}{2} \left(1 - \left(\frac{\bar{x}}{bS^\sigma} \right)^{\frac{1}{\sigma}} \right)$. With such N, K_c , we have $bN^\sigma > \bar{x}$, and hence, we can verify that

$$g(K_c, N, H) = \left[A_c + a \left(\left(b^{\frac{1}{\sigma}} \frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2} \right)^\sigma - \bar{x} \right) \right] \frac{1}{p^\alpha} \left(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}} \right)^\alpha \quad (\text{A.2})$$

$g(K_c, N, H)$ is increasing in a and b . It will be higher than $F(S)$ when a and b are high enough because $F(S)$ does not depend on (a, b) .

□

Proof of Proposition 3. We need an intermediate step.

Claim 1. Assume that $a\bar{x} > A_c$. Denote $N_1 \equiv (\bar{x}/b)^{1/\sigma}$, $x_1 \equiv \left(\frac{\alpha+\sigma}{\sigma} - \frac{\alpha}{\sigma} \frac{a\bar{x}-A_c}{a\bar{x}}\right)N_1$ and

$$G_1(x) = \max\{(A_c + a(bN^\sigma - \bar{x}))K_c^\alpha : K_c, N \geq 0, pK_c + N \leq x, bN^\sigma \geq \bar{x}\}.$$

We have that $x_1 \geq N_1$. The function G_1 is well-defined on the interval $[N_1, \infty)$, and $G_1(N_1) = 0$. On the interval (N_1, ∞) , the function G_1 is strictly increasing, continuously differentiable and $G_1'(x) > \frac{\alpha Ax^{\alpha-1}}{p^\alpha}$. $G_1(x) - A_c x^\alpha/p^\alpha$ is strictly increasing in x and there exists a unique x_2 such that $G_1(x_2) = A_c x_2^\alpha/p^\alpha$. Moreover, $bx_2^\sigma - \bar{x} > 0$.

Proof of Claim 1. If $x \leq N_1$, then condition $bN^\sigma \geq \bar{x}$ implies that $N \geq N_1 \geq x \geq N$. Then $x = N$, $pK_c = x - N = 0$ and hence $G_1(x) = 0$.

Consider the case $x > N_1$. Let (K_c, N) be an optimal point. It is clear that $K_c > 0$ and $N < x$.

If $N \leq N_1$, then $N = N_1$, $G_1(x) = A_c K_c^\alpha = A_c(x - N)^\alpha/p^\alpha = A_c(x - N_1)^\alpha/p^\alpha$ and $G_1'(x) = \alpha A_c(x - N_1)^{\alpha-1}/p^\alpha > \alpha A_c x^{\alpha-1}/p^\alpha$.

If $N \in (N_1, x)$ at optimal, we write FOCs

$$\sigma abN^{\sigma-1}K_c^\alpha = \lambda \tag{A.3}$$

$$(A_c + a(bN^\sigma - \bar{x}))\alpha K_c^{\alpha-1} = p\lambda = p\sigma abN^{\sigma-1}K_c^\alpha. \tag{A.4}$$

It follows that $(A_c + a(bN^\sigma - \bar{x}))\alpha = \sigma abN^{\sigma-1}pK_c$ or equivalently

$$\sigma ab \frac{x - N}{N} + \frac{\alpha(a\bar{x} - A_c)}{N^\sigma} - \alpha ab = 0. \tag{A.5}$$

The left hand side (LHS) is strictly decreasing in N because $a\bar{x} - A_c > 0$. When $N = x$, the LHS equals $\frac{\alpha(a\bar{x}-A_c)}{x^\sigma} - \alpha ab < 0$ because $x > N_1$. When $N = N_1$, the LHS equals

$$LHS(N_1) \equiv \sigma ab \frac{x - N_1}{N_1} + \frac{\alpha(a\bar{x} - A_c)}{(N_1)^\sigma} - \alpha ab. \tag{A.6}$$

Observe that $LHS(N_1) \geq 0$ if and only if

$$x \geq x_1 \equiv \frac{\alpha + \sigma}{\sigma} N_1 - \frac{\alpha}{\sigma} N_1 \frac{a\bar{x} - A_c}{a\bar{x}} = N_1 + \frac{\alpha(N_1)^{1-\sigma}}{\sigma ab} \left(\alpha ab(N_1)^\sigma - a\bar{x} + A_c \right).$$

Therefore, we get that:

1. If $x \leq x_1$, then $N = N_1$ and $G_1(x) = A_c(x - N_1)^\alpha/p^\alpha$. In this case, we have

$$G'_1(x) = \frac{\alpha A_c(x - N_1)^{\alpha-1}}{p^\alpha} > \frac{\alpha A x^{\alpha-1}}{p^\alpha}$$

because $N_1 > 0$ and $\alpha - 1 < 0$.

2. If $x > x_1$, then the equation (A.5) has a unique N_x in the interval (N_1, x) . The optimal point (K_c, N) is given by $N = N_x$ and $pK_c + N = S$. Moreover, when x increases, we have N_x , $x - N_x$ and $\frac{x - N_x}{N_x}$ increase.

We have $G_1(x) = (A_c + a(bN^\sigma - \bar{x}))K_c^\alpha$ where N is uniquely given by (A.5) and $pK_c = S - N$. By computing directly or using the envelop theorem, we have

$$G'_1(x) = (A_c + a(bN^\sigma - \bar{x}))\alpha K_c^{\alpha-1} > \frac{\alpha A x^{\alpha-1}}{p^\alpha} \quad (\text{A.7})$$

because $bN^\sigma - \bar{x} > 0$ and $pK_c < S$.

3. When x tends to x_1 , we have N_x tends to N_1 and therefore $G'_1(x) = (A_c + a(bN^\sigma - \bar{x}))\alpha K_c^{\alpha-1}$ tends to $\frac{\alpha A_c(x - N_1)^{\alpha-1}}{p^\alpha} > \frac{\alpha A_c x^{\alpha-1}}{p^\alpha}$ because $\alpha < 1$.

To sum up, the function G_1 is continuously differentiable and $G'_1(x) > \frac{\alpha A x^{\alpha-1}}{p^\alpha}$. \square

We now prove Proposition 3. Observe that

$$G_0(S) \equiv \max_{K_c, N, H} \left\{ G_1(x) + wA_h H^{\alpha_h} : x + H \leq S; x, H \geq 0 \right\}. \quad (\text{A.8})$$

Let (x_g, H_g) be an optimal point. Since G_1 is differentiable, we have the FOC

$$G'_1(x_g) - \alpha_h w A_h (S - x_g)^{\alpha_h - 1} = 0.$$

Let $(x, S - x)$ be the unique pair such that $A_c(\frac{x}{p})^\alpha + wA_h H^{\alpha_h} = F(S)$. Then, we have $\frac{\alpha A x^{\alpha-1}}{p^\alpha} - \alpha_h w A_h (S - x)^{\alpha_h - 1} = 0$.

Since $G'_1(x_g) > \frac{\alpha A (x_g)^{\alpha-1}}{p^\alpha}$, we have $0 > \frac{\alpha A (x_g)^{\alpha-1}}{p^\alpha} - \alpha_h w A_h (S - x_g)^{\alpha_h - 1}$. Therefore, we have $x_g > x$ and hence $H_g < H$. It follows that

$$G'_0(S) = \alpha_h w A_h H_g^{\alpha_h - 1} > \alpha_h w A_h H^{\alpha_h - 1} = F'(S). \quad (\text{A.9})$$

So, $G_0(S) - F(S)$ is strictly increasing.

When S is small enough, $G_0(S) - F(S)$ is negative. When S is high enough, $G_0(S) - F(S)$ is positive (see, for example, point (ii) of Proposition 2). So, there exists a unique S^* such that $G_0(S^*) - F(S^*) = 0$.

According to (11) and $G(S) \geq G_0(S) \forall S$, we have $G(S) - F(S) = 0 \forall S \leq S^*$, and $G(S) - F(S) > 0 \forall S > S^*$. □

A.2 Dynamic analysis

Proof of Proposition 4. Since the function $G(\cdot)$ is continuous, strictly increasing, we can use the standard argument in dynamic programming (Amir, 1996) to prove that the optimal path $(S_t)_t$ is monotonic.

We now prove that S_t does not converge to zero. First, following Kamihigashi and Roy (2007), we have Euler condition in the form of inequality

$$\beta u'(c_{t+1})D^- f(S_{t+1}) \geq u'(c_t) \geq \beta u'(c_{t+1})D^+ f(S_{t+1}). \quad (\text{A.10})$$

where the Dini derivatives of function f are defined by $D^+ f(x) = \limsup_{\epsilon \downarrow 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$

and $D^- f(x) = \liminf_{\epsilon \downarrow 0} \frac{f(x) - f(x-\epsilon)}{\epsilon}$.

Suppose that $\lim_{t \rightarrow \infty} S_t = 0$. According to budget constraints and the fact that $G(0) = 0$, we have $\lim_{t \rightarrow \infty} c_t = 0$. Since $\lim_{t \rightarrow \infty} S_t = 0$, there exists t_0 such that $\beta D^+ G(S_{t+1}) > 1$ for every $t \geq t_0$. Consequently, $u'(c_t) \geq u'(c_{t+1})$ and hence $c_t \leq c_{t+1}$ for every $t \geq t_0$. This leads to a contradiction to the fact that $\lim_{t \rightarrow \infty} c_t = 0$. □

Proof of Lemma 1. (1) If $x < x^*$, then $F(x) < F(x^*) = x^*$. If $x > x^*$, then $\frac{F(x)}{x} \leq \frac{F(x^*)}{x^*} = 1$ since F is concave.

(2) It is obvious that $S_t \leq x_t \forall t$. We prove $x_t \leq \bar{S} \forall t$ by induction argument. First, we see that $x_0 \leq \bar{S}$. Second, assume that $x_s \leq \bar{S} \forall s \leq t$. If $X_0 \leq x^*$, then $x_t \leq \bar{S} = x^*$, then $x_{t+1} = F(x_t) \leq F(x^*) = x^* = \bar{S}$. If $X_0 > x^*$, then $x_t \leq \bar{S} = X_0$ and hence $x_{t+1} = F(x_t) = F(x_0) \leq x_1 \leq \bar{S}$. □

Proof of Proposition 5. We will prove, by induction argument, that $b\bar{x}_t^\sigma \leq \bar{x}$ and $S_t \leq x_1 \forall t \geq 1$.

When $t = 1$. We have $N_1 \leq S_1 \leq X_0 \leq x_1$, So, $bN_1^\sigma \leq b\bar{S}_1^\sigma \leq \bar{x}$.

Assume that $b\bar{x}_t^\sigma \leq \bar{x}$ and $S_t \leq x_1 \forall t \leq T$. This implies that $N_T = 0$, because otherwise we can reduce N_T and augment $K_{c,T}$ in order to get a higher utility, which is a contradiction.

Since $N_T = 0$, we have that $G(S_T) = F(S_T)$. Since $S_T \leq x_1$, we have $F(S_T) \leq F(x^*) = x^*$.

Hence, $S_{T+1} \leq G(S_T) \leq x^*$ and therefore $b\bar{x}_{T+1}^\sigma \leq bS_{T+1}^\sigma \leq b(x^*)^\sigma \leq \bar{x}$. We have finished our proof. \square

Proof of Proposition 6. We need intermediate steps.

Lemma 2. *Assume that $\alpha + \sigma \geq 1$. For any solution K_c, N, H of the problem (G_S) , there exists the following limits*

$$\lim_{S \rightarrow \infty} \theta_c = \frac{\alpha}{\alpha + \sigma}, \lim_{S \rightarrow \infty} \theta_n = \frac{\sigma}{\alpha + \sigma}, \lim_{S \rightarrow \infty} \theta_h = 0. \quad (\text{A.11})$$

Proof of Lemma 2. Observe that, when S is high enough, we have $bN^\sigma - \bar{x} > 0$ at optimal. It is also to see that $\theta_c, \theta_h > 0$. By consequence, we can write FOCs for the problem (G') as follows (we have FOCs even the objective function is not concave):

$$\alpha_h w A_h S^{\alpha_h} \theta_h^{\alpha_h - 1} = \lambda \quad (\text{A.12})$$

$$\left(A_c + a(bS^\sigma \theta_n^\sigma - \bar{x})^+ \right) \frac{\alpha}{p^\alpha} \theta_c^{\alpha - 1} S^\alpha = \lambda \quad (\text{A.13})$$

$$ab\sigma S^{\sigma + \alpha} \theta_n^{\sigma - 1} \left(\frac{\theta_c}{p} \right)^\alpha = \lambda \quad (\text{A.14})$$

where λ is the multiplier associated to the constraint $\theta_c + \theta_n + \theta_h \leq 1$. The first and the third equations imply that

$$\frac{\alpha_h w A_h p^\alpha}{ab\sigma} = S^{\sigma + \alpha - \alpha_h} \theta_n^{\sigma - 1} \theta_c^\alpha \theta_h^{1 - \alpha_h} = (S\theta_n)^{\sigma - 1} (S\theta_c)^\alpha (S\theta_h)^{1 - \alpha_h} \quad (\text{A.15})$$

while the second and third conditions imply that

$$\left(A_c + a(bS^\sigma \theta_n^\sigma - \bar{x})^+ \right) \alpha = ab\sigma S^\sigma \theta_n^{\sigma - 1} \theta_c. \quad (\text{A.16})$$

By consequence, we obtain

$$\theta_c = \frac{\alpha}{\sigma} \theta_n + \frac{\alpha \theta_n^{1 - \sigma} (A_c - a\bar{x})}{ab\sigma S^\sigma} \quad (\text{A.17})$$

$$\text{or equivalently } \frac{S\theta_c}{S\theta_n} = \frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S\theta_n)^\sigma}. \quad (\text{A.18})$$

From this, we get $\lim_{S \rightarrow \infty} (\frac{\sigma \theta_c}{\alpha \theta_n} - 1) \theta_n^\sigma = 0$. By combining this with the fact that $\sigma \leq 1$, we obtain $\lim_{S \rightarrow \infty} (\theta_c - \frac{\alpha}{\sigma} \theta_n) = 0$.

Notice that $b(S\theta_n)^\sigma > N$ for S high enough.

We will prove that when S tends to infinity, $S\theta_h$ is bounded from above, and hence $\lim_{S \rightarrow \infty} \theta_h = 0$. To do so, we firstly prove that $\liminf_{S \rightarrow \infty} \frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} > 0$. Indeed, according to (A.18), we have

$$\frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} = (S\theta_n)^{\alpha+\sigma-1} \left(\frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S\theta_n)^\sigma} \right)^\alpha \quad (\text{A.19})$$

Suppose that there is a sequence (S_k) tends to infinity such that $\lim_{k \rightarrow \infty} \frac{(S_k \theta_c)^\alpha}{(S_k \theta_n)^{1-\sigma}} = 0$. Notice that

$$\frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} = \frac{1}{(S\theta_n)^{(1-\sigma)(1-\alpha)}} \left(\frac{\alpha}{\sigma ab} [A_c + a(b(S\theta_n)^\sigma - \bar{x})] \right)^\alpha \geq \frac{1}{(S\theta_n)^{(1-\sigma)(1-\alpha)}} \left(\frac{\alpha}{\sigma ab} A_c \right)^\alpha$$

for any S high enough, which implies that $\lim_{k \rightarrow \infty} S_k \theta_n = \infty$. However, this will follow that

$$\frac{(S_k \theta_c)^\alpha}{(S_k \theta_n)^{1-\sigma}} = (S_k \theta_n)^{\alpha+\sigma-1} \left(\frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S_k \theta_n)^\sigma} \right)^\alpha \quad (\text{A.20})$$

is bounded away from zero (because $\alpha + \sigma \geq 1$), a contradiction.

So, we get that $\liminf_{S \rightarrow \infty} \frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} > 0$. By combining this with (A.15), we have that $S\theta_h$ is bounded from above and hence $\lim_{S \rightarrow \infty} \theta_h = 0$. Combining with (A.17), we obtain (A.11). \square

Lemma 3. *Assume that $a\bar{x} - A_c \geq 0$. We have*

$$D^+G(S) = \limsup_{\epsilon \downarrow 0} \frac{G(S + \epsilon) - G(S)}{\epsilon} \geq \min \left(F'(S^*), \Gamma(a, b, \bar{x}) \right) \quad (\text{A.21})$$

$$\text{where } \Gamma(a, b, \bar{x}) \equiv \frac{\left(\frac{\alpha A_c}{p\sigma} \right)^\alpha \bar{x}^{-\frac{(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma (\alpha A_c)^\alpha} \right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h - \alpha}{\sigma(1-\alpha_h)}}} \right)^\alpha} \quad (\text{A.22})$$

By consequence, when a and b are high enough and $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\beta D^+G(S) > 1 \forall S > 0$.

Proof of Lemma 3. Part 1. We prove (A.21). Let $S > S^*$. Consider the function

$$(G_0) : \quad G_0(S) \equiv \max_{K_c, N, H} \left\{ (A_c + a(bN^\sigma - \bar{x}))K_c^\alpha + wA_hH^{\alpha_h} \right\} \quad (\text{A.23a})$$

$$\text{subject to: } pK_c + N + H \leq S, \quad bN^\sigma \geq \bar{x} \text{ and } K_c, N, H \geq 0. \quad (\text{A.23b})$$

When $S > S^*$, we have $G(S) = G_0(S)$ and $bN^\sigma > \bar{x}$ at optimal. We will quantify $G'_0(S)$.

Let λ be the multiplier associated to the constraint $pK_c + N + H \leq S$, we have FOCs

$$(abN^\sigma - (a\bar{x} - A_c))\alpha K_c^{\alpha-1} = p\lambda \quad (\text{A.24})$$

$$ab\sigma N^{\sigma-1}K_c^\alpha = \lambda \quad (\text{A.25})$$

$$\alpha_h w A_h H^{\alpha_h-1} = \lambda. \quad (\text{A.26})$$

FOCs imply that $\alpha(abN^\sigma - (a\bar{x} - A_c)) = pab\sigma N^{\sigma-1}K_c$ and hence

$$\frac{\alpha}{\sigma}N \geq pK_c = \frac{\alpha}{\sigma}N \left(1 - \frac{a\bar{x} - A_c}{abN^\sigma}\right) > N \frac{\alpha A_c}{\sigma a\bar{x}} \quad (\text{A.27})$$

because $a\bar{x} - A_c \geq 0$ and $bN^\sigma \geq \bar{x}$.

Since $ab\sigma N^{\sigma-1}K_c^\alpha = \alpha_h w A_h H^{\alpha_h-1}$, we have

$$\frac{H^{1-\alpha_h}}{N^{1-\alpha_h}} = \frac{\alpha_h w A_h}{ab\sigma} N^{1-\sigma-(1-\alpha_h)} K_c^{-\alpha} \quad (\text{A.28})$$

$$\leq \frac{\alpha_h w A_h}{ab\sigma} N^{1-\sigma-(1-\alpha_h)} \left(N \frac{\alpha A_c}{p\sigma a\bar{x}}\right)^{-\alpha} = \frac{\alpha_h w A_h}{ab\sigma} \left(\frac{p\sigma a\bar{x}}{\alpha A_c}\right)^\alpha N^{-(\alpha+\sigma-\alpha_h)} \quad (\text{A.29})$$

$$\leq \frac{\alpha_h w A_h}{ab\sigma} \left(\frac{p\sigma a\bar{x}}{\alpha A_c}\right)^\alpha \left(\frac{\bar{x}}{b}\right)^{-\frac{\alpha+\sigma-\alpha_h}{\sigma}} = \frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha} \frac{1}{a^{1-\alpha} b^{\frac{\alpha_h-\alpha}{\sigma}}} \quad (\text{A.30})$$

Thus, we get that

$$H \leq N \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}} \quad (\text{A.31})$$

Since $S = N + pK_c + H$, we have

$$S \leq N + N \frac{\alpha}{\sigma} + N \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}} \quad (\text{A.32})$$

which implies that

$$N\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\right) \geq S \geq \left(\frac{\bar{x}}{b}\right)^{\frac{1}{\sigma}} \quad (\text{A.33})$$

Denote $d \equiv a\bar{x} - A_c \geq 0$. We have

$$\begin{aligned} G_0(S) &= (abN^\sigma - d)K_c^\alpha + wA_h H^{\alpha_h} \\ G'_0(S) &= (abN^\sigma - d)\alpha K_c^{\alpha-1} K'_c(S) + \sigma abN^{\sigma-1} K_c^\alpha N'(S) + \alpha_h w A_h H^{\alpha_h-1} H'(S) \\ &= \sigma abN^{\sigma-1} K_c^\alpha \end{aligned}$$

because $pK'_c(S) + N'(S) + H'(S) = 1$.

By combining this with $K_c \geq \frac{\alpha A_c}{p\sigma a\bar{x}} N$ and $\sigma < 1$, we have

$$G'_0(S) = abN^{\sigma-1} K_c^\alpha \geq abN^{\sigma+\alpha-1} \left(\frac{\alpha A_c}{p\sigma a\bar{x}}\right)^\alpha \quad (\text{A.35})$$

$$\begin{aligned} &\geq \left(\frac{\alpha A_c}{p\sigma \bar{x}}\right)^\alpha a^{1-\alpha} b \left(\frac{\bar{x}}{b}\right)^{\frac{\sigma+\alpha-1}{\sigma}} \frac{1}{\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\right)^\alpha} \\ &\quad (\text{A.36}) \end{aligned}$$

$$\begin{aligned} &= \frac{\left(\frac{\alpha A_c}{p\sigma}\right)^\alpha \bar{x}^{-\frac{(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\right)^\alpha} \equiv \Gamma(a, b, \bar{x}) \\ &\quad (\text{A.37}) \end{aligned}$$

At point S^* , the right Dini derivative $D^+G(S^*)$ of G is

$$D^+G(S^*) = \limsup_{\epsilon \downarrow 0} \frac{G(S^* + \epsilon) - G(S^*)}{\epsilon} \geq \limsup_{\epsilon \downarrow 0} \frac{F(S^* + \epsilon) - F(S^*)}{\epsilon} = F'(S^*). \quad (\text{A.38})$$

When $S < S^*$, we have $G(S) = F(S)$ and hence $G'(S) = F'(S) \geq F'(S^*)$ because F' is decreasing.

Part 2. We prove that, when a and b are high enough and $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\beta D^+G(S) > 1 \forall S > 0$.

Observe that $\Gamma(a, b, \bar{x})$ is increasing in a and $\beta\Gamma(a, b, \bar{x}) > 1$ when a is high enough.

When $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\frac{1-\alpha}{\sigma} + \frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)} \geq 0$ and therefore $\Gamma(a, b, \bar{x})$ is strictly increasing in b . In this case, it is easy to see that $\beta\Gamma(a, b, \bar{x}) > 1$ when b is high enough.

We now prove that $\beta F'(S^*) > 1$ when a or b is high enough. As in proof of point (ii) of Proposition 2, we have that: If $bS^\sigma > \bar{x}$, then

$$G(S) \geq \left[A_c + a \left(\left(b^{\frac{1}{\sigma}} \frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2} \right)^\sigma - \bar{x} \right) \right] \frac{1}{p^\alpha} \left(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}} \right)^\alpha > 0 \quad (\text{A.39})$$

At point S^* which depends on a and b , we have $F(S^*) = G(S^*)$ and hence

$$\left[A_c + a \left(\left(b^{\frac{1}{\sigma}} \frac{S^*}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2} \right)^\sigma - \bar{x} \right) \right] \frac{1}{p^\alpha} \left(\frac{S^*}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}} \right)^\alpha \leq F(S^*) \leq \max(A_c(S^*)^\alpha, w_h A_h S^{\alpha_h}).$$

We prove that S^* tends to zero when a or b goes to infinity. Indeed, let, for example, b tend to infinity. If $\liminf_{b \rightarrow \infty} S^* > 0$, by using the property $\alpha + \sigma \geq 1 > \max(\alpha, \alpha_h)$, we get that

$$\left[A_c + a \left(\left(b^{\frac{1}{\sigma}} \frac{S^*}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2} \right)^\sigma - \bar{x} \right) \right] \frac{1}{p^\alpha} \left(\frac{S^*}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}} \right)^\alpha > \max(A_c(S^*)^\alpha, w_h A_h S^{\alpha_h}).$$

where b is high enough, a contradiction.

So, when a or b is high enough, S^* is low enough and hence $\beta F'(S^*) > 1$ since $F'(0) = \infty$. □

We are now ready to prove Proposition 6. According to Lemma 3, we have $\beta D^+ G(S) > 1 \forall S > 0$ when a and b are high enough. According to Proposition 4.6 in Kamihigashi and Roy (2007), we have that $\lim_{t \rightarrow \infty} S_t = \infty$. According to Lemma 2, we obtain point 2 of Proposition 6. □

Proof of Proposition 7. We observe that

$$G(S) \leq (A_c + abS^\sigma) \frac{1}{p^\alpha} S^\alpha + wA_h S^{\alpha_h} \leq \begin{cases} \frac{A_c + ab}{p^\alpha} + wA_h & \text{if } S \leq 1 \\ \left(\frac{A_c + ab}{p^\alpha} + wA_h \right) S^{\max(\alpha + \sigma, \alpha_h)} & \text{if } S \geq 1. \end{cases}$$

By using $\max(\alpha + \sigma, \alpha_h) < 1$, it is easy to prove that S_t is bounded from above. Since S_t is monotonic, it must converge to a finite value, say S_d . So, we have that $\beta D^- G(S_d) \geq 1 \geq \beta D^+ G(S_d)$, where the Dini derivatives of a function f are defined by $D^+ f(x) = \limsup_{\epsilon \downarrow 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$ and $D^- f(x) = \liminf_{\epsilon \downarrow 0} \frac{f(x) - f(x-\epsilon)}{\epsilon}$

(see, for instance, Kamihigashi and Roy (2007)).

We consider three cases:

1. If $S_d < S^*$, then G is differentiable at S_d and $\beta G'(S_d) = 1 = \beta F'(S_b)$ which in turn implies that $S_d = S_b$.
2. If $S_d > S^*$, then G is differentiable at S_d and $\beta F'(S_b) = 1 = \beta G'(S_d) > \beta F'(S_d)$ which in turn implies that $S_d > S_b$ (because $F'(S)$ is decreasing).
3. If $S_d = S^*$, then we have $\beta F'(S_d) = \beta D^-G(S_d) \geq 1 \geq \beta D^+G(S_d) \geq \beta F'(S_d)$. So, $S = S^* = S_b$.

Suming up three cases, we have that $S_d \geq S_b$ in any case. Since $X_0 < S_b$, we have $S_1 < S_b \leq S$. Hence S_t is increasing in t . Moreover, when a and b are high enough, we have $S_b > S^*$. In this case, we have $S_d > S^*$. According to the three cases mentioned above, we must have $S_d > S_b$. □

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