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A polynomial chaos expansion model for sensitivity analysis and reliability assessment of reinforced concrete structures subjected to cyclic loading

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ABSTRACT

Cyclic loading is one of the main causes of degradation of reinforced concrete structures. The behavior of a reinforced concrete structure in fatigue can be represented by mechanical models, based on different theories (damage, fracture, plasticity ...), whose development requires knowledge of the characteristic parameters of the material and uncertainties associated. In this paper, we propose a polynomial chaos expansion (PCE) methodology to propagate parameter uncertainties in a damage model. The PCE will also be used to perform a sensitivity analysis of the model. Sensitivity analysis allows quantifying the effects of input variables or their combinations on the output variables of a model. The methodology is illustrated with a reinforced concrete beam subjected to cyclic loading. The results indicate that the deterministic model and PCE are close to the experimental results. The sensitivity analysis was also useful to determine which are the most influencing parameters for a reliability analysis.

KEYWORDS: reinforced concrete; cyclic loading; polynomial chaos expansion; sensitivity analysis; reliability.

1 INTRODUCTION

During their lifetime, reinforced concrete structures are subjected to stresses of various kinds which lead to their early degradation. Many digital models have been developed in order to represent with the greatest precision the behavior of concrete under mechanical loading. In the case of cyclic loads, several models based on damage theory allow the effects of micro-cracking in concrete to be evaluated after each loading cycle (load / unload). Their use requires, among other things, knowledge of the intrinsic parameters of the material which could be determined from experimental tests. These can contain uncertainties related to the implementation of the tests and the calculation methods used. However, models take time and experimental data is scarce. The work we propose

consists of a meta-modeling of the mechanical behavior of a reinforced concrete beam subjected to cyclic loading. The meta-model, based on the polynomial chaos expansion method, allows the uncertainties of the input parameters to be propagated in the mechanical model and is used to perform a sensitivity analysis of the inputs to the outputs.

The paper is structured as follows. The first part describes the damage model used with the constitutive equations (section 2). The PCE method is presented in section 3. The section 4, present the numerical implementation. Finally, the results are presented and discussed in the last section.

2 MECHANICAL MODEL

To model the concrete, we used the isotropic damage model developed by [1], and implemented in the finite element CAST3M software. In the latter, the damage variable is a scalar which varies between 0 for sound concrete and 1 for cracked concrete. The model takes into account the residual strains and makes it possible to represent the hysteretic behavior as well as the unilateral effects (opening - reclosing of the crack) of concrete.

The different state laws are expressed by the following differential equations:

$$\begin{cases} \boldsymbol{\sigma} = E((1 - D) \langle \varepsilon_x \rangle_+ + \langle \varepsilon_x \rangle_-) + 2(1 - D)\mu\boldsymbol{\varepsilon} + 2D\mu(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^\pi) \\ \boldsymbol{\sigma}^\pi = 2D\mu(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^\pi) \\ \mathbf{Y} = \frac{E}{2} \langle \varepsilon_x \rangle_+^2 + \mu\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon} - \mu(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^\pi) \cdot (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^\pi) \end{cases} \quad (1)$$

where μ is a shear coefficient, $\boldsymbol{\sigma}^\pi$ is the frictional tensor, $\boldsymbol{\varepsilon}^\pi$ is the sliding tensor and \mathbf{Y} is The damage energy released rate.

Equation (2) gives the expression of the scalar damage variable.

$$D = 1 - \frac{1}{1 + (A_{Dir}H(\langle \varepsilon_{ij} \rangle_+ \langle \sigma_{ij} \rangle_+) + A_{Ind}(1 - H(\langle \varepsilon_{ij} \rangle_+ \langle \sigma_{ij} \rangle_+)))(Y - Y_0)} \quad (2)$$

where A_{Dir} represent the brittleness in tension, A_{Ind} the brittleness in compression, Y_0 the damage initial threshold and H is Heaviside.

3 EXPANSION OF POLYNOMIAL CHAOS

The PCE is a meta-modeling technique which aims to approximate a computational model in the form of polynomial functions. The general decomposition of a PCE is written as follows:

$$\hat{Y} = M^{PCE}(\mathbf{X}) = \sum_{\alpha=0}^P y_\alpha \psi_\alpha(\mathbf{X}) \quad (3)$$

Where y_α is the coefficient of the PCE and $\psi_\alpha(\mathbf{X})$ a multivariate polynomial of degree α .

The post-processing of a PCE for the sensitivity analysis proposed by [2] facilitates the evaluation of expensive models in computation. The sensitivity analysis quantifies the influence of the variability of the input parameters (individually or combined) of a model on the output variables using the sensitivity indices [3, 4]. To quantify the impact of the selected input variables on the model, we calculate the Sobol's sensitivity indices directly deduced from the PCE coefficients, with the assumption that there is no correlation between the different variables.

4 NUMERICAL IMPLEMENTATION

To perform the various analyzes, we model a reinforced concrete beam subjected to a cyclic loading (10,000 cycles) of three-point bending with an alternating point load varying between 5.4 kN and 18 kN (Fig. 1).

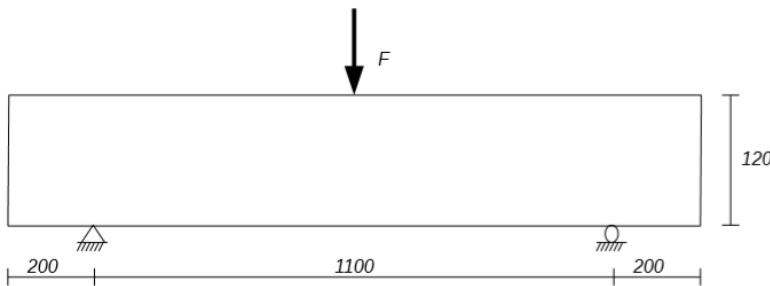


Figure 1 – Beam geometry configuration

The studies were carried out for a C40 concrete with 90 kg.m⁻³ of admixture and a w / c ratio of 0.44 [5]. The following table presents the variability retained for the generation of the random variables.

Table 1 – Variability of input parameters

Parameters		Notation	Value
Elasticity modulus	(MPa)	E	Lognormal ($\mu = 35000$; $COV = 0.12$)
Tensile strength	(MPa)	f_t	Lognormal ($\mu = 3.5$; $COV = 0.13$)
Concentrate load	(N)	F	Uniform on the interval $[5.4; 18] \times 10^3$

Figure 2 describes the methodology used to perform the sensitivity and reliability analyzes. 200 LHS simulations of the three input variables are performed and fed into the finite element code to calculate deflection and tensile stress in concrete. The input and output values are then injected into the PCE; the PCE coefficients which make it possible to deduce the Sobol's indices are determined. At the exit of the meta-model we also obtain the values of the deflection and the stress, of which those of the deflection are used to calculate the reliability of the structure.

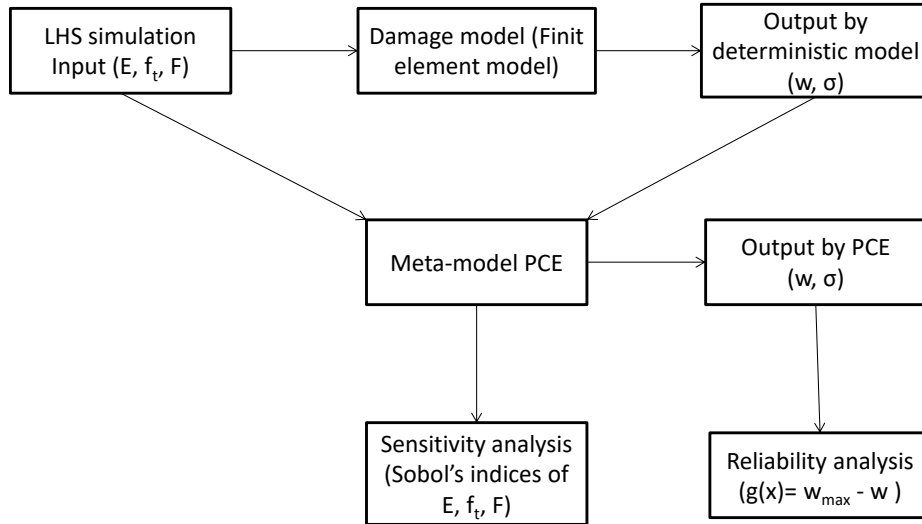


Figure 2 – Flowchart of the methodology

5 RESULTS AND DISCUSSIONS

Table 2 presents the Sobol sensitivity indices of the input random variables on the damage model. One can thus establish an order of influence of the various input parameters on the deflection and the tensile stress in the concrete. The ultimate tensile stress (f_t) has negligible influence on the two outputs; the loading has a stronger influence on the stress which is logical. The sensitivity indices on the deflection of the Young's modulus and the charge intensity are very close (see Fig. 3). This analysis provides to know the most influential parameters to prioritize in order to estimate the lifespan of a structure.

Table 2 – Sensitivities of the input parameters

Parameters	Deflection	Stress
E	0.594	3.071×10^{-4}
f_t	1.793×10^{-6}	3.724×10^{-6}
F	0.412	0.9997

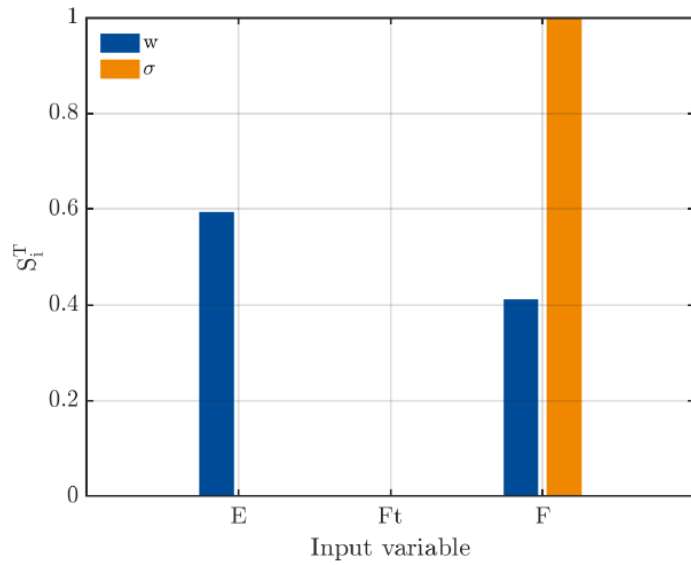


Figure 3 – Sobol's total indices

The probability of failure of the beam was calculated from the Monte Carlo simulation method. This method aims to estimate the probability of failure by probability sampling of the inputs of a model. For our example, the failure criterion is the maximum deflection taken equal to 0.8 mm and we obtain a failure probability of 0.402 with a reliability index of $\text{Beta} = 0.2468$.

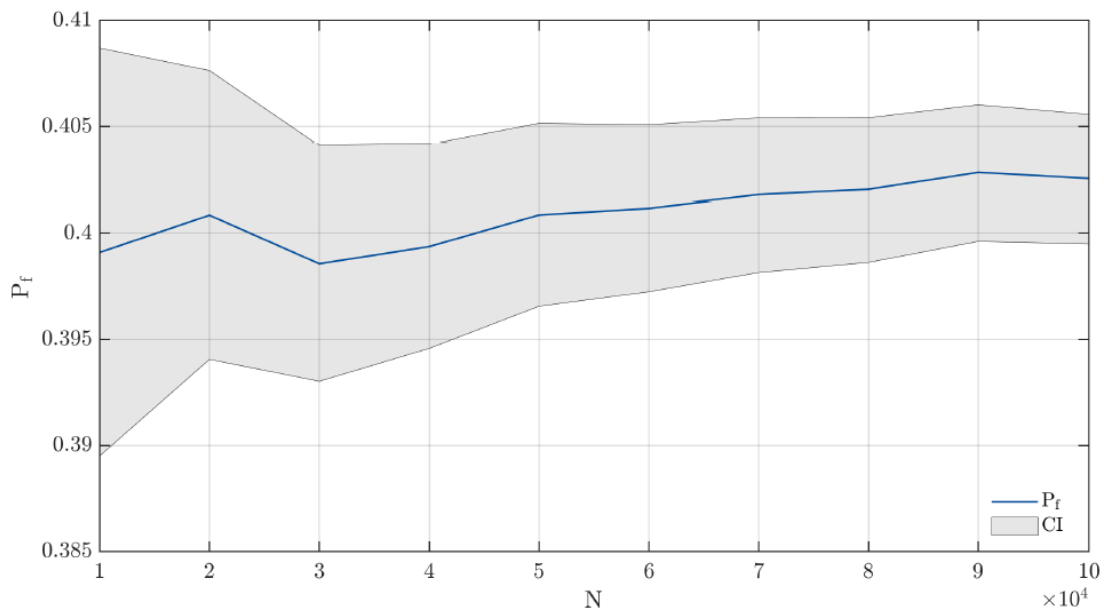


Figure 4 – Convergence of failure probability P_f

6 CONCLUSION

The methodology used made it possible to highlight the influence of certain input variables on the damage model. It provides information on the variables whose variability will have the most impact on determining the lifespan of a structure. The use of the polynomial chaos expansion is quite interesting in terms of computation time which is relatively short for the number of cycles which is quite high. For a more complete study, the parameters of the damage model considered as deterministic in this study and whose values are extracted from the literature, will be modeled as random variables in order to evaluate the effects of their uncertainties on the output parameters of the model.

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