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PATCH NORMALIZING FLOW REGULARIZATION FOR HYPERSPECTRAL PANSHARPENING

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ABSTRACT

This paper presents an unsupervised neural network-based framework for fusing hyperspectral (HS) and multispectral (MS) images, addressing their inherent resolution trade-offs. Unlike supervised HS-MS fusion methods that require large training datasets, our approach is model-based and fully unsupervised. It is based on Principal Component Analysis (PCA) for spectral subspace identification and on an innovative Patch Normalizing Flow (Patch-NF) for spatial regularization. Experiments show that the proposed method offers a nice trade-off in terms of performance and computation time, when compared to alternative unsupervised baselines from the literature.

Index Terms— Hyperspectral and Multispectral image fusion, Spatial Regularization, Generative models, Patch Normalizing Flow

1. INTRODUCTION

Fusion of multisensor images is an important and active research area in hyperspectral imaging. In this setting, the fusion of hyperspectral (HS) and multispectral (MS) images addresses the inherent trade-offs between these imaging modalities: HS images offer extensive spectral information yet suffer from low spatial resolution, whereas MS images provide superior spatial resolution but with reduced spectral details. In many applications, such as in Earth observation, astronomy or microscopy, both images of the same scene can be acquired simultaneously, and the fusion process is required to achieve accurate analyses at high spatial and spectral resolutions.

The core challenge in HS-MS fusion lies in effectively inverting the degradation processes inherent to image acquisition, a task that traditionally employs model-based inversion techniques with regularization [1, 2]. Conventional methods rely on simple assumptions such as spectral low-rankness and spatial smoothness but often fall short in complex scenarios due to their simplistic nature. The advent of deep learning presents new opportunities, yet its efficacy is hampered by the

extensive need for training data, a limitation in many practical applications.

Addressing these challenges, this work introduces an innovative unsupervised approach leveraging neural networks for spatial regularization without the prerequisite of training datasets. We propose to learn a patch generative model from the MS image, that will serve as a prior or regularization in the fusion problem. Our methodology employs another common mechanism: an identification of the spectral subspace identification via Principal Component Analysis (PCA) on the HS images. To demonstrate the efficacy of our approach, we conduct comparative analyses against established fusion techniques, including FUSE [3] and CNMF [4], and a more recent unsupervised deep learning model [5], on two well-known images from Pavia University and Indian Pines.

This paper is organised as follows. Section 2 presents a sample of state-of-the-art methods for HS-MS fusion, together with an overview of the recent literature dedicated to neural-networks-based regularization. Section 3 describes the proposed new method and its implementation. Experiments are presented in Section 4, while Section 5 summarizes the contribution and draws some perspectives.

2. RELATED WORKS

2.1. Image Fusion as an inverse problem

HS-MS image fusion methods are traditionally categorised into four groups [1]: component substitution, multi-resolution analysis (MRA), Bayesian approaches, and spectral unmixing (SU). In this short paper we will focus on the Bayesian (or inverse problem) approaches, that assume the forward models of the MS and HS images to be known [6]. The reconstruction of the full-resolution datacube can be cast as a maximum-a-posteriori (MAP) estimation problem, that seeks the minimum of a data-fidelity term involving the two images, and additional regularizations reflecting the prior on the solution.

To spectrally regularize the problem, some works use classical low-rank penalties such as nuclear norm minimization. Indeed, HS images are known to have highly correlated spectral bands, suggesting that spectral vectors lie in low-dimensional manifolds or subspaces [7, 8]. But it is more

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common and more efficient to identify a spectral subspace beforehand from the HS image [6, 9]: this amounts to reducing the dimension and thus the computation time, and it avoids the bias caused by the nuclear norm, which is only a proxy for the rank [10].

Concerning spatial regularization or prior, many works use classical spatial image regularizations such as vector total variation [9]. Several works proposed instead *informed spatial regularizations* learnt from the MS image. They can be based on dictionary learning and sparsity [6], Gaussian priors [3], or weighted Tikhonov [11]. Assuming that the spatial structures of interest are visible in the MS image, such informed approaches can outperform standard priors, and can be viewed as the spatial counterparts of the spectral subspace methods.

Some unmixing-based approaches can also be cast as inverse problem formulations. The most famous method in this area is CNMF [12, 4], that seeks a hyperspectral image compatible with the forward model and a linear mixing model, with an algorithm inspired by the multiplicative updates of non-negative matrix factorization [13]. From an inverse problem perspective, this approach is similar to low-rank regularization, with no spatial regularization.

More recently, numerous techniques have leveraged the power of deep neural networks for HS-MS image fusion [2]. Most of them are supervised approaches [14, 15, 16], trained on large datasets with known ground-truth. While they can achieve impressive performance, they can not be used in some contexts where ground-truth data is not available, or for out-of-distribution samples. But neural networks can also be used differently, in an semi- or un-supervised way, as described in the next section.

2.2. Regularization with neural networks

Inspired by the plug-and-play paradigm [17], which uses image denoisers as regularizations in splitting optimization algorithms, many works considered deep neural denoisers learnt beforehand, which can lead to impressive results [18]. Such approaches have been successfully applied in hyperspectral imaging [19, 20]. Another line of works consider generative models instead of denoisers, that can be plugged into the restoration process in a Bayesian way [21, 22]. Yet, both frameworks still require a dataset of clean HS images to train the denoiser or the generative model.

Adapting these ideas to our context requires to consider patch-based models, that can be fully trained from the MS images. Similar ideas have been used in the past with sparse representation in dictionaries [6], and more recent works consider more general patch-based representations, based on optimal transport [23] or generative models [24]. This last approach learns the density probability of patches with a normalizing flow (Patch-NF) from few data, and use it as a spatial prior similarly to [21]. Although it has never been applied

to HS-MS image fusion, it seems particularly suited to this context because the patch-NF can be learned only from the MS image, resulting in a fully unsupervised fusion technique.

3. PROPOSED METHOD

3.1. Problem formulation

Let the unknown target image be denoted by \mathbf{X} , a $N \times B$ matrix, with N pixels and B spectral bands. Let denote the MS image by matrix \mathbf{Y}_M (of size $N \times b$, with $b < B$) and the image HS by \mathbf{Y}_H (of size $n \times B$, with $n < N$). The forward (observation) model writes:

$$\mathbf{Y}_M = \mathbf{X}\mathbf{R} + \mathbf{N}_M, \quad (1)$$

$$\mathbf{Y}_H = \mathbf{L}\mathbf{X} + \mathbf{N}_H. \quad (2)$$

The left and right multiplying matrices \mathbf{R} and \mathbf{L} act as spectral and spatial degradation operators, while \mathbf{N}_M and \mathbf{N}_H denote the MS and HS noise terms, respectively. We assume here white Gaussian noise of respective variances σ_M^2 and σ_H , although more complicated models can be chosen such as multivariate Gaussians with diagonal covariance matrices [6].

To address the ill-posed or ill-conditioned nature of the fusion problem, a common strategy is to add a regularization $\phi(\mathbf{X})$ that favors some desired properties of the solution. From a statistical point of view, it amounts to introducing a prior distribution for the unknown parameter \mathbf{X} and derive Bayesian estimators. Both approaches are equivalent for the well-known Maximum A Posteriori (MAP) when choosing $p(\mathbf{X}) \propto \exp(-\lambda\phi(\mathbf{X}))$, which leads to:

$$\begin{aligned} \min_{\mathbf{X}} \frac{1}{2\sigma_M^2} \|\mathbf{Y}_M - \mathbf{X}\mathbf{R}\|_F^2 + \frac{1}{2\sigma_H^2} \|\mathbf{Y}_H - \mathbf{L}\mathbf{X}\|_F^2 + \lambda\phi(\mathbf{X}) \\ \Leftrightarrow \min_{\mathbf{X}} -\log p(\mathbf{Y}_M|\mathbf{X}) - \log p(\mathbf{Y}_H|\mathbf{X}) - \log p(\mathbf{X}) \end{aligned} \quad (3)$$

Similar to [6, 7], we propose to compute a PCA beforehand to identify the spectral subspace from the HS image. This writes $\mathbf{Y}_H = \bar{\mathbf{Z}}\mathbf{U} + \mathbf{1}_N\mu^T$, with \mathbf{U} the $K \times B$ matrix that defines the K principal components. The problem is then solved in the subspace, with the change of variable $\mathbf{X} \simeq \mathbf{Z}\mathbf{U} + \mathbf{1}_N\mu$. Defining $\bar{\mathbf{Y}}_H = \mathbf{Y}_H\mathbf{U}^T - \mathbf{L}(\mathbf{1}_N\mu^T)\mathbf{U}^T$ and $\bar{\mathbf{Y}}_M = \mathbf{Y}_M - (\mathbf{1}_N\mu^T)\mathbf{R}$, equation (3) can be simplified into

$$\begin{aligned} \hat{\mathbf{Z}}_{MAP} \in \arg \min_{\mathbf{Z}} \frac{1}{2 \cdot \sigma_M^2} \|(\bar{\mathbf{Y}}_M - \mathbf{Z}\mathbf{U}\mathbf{R})\|_F^2 \\ + \frac{K}{2 \cdot B \cdot \sigma_H^2} \|(\bar{\mathbf{Y}}_H - \mathbf{L}\mathbf{Z})\|_F^2 + \lambda\phi(\mathbf{Z}) \end{aligned} \quad (4)$$

3.2. Patch-NF regularization

In this subsection, we describe the major contribution of this paper: the Patch-NF regularization for image fusion, which

has some similarities with [24]. We assume that \mathbf{Y}_M contains all relevant spatial information, so that we can learn a normalizing flow E_θ from the patches of \mathbf{Y}_M , and use it in a second step as a prior in the fusion process.

Denoting by $\mathcal{P}(\cdot)$ the patch extraction operator, the proposed spatial regularization is simply a squared ℓ_2 -norm in the latent space of the normalizing flow, since the latent prior is a Gaussian distribution:

$$\phi(\mathbf{Z}) = \sum_{i=1}^K \lambda_i \phi(Z_i) \text{ with } \phi(Z_i) = \frac{1}{2} \|E_\theta(\mathcal{P}(Z_i))\|_F^2. \quad (6)$$

The parameters λ_i control the regularization trade-off for each principal component Z_i of \mathbf{Z} , and need to be carefully chosen as detailed below. Note that this regularization leads to a MAP in the latent space (MAP-z) that maximizes $p(\mathbf{Z}|\mathbf{Y}_M, \mathbf{Y}_H)$. We also consider the MAP-x alternative, that maximizes $p(\mathbf{X}|\mathbf{Y}_M, \mathbf{Y}_H)$ with $\mathbf{Z} = E_\theta(\mathbf{X})$, and leads to the following regularization [25]

$$\phi(Z_i) = \frac{1}{2} \|E_\theta(\mathcal{P}(Z_i))\|_F^2 - \log |\det \nabla_{Z_i} E_\theta(\mathcal{P}(Z_i))|. \quad (7)$$

We also investigate a third variant, which exploits the encoding of the MS image patches, and impose to the latent representations of the sought image patches to be close to their corresponding counterparts in the MS image:

$$\phi(Z_i) = \frac{1}{2} \|E_\theta(\mathcal{P}(Z_i))\|_F^2 - \log |\det \nabla_{Z_i} E_\theta(\mathcal{P}(Z_i))| + \frac{1}{2} \left\| E_\theta(\mathcal{P}(Z_i)) - \frac{1}{b} \sum_{j=1}^b E_\theta(\mathcal{P}(\mathbf{Y}_M^j)) \right\|_F^2. \quad (8)$$

3.3. Implementation details

We use the GLOW model [26], which achieves state-of-the-art results among normalizing flow techniques. GLOW maps the image patches to a latent space with a succession of layers (or steps of flow), that are invertible with triangular Jacobian. Each layer is composed of a normalisation with scaling and bias, 1×1 convolutions, and Affine Coupling Layers [27]. We use 5 steps of flow, a subnet of size 512 and 7×7 patches, for a total of 3 008 980 trainable weights. We use the Adam optimizer for 2000 steps with learning rate of $1e-4$. each batch is composed of 2000 random patches taken from the total 75 076 available.

Once the network is trained, we can use E_θ to regularize the image fusion problem according to equations (4) and (6). The Adam optimizer, set with a learning rate of 0.005, was utilized for the experiments documented in this paper. For this paper a fixed number of steps was used for each experiment scenario.

Furthermore, to allow the regularization of the PCA components as images, a specific scaling was applied. This scaling adjusted the values of each component to range between 0 and 1, based on its minimum and maximum values.

The tuning of the parameters λ_i is not trivial, a dynamic strategy adapted from the discrepancy principle was tested but for this paper the value of these parameters were chosen empirically.

4. EXPERIMENTS

This section evaluates the performance of our proposed PatchNF regularization, and compares the results with several state-of-the-art baselines. We will first describe the experimental setting, then give the main results and conclude with an ablation study. More details and experiments will be available at [28].

4.1. Experimental setting

We use two classical hyperspectral images: Pavia University [29] acquired by the ROSIS-3 sensor, and Indian Pines Dataset [30] acquired by AVIRIS. Both have 93 spectral bands ranging from visible to short-wave infrared and excluding the water vapour absorption bands. Pavia has 256×256 pixels, while Indian Pines is of size 145×145 . For both reference images, we simulated a pair of HS and MS images by using the spectral responses of IKONOS as in [6], and spatial Gaussian filtering followed by a subsampling of factor 4. We added white Gaussian noise with SNR 20 and 30 dB.

Working with simulated images with known ground-truth allows us to use standard quality metrics. We will consider the usual PSNR, SSIM and SAM, whose definitions can be found in [1]. We will compare our method with several classical baselines adapted to our context, i.e., a knowledge of the forward model but no supervision from an external dataset. We consider CNMF [4], FUSE [3] and Deep-SURE-Fusion [5]. Note that all methods have been modified to explicitly exploit the forward models, to allow for fair comparison.

Regarding the parameters, we opted to retain 9 components in the PCA. We used patches of size 7×7 , with a stride of 2 between patches. To help convergence of Patch-NF, we initialized it with the solution of FUSE.

4.2. Results

Table 1 shows the obtained metrics for the different methods on the two images. Overall, Patch-NF achieves intermediary results between FUSE and Deep SURE, both in performance and in computation time. Our approach obtains slightly worse results than Deep-SURE, but it is way faster. This seems mostly due to convergence issues and to our rough tuning of the 9 parameters λ_i , and we believe it is not because of the Patch-NF regularization, which seems accurate. To investigate this, we also try to initialize Patch-NF with the output of Deep-SURE. The results, shown in Table 2, confirm this intuition: with this enhanced initialization, Patch-NF is able to improve all metrics in all scenarios.

Table 1. Performance Comparison on Pavia and Indian Pines Datasets

	SNR	Method	PSNR	SSIM	SAD	Time
Pavia	20	CNMF	32.58	0.989	5.86	14s
		FUSE	34.5	0.993	4.38	0.41s
		SURE	36.6	0.996	3.05	40min
		Patch-NF	35.1	0.994	4.18	146s
	30	CNMF	31.4	0.986	4.27	14s
		FUSE	40.7	0.998	2.28	0.40s
		SURE	41.6	0.999	1.94	40min
		Patch-NF	41.1	0.999	2.21	105s
Indian Pines	20	CNMF	26.9	0.952	6.07	3.1s
		FUSE	30.0	0.976	3.81	0.16s
		SURE	31.3	0.982	3.65	14min
		Patch-NF	30.4	0.979	3.77	114s
	30	CNMF	33.2	0.988	3.25	4.9s
		FUSE	34.5	0.991	2.74	0.35s
		SURE	35.0	0.992	2.59	14min
		Patch-NF	34.7	0.992	2.74	111s

Table 2. Performance Comparison on PaviaU and Indian Pines Datasets (Deep SURE and Patch-NF Methods)

	SNR	Method	PSNR	SSIM	SAD	Time
Pavia	20	SURE	36.6	0.996	3.05	40min
		Patch-NF	36.8	0.996	2.88	40min+52s
	30	SURE	41.6	0.999	1.94	40min
		Patch-NF	42.2	0.999	1.87	40min+78s
Indian Pines	20	SURE	31.3	0.982	3.65	14min
		Patch-NF	32.3	0.986	3.29	14min+94s
	30	SURE	35.0	0.992	2.59	14min
		Patch-NF	35.2	0.993	2.55	14min+67s

4.3. Ablation study

We here compare the three proposed variants of Patch-NF detailed in equations (7), (6) and (8). We consider the scenario of Pavia University with SNR=20, and display the results in Table 3. In Table 3, a comparison is made between the three studied loss functions, made with a stride of 3 in the patch operator, on Pavia University, and with SNR=20. While the three methods perform correctly, it seems that the regularization with Encoding of MS, of equation (8) outperforms its counterparts. Despite its superior performance, we chose for this paper the solution from equation (7) due to its time efficiency. However, since the encoded solution used a rough average of all the bands, there is room for further improvement, which should be explored in future works.

5. CONCLUSION

We proposed in this short paper to use patch-normalizing flow as a spatial prior in HS-MS fusion, which leads to a simple,

Table 3. Different Patch-NR regularizations on Pavia for SNR=20 and SNR=30.

SNR	Method	PSNR	SSIM	SAD	time(s)
20	MAP Z (6)	35.1	0.993	4.19	139
	MAP X (7)	35.1	0.994	4.18	146
	Encoded (8)	36.0	0.995	3.51	626
	MAP Z (6)	41.1	0.999	2.21	104s
30	MAP X (7)	41.1	0.999	2.21	105s
	Encoded (8)	41.6	0.999	2.04	765s

fully-unsupervised method. The experiments show that it offers an interesting trade-off between performance and computation time compared to state-of-the-art unsupervised methods. Future works are needed to further improve the performance of the method, for instance by better tuning the regularization parameters and developing more optimal stopping criteria. Deeper experimental results will also be conducted, by considering larger datasets and by evaluating the robustness to a potential error in the forward models.

6. REFERENCES

- [1] L. Loncan, L. B. de Almeida, J. M. Bioucas-Dias, X. Briottet, and et al., “Hyperspectral pansharpening: A review,” *IEEE Geoscience and Remote Sensing Magazine*, vol. 3, no. 3, pp. 27–46, Sept. 2015.
- [2] G. Vivone, “Multispectral and hyperspectral image fusion in remote sensing: A survey,” *Information Fusion*, vol. 89, pp. 405–417, 2023.
- [3] Q. Wei, N. Dobigeon, and J.-Y. Tourneret, “Fast fusion of multi-band images based on solving a Sylvester equation,” *IEEE Transactions on Image Processing*, vol. 24, no. 11, pp. 4109–4121, 2015.
- [4] N. Yokoya and T. Yairi, “A coupled nonnegative matrix factorization unmixing for hyperspectral and multispectral data fusion,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 50, pp. 528–537, 2012.
- [5] H. V. Nguyen, M. O. Ulfarsson, J. R. Sveinsson, and M. Dalla Mura, “Deep SURE for unsupervised remote sensing image fusion,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 60, 2022, 5412613.
- [6] Q. Wei, J. Bioucas-Dias, N. Dobigeon, and J. Y. Tourneret, “Hyperspectral and multispectral image fusion based on a sparse representation,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 53, no. 7, pp. 3658–3668, 2015.
- [7] J. Bioucas-Dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader, and J. Chanussot, “Hyperspectral unmixing overview: Geometrical, statistical, and sparse

- regression-based approaches,” *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 5, no. 2, pp. 354–379, Apr. 2012.
- [8] J. M. Bioucas-Dias and J. M. Nascimento, “Hyperspectral subspace identification,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 46, no. 8, pp. 2435–2445, Aug. 2008.
- [9] M. Simoes, J. Bioucas-Dias, L. Almeida, and J. Chanussot, “A convex formulation for hyperspectral image super resolution via subspace-based regularization,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 53, pp. 3373–3388, 2015.
- [10] K. Chen, H. Dong, and K-S. Chan, “Reduced rank regression via adaptive nuclear norm penalization,” *Biometrika*, vol. 100, no. 4, pp. 901–920, 2013.
- [11] C. Guilloreau, T. Oberlin, O. Berné, and N. Dobigeon, “Hyperspectral and multispectral image fusion under spectrally varying spatial blurs—application to high dimensional infrared astronomical imaging,” *IEEE Transactions on Computational Imaging*, vol. 6, pp. 1362–1374, 2020.
- [12] O. Berné, A. Helens, P. Pilleri, and C. Joblin, “Non-negative matrix factorization pansharpening of hyperspectral data: An application to mid-infrared astronomy,” in *IEEE WHISPERS*, 2010.
- [13] D. D. Lee and H. S. Seung, “Learning the parts of objects by non-negative matrix factorization,” *Nature*, vol. 401, no. 6755, pp. 788–791, 1999.
- [14] W. Song, S. Li, L. Fang, and T. Lu, “Hyperspectral image classification with deep feature fusion network,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 56, no. 7, pp. 3173–3184, 2018.
- [15] Z. Shao and J. Cai, “Remote sensing image fusion with deep convolutional neural network,” *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 11, no. 5, pp. 1656–1669, 2018.
- [16] L. Chen, Z. Wei, and Y. Xu, “A lightweight spectral–spatial feature extraction and fusion network for hyperspectral image classification,” *Remote Sensing*, vol. 12, pp. 1395, 2020.
- [17] S. V. Venkatakrisnan, C. A. Bouman, and B. Wohlberg, “Plug-and-play priors for model based reconstruction,” in *IEEE GlobalSIP*, 2013, pp. 945–948.
- [18] K. Zhang, Y. Li, W. Zuo, L. Zhang, L. Van Gool, and R. Timofte, “Plug-and-play image restoration with deep denoiser prior,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 44, no. 10, pp. 6360–6376, 2021.
- [19] Z. Lai, K. Wei, and Y. Fu, “Deep plug-and-play prior for hyperspectral image restoration,” *Neurocomputing*, vol. 481, pp. 281–293, 2022.
- [20] X. Wang, J. Chen, and C. Richard, “Tuning-free plug-and-play hyperspectral image deconvolution with deep priors,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 61, pp. 1–13, 2023.
- [21] A. Bora, A. Jalal, E. Price, and A. G. Dimakis, “Compressed sensing using generative models,” in *ICML*, 2017, pp. 537–546.
- [22] M. Biquard, M. Chabert, and T. Oberlin, “Variational Bayes image restoration with compressive autoencoders,” *arXiv preprint arXiv:2311.17744*, 2023.
- [23] J. Hertrich, A. Houdard, and C. Redenbach, “Wasserstein patch prior for image superresolution,” *IEEE Transactions on Computational Imaging*, vol. 8, pp. 693–704, 2022.
- [24] F. Altekrüger, A. Denker, P. Hagemann, J. Hertrich, P. Maass, and G. Steidl, “PatchNR: Learning from very few images by patch normalizing flow regularization,” *Inverse Problems*, vol. 39, no. 6, pp. 1–31, 2023.
- [25] T. Oberlin and M. Verm, “Regularization via deep generative models: an analysis point of view,” in *IEEE ICIP*, 2021, pp. 404–408.
- [26] D. P. Kingma and P. Dhariwal, “Glow: Generative flow with invertible 1x1 convolutions,” 2018, vol. 31.
- [27] L. Dinh, J. Sohl-Dickstein, and S. Bengio, “Density estimation using real NVP,” in *ICLR*, 2017.
- [28] Afonso Carvalho, “Hyperspectral/multispectral image fusion with a neural network regularisation,” <https://drive.google.com/drive/folders/1aYppFlHks3CCtBzmKCIXJd0lXRYkgnyf?usp=sharing>, 2024.
- [29] P. Gamba, “Pavia hyperspectral remote sensing scenes,” https://www.ehu.eus/ccwintco/index.php/Hyperspectral_Remote_Sensing_Scenes, Pavia scenes provided by the Telecommunications and Remote Sensing Laboratory, University of Pavia, Italy.
- [30] M. F. Baumgardner, L. L. Biehl, and D. A. Landgrebe, “220 band aviris hyperspectral image data set: June 12, 1992 indian pine test site 3,” Sep 2015.