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# A Modified PUMA/EPUMA for Direction-of-Arrival Estimation of Coherent Sources<sup>★</sup>

Diyuan XU<sup>a</sup>, Yide WANG<sup>b</sup>, Biyun MA<sup>a,\*</sup>, Qingqing ZHU<sup>a</sup> and Julien SARRAZIN<sup>c</sup>

<sup>a</sup>School of Electronics and Information Engineering, South China University of Technology, 510640 Guangzhou, China

<sup>b</sup>Institut d'Electronique et des Technologies du Numérique (IETR), CNRS UMR6164, Nantes University, 44306 Nantes, France

<sup>c</sup>Laboratoire de Génie Electrique et Electronique de Paris, CNRS, Sorbonne Université, 75252, Paris, France; CentraleSupélec, CNRS, Université Paris-Saclay, 91192, Gif-sur-Yvette, France

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## ABSTRACT

The principal-singular-vector utilization modal analysis (PUMA) related algorithms have been proposed to address the problem of insufficient robustness of the method of direction estimation (MODE) related algorithms, which are sensitive to the parity of the number of sources due to the additional assumption and constraints on the symmetry of the root polynomial coefficients. Moreover, the MODE-related algorithms do not have severe performance degradation when the source covariance matrix is rank deficient, however, the initial PUMA-related algorithms will have a degraded performance under such circumstances. The initial PUMA is developed using a full rank source covariance matrix hypothesis, which is not valid for coherent sources. In this paper, a rigorous extension of the PUMA and enhanced-PUMA (EPUMA) is proposed to handle the case where the source covariance matrix may be rank deficient. The modified PUMA/EPUMA (Mod-PUMA/EPUMA) can be applied rigorously in the case of multiple coherent sources. In addition, it has lower computational complexity and faster convergence than the initial PUMA/EPUMA. The effectiveness of the Mod-PUMA/EPUMA is shown by experimental comparison with the initial PUMA-related algorithms and MODE-related algorithms.

## 1. Introduction

The subspace based direction-of-arrival (DOA) finding methods, such as MUSIC [1], ESPRIT [2, 3], and their improvements [4], have competitive advantages in computational complexity and resolution power, with respect to the maximum likelihood method and beamforming based methods, respectively. However, for coherent sources, the above algorithms require a preprocessing by some decorrelation techniques, e.g., spatial smoothing (SS) [5, 6], modified spatial smoothing preprocessing (MSSP) [7, 8], which significantly reduces the effective aperture of the sensor array. Unitary root-MUSIC is found to be equivalent to forward-backward spatial smoothing (FBSS) root-MUSIC [9]. Although it has a better decoherence performance compared with the conventional root-MUSIC, it is not suitable for the case where there are more than three coherent sources [10]. The method of direction estimation (MODE) can deal with correlated sources with additional constraints on the symmetry of the root polynomial coefficients [11]. However, these constraints result in some performance degradation in the case of odd source number, even for single source case [10]. To improve the performance breakdown of MODE at low SNR region, MODE with extra-roots (MODEX) is proposed in [12]. Suppose that there are  $K$  incoming


sources, MODEX runs the MODE algorithm twice with the source number being  $K$  and  $P$  ( $P > K$ ), respectively, to generate  $(P + K)$  DOA candidates, then selects  $K$  of them by a maximum likelihood (ML) based cost function [12]. It is shown that MODEX can improve the threshold performance of MODE in the case of  $K$  being odd and  $P = K + 1$  being even [10].

The principal-singular-vector utilization for modal analysis (PUMA, called classical PUMA in the rest of this paper) proposed in [13], has further reduced the computational complexity compared to MUSIC and ESPRIT [14, 15]. Besides, its estimation performance reaches the Cramér-Rao bound (CRB) [16] under conditions of sufficiently high SNR and/or a large number of samples. Even in the case of low SNR and a small number of samples, the classical PUMA surpasses MODE due to the absence of additional constraints on the polynomial coefficients [10, 17]. With a similar idea as MODEX, [18] introduces enhanced-PUMA (EPUMA, called classical EPUMA in the rest of this paper) to improve the performance breakdown in the situations of low SNR and/or a small number of samples, especially when there are coherent sources. Interestingly, [19] has shown that EPUMA and MODEX have equivalent cost functions based on different theories.

During our investigation into the DOA estimation of coherent signals with frequency beam scanning leaky-wave antennas (FBS-LWA) [20] and the time delay estimation of coherent echoes with a ground penetrating radar (GPR) [17], the classical PUMA and EPUMA in the initial papers [10] and [18] are mathematically based on a full rank source covariance matrix hypothesis. They use the eigenvectors associated with the  $K$  (the true number of sources) largest

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\*Corresponding author

 eebyma@scut.edu.cn (B. MA)

eigenvalues to reconstruct the signal subspace, which is no longer true in the coherent sources situation. Actually, the classical PUMA/EPUMA cannot work correctly if the source covariance matrix is rank deficient. Therefore, rigorously, the classical PUMA cannot handle coherent sources. However, thanks to the presence of noise and limited number of samples, the appropriate DOA estimation steps could still be conducted in simulations.

To solve this problem, the modified PUMA (Mod-PUMA) and the modified EPUMA (Mod-EPUMA) are proposed. The proposed Mod-PUMA uses the eigenvectors associated with the  $N_r$  (the actual rank of the sources covariance matrix) largest eigenvalues to reconstruct the signal subspace, which explicitly takes into account the possible rank deficiency of the source covariance matrix. The proposed Mod-EPUMA is a two-step selection strategy of the proposed Mod-PUMA based on the stochastic ML (Sto-ML) criteria to further improve the performance in the case of multiple coherent sources, while the classical EPUMA used the deterministic ML (Det-ML) criteria [18]. Additionally, due to  $K > N_r$  in multiple coherent sources case, the computational complexity of the proposed Mod-PUMA/EPUMA, using fewer signal eigenvectors during the processes, is smaller than that of the classical PUMA/EPUMA. The proposed Mod-PUMA/EPUMA is used in [20], which shows a promising estimation performance for coherent signals, without a comprehensive analysis and comparison with classical PUMA/EPUMA, which constitutes the primary focus of this paper. So this paper highlights the reason behind the improvement and emphasizes the advantages of the proposed algorithms.

The rest of the paper is organized as follows. Section 2 introduces the data model, Section 3 presents the detailed steps of the proposed Mod-PUMA/EPUMA, Section 4 provides simulation results, and Section 5 concludes the paper.

## 2. System Model

Suppose that there are  $K$  far field narrowband sources impinging on a uniform linear array (ULA) with  $N$  ( $K < N$ ) half wavelength spaced sensors from distinct directions  $\{\theta_1, \dots, \theta_K\}$ , and  $K$  is assumed to be known.

The received signals can be expressed as

$$\mathbf{y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad 1 \leq t \leq M \quad (1)$$

where  $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_N(t)]^T \in \mathbb{C}^{N \times 1}$  represents the received signal vector,  $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \in \mathbb{C}^{K \times 1}$  denotes the source signal vector,  $\mathbf{n}(t) = [n_1(t), n_2(t), \dots, n_N(t)]^T \in \mathbb{C}^{N \times 1}$  is the noise vector with zero mean and covariance matrix  $\sigma^2 \mathbf{I}_N$ , where  $\sigma^2$  is the noise power and  $\mathbf{I}_N \in \mathbb{C}^{N \times N}$  is the identity matrix. The noise is assumed to be uncorrelated with  $\mathbf{s}(t)$ .  $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{N \times K}$  is the steering matrix, where  $\mathbf{a}(\theta_i) = [1, e^{j\pi \sin(\theta_i)}, \dots, e^{j\pi(N-1)\sin(\theta_i)}]^T \in \mathbb{C}^{N \times 1}$  denotes the steering vector.  $M$  represents the number of snapshots.

The covariance matrix of  $\mathbf{y}(t)$  can be written as

$$\mathbf{\Gamma} = E[\mathbf{y}(t)\mathbf{y}(t)^H] = \mathbf{A}\mathbf{\Gamma}_s\mathbf{A}^H + \sigma^2\mathbf{I}_N \quad (2)$$

where  $\mathbf{\Gamma}_s = E[\mathbf{s}\mathbf{s}^H] \in \mathbb{C}^{K \times K}$  denotes the source covariance matrix. The rank of  $\mathbf{\Gamma}_s$  is

$$1 \leq \text{rank}(\mathbf{\Gamma}_s) = N_r \leq K \quad (3)$$

when  $N_r = K$ , the sources are uncorrelated, when  $N_r < K$ , some sources are coherent, especially, when  $N_r = 1$ , all sources are fully coherent. The classical PUMA [10] uses the eigenvectors associated with the  $K$  largest eigenvalues of  $\mathbf{\Gamma}$  to reconstruct the signals subspace, or more precisely, the mathematical development of the classical PUMA is based on a non-coherent sources hypothesis.

In this paper, we modify the classical PUMA/EPUMA methods to directly and explicitly take into account the situation when the source covariance matrix may be rank deficient.

## 3. Modified PUMA/EPUMA Algorithms

### 3.1. Modified PUMA and EPUMA

The eigenvalue decomposition of  $\mathbf{\Gamma}$  (2) can be written as [20]

$$\begin{aligned} \mathbf{\Gamma} &= \mathbf{A}\mathbf{\Gamma}_s\mathbf{A}^H + \sigma^2\mathbf{I}_N = \sum_{i=1}^{N_r} (\lambda_i + \sigma^2) \mathbf{u}_i \mathbf{u}_i^H + \sum_{i=N_r+1}^N \sigma^2 \mathbf{u}_i \mathbf{u}_i^H \\ &= \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \sigma^2 \mathbf{I}_N = \mathbf{E}_s \mathbf{D}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{D}_n \mathbf{E}_n^H \end{aligned} \quad (4)$$

where the columns of  $\mathbf{E}_s = [\mathbf{u}_1, \dots, \mathbf{u}_{N_r}]$  are the eigenvectors associated with the  $N_r$  largest eigenvalues in  $\mathbf{D}_s = \text{diag}(\lambda_1 + \sigma^2, \dots, \lambda_{N_r} + \sigma^2)$  and  $\mathbf{\Lambda}_s = \text{diag}(\lambda_1, \dots, \lambda_{N_r})$ , they span the signal subspace. The columns of  $\mathbf{E}_n = [\mathbf{u}_{N_r+1}, \dots, \mathbf{u}_N]$  are the eigenvectors associated with the  $N - N_r$  smallest eigenvalues in  $\mathbf{D}_n = \text{diag}(\sigma^2, \dots, \sigma^2)$ , they span the noise subspace.

Then from (4),

$$\mathbf{A}\mathbf{\Gamma}_s\mathbf{A}^H = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H \quad (5)$$

as the rank of  $\mathbf{\Gamma}_s$  is  $N_r$ , it can be decomposed by  $\mathbf{\Gamma}_s = \mathbf{T}\mathbf{T}^H$ , with  $\mathbf{T} \in \mathbb{C}^{K \times N_r}$ , then we have,

$$\mathbf{A}\mathbf{T}\mathbf{T}^H\mathbf{A}^H = \mathbf{E}_s \mathbf{\Lambda}_s^{1/2} \mathbf{\Lambda}_s^{H/2} \mathbf{E}_s^H \quad (6)$$

where  $\mathbf{\Lambda}_s^{1/2} \in \mathbb{C}^{N_r \times N_r}$ . From (6),

$$\mathbf{E}_s = \mathbf{A}\mathbf{T}_1 \quad (7)$$

where  $\mathbf{T}_1 \in \mathbb{C}^{K \times N_r}$ , (7) implies  $\text{span}(\mathbf{E}_s) = \text{span}(\mathbf{A}\mathbf{T}_1)$ .

The Mod-PUMA is based on the following orthogonal relation obtained from the linear prediction theory as in [18],

$$\mathbf{B}\mathbf{A} = \mathbf{0} \quad (8)$$

where  $\mathbf{B} \in \mathbb{C}^{(N-K) \times N}$  is a Toeplitz matrix, given by

$$\mathbf{B} = \begin{bmatrix} c_K & c_{K-1} & \cdots & c_0 & 0 & \cdots & 0 & 0 \\ 0 & c_K & \cdots & \cdots & c_0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 0 & c_K & \cdots & \cdots & c_0 \end{bmatrix} \quad (9)$$

(8) means that for the  $\tilde{k}^{th}$  column in  $\mathbf{A}$ , the  $l^{th} \geq K+1$  row is a linear combination of the  $K$  previous rows, which can be expressed as

$$z_{\tilde{k}}^l + \sum_{i=1}^K c_i z_{\tilde{k}}^{l-i} = 0, \quad 1 \leq \tilde{k} \leq K, K+1 \leq l \leq N \quad (10)$$

where  $c_0 = 1$ ,  $z_{\tilde{k}} = e^{j\pi \sin(\theta_{\tilde{k}})}$ . Now, the objective is to find the value of  $c_i$ , then the directions  $\theta_1, \dots, \theta_K$  can be obtained by the roots of the following polynomial of order  $K$ ,

$$\sum_{i=0}^K c_i z^{K-i} = 0 \quad (11)$$

From (7) and (8),

$$\mathbf{B}\mathbf{A}\mathbf{T}_1 = \mathbf{B}\mathbf{E}_s = \mathbf{B} \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{N_r} \end{bmatrix} = \mathbf{0}_{(N-K, N_r)} \quad (12)$$

thus, for the  $k^{th}$  column of  $\mathbf{E}_s$ ,

$$[\mathbf{u}_k]_l + \sum_{i=1}^K c_i [\mathbf{u}_k]_{l-i} = 0, \quad 1 \leq k \leq N_r, K+1 \leq l \leq N \quad (13)$$

Note, the maximum value of  $k$ , which is also the number of the retained signal subspace vectors, is not the true number of sources  $K$  as used in [18], but the actual rank  $N_r$  of the source covariance matrix, which is the main difference between the proposed modified PUMA and the original one, making the proposed method rigorous for estimating the DOAs of coherent sources.

So (13) can be rewritten as

$$\mathbf{F}_k \mathbf{c} - \mathbf{g}_k = \mathbf{0}_{(N-K, 1)}, \quad 1 \leq k \leq N_r \quad (14)$$

where

$$\mathbf{F}_k = \begin{bmatrix} [\mathbf{u}_k]_K & [\mathbf{u}_k]_{K-1} & \cdots & [\mathbf{u}_k]_1 \\ [\mathbf{u}_k]_{K+1} & [\mathbf{u}_k]_K & \cdots & [\mathbf{u}_k]_2 \\ \vdots & \vdots & \ddots & \vdots \\ [\mathbf{u}_k]_{N-1} & [\mathbf{u}_k]_{N-2} & \cdots & [\mathbf{u}_k]_{N-K} \end{bmatrix}_{(N-K, K)} \quad (15)$$

$$\mathbf{g}_k = - \begin{bmatrix} [\mathbf{u}_k]_{K+1} \\ [\mathbf{u}_k]_{K+2} \\ \vdots \\ [\mathbf{u}_k]_N \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_K \end{bmatrix}$$

and

$$\text{vec}(\mathbf{B}\mathbf{E}_s) = \mathbf{F}\mathbf{c} - \mathbf{g} = \mathbf{0}_{((N-K)N_r, 1)} \quad (16)$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_{N_r} \end{bmatrix}_{((N-K)N_r, K)}, \quad \mathbf{g} = \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \vdots \\ \mathbf{g}_{N_r} \end{bmatrix}_{((N-K)N_r, 1)} \quad (17)$$

Due to the presence of noise, (16) is only an approximation. As proposed in [18],  $\mathbf{c}$  can be obtained by the following weighted least squares (WLS) solution,

$$\hat{\mathbf{c}}_{wls} = (\mathbf{F}^H \mathbf{W} \mathbf{F})^{-1} \mathbf{F}^H \mathbf{W} \mathbf{g} \quad (18)$$

with  $\mathbf{W} \cong \hat{\mathbf{T}} \otimes (\mathbf{B}\mathbf{B}^H)^{-1}$  and the diagonal matrix  $\hat{\mathbf{T}}$  defined as

$$\hat{\mathbf{T}} = \text{diag} \left( \frac{\lambda_1^2}{\lambda_1 + \sigma^2}, \frac{\lambda_2^2}{\lambda_2 + \sigma^2}, \dots, \frac{\lambda_{N_r}^2}{\lambda_{N_r} + \sigma^2} \right) \quad (19)$$

with  $\sigma^2 = \frac{\text{tr}(\hat{\mathbf{D}}_n)}{N-N_r}$ .

If after a preprocessing by some decorrelation techniques, the rank of the source covariance is fully recovered, then  $N_r$  is equivalent to  $K$ , as in [18]. However, if the rank of the source covariance matrix is partially recovered ( $N_r < K$ ), then  $\lambda_{N_r+1} = \dots = \lambda_K = 0$ , making  $\mathbf{T}_c = \text{diag} \left( \frac{(\lambda'_1 - \sigma^2)^2}{\lambda'_1}, \frac{(\lambda'_2 - \sigma^2)^2}{\lambda'_2}, \dots, \frac{(\lambda'_K - \sigma^2)^2}{\lambda'_K} \right)$ , where  $\lambda'_i = \lambda_i + \sigma^2$ , as in [18], a singular matrix (non-invertible matrix) under no noise and infinite number of snapshots situation ( $\lambda'_i = \sigma^2 = 0, i = N_r + 1, \dots, K$ ).

In practice, thanks to the presence of noise and the finite number of snapshots, the last  $K - N_r$  elements on the diagonal of  $\mathbf{T}_c$  are not exactly zero, then  $\mathbf{T}_c$  is still invertible to conduct the subsequent calculations, even it is very ill-conditioned.

Moreover, in coherent sources case, the eigenvectors associated with the  $N_r$  largest eigenvalues,  $\mathbf{u}_1, \dots, \mathbf{u}_{N_r}$ , span the signal subspace, so only  $N_r$  eigenvectors are needed to reconstruct  $\mathbf{F}$ . Therefore the proposed modification can reduce the computational complexity due to not only the decreased dimensions of the matrices  $\mathbf{F}$ ,  $\mathbf{g}$ ,  $\mathbf{T}$ , and  $\mathbf{W}$  by a factor of  $K/N_r$ , but also the reduced number of iterations  $n_{iter}$  by avoiding the possibility of mixing signal and noise eigenvectors [21].

It can be seen from (9) and (18) that the values of  $\mathbf{W}$  and  $\mathbf{c}$  depend on each other. Consequently, the Mod-PUMA is summarized in the following steps to obtain the  $K$  DOA estimates.

1. Estimate the covariance matrix  $\mathbf{\Gamma}$  by  $\hat{\mathbf{\Gamma}} = \frac{1}{M} \sum_{t=1}^M [\mathbf{y}(t)\mathbf{y}(t)^H] \in \mathbb{C}^{N \times N}$ .
2. Let  $\mathbf{\Gamma} = \hat{\mathbf{\Gamma}}$ , then make the eigenvalue decomposition of  $\mathbf{\Gamma}$  and find  $\mathbf{E}_s$  by (4).
3. Obtain  $\mathbf{F}$  and  $\mathbf{g}$  according to (13)-(17).
4. Calculate the initial  $\mathbf{c}$  by the LS method,  $\hat{\mathbf{c}} = \hat{\mathbf{c}}_{ls} = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{g}$ .
5. Find the weighting matrix  $\mathbf{W}$  with  $\hat{\mathbf{c}}$  by  $\mathbf{W} \cong \hat{\mathbf{T}} \otimes (\mathbf{B}\mathbf{B}^H)^{-1}$  and (9), (18).
6. Calculate  $\hat{\mathbf{c}}_{wls}$  with  $\mathbf{W}$  by (18).
7. Determine whether  $\|\hat{\mathbf{c}} - \hat{\mathbf{c}}_{wls}\|_2$  becomes stable, if not let  $\hat{\mathbf{c}} = \hat{\mathbf{c}}_{wls}$ , and repeat steps 5 and 6 until the stable criterion is satisfied, then obtain  $\hat{\mathbf{c}}$ .

**Table 1**

Comparison of computational complexity between the classical EPUMA and the Mod-EPUMA

Equation	Classical EPUMA computational complexity	Mod-EPUMA computational complexity
$\mathbf{W}$	$\mathcal{O}(K(N - P)^3)$	$\mathcal{O}(N_r(N - P)^3)$
$\mathbf{c}_{WLS}$	$\mathcal{O}(2P^2K(N - P) + 2PK^2(N - P)^2 + P^3 + PK(N - P))$	$\mathcal{O}(2P^2N_r(N - P) + 2PN_r^2(N - P)^2 + P^3 + PN_r(N - P))$
Total complexity	$\mathcal{O}(N^2M + N^3 + G(3N^3 + 4NK^2 + 2N^2K + 2K^3) + n_{iter}(N^3K^3 - 11NK^2 - 2N^2K + 4K^3N + 4K^2N^2))$	$\mathcal{O}(N^2M + N^3 + G(3N^3 + 4NK^2 + 2N^2K + 2K^3) + n_{iter}(N^3N_r^3 - 11NN_r^2 - 2N^2N_r + 4N_r^3N + 4N_r^2N^2))$

The Sto-ML cost function as shown in (21) is applied in the classical EPUMA and the Mod-EPUMA.

8. Find the  $K$  roots  $z_{\tilde{k}}$  of the polynomial (11), then the  $K$  DOA candidates are obtained by

$$\hat{\theta}_{\tilde{k}} = \sin^{-1} \left( \frac{\angle z_{\tilde{k}}}{\pi} \right), \quad \tilde{k} = 1, 2, \dots, K \quad (20)$$

The Mod-EPUMA is a two step DOA selection strategy of the Mod-PUMA based on the Sto-ML criteria to improve the estimation performance, while the initial EPUMA used the Det-ML criteria [18]. So the steps of the Mod-EPUMA are summarized as follows,

1. Apply the Mod-PUMA twice, first with the actual source number  $K$ , then with a supposed larger source number  $P$  ( $P > K$ ), to generate  $(K + P)$  DOA candidates.

2. Select the  $K$  DOAs from the  $(K + P)$  DOA candidates based on the Sto-ML cost function as follows,

2.1. Divide the  $(K + P)$  DOA candidates into  $G = \frac{(P+K)!}{K!P!}$  different groups with  $K$  DOAs in each group, represented by  $\Theta_1, \dots, \Theta_G$ , corresponding to  $G$  different  $\mathbf{A}$ , i.e.,  $\mathbf{A}(\Theta_1), \dots, \mathbf{A}(\Theta_G)$ .

2.2. Calculate the Sto-ML cost function  $L(\Theta_i)$  of each  $\mathbf{A}(\Theta_i)$ ,  $i \in \{1, \dots, G\}$ ,

$$L(\Theta_i) = \log \left\{ \det \left( \mathbf{P}_A \mathbf{\Gamma} \mathbf{P}_A + \frac{\text{tr}(\mathbf{P}_A^\perp \mathbf{\Gamma}) \mathbf{P}_A^\perp}{N - K} \right) \right\} \quad (21)$$

with  $\mathbf{P}_A = \mathbf{A}(\Theta_i)(\mathbf{A}^H(\Theta_i)\mathbf{A}(\Theta_i))^{-1}\mathbf{A}^H(\Theta_i)$ , and  $\mathbf{P}_A^\perp = \mathbf{I}_N - \mathbf{P}_A$ .

2.3 Choose the group  $\Theta_i$  with the minimum  $L(\Theta_i)$ .

### 3.2. Complexity analysis

Compared with the classical PUMA/EPUMA, the Mod-PUMA/EPUMA can approximately reduce the dimensions of the matrices  $\mathbf{F}$ ,  $\mathbf{g}$ ,  $\mathbf{T}$  and  $\mathbf{W}$  by a factor of  $K/N_r$ . Consequently, the complexity of the Mod-PUMA/EPUMA is reduced. With the number of iterations  $n_{iter}$ , the computational complexity of some key steps and the total scheme of the Mod-EPUMA and the classical EPUMA are detailed in Table 1.

## 4. Simulation Results

The performance of the Mod-PUMA/EPUMA, the classical PUMA/EPUMA, and the MODE/MODEX is compared in the simulations. We consider a 10 half wavelength spaced elements ULA receiving  $K$  coherent sources, with

coherence coefficient<sup>1</sup>  $\rho = \mathbf{1}$  unless stated otherwise. The number of iterations  $n_{iter}$  in the MODE/PUMA-related algorithms is 3, the number of snapshots  $M$  is 50 and the number of Monte Carlo test  $I$  is 100. MODEX/EPUMA employs the MODE/PUMA twice with  $P = K$  and  $P = K + 1$ , respectively. The remaining simulations use the same parameters, unless otherwise specified. Additionally, the rank of the source covariance matrix  $N_r$  is set to 1 and 2 after employing FBSS for coherent sources. RMSE is used to assess the performance of the compared algorithms, which is defined as

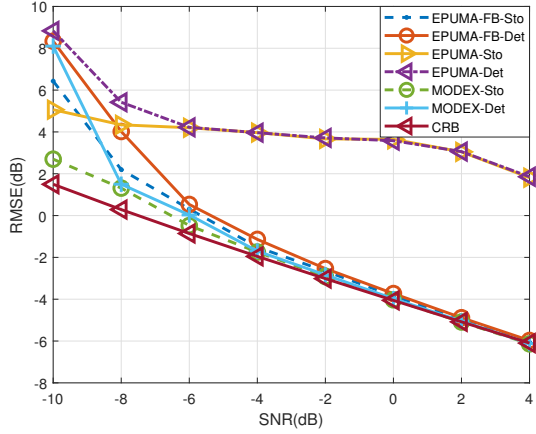
$$RMSE = \left( \frac{1}{KI} \sum_{\tilde{k}=1}^K \sum_{i=1}^I (\hat{\theta}_{\tilde{k},i} - \theta_{\tilde{k}})^2 \right)^{1/2} \quad (22)$$

where  $\hat{\theta}_{\tilde{k},i}$  is the  $\tilde{k}^{th}$  estimated angle obtained from the  $i^{th}$  Monte Carlo test,  $\theta_{\tilde{k}}$  is the  $\tilde{k}^{th}$  true angle.

In the first simulation, we compare the performance of the MODEX and the classical EPUMA under rank deficiency of the source covariance matrix situations and different ML cost functions. Three coherent sources come from the directions  $[1^\circ, 8^\circ, 35^\circ]$ , and SNR is defined as the ratio between the sum of signal powers and the complex additive white Gaussian noise power. In the case of fully coherent sources, the rank of the source covariance matrix is 1. Because the matrix  $\mathbf{F}^H \mathbf{W} \mathbf{F}$  in the classical algorithms is ill-conditioned, the classical EPUMA still suffers from rank deficiency. The  $K$  eigenvectors used in EPUMA actually involve the noise subspace, making it sensible for the presence of the noise. Therefore, the performance of EPUMA-Sto and EPUMA-Det is poor in Fig.1. With FBSS<sup>2</sup> method, the rank of the source covariance matrix has been partly recovered and becomes to 2. At this point, less information about the noise interferes with the DOA estimation and thus the performance of EPUMA-FB-Sto and EPUMA-FB-Det is greatly improved. In conclusion, the classical EPUMA cannot efficiently handle the case of fully coherent sources. However,

<sup>1</sup>In terms of correlated signals,  $\mathbf{s}(t)$  in (1) can be expressed as  $\mathbf{s}(t) = \rho \mathbf{s}_0(t)$  with generation source  $\mathbf{s}_0(t)$  and coherence coefficient vector  $\rho = [\rho_1, \rho_2, \dots, \rho_K]^T$ .

<sup>2</sup>Note that FBSS is expressed by  $\mathbf{\Gamma}_{FB} = \frac{1}{2}(\mathbf{\Gamma} + \mathbf{J}_1 \mathbf{\Gamma}^* \mathbf{J}_1)$ , where  $\mathbf{J}_1$  is the  $(N \times N)$  anti-identity matrix (exchange matrix). Since FBSS is equivalent to MSSP with only one subarray, in  $K \geq 3$  fully coherent sources case, FBSS can only partially recover the rank of the source covariance matrix. Note, after preprocessing by FBSS,  $N_r = 2$  for the full multiple coherent sources when  $K \geq 2$ .



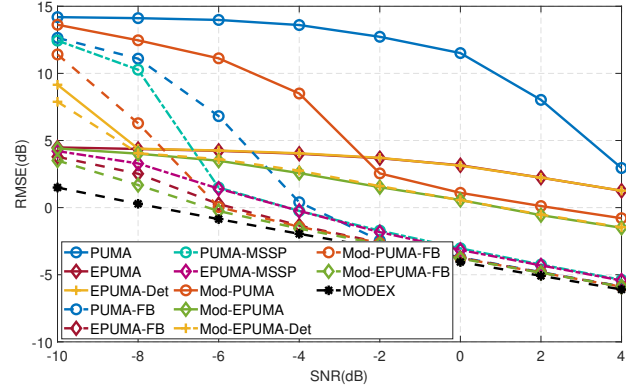
**Figure 1:** DOA estimation of EPUMA/MODEX for three fully coherent sources from directions  $1^\circ, 8^\circ, 35^\circ$

the presence of noise and limited number of snapshots allow the classical methods to continue to work. In fact, for limited snapshots, the noise free sources covariance matrix is not strictly singular, even if it is very ill-conditioned, it behaves like that the sources are not fully coherent.

Despite the incorporation of the FBSS method, EPUMA-FB continues to exhibit the rank-deficiency issue, thereby rendering its performance at low SNR susceptible to the interference of noise. In contrast, MODEX is not subject to this limitation and therefore exhibits superior performance compared to EPUMA-FB. Furthermore, the addition of FBSS does not result in any additional enhancement in the performance of MODEX [22]. Therefore, in the following simulations, FBSS is only applied to PUMA-related algorithms.

Moreover as shown in Fig.1, for either MODEX or classical EPUMA algorithms, Sto-ML always has a clearly distinct advantage over Det-ML. Despite the higher computational complexity of Sto-ML, it greatly improves the performance in the case of coherent sources, which has been proven to make MODEX more performant, especially in low SNR regions [12]. Therefore, although the classical EPUMA adopts the Det-ML cost function in [10] and [18], the Sto-ML cost function is used in the MODEX, EPUMA, and Mod-EPUMA in the following coherent sources scenarios.

In the second simulation, the performance of the classical PUMA/EPUMA and Mod-PUMA/EPUMA is compared with three coherent sources  $[1^\circ, 8^\circ, 35^\circ]$ . As shown in Fig. 2, the threshold performance of the classical PUMA is poor, but greatly improved when combined with the FBSS method. It is clear that  $N_r = 1$  for fully coherent sources and  $N_r = 2$  after adopting FBSS. At this point, for the classical PUMA and the classical PUMA with FBSS,  $\mathbf{T}_c$  is ill-conditioned, and  $\mathbf{F}$  is reconstructed by the eigenvectors  $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ . So the classical PUMA-related algorithms still face the problem of rank deficiency, however thanks to the limited snapshots (making the source covariance matrix not strictly rank deficient) and noise, they can still work. Compared



**Figure 2:** DOA estimation of classical PUMA/EPUMA and Mod-PUMA/EPUMA for three fully coherent sources from directions  $1^\circ, 8^\circ, 35^\circ$

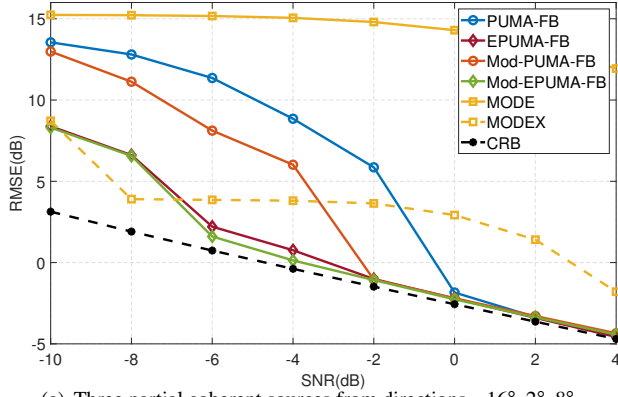
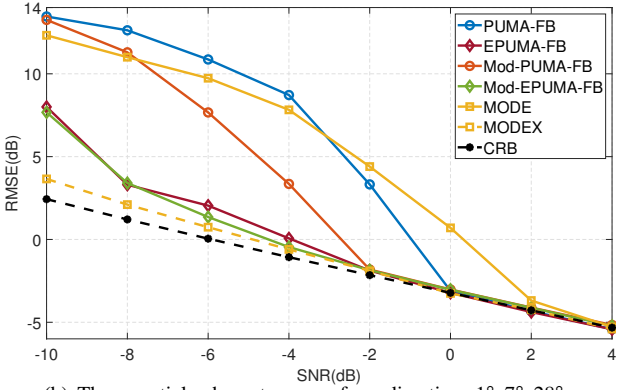
with the classical PUMA, the performance of the proposed Mod-PUMA, where  $\mathbf{F}$  is reconstructed only by  $\mathbf{u}_1$ , shows a significant improvement at high SNR region, which matches the aforementioned motivation of the proposed modification.

Meanwhile, the impact of the MSSP<sup>3</sup> and FBSS is also compared. The MSSP method could fully recover the rank of the source covariance matrix to make the classical PUMA and EPUMA work properly but with the cost of a reduced effective antenna aperture. Due to its higher dimensionality of the covariance matrix compared to MSSP, the same algorithm with FBSS could perform better. Note that after adopting FBSS, the classical PUMA and EPUMA algorithms still have the problem of rank deficiency for  $K = 3$ , and (18) in classical PUMA/EPUMA algorithms still suffer from ill-condition problems for subsequent computations. So compared with the PUMA-related algorithms, due to the consideration of rank deficiency, the Mod-PUMA/Mod-PUMA-FB shows a significant improvement over PUMA/PUMA-FB at low SNR region, while the Mod-EPUMA-related algorithms perform better than the classical EPUMA-related algorithms, especially in low SNR region.

Besides, the effect of the Sto-ML cost function is also considered in Fig.2. Compared with Mod-EPUMA-Det (with Det-ML cost function), the performance of Mod-EPUMA is improved owing to the noise resistance of the Sto-ML cost function when  $\text{SNR} = 10\text{ dB}$ . However, the gain of estimate accuracy at high SNR is still attributed to the proposed modification as illustrated by the comparison of the RMSE of EPUMA/EPUMA-Det and Mod-EPUMA/Mod-EPUMA-Det. Overall, the Mod-EPUMA with FBSS outperforms the other PUMA-related algorithms by a considerable margin.

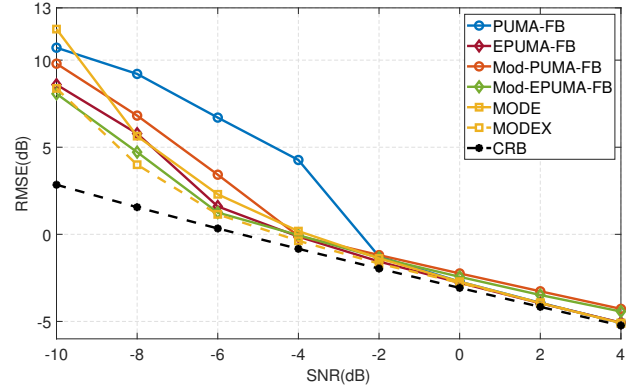
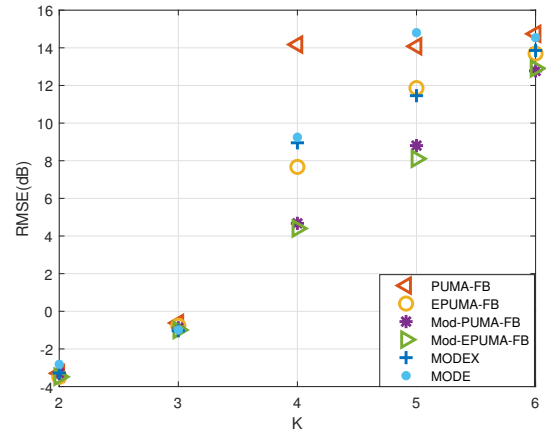
In the third simulation, the performance of MODE-related algorithms and Mod-PUMA-related algorithms is

<sup>3</sup>MSSP partitions  $N$  sensors into  $Q$  overlapping subarrays, each consisting of  $L$  elements. The equation of MSSP is  $\mathbf{\Gamma}_{MSSP} = \frac{1}{2Q} \sum_{i=1}^Q (\mathbf{\Gamma}_i + \mathbf{J}_2 \mathbf{\Gamma}_i^* \mathbf{J}_2)$ , where  $\mathbf{J}_2$  is the anti-identity matrix with dimension  $(L \times L)$ ,  $\mathbf{\Gamma}_i$  is the covariance matrix of the subarrays with dimension  $(L \times L)$ . When  $L \geq K$ ,  $2Q \geq K$ , MSSP can totally recover the rank of the source covariance matrix. Here,  $L = 8$ .


(a) Three partial coherent sources from directions  $-16^\circ, 2^\circ, 8^\circ$ 

(b) Three partial coherent sources from directions  $1^\circ, 7^\circ, 28^\circ$ 
**Figure 3:** DOA estimation of Mod-PUMA/EPUMA and MODE/MODEX for three partial coherent sources with  $\rho = [1 \ 0.9 \ 0.9]^T$ 

compared with an odd number of coherent sources. Considering the commonly partial coherent scenarios, three partial coherent sources come from  $[-16^\circ, 2^\circ, 8^\circ]$  in Fig.3 (a) and  $[1^\circ, 7^\circ, 28^\circ]$  in Fig.3 (b) with coherence coefficient  $\rho = 0.9$ , respectively. Compared with Fig.3(a) and Fig.3(b), Mod-PUMA continues to improve on the classical PUMA and Mod-EPUMA-FB always maintains good performance. Despite the performance being changed with different parameters, the difference in the RMSE with the same SNR for those of Mod-PUMA-related algorithms is not as noticeable as those obtained by MODE-related algorithms. It indicates that the MODE-related algorithms are not as robust as the Mod-PUMA-related algorithms when the number of sources is odd, which matches the statement mentioned in [10]. Besides, owing to less noise interference, Mod-PUMA-related methods, the modification of PUMA-related algorithms, tend to be consistent and converge faster than MODE-related algorithms and their original at high SNR.

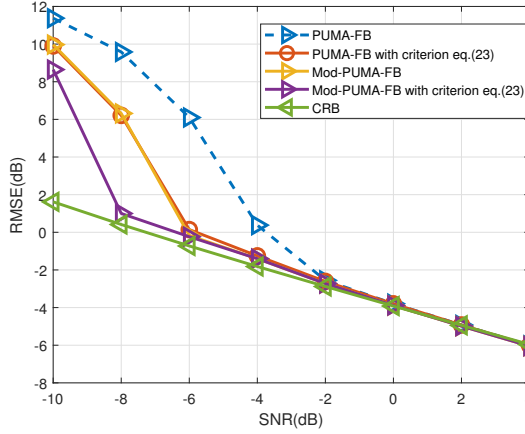
In the fourth simulation, the performance of MODE/MODEX, classical PUMA/EPUMA, and Mod-PUMA/EPUMA is compared in the case of an even number of coherent sources with one incoherent source coming from  $1^\circ$  and three partial from  $[8^\circ, 35^\circ, 56^\circ]$ . As shown in Fig.4, as the conclusions obtained by the second simulation, Mod-PUMA/EPUMA-FB retains an improvement over


**Figure 4:** DOA estimation of Mod-PUMA/EPUMA and MODE/MODEX for four partial coherent sources from directions  $1^\circ, 8^\circ, 35^\circ, 56^\circ$  with  $\rho = [0 \ 1 \ 0.9 \ 0.9]^T$ 

**Figure 5:** RMSE performance versus number of sources with SNR=  $-5$ dB

PUMA/EPUMA-FB in the mixed coherent incoming wave scenario while MODE/MODEX achieves a more accurate estimate at low SNR regions. The experimental results validate the effectiveness of the proposed modification to PUMA-related algorithms as mentioned in Section 3. In addition, with the modification of PUMA/EPUMA, the performance of the Mod-PUMA/EPUMA-FB is unaffected by the number of sources and more robust than the case of the MODE-related algorithms.

In the fifth simulation, we compare the RMSE performance versus the number of coherent sources  $K$  with SNR=  $-5$ dB. As the error is considered significant when RMSE is greater than 10dB, the cases when  $K > 6$  are not shown in Fig.5.

As shown in Fig.5, there is little difference between MODEX, EPUMA, and Mod-PUMA algorithms in the cases where  $K = 2, 3$ . However, MODE starts to deteriorate when  $K \geq 4$ . Furthermore, the Mod-PUMA and Mod-EPUMA algorithms show significant advantages when  $K$  is between 2 and 5, consistent with the previous results, which is attributed to less interference from the noise. The performance of Mod-PUMA-related algorithms is relatively robust



**Figure 6:** RMSE performance versus SNR with different scenarios to stop iteration, three coherent sources from directions  $1^\circ, 8^\circ, 35^\circ$

with different numbers of sources, while the performance of MODE-related algorithms and classical PUMA-related algorithms deteriorates rapidly when there are 5 sources. Similarly, the performance improvement between the Mod-PUMA and classical PUMA is more significant than that between Mod-EPUMA and classical EPUMA. Overall, the performance of Mod-PUMA-related algorithms is more robust than MODE-related algorithms.

In the sixth simulation, we compare two scenarios to stop the iterations in the PUMA-related algorithms, which greatly affect the values of  $n_{iter}$  and  $\mathbf{c}_{wls}$ . Firstly, the iteration stops when the following equation is satisfied,

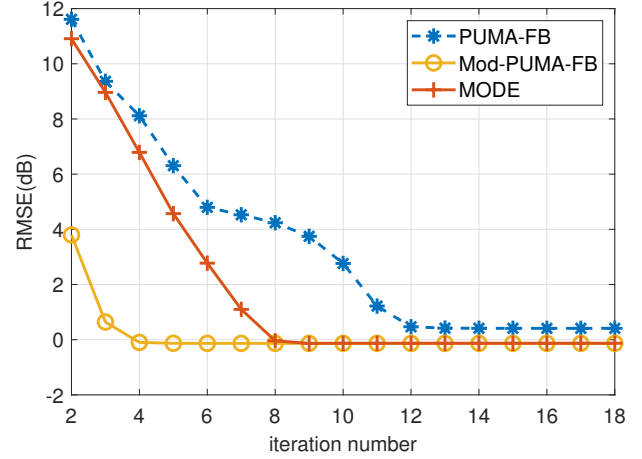
$$\|\mathbf{c}_{wls}[n_{iter}] - \mathbf{c}_{wls}[n_{iter} - 1]\|_2 < \epsilon \quad (23)$$

where  $\mathbf{c}_{wls}[n_{iter}]$  represents the  $\mathbf{c}_{wls}$  acquired at the  $n_{iter}^{th}$  iteration. When we set  $\epsilon = 10^{-4}$ , the Mod-PUMA always has fewer iteration numbers than the classical PUMA.

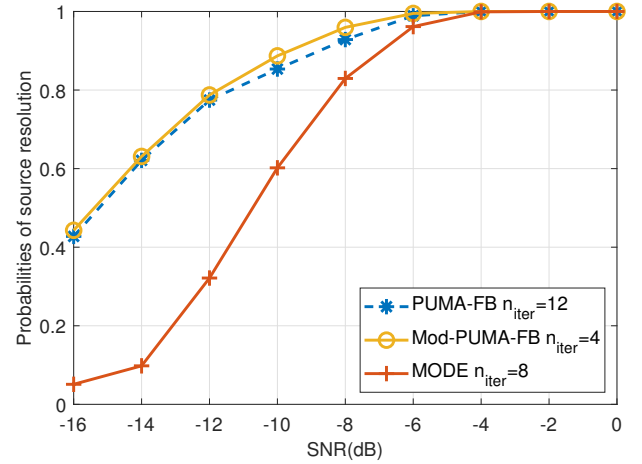
Secondly,  $n_{iter}$  is set to 3 directly as suggested in [18]. Fig.6 plots the RMSE performance versus SNR with different stopping conditions. The criterion equation (23) outperforms the second scenario due to bigger iteration numbers. Meanwhile, it shows that the proposed Mod-PUMA has better threshold performance in low SNR cases than the classical PUMA with the same scenario to stop iteration.

As EPUMA is simply a two-step strategy for PUMA, the different stopping conditions with EPUMA are not discussed. Therefore, The seventh simulation shows the RMSE performance of the classical PUMA, Mod-PUMA and MODE versus iteration number  $n_{iter}$ , which greatly affects the value of  $\mathbf{c}_{wls}$ .

As shown in Fig. 7, the performance of all algorithms can be improved by increasing the iteration number. When  $n_{iter}$  is greater than 4, 12 and 8, the RMSE of the Mod-PUMA-FB, PUMA-FB and MODE tend to be stable, respectively. It shows that the PUMA-FB and MODE are more sensitive to the iteration number than the Mod-PUMA-FB, especially in low SNR regions. In small iteration number cases, the



**Figure 7:** RMSE performance versus iteration number, three coherent sources from directions  $1^\circ, 7^\circ, 28^\circ$ , SNR =  $-5$ dB



**Figure 8:** Probabilities of source resolution with three coherent sources from directions  $1^\circ, 7^\circ, 28^\circ$ ,  $I = 2000$ .

Mod-PUMA-FB always outperforms the PUMA-FB and MODE. In other words, the Mod-PUMA is more performant and converges faster than the classical PUMA and MODE. This simulation shows that the proposed modification, using the eigenvectors associated with the  $N_r$  largest eigenvalues to reconstruct  $\mathbf{F}$ , can reduce the iteration number  $n_{iter}$  to achieve similar performance by avoiding the possibility of mixing signal and noise eigenvectors.

The last simulation shows the probabilities of source resolution  $P_r$  with three coherent sources from directions  $[1^\circ, 7^\circ, 28^\circ]$  of different algorithms, which is calculated by  $P_r = I_s/I$ , where  $I_s$  represents the number of successful separations of sources, and the number of Monte Carlo iterations  $I$  is 2000. According to Fig.7, the iterative number of MODE  $n_{iter-M}$  is 8, and the iterative number of PUMA and Mod-PUMA is 12 and 4, respectively. When  $|\hat{\theta}_i - \theta_i| < \frac{\Delta\theta}{2}$ ,  $i = 1, 2, 3$  is met, it counts a successful separation of sources, where  $\Delta\theta = \min |\theta_m - \theta_n|$ ,  $1 \leq n < m \leq 3$ , and  $\hat{\theta}_i$  is the estimated angle of  $\theta_i$ .



As shown in Fig.8, MODE performs the worst, the probability of source resolution of Mod-PUMA-FB is slightly better than that of the classical PUMA-FB. Note that, the iterative number of the classical PUMA is bigger than Mod-PUMA, so the computational complexity of the classical PUMA is higher than that of Mod-PUMA to achieve similar performance. Additionally, PUMA-related algorithms are significantly superior to MODE-related algorithms in terms of resolution for this simulation.

## 5. Conclusion

In this paper, we have discussed the performance of the MODE-related algorithms and PUMA-related algorithms in deficient rank source covariance matrix situations. We have explicitly taken into account the possible rank deficiency of the source covariance matrix in the development of the PUMA algorithm, which is ignored in the initial paper. The simulation results show that the proposed Mod-PUMA/EPUMA have higher estimation accuracy, faster convergence speed and lower computational complexity than the classical PUMA/EPUMA, respectively, in the case of multiple coherent sources. Moreover, compared with the MODE and MODEX algorithms, the Mod-PUMA/EPUMA, which evolved from PUMA/EPUMA, is also not sensitive to the parity of the number of sources.

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**DIYUAN XU** received the B.S. degree in railway traffic signaling and control from Southwest Jiaotong University, Chengdu, China, in 2020. She received the M.S. degree in wireless embedded technologies from the Nantes University, France, in 2022. She is currently pursuing the M.S. degree in information engineering with South China University of Technology, Guangzhou, China. Her research interests include array signals processing, signal processing, and wireless communication.

**YIDE WANG** received the B.S. degree in electrical engineering from the Beijing University of Post and Telecommunication, Beijing, China, in 1984, and the M.S. and the Ph.D. degrees in signal processing and telecommunications from the University of Rennes, France, in 1986 and 1989, respectively. He is currently a Professor with the Ecole Polytechnique de Nantes Université. His research interests include array signal processing, spectral analysis, mobile wireless communication systems, signal processing for fault detection and diagnosis in electrical machines.

**BIYUN MA** received the B.S. degree in information engineering from the South China University of Technology, Guangzhou, China, in 2004, and the M.S. and the Ph.D. degrees in electronic engineering from the Nantes University, France, in 2007 and 2010, respectively. She is currently an associate professor with the School of Electronic and Information Engineering of South China University of Technology. Her research interests include

array signals processing, mobile wireless communication systems, signal processing for ultrasonic detection and communication.

**QINGQING ZHU** received the B.S. degree in information engineering from South China University of Technology, Guangzhou, China, in 2022, where she is currently pursuing the M.S. degree in electronic information. Her current research interests include array signal processing, spectral estimation, and signal processing for communications.

**JULIEN SARRAZIN** received his Master and PhD degrees from the University of Nantes in France, in 2005 and 2008 respectively. In 2009 and 2010, he worked with the BK Birla Institute of Technology of Pilani, India. In 2011 and 2012, he was a research engineer at Telecom ParisTech in Paris. Since September 2012, he is an Associate Professor at Sorbonne Université in Paris. His research interests include antenna design, spatial data focusing and channel modeling.

**Diyuan XU:** Conceptualization, Methodology, Software, Formal analysis, Writing - Original Draft.

**Yide WANG:** Methodology, Formal analysis, Writing - Review & Editing, Funding acquisition.

**Biyun MA:** Methodology, Formal analysis, Writing - Review & Editing, Funding acquisition.

**Qingqing ZHU:** Formal analysis, Writing - Review & Editing.

**Julien SARRAZIN:** Methodology, Formal analysis.

**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

DIYUAN XU received the B.S. degree in railway traffic signaling and control from Southwest Jiaotong University, Chengdu, China, in 2020. She received the M.S. degree in wireless embedded technologies from the Nantes University, France, in 2022. She is currently pursuing the M.S. degree in information engineering with South China University of Technology, Guangzhou, China. Her research interests include array signals processing, signal processing, and wireless communication.

YIDE WANG received the B.S. degree in electrical engineering from the Beijing University of Post and Telecommunication, Beijing, China, in 1984, and the M.S. and the Ph.D. degrees in signal processing and telecommunications from the University of Rennes, France, in 1986 and 1989, respectively.

He is currently a Professor with the Ecole Polytechnique de Nantes Université. His research interests include array signal processing, spectral analysis, mobile wireless communication systems, signal processing for fault detection and diagnosis in electrical machines.

BIYUN MA received the B.S. degree in information engineering from the South China University of Technology, Guangzhou, China, in 2004, and the M.S. and the Ph.D. degrees in electronic engineering from the Nantes University, France, in 2007 and 2010, respectively.

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