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Stock-Flow-Consistent Macroeconomic Dynamics in Continuous Time

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1 Introduction

The literature on social-ecological systems now widely recognizes that such systems can be characterized as intertwined, complex adaptive systems (Folke et al. 2003; Norberg and Cumming 2008, Schoon and Van Der Leeuw 2015; Preiser et al. 2018; De Vos et al. 2019; Schlüter et al. 2019). This implies, among other things, that social-ecological systems need to be understood as having context-sensitive adaptive capacities, which come about as the emergent outcomes of multiple non-linear, recursive causal interactions that cause spill-over effects across different spatial and temporal scales (Levin and Heshaimer 2000).

Despite the growing scientific understanding that nature and humans in the Anthropocene are interwoven, the tools and technologies we have to analyze the human influence on natural environments fall short when dealing with macroeconomics. One symptom of this difficulty is the stream of literature in environmental economics centered on Integrated Assessment Models (IAMs), which, as forcefully argued by Stern et al. 2022, tend to be misleading. Indeed, they argue that

using standard IAMs has led some prominent economists to conclude that “societal optimization” entails accepting an increase in temperature of around 3.5 to 4°C (Nordhaus 2018)¹, an increase seen as catastrophic by many scholars, especially climate scientists. In the same vein, several economists, using such models, have argued that public interventions should solely focus on setting the correct price of carbon emissions and argue for a price (reflecting the “Social Cost of Carbon”) of around US\$ 50 a ton of carbon by 2030² (see [Interagency Working Group on Social Cost of Greenhouse Gases 2016](#)). This value is much lower than the numbers advocated by many experts.³ Moreover, several authors suggest the use of a far wider range of instruments, such as public spending, green accounting, etc. (see [Stiglitz et al. 2017](#)).

This divorce between climate science and macroeconomics is feeding a growing dissatisfaction within the community of economists ([Fankhauser and Stern 2016](#), [Giraud 2017](#), [Stern et al. 2022](#) *inter alia*). We can view this disagreement as being one aspect, among several others, of the epistemological crisis where macroeconomics has been struggling since the Great Financial Crisis (GFC) of 2007-2009 ([Romer 2016](#), [Blanchard 2017](#), [Stiglitz et al. 2016](#)). It seems necessary if one concludes these epistemological impasses, to provide different economic models which better capture the complex interactions, the adaptation, and the dynamics of socio-ecological systems. This paper presents a series of models that attempt to meet this challenge.

It is organized as follows: section 2 deals with methodological issues that are central to developing complex macroeconomic dynamics. Building on these premises, section 3 sets the scene by starting with the seminal stock-flow-consistent predator-prey models. It emphasizes the surprisingly good empirical performance of this basic approach compared to anything produced within the conventional viewpoint. It also introduces other basic models that vary around the same core structure, particularly the putty-clay capital structure of [Akerlof and Stiglitz 1969](#) and the extension introduced by [Keen 1995](#) through the addition of corporate debt dynamics. Section 4 adds inflation dynamics, drops Say’s law, and considers oligopoly pricing. It also introduces climate change and its interaction with the economic sphere. We will take a step back to consider the economy as a dissipative structure in section 5. Section 6 adds complexity to the model by shifting from a one-sector economy to the consideration of multi-sectorality. We then conclude in section 7.

2 Methodological Preliminaries

2.1 The Importance of Time

A first step to better deal with the complexity of the economy consists in departing from the postulate that economic agents automatically and instantaneously coordinate on a general equilibrium. Instead, we aim to study under which conditions an economy may reach a steady state and along what path. Therefore, time is of the essence. For several years, trying to view economic kinetics as a succession of *short-run equilibria* was the strategy followed by the “temporary equilibrium” concept forged by [Grandmont 1977](#). With a different formalism, it was continued by [Champsaur et al. 1990](#) and [Giraud and Tsomocos 2010](#), but with little follow-up in the literature.⁴ As a matter of fact, learning to walk upright is definitely *not* a matter of collecting a series of successive balancing positions but

¹“In the DICE model, [...] another finding, much more controversial, is that the cost-benefit optimum rises to over 3°C in 2100” ([Nordhaus 2018](#), p. 452).

²In 2007 US\$ and assuming a 3% discount rate.

³See section 4.4.

⁴It is also at the heart of the multisectoral IMACLIM model by [Sassi et al. 2010](#). For early contributions in non-*tâtonnement* processes for which out-of-equilibrium exchanges are allowed, see [Negishi 1961](#), [Uzawa 1962](#), [Hahn 1962](#), [Hahn and Negishi 1962](#), [Morishima 1962](#), chapter 13 of [Arrow et al. 1984](#), [Smale 1976](#) and [Tulkens and Zamir 1979](#).

of risking a position of imbalance, which, *perhaps*, will lead to a new equilibrium state—one step further. This point has long been advocated by many economists of different backgrounds. For instance, Joan Robinson argued in 1974:

The real source of trouble is the confusion between comparisons of equilibrium positions and the history of a process of accumulation. We might suppose we can take several still photographs of economies in stationary equilibrium [...]. This is an allowable thought experiment. But it is not allowable to flip the stills through a projector to obtain a moving picture of a process of accumulation. [Robinson 1974](#)

Robinson’s statement—which amounts to calling for the modeling of non-equilibrium dynamics—, however, was only a distant echo of calls already made much earlier. At the turn of the twentieth century (to avoid going further back in time), for instance, an Austrian economist like Eugen von Böhm-Bawerk and an American one like Thorstein Veblen expressed concerns about the very concept of static equilibrium embodied in the then-young marginalist school of economic thought launched by Jevons, Menger, and Walras (see [Böhm-Bawerk 1907](#) and [Veblen 1908](#)). One generation later, Friedrich von Hayek’s concept of “dynamics” emphasized historical causation over mutual interdependence (see [Hayek 1941](#)). Nicholas Kaldor argued against comparative statics and for a “process of change” (see [Kaldor 1938](#), and [Kaldor 1984](#)). Frank Knight claimed that capital and growth are “long run historical changes [that] must be faced as problems of historical causality and treated in terms of concepts very different from those of given supply and demand functions and a tendency toward equilibrium under given conditions” (see [Knight 1931](#)). Eventually, Paul Samuelson (see [Samuelson 1966](#)) pleads in favor of the Austrian conception of capital as time so that capital productivity is the productivity of time itself. But if this is so, an economic theory of capital must rely on a proper modeling of time and not on its methodological concealment, as induced by the equilibrium postulate with complete markets in the standard Arrow-Debreu general equilibrium theory (GET) (see [Debreu 1959](#)).

Therefore, a major feature and a crucial contribution of complexity economics are widening the usual macroeconomic focus from equilibrium-based statics to far-from-equilibrium dynamics. This is challenging in many ways, and we should now see what precautions one should observe in doing so and how one can proceed as simply as possible. Let us start with the crucial requirement of *stock-flow consistency*.

2.2 Stock-Flow Consistency

One criterion that should be satisfied by out-of-equilibrium trajectories is *stock-flow consistency* ([Godley and Lavoie 2007](#), [Taylor 2021](#) chapter 1), which states that, at any point in time, the stocks in the system are the exact outcome of the history of flows. This requirement might sound elementary, but it is not.⁵ To begin with, it is hard to assess whether it is satisfied (or, for that matter, violated) in a standard GET model. Indeed, everything is supposed to be taking place *at equilibrium* makes the very distinction between stocks and flows blur. Consider, for instance, final consumption vectors or initial endowments: are these flows or stocks? It is hard to tell as the unfolding of time itself is not easy to identify, especially in a complete market set-up where the future is *entirely* encapsulated in the present ([Debreu 1959](#)). Out-of-equilibrium, stock-flow consistency (SFC) makes perfect sense and does provide some bite, ensuring that, across time, no monetary aggregate, nor matter, nor energy is created out of nowhere or disappears. As the first rule we should observe, SFC provides the skeleton

⁵Especially when embedded into the first two laws of thermodynamics; see section [5.2](#).

of any macroeconomic model, in the form of a set of structural and accounting conditions on its variables, on top of which differential equations describing the dynamics of those variables can be laid out.

As an example, consider the Social Accounting Matrix (SAM) of table ??, which describes the balance sheet, transactions, and flow of funds in a mono sectoral, closed economy, \mathcal{E} , comprising households, private firms, and private banks.

INSERT TABLE 1

To be as comprehensive as possible, we include inventories in table ??, even though we will have to wait until section 4 before explicitly introducing inventories in our dynamics. This is a way that will later prove useful to relax Say’s law, as inventories of unsold items, therefore, serve as a buffer on the commodity market. As a consequence, the production gross of intermediate inputs is given by the sum of consumption C , investment I , and change in inventories V , which can be written as $Y(t) = C(t) + I(t) + \dot{V}(t)$. If $p(t)$ stands for some price deflator, nominal income flows is given by $Y_n(t) := p(t)(C(t) + I(t)) + c(t)\dot{V}(t)$, where c is the unitary production cost: $c(t) := W(t)/Y(t)$, with $W(t) := wL(t)$ the wage bill of the economy.

Households’ assets consist of firms’ equity, $E_f(t)$, bank equity, $E_b(t)$, and current money deposits, $M_h(t)$. Their liabilities consist of loans, $L_h(t)$, resulting in net worth (i.e., wealth) $X_h(t) := E(t) + M_h(t) - L_h(t)$, with $E(t) := E_f(t) + E_b(t)$. Firms’ assets are capital stock, $p(t)K(t)$, plus inventories, $c(t)V(t)$ (where $c(t)$ stands for the unitary production cost), and current money deposits, $M_f(t)$, whereas their liabilities are their loans, $L_f(t)$, yielding equity $E_f(t) := p(t)K(t) + c(t)V(t) + M_f(t) - L_f(t)$.⁶ Similarly, loans, $L(t) := L_h(t) + L_f(t)$ are the assets of the banking sector. Its liabilities are deposits, $M(t) := M_h(t) + M_f(t)$, whereas bank equity is $E_b(t) = L(t) - M(t)$. Hence, the net worth of firms and banks remains $X_f(t) = X_b(t) = 0$. Again, for simplicity, we assume that firms do not keep positive deposits, preferring to use any balances to repay their loans instead. As already noted, all their net profits minus dividends are reinvested: $\Pi_f(t) - \Delta_f(t) = S_f = I_f$. Savings of the banking sector are given by⁷

$$S_b(t) = r(t)L(t) - r(t)M(t) - \Delta_b(t) = r(t)(D_f(t) + D_h(t)) - \Delta_b(t) = 0.$$

Finally, the financial balance row in table ?? obeys this *ex post* accounting identity between total nominal savings and investment in the economy \mathcal{E} :

$$S(t) := S_h(t) + S_f(t) + S_b(t) = p(t)(I(t) - \delta K(t)) + c(t)\dot{V}(t) \quad (1)$$

where $K(t)$ denotes capital and $\delta > 0$ is the constant depreciation rate of capital. To write it familiarly, the capital accumulation dynamics is given by the equation:⁸

$$\dot{K} = I - \delta K. \quad (2)$$

This equation can hardly be dispensed with, as it is the driving equation of the Perpetual Inventory Method used by accountants to build capital time series. Of course, in “reality”. We should account for some delay between investment and the production of capital — as was forcefully underlined by [Kalecki 1968](#). Adding a delay quickly leads to chaotic behavior ([Matsumoto and Szidarovszky 2020](#)). It remains a wide open area of research at the time of writing this chapter.

⁶Here, equity is treated as a balancing variable, so that the net worth of non-financial firms is always $X_f(t) = 0$.

⁷To save notations, we assume that banks have no consumption. Several alternative specifications of banks’ behavior would be conceivable. Details can be found in [Giraud and Grasselli 2021](#).

⁸The equation is assumed a.e. in t .implies that $K(t)$ is assumed a local Perron primitive, and $I(t)$ is Perron integrable. We do not strive for the utmost generality here.

For now, we eventually get an expression of the total wealth, X , in \mathcal{E} :

$$\begin{aligned} X(t) &= X_h(t) = E_f(t) + E_b(t) + M_h(t) - L_h(t) \\ &= p(t)K(t) + c(t)V(t). \end{aligned}$$

In words, total wealth equals the wealth of households, which in turn is equivalent to the productive assets of the corporate sector. From now on, we shall drop the time index since, unless otherwise stated, *all* quantities are functions of time.

Equation (1) has been the source of many confusions about the origin of money (see [Werner 2016](#) and the references therein) that we do not tackle in this paper. Suffice it to say, here, that it is the gist of *emerging phenomena* in macroeconomics. Indeed, as observed (e.g., by [Taylor 2021](#), savings and investment flows must be equilibrated *independently* of the (free) will of economic actors. In *The General Theory*, John Maynard Keynes noticed that:

The reconciliation of the identity between saving and investment with the apparent ‘free-will’ of the individual to save, irrespective of what he or others may be investing essentially depends on saving being [...] a two-sided affair. For [...] the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums. Every [...] attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself.”
[Keynes 1936](#)

It should be clear now that stock-flow consistency is more than a methodological provision: it captures a deep emergent phenomenon and therefore is a key tool of complexity economics. However, as remarked earlier in this paper, this requirement only provides the skeleton for our economy, which still lacks flesh, namely dynamic relations between variables. The next step is to introduce the time-varying relationships between our stocks and flows.

2.3 On Microeconomic Foundations of Modern Macroeconomics

Many macroeconomic theories strive for “microeconomic foundations.” These attempts aim at bringing the theory back to the familiar soil of lifetime utility-maximizing consumption decisions of individuals as the driving force of economic activity, with the allocation of given, scarce resources as the fundamental economic problem. Unfortunately, it has been shown that, once market prices are normalized into the (positive part of) the unit sphere of the commodity space,⁹ *any* continuous and inward-pointing vector field on the sphere can be viewed as the aggregate excess demand of a well-chosen Arrow-Debreu economy.¹⁰ This implies, in particular, that the set of zeroes of this vector field (i.e., the set of Arrow-Debreu equilibria) can be quite arbitrary, while the behavior of aggregate demand around such equilibria can exhibit all sorts of pathologies. A decade later, [Mas-Colell 1986](#) showed the same result true for aggregate supply.

A first consequence is that, from an empiricist’s viewpoint, data will never be able to falsify any statement formulated within GET. The latter can, therefore hardly qualify as a falsifiable science

⁹Which is possible because, in an Arrow-Debreu economy, all equilibrium conditions are homogeneous of degree zero concerning prices.

¹⁰Sonnenschein-Mantel-Debreu theorem; see [Debreu 1975](#). In plain English, “anything can happen at the aggregate level.” For instance, aggregate demand can exhibit patterns that are incompatible with utility maximization under some budget constraint so that the common assumption of a representative consumer actually denies the possibility of such emerging phenomena. This result has been further refined, on the one hand, by allowing for incomplete financial markets ([Bottazzi and Hens 1996](#)), on the other, by strengthening the original theorem ([Chiappori and Ekeland 2009](#)).

in the Popperian sense of the word (see [Popper 1959](#)). A second consequence is that, once again, emergence inevitably enters the economic picture: there exist properties of the aggregate economic system that cannot be deduced from the duplication of microeconomic properties but only from the complex interactions of microeconomic units within the system.

Another important result given by [Cass and Stiglitz 1970](#) is that rational expectations are not eductively stable. Let us explain what that means. An overwhelmingly common assumption made by neo-classical macro- and microeconomic models is that, at every point in time, economic agents perfectly forecast future scenarios and environments given the information at hand. Therefore, they can solve inter-temporal optimization problems under uncertainty by maximizing, say, the expected discounted value of the stream of utility levels a given action will provide them with. Such expectations are called “rational.” Using the dynamics induced by a putty-clay capital structure *à la* Akerlof-Stiglitz, Cass and Stiglitz show that, even if individual agents are optimizers sharing rational expectations endowed with unbounded computing power, the optimal aggregate trajectory followed by the economy is not stable concerning the slightest deviation, by a single agent, from rational expectations: as soon as at least one agent, at one point of time, deviates from perfectly anticipating the future conditionally on the past information she has access to, the resulting trajectory of the economy never falls back on an optimal path. Otherwise stated, if one considers the “enlarged phase space” of an economy obtained by adding the space of expectations to that of the main variables of the economy,¹¹ solutions of the dynamics that are compliant with rational expectations are not locally stable. On the contrary, locally stable trajectories in such an enlarged phase space are said “eductively stable” (see [Evans et al. 2019](#)).

From the complexity-sensitive standpoint adopted here, there is not much reason to prioritize micro-foundations that would risk neglecting important emergent phenomena nor rational expectations that would threaten to push the analysis into an eductively unstable corner of the enlarged phase space of our economy.

2.4 A Key Element, the short-run Phillips Curve

Let us now add some flesh to the skeleton given by our Social Accounting Matrix (SAM) by describing how flows will affect stocks. Besides capital accumulation, we need several other dynamical equations for which there are more degrees of maneuver. Let us start with the basic Lotka-Volterra dynamics, as they are underlying the first three macroeconomic models that constitute the cornerstone of the approach to economic complexity explored in this chapter: [Goodwin 1967](#), [Akerlof and Stiglitz 1969](#), and [Van der Ploeg 1985](#). These dynamics are often referred to as predator-prey dynamics, as they are also used in biology to describe how the populations of two species evolve. In our case, employment is the prey, and the wage share in the economy (i.e., the fraction of total incomes accrued as wages) is the predator, which means that these two variables have a partially antagonistic relationship. The rationale behind this interplay goes as follows. As activity growth increases, hires accelerate. The growing strain on the labor market makes it easier for workers to negotiate an increase in money wages. Thus, wages should increase more rapidly so that the wage share, say $\omega := wL/Y \in [0, 1]$, will rise. Consequently, profit share, $\pi := 1 - \omega$, should dwindle. As investment turns out to be less profitable, private firms will adjust their investment strategy downward. Hence, growth will slow down. The employment rate should therefore decrease, making it harder for employees to get a pay raise. Consequently, the wage share, ω , should drop. Up to the point where profits are restored, the investment will accelerate again — the economy then enters a new business cycle.

This simple but appealing narrative relies on a relationship between the employment rate,

¹¹Such as ω , λ , and d , as we shall see *infra*.

$\lambda := L/N \in [0, 1]$,¹² and wages, known as the short-run Phillips curve (Phillips 1958, Samuelson and Solow 1960). It can be formalized by assuming that there exists some continuous, increasing function, ϕ , such that

$$\frac{\dot{w}}{w} = \phi(\lambda)$$
¹³ (3)

There is an abundant and controversial literature on the Phillips curve (see, e.g., Phillips 1958 and Samuelson and Solow 1960, as well as Gordon 2011 and Gordon 2013 for a historical perspective). On the one hand, it has long been a workhorse model of cost-pushed inflation. Still, today, when a one-year lag is accounted for in the adjustment of wages, the correlation between employment and wage growth is empirically patent (see figure ??). On the other hand, however, being largely interpreted as leading to a relationship between employment and *prices* (not wages), the Phillips curve is blamed for having failed to predict the stable inflation seen in the aftermath of the GFC during 2007-9 period, dubbed the “missing deflation” puzzle. Next, the Phillips curve failed to predict stable inflation during the recovery from the GFC—a paradox which has been called “the missing inflation” puzzle. The two puzzles together suggest that developments in prices and wages have been disconnected from developments in real activity. Several economists and commentators of different backgrounds have gone so far as to declare the death of the Phillips curve.

As argued by others, nonetheless, the main idea underlying the short-run Phillips curve is still relevant if one grant that (i) the channel between wages and prices is not a trivial one and might play a role in the two puzzles just alluded to; (ii) the graph of ϕ can be deformed by institutional changes as well as shifts in the bargaining power of, say, trade unions (Nalewaik 2016 and Ratner and Sim 2022).¹⁴

INSERT FIGURE 1

One way to account for these microeconomic phenomena and for the variety of institutional changes that affect the social fabric of an economy—namely wage bargaining—is to allow context-dependent structural breaks to affect the shape of ϕ across time. This is, for instance, what McIsaac 2021 advocates for by providing empirical evidence that the US labor market has experienced two structural breaks since the postwar period. This dependency is a way to let the “society enter the model.” It forces us to relinquish the quest for ergodic economic dynamics. And, of course, it forces us to partially give up abstract, general economic “laws” to enter into the nitty-gritty of a country’s or a region’s institutional specifics. However, is this not the hallmark of social-ecological systems as complex systems?¹⁵

3 The Basic Framework

We now have all the major ingredients required for a stock-flow-consistent, predatory-prey macrodynamics. To begin with actual models, we shall go back in time and start with the pioneering models built in this spirit.

¹²Where N is the size of the working population.

¹³Downward rigidity of real wages may be allowed for by having $\phi'(\lambda) \rightarrow 0^+$ for low values of λ .

¹⁴A famous example would be the Hartz agreement (2003) in Germany, which has greatly contributed to wage repression. Indeed, the legally shortened period of unemployment benefits led to the rise of “mini-jobs” in Germany. Hence, when a company enters liquidation, most employees find themselves obliged to accept one or two mini-jobs to avoid leaving the labor market and relying on social assistance. At Schlecker, the saleswomen used to earn € 1,300 for 35 hours a week; now, with two mini-jobs, they received € 900 for 40 hours a week. This contributed to the appearance of the German “working poor,” whose birth coincides with the apparent death of the Phillips curve.

¹⁵For an even more radical Post-Keynesian viewpoint that advocates the abandonment of the Phillips curve in favor of a sociological, institutional and historical study of the labor “market,” see Forder 2014.

3.1 Leontief Technology: The Goodwin Model

The predator-prey dynamics first introduced by [Goodwin 1967](#) relies on the following assumptions: (i) prices are all normalized to $p \equiv 1$ (equivalently, the unique commodity serves as a *numéraire*); (ii) the supply side is a two-factor production sector with a Leontief technology:

$$Y = \min\left\{\frac{K}{\nu}, aL\right\}, \quad (4)$$

where $\nu := K/Y$ is the capital-output ratio —assumed to be constant— and $\dot{a}/a = \alpha > 0$ is the growth rate of Harrod-neutral labor-enhancing technological progress; (iii) profits, $\Pi := Y - W$, are entirely reinvested, so that investment is $I = \Pi$; (iv) Say's law is in force¹⁶. Hence, consumption is supposed to adjust to supply and investment: $C := Y - I$ automatically.

INSERT FIGURE 2

Together with a basic profit-maximization requirement saying that firms will recruit exactly the minimal number of workers (so that $Y = K/\nu = aL$), these assumptions lead us to the following system of differential equations:

$$\begin{cases} \dot{\omega} = \omega(\phi(\lambda) - \alpha) \\ \dot{\lambda} = \lambda\left(\frac{1-\omega}{\nu} - (\alpha + n + \delta)\right) \end{cases} \quad (5)$$

where, $n = \dot{N}/N > 0$ is the growth rate of labor forces. The first ordinary differential equation (ODE) means that the wage share increases as long as wages grow more rapidly than labor productivity. The second ODE says that the employment rate will rise if the growth rate of output,

$$g := \frac{\dot{Y}}{Y} = \frac{1-\omega}{\nu} - \delta,$$

remains higher than the sum of labor productivity and labor force growth rates. A graphic representation of the causal links between variables is provided by figure ??.

One criticism against this first version targets the choice of a Leontief production function, which implies that no short-run substitutability can take place between capital and labor. One consequence of this restriction is that the capital-output ratio, ν , is constant, while empirical evidence amply shows that this is not the case. Moreover, the dynamic system (5) appears to be conservative, so that trajectories reduce either to a (unique) fixed point or to closed orbits indefinitely circling this fixed point (as illustrated by figure ??). While the endogenous emergence of real business cycles is confirmed by historical data, the periodic nature of solution paths is utterly unrealistic. To prevent such downsides, let us consider a Constant Elasticity of Substitution (CES) production function instead, as in the Van der Ploeg model.

INSERT FIGURE 3

3.2 CES Technology: The Van der Ploeg Model

Allowing for some short-run substitutability between input factors enables to retrieve of an endogenous capital-output ratio, as shown by [Van der Ploeg 1985](#) (see also [Bovari et al. 2020](#)). Instead of (4), let us therefore consider a CES production function

$$Y = A\left[bK^{-\eta} + (1-b)(e^{\alpha t}L^{-\eta})\right]^{-\frac{1}{\eta}}, \quad (6)$$

¹⁶We shall soon drop this restriction by introducing inventories, as described earlier in section (2).

where $A > 0$ is a (constant) total factor productivity, $b \in [0, 1]$ reflects the capital intensity of production, and the short run elasticity between capital and labor is captured by $\zeta := 1/(1+\eta)$.¹⁷

At the time t , firms take prevailing wages, w , as given, as well as the stock, K , of installed capital. They choose the quantity L of employed labor to maximize their profit. As is usual in neo-classical models, profit maximization is captured by the first-order condition, which entails that labor productivity should be equal to the level of (real) wages:¹⁸

$$\frac{\partial Y}{\partial L} = w. \quad (7)$$

This yields the following endogenous capital-output ratio:

$$\nu_\eta(t) = \frac{K(t)}{Y(t)} = \frac{1}{A} \left(\frac{1 - \omega(t)}{b} \right)^{-\frac{1}{\eta}}.$$

Switching from a Leontief to a CES production structure does change the dynamics of the economy. Indeed, the analog of (5) becomes dissipative: when workers manage to obtain a larger share of the output *via* wages, firms find it less profitable to hire employees and therefore switch away from labor to machinery, as shown in figure ??.¹⁹

INSERT FIGURE 4

Consequently, employers and workers no longer have the antagonistic relationship of species in perpetual conflict with each other: solution trajectories still exhibit cycles, but now, oscillations are damped and, in the long run, the economy converges towards some globally asymptotically stable long-run steady state (this change is illustrated in a schematic way by ??).

INSERT FIGURE 5

The CES and the Leontief technologies are extreme cases. In the first case, capital is somewhat unrealistically viewed as a Meccano set, “the pieces of which can be timelessly and costlessly reshaped into appropriate quantities of “capital” in response to the pull of relative factor prices” (Harcourt 2012, p.205). But, as already said, the symmetrical extreme case envisioned by the Leontief technology lacks realism as well. An interesting middle ground between these two polar cases is provided by the vintage production structure called putty-clay capital. As Hu 1972 has it: “Solow assumed that capital is pure putty: even after installation a machine can always be reshaped to accommodate any number of workers. Thus both the *ex ante* and the *ex post* production functions are smooth and are coincident. On the other hand, L. Johansen assumed that only before construction can a machine be designed to utilize any number of workers; once it has been built and put in use, it can only be combined with labor in a fixed proportion. Thus capital is putty *ex ante* and clay *ex post*.”

¹⁷Following Bastidas et al. 2019, we restrict ourselves to the economically more relevant case, where $\eta \geq 0$, so that $\zeta \in [0, 1]$.

¹⁸More on this equation in section 4.3.

¹⁹The CES technology exhibits a characteristic quantity of labor, $L_c = (b/(1-b))^{-1/\eta} K/e^{\alpha t}$. Profit optimization induces an over- or under-recruitment concerning L_c , depending upon the wage share :

$$\ell := L/L_c = \left(\omega_c^{-\frac{\eta}{(1+\eta)}} - 1 \right)^{\frac{1}{\eta}}$$

with $\omega_c = wL_c/Y_c$ and $Y_c = AKb^{-1/\eta}$ the corresponding characteristic output. The CES technology exhibits a characteristic quantity of labor, $L_c = (b/(1-b))^{-1/\eta} K/e^{\alpha t}$, equal to the quantity of labor of a comparable Leontief framework. Thanks to substitutability, firms can shift away from this level depending upon the wage share from a profit-optimization perspective. Figure ?? computes the ratio between the actual quantity of labor and the characteristic one as a function of the wage share: $\ell := L/L_c = \left(\omega_c^{-\eta/(1+\eta)} - 1 \right)^{1/\eta}$, where $\omega_c = wL_c/Y_c$ and $Y_c = AKb^{-1/\eta}$ is the characteristic output.

Implicit in the putty-clay model are that while the *ex ante* production function is smooth the *ex post* production function exhibits fixed proportions, and that the *ex ante* production function is in fact the envelope of a continuum of *ex post* production functions” (see also [Akerlof 1967](#) on this issue). Figure ?? illustrates the relation between *ex ante* and *ex post* isoquants.

INSERT FIGURE 6

Therefore, an interesting theoretical “third way” between the putty-putty structure of the Van der Ploeg dynamics and the clay-clay framework of Goodwin is provided by [Akerlof and Stiglitz 1969](#). To illustrate the macro-dynamics that results from a ‘putty-clay’ capital structure, let us recall the basics of the model developed by [Akerlof and Stiglitz 1969](#), except that we use a differential rather than integral formulation and we assume that wage inflation is determined by a short run Phillips curve rather than as a direct function of unemployment —see [Arran and Giraud 2022](#) for details.

We hence consider an economy with a single produced good, used as *numéraire*, and suppose, with [Akerlof and Stiglitz 1969](#), that productive technologies exist with (i) a range of ratios k between capital cost and the number of workers who can be employed in utilising that capital, and (ii) respective outputs per person-year $f(k)$ when utilized, increasing with k . We label capital vintage by the ratio k ,²⁰ so that a firm will employ a worker to use available capital k if and only if surplus per person-year $f(k) - w(t)$ is positive. If $l(k, t)$ is the density of labor employable on the capital k that exists at time t , total employment will therefore be

$$L(t) = \int_{f^{-1}(w(t))}^{\infty} l(k, t) dk \quad (8)$$

and total annual output

$$Y(t) = \int_{f^{-1}(w(t))}^{\infty} f(k) l(k, t) dk. \quad (9)$$

For the sake of simplicity, all utilization costs are assumed to be consumed and all surplus invested, with investment at time t concentrated in the capital $k^*(t)$ that maximizes the return associated with the then-expected future wage trajectory $\hat{w}(t'; t)$. Supposing capital to decay at constant rate δ , this maximization implies that k^* satisfies

$$f(k^*) - k^* f'(k^*) = \frac{\int_t^{\infty} \hat{w}(t') H(f(k^*) - \hat{w}(t')) e^{-\delta(t'-t)} dt'}{\int_t^{\infty} H(f(k^*) - \hat{w}(t')) e^{-\delta(t'-t)} dt'}, \quad (10)$$

for Heaviside step function H , with $f''(k^*) < 0$. Over any technologies of interest, k^* will increase with increasing \hat{w} , so that $f''(k) < 0$ in general and k^* is uniquely defined by \hat{w} . Investment then implies that, for Dirac delta function δ_D ,

$$\frac{\partial}{\partial t} l = \frac{Y - wL}{k} \delta_D(k - k^*) - \delta l. \quad (11)$$

Finally, let us suppose that wage dynamics are determined by a short run Phillips curve and take wage expectations to be adaptive and consistent with these dynamics. Firms are therefore be supposed to base wage expectations on past wages and past unemployment, down-weighted over some timescale τ , so that writing

²⁰Note that this can include costs of raw materials as well as labor, but shall be referred to as wages

$$\begin{aligned}\hat{w}_0(t) &= \frac{1}{\tau} \int_{-\infty}^t w(t') e^{-(t-t')/\tau} dt', \\ \hat{L}_0(t) &= \frac{1}{\tau} \int_{-\infty}^t L(t') e^{-(t-t')/\tau} dt',\end{aligned}$$

we have

$$\hat{w}(t'; t) = \hat{w}_0 + \xi \phi(\hat{L}_0/N)(t' - t)$$

where $\xi \in [0, 1]$ corresponds to firms' confidence in extrapolating historic wage trends. As shown by [Akerlof and Stiglitz 1969](#), the introduction of a putty-clay structure of capital leads us to the domain of Partial Differential Equations (PDE) as opposed to mere ODE. Analysis of the PDE relevant for our economy demonstrates that a fixed point of the economic dynamics exists and is both Solovian and an attractor. Thus, at variance with the putty-putty version *à la* Goodwin, the putty-clay structure leads to a dissipative dynamical system whose phase space resembles that of a clay-clay technology.

For the sake of simplicity, in the sequel, we shall switch back to either of the two extreme cases (CES and Leontief). An open avenue of research consists in extending the rest of this chapter to the PDE framework of putty-clay capital structures briefly sketched in this section.

3.3 Corporate Debt: The Keen Model

It is now possible to refine this basic prey-predator setting by adding other dynamics that capture additional stylized facts beyond the mere interplay between wages and underemployment. One striking restriction of the three models studied so far is that they all assume investment to be entirely and exclusively financed through current profits. This is rarely true and ignores the role played by the banking sector in financing investment—a role already strongly emphasized by ([Schumpeter 1939](#), vol.1, pp.79-80) in the early 20th century. In order to alleviate this difficulty, [Keen 1995](#) amended the Goodwin model by defining investment as $I := \kappa(\pi)Y$, with κ , a continuous, increasing function. Investment is now allowed to be larger (or lower) than profits. Corporate's debt, D , fills the gap between investment costs and current profits:

$$\dot{D} = \kappa(\pi)Y - \pi Y.$$

Profits, now, have to take due account of debt servicing: $\Pi = Y - W - rD$, where W is the wage bill and with $r \geq 0$, the short run interest rate. In the CES case, the following 3-dimensional dynamical system obtains (cf. [Bastidas et al. 2019](#)), noting $d := D/Y$, the debt-to-output ratio:

$$\begin{cases} \dot{\omega} = \omega \left(\frac{\eta}{1+\eta} (\phi(\lambda) - \alpha) \right) \\ \dot{\lambda} = \lambda \left(\kappa(\pi) A \left(\frac{1-\omega}{b} \right)^{\frac{1}{\eta}} - \delta - \alpha - n - \frac{1}{\eta(1-\omega)} \frac{\dot{\omega}}{\omega} \right) \\ \dot{d} = d \left(r - \kappa(\pi) A \left(\frac{1-\omega}{b} \right)^{\frac{1}{\eta}} + \delta + \frac{\dot{\omega}}{\eta(1-\omega)} \right) + \kappa(\pi) - (1 - \omega) \end{cases} \quad (12)$$

Having switched from 2- to 3-dimensional models not only increases their size but also deepens their complexity: as is well-known, indeed, 2-dimensional dynamical systems cannot exhibit chaotic behavior while 3-dimensional (or more) ones are no longer immune to this kind of property.

3.4 Empirical and Theoretical Support for Predator-Prey Models

Having now in mind an overview of the main basic models, we can explore their empirical and theoretical merits. From an empirical point of view, the predator-prey models generate highly correlated cycles in ω and λ . Correcting mistakes from the preceding literature, [Grasselli and Maheshwari 2017](#) and [Grasselli and Maheshwari 2018](#) showed in particular that a slight modification of the Goodwin model²¹ enables these dynamics to fit rather well employment and wage data of a dozen major economies of the OECD.

INSERT FIGURE 7

Back-testing their relative performance against a multivariate Vector Auto Regression (VAR) specification, as reported in [McIsaac 2021](#), yields overwhelmingly better results for both Leontief and CES predator-prey models, for time horizons ranging up to 8 years ahead (??). As an important point of comparison, the performance of most conventional macroeconomic models further than 2 years into prediction is often very unsatisfactory, leading macro-economists to rely heavily on VAR models that lack any formal modeling of the non-linear part of macrodynamics.

INSERT FIGURE 8

Turning to an analytical perspective, the models we just introduced share a set of interesting properties, beyond generating endogenous business cycles.

(i) First, they display multiple equilibria —a feature that is hard to get in a GET framework, and nevertheless viewed as a must (see, e.g., [Vines and Wills 2020](#)). Among these, two types of equilibria show up as being most interesting —both because they are locally asymptotically stable under reasonable conditions and economically meaningful. The first one is an interior steady state, analogous to the Solovian equilibrium and sometimes referred to as the “good equilibrium,” where the economy stabilizes in the long run at a finite debt ratio, positive wage share, and positive employment rate. In general, this equilibrium satisfies the Golden rule balanced growth property that was the hallmark of the (unique and globally stable) fixed point of the Solow model. But, at variance with the Solow set-up, there is also a second steady state, namely, a catastrophic equilibrium displaying an infinite debt ratio along with vanishing production and wages. Around this Fisherian debt-deflationary equilibrium, private debt accumulates too fast, to the point where the productive sector sinks into an unescapable debt overhang. Figure ?? gives an overview of the diversity of trajectories around these equilibria. What mostly changes depending on the precise modelling features is the stability properties of each equilibrium, the geometry of their basin of attraction, and the speed at which an economy converges to each of them. For instance, and rather unsurprisingly, the basin of attraction of the Solovian equilibrium in the Van der Ploeg model (based on a CES technology) is usually larger than in the case with complementary factors of production (embodied by the Leontief technology of the Goodwin model).

INSERT FIGURE 9

(ii) At variance with the static equilibrium approach, these models enable us to understand how the state of an economy can wander from one steady state to *another* within the

²¹Which consists in “allowing the ratio of investment to profit to be given by a parameter k , which should then be estimated from the data along with the other parameters in the model, rather than assumed to be identically equal to one as in the original Goodwin model” ([Grasselli and Maheshwari 2018](#)).

phase space: we shall see an example of this behavior linked to climate change in section 4.4 *infra*.

(iii) All these models put private debt (introduced by Keen 1995) into the spotlight, whereas traditional macroeconomics has long neglected private debts under the pretext that when there is a debtor in an economy, there must be a creditor, so that at the aggregate level, the liabilities and assets of both cancel each other out. It is only as late as 2012, after the GFC of 2007-9, that the profession realised its mistake (Eggertsson and Krugman 2012).

(iv) But most important perhaps, at least from a political economy viewpoint, this family of models brings an end to the debate initiated in the thirties by Harrod 1939. To understand this, it is worthwhile to recall some piece of the history of macroeconomics. In Harrod's model (later on developed by Domar 1957), three types of growth rates were distinguished: the actual real growth rate, g , of the economy, the maximal full-employment rate, G , made possible by installed capital capacity, and the "warrented" rate, g_w , which leaves entrepreneurs satisfied with their productive capacity as measured by the ratio \dot{K}/\dot{Y} . Harrod's remark was that, in a world where the capital-output ratio, $\nu := K/Y$, is constant, as soon as these three rates differ, there is no way for them to reconcile. Suppose, indeed, that $g > g_w$: This means that the actual ratio \dot{K}/\dot{Y} is lower than the one targeted by entrepreneurs (which is constant, since ν is). Therefore, firms will tend to invest more, but this will only increase the gap between g and g_w . And similarly, if $g < g_w$. Instead of being self-righting, any departure from equilibrium is self-aggravating. In the context of the Great Depression of the 1930s, this had been interpreted as a theoretical manifestation of the fact that "capitalism" is precariously balanced on a "knife-edge."

Twenty years later, Solow provided a counterargument to this grim outlook (Solow 1956) by allowing ν to become endogenous. In his model, capital and labor are substitutable, so that ν is determined by capital-labor substitution undertaken by firms as a response to variations in the relative costs of these two inputs, which are themselves endogenously determined by the marginal productivity of each factor. The consequence was striking: now, the phase space of the economic dynamical system would admit a single equilibrium, which would be globally asymptotically stable. No such thing as social conflicts should ever appear in a "capitalist" economy as the collaboration between labor and capital could only result in the best of all possible worlds. The debate, however, was not closed: empirically, ν is obviously not constant. What was the best modelling approximation? To take ν constant or variable?

The family of models around (12) suggests that this is actually *not* the overriding issue. Indeed, in both cases, the phase space admits at least two equilibria, one "good," one "bad," and in either case, each of these steady states is (locally, asymptotically) stable under standard conditions. "Capitalism" *per se* need not lead to self-destruction, nor to a Panglossian never ending growth: it all depends upon the geometry of the basins of attraction and on the initial conditions of the economic dynamics. Moreover, whether a given economy is prone to converge towards the Solovian or the Fisherian equilibrium is not an immediate, one-way consequence of whether input factors are substitutable or not: substitutability only broadens the basin of attraction of the interior equilibrium and sharpens the dampening of endogenous cycles as an economy converges towards the latter. It does not eliminate the catastrophic attractor, nor does it destabilize it.

This should encourage us to go a few step further and craft more extended versions of the basic models introduced so far. As a matter of fact, a number of papers study several variations of these predatory-prey dynamics: [Grasselli and Costa Lima 2012](#) add a financial Ponzi scheme and explore its destabilizing effects on the “good” equilibrium, [Nguyen-Huu and Costa-Lima 2014](#) study a stochastic version of the Goodwin dynamics, [Grasselli and Nguyen Huu 2015](#) introduce a price dynamics and study the impact of inflation, [Grasselli and Nguyen-Huu 2018](#) drop Say’s law by allowing for inventories, [Giraud and Grasselli 2021](#) consider heterogeneous households to focus on inequality. The next section reviews some of these developments, without claiming to be exhaustive.

4 First Extensions

4.1 Inventories and Inflation

A first range of extensions of the basic model consists in dropping Say’s law and relaxing the assumption that the (unique) commodity is a *numéraire*. As already evoked earlier in this paper, a way to relax the first restriction is to introduce an autonomous aggregate consumption function together with inventories, such that changes in inventories at each period equal the difference between total production and effective demand (the sum of private consumption and investment). An argument supporting this modeling choice and underlined by [Grasselli and Nguyen-Huu 2018](#) is that changes in inventories seem to account for a disproportionately large fraction of changes in output, and therefore ought to be taken into proper consideration. Following these authors, we denote inventories $V(t)$. Their dynamics is given by:

$$\dot{V} := Y - Y_d = Y - C - I, \quad (13)$$

where Y_d denotes total effective demand. By contrast, firms expect to sell all their production, i.e., expected demand is equal to Y . It is this value that firms use to compute their expected profitability and take their investment decisions. On the other hand, a simple way to model consumption is, for instance, to set:

$$C = \mathcal{C}(\omega, d)Y \quad (14)$$

where \mathcal{C} is a smooth, increasing function of the wage share, ω , and debt ratio, d . This is only one option among many others, though —as one could chose to make consumption depend only on w or ω , for instance— but we do not wish to discuss the best fit for an aggregate consumption function here.

Next, to deal with inflation, we can follow [Grasselli and Nguyen Huu 2015](#) and [Grasselli and Nguyen-Huu 2018](#). Before anything else, let us redefine the wages’ dynamics in order to account for the effect of prices, by introducing a degree of monetary illusion, $0 \leq \gamma \leq 1$ (whenever γ is equal to one, workers perfectly incorporate inflation in their bargaining). Denoting inflation by $i := \dot{p}/p$, equation (3) now writes:

$$\frac{\dot{w}}{w} = \phi(\lambda) + \gamma i.$$

Inflation in modern economies is both cost-pushed and demand-driven. As far as the cost side is concerned, we can consider in accordance with the post-Keynesian tradition that the long run equilibrium price is given by a constant markup, $\mu > 1$, times the unit labor cost $c = wL/Y$, and that observed prices relax towards this value with speed $\eta_p > 0$. Turning to the demand side of inflation, when setting their price strategy, firms take into account unexpected changes in inventories

(resulting from larger or lower sales than expected), with parameter η_v . Therefore, the inflation rate is:

$$i := \frac{\dot{p}}{p} = -\eta_p \left(1 - \mu \frac{c}{p}\right) - \eta_v \frac{\dot{V}}{Y} = \eta_p(\mu\omega - 1) - \eta_v \left(1 - \mathcal{C}(\omega, d) - \kappa(\pi)\right). \quad (15)$$

After a few calculations, the whole system boils down to the following dynamics in the Leontief case:

$$\begin{cases} \dot{\omega} = \omega(\phi(\lambda) - \alpha - (1 - \gamma)i) \\ \dot{\lambda} = \lambda \left(\frac{\kappa(\pi)}{\nu} - (\alpha + n + \delta)\right) \\ \dot{d} = d \left(r - \frac{\kappa(\pi)}{\nu} + \delta - i\right) - \mathcal{C}(\omega, d) + \omega, \end{cases} \quad (16)$$

where, as before, $\pi = 1 - \omega - rd$. This systems corresponds approximately to the one obtained by [Grasselli and Nguyen-Huu 2018](#) in their “long run dynamics, monetary version” section. They find that (16) exhibits three types of meaningful equilibria (and one irrelevant, trivial equilibrium), which are in line with the equilibria usually obtained with predator-prey models: a “good” equilibrium (positive employment rate and wage share, finite debt ratio), a debt crisis equilibrium (finite debt ratio but shrinking wages and employment), and a deflationary steady state (vanishing real wages and deflation) that can be associated with a credit explosion.

So far, we manage to explore instability and economic cycles, but mostly as coming from firms. Yet, household debt and inequality are a major source of economic turbulences, so which should account for these in the next section (4.2).

4.2 Income Inequalities

A second series of possible extensions regards inequalities. Do large differences between households in terms of income and wealth have an impact on growth? The conventional answer is negative: the size of the cake is claimed to be independent from the way it is distributed. A positive answer has been provided by [Giraud and Grasselli 2021](#) using a predator-prey dynamics and highlighting the channel of households’ private debt as the missing link between inequalities and growth. The authors consider two categories of households: workers and investors. Workers only derive their income from wages and from the difference between the interest received on deposits and paid on loans (for simplicity, the public sector is absent). Investors privately own banks and non-financial firms, so that their income consists of dividends paid by firms and banks and of the interest rate differential.

This extended model displays equilibria comparable to those of the models already alluded to: one interior “good” equilibrium and four classes of catastrophic equilibria that exhibit infinite levels of workers’ or investors’ debt together with vanishing wage share and employment rate. However, an important difference with the previous literature lies in the stability analysis of these steady states. Indeed, considering two classes of households enables to assess the impact of inequality (as measured by capital-to-income ratio in particular) on the (in-)stability of steady states. It appears in particular that, in most cases, increasing the capital-to-income ratio reinforces the stability of the catastrophic long run steady states: the higher the inequality level, the greater the attraction basin of those asymptotic crisis states. Conversely, the good equilibrium is associated with a balanced growth path, along which the level of inequalities remains constant. Therefore efficiency and fairness are more entangled than is commonly supposed in economics: more egalitarian economies are more efficient in the long run.

Moreover, [Giraud and Grasselli 2021](#) show that a necessary condition for the asymptotic stability of the balanced growth path is $r > g + i$, where, as before, $g + i$ is the nominal growth rate of the economy. This is, of course, reminiscent of the controversial condition $r > g$ emphasized in [Piketty 2014](#). Some differences, however, are worth mentioning.

(i) They deal with a non-linear predator-prey dynamical system with endogenous saving rate and private debt, while [Piketty 2014](#) relies on a Solow model with *numéraire*, exogenous saving rate, and no debt, where in fact the condition $r > g$ does not imply a divergence between the incomes from work and capital, as observed in [Acemoglu 2015](#).

(ii) In [Piketty 2014](#), r denotes the average return of capital, which includes the interest paid on government bonds, but also the return on many other types of financial assets. Here, r denotes the average rate of interest paid on private debt in the form of bank loans and cannot be confused with the marginal productivity of capital.²²

(iii) [Piketty 2014](#) claims that money is neutral in the long run, even though no argument is provided that would sustain this statement. In [Giraud and Grasselli 2021](#), by contrast, money is neither neutral in the short run, nor in the long run. Indeed, the nominal rate, r , and inflation, i , deeply shape the configuration space of the economic vector field. Hence, the entire dynamical landscape of an actual economy is likely to be affected by monetary policy.

As observed by [Giraud and Grasselli 2021](#), the interior equilibrium exhibits some degree of inflation in the long run, while the debt-deflationary steady state exhibits a huge, asymptotic rate of unemployment. Because of this non-trivial trade-off between long-term inflation and employment, the analog of a “long run Phillips curve” cannot be vertical in this setting. Hence, the structural unemployment rate at the interior long run equilibrium is not to be confused with the “natural rate of unemployment” introduced in [Friedman 1968](#) and [Phelps 1968](#). Moreover, as becomes clear from the stability analysis of the balanced-growth path, monetary policy (captured here through the setting of r , the short run nominal interest rate) does play a role, even in the long run, since it influences the asymptotic local stability of the interior steady state. Similarly, if the markup μ is linked to the monopoly power of firms (as suggested, among others, by [Kalecki 1971](#)), the institutional rules governing the consumption good market will have an influence on employment (via the impact of μ on the long run value of λ). This contrasts with the very concept of “natural unemployment,” which embodies the idea that monetary policy is ineffective in reducing underemployment. Actually, the structural unemployment rate, $1 - \lambda$, prevailing at the interior steady state is closer to the NAIRU (“Non Accelerating Inflation Rate of Unemployment”) introduced in [Tobin et al. 1980](#), since, at the balanced-growth path, inflation remains constant. As for the NIRU (“Non Inflationary Rate of Unemployment”) of [Modigliani and Papademos 1975](#), it is given by $1 - \lambda^*$, where $\lambda^* := \phi^{-1}(\alpha)$, and corresponds to the special case $\omega^* = 1/\mu$. Consequently, the concept of “structural unemployment” (i.e., the unemployment rate prevailing at the interior equilibrium) departs from the hybrid New Keynesian Phillips curve (NKPC) as developed in [Galí and Gertler 1999](#), where the inflation rate is related to lagged inflation, inflation expectations and a measure of excess demand, which results in a short run trade-off between inflation and unemployment but a unique long run equilibrium, with steady inflation.

To conclude this section, let us eventually mention a radical criticism against equation (7), and a possible answer to it.

²²For reasons to be explained in section 6.2.

4.3 Oligopoly Pricing

As recalled by [Keen and Standish 2006](#), George J. Stigler (in [Stigler 1957](#)) had made a striking argument against the way our textbooks formalize the behavior of profit-maximizing firms in a perfectly competitive environment. If each firm is price-taker, so goes Stigler’s argument, it does not react to the behavior of its opponents. In an m firms industry, where y_i designates the output of firm i , this means that $dy_i/dy_j = 0, \forall i \neq j$. Therefore, writing $Y := \sum_{i=1}^m y_i$ for the aggregate output, one gets $dY/dy_i = 1$ so that, if p is the consumption price and $Y \mapsto p(Y)$ is the inverse demand function,

$$\frac{dp}{dy_i} = \frac{dp}{dY}, \quad \forall i. \quad (17)$$

The unfortunate consequence is that one cannot have, at the same time, an aggregate demand function, $p \mapsto Y(p)$, with a negative slope *and* perfect competition. Indeed, the first requirement implies that $dp/dY < 0$, and the second, $dp/dy_i = 0$ —a contradiction with (17). This means that the usual way perfect competition is formalized — namely that atomistic firms are price-takers, hence that they should have no impact on prices— enters in conflict with the equally usual assumption that aggregate demand is downward sloping with respect to prices.

As already seen, the Sonnenschein-Mantel-Debreu theorem implies that, in fact, $p \mapsto Y(p)$ need not have a negative slope everywhere. This might partially solve the conundrum discovered by Stigler but at the cost of dismantling the familiar Marshallian supply-demand diagrams on which our textbook understanding of economic equilibrium is based. Moreover, we might also have to give up even the second option, namely $dp/dy_i = 0$ —despite its prominent role in the conventional treatment of perfect competition.

Indeed, consider, for simplicity a Leontief economy of the type studied earlier in this paper, with m firms and where the profit of firm i reads $\Pi_i = p(Y)y_i - w_i L_i$, with $y_i = a_i L_i$. Obviously, Π_i not only depends upon y_i but also upon Y *via* the inverse demand function, $p(\cdot)$. As argued by [Keen and Standish 2006](#), the profit-maximization program of firm i should therefore not be obtained by taking the zero of the partial derivative of Π with respect to y_i , but by considering the *total* derivative of Π with respect to Y , and then finding a zero by adjusting y_i . This yields

$$\begin{aligned} \frac{d\Pi_i}{dY} &= \sum_{j=1}^m \frac{\partial}{\partial y_j} \Pi_i = \sum_j \frac{\partial}{\partial y_j} (p(Y)y_i - w_i L_i) \\ &= \sum_j \left(p(Y) \frac{\partial y_i}{\partial y_j} + y_i \frac{\partial p(Y)}{\partial y_j} - \frac{w_i}{a_i} \right) = p(Y) + m y_i \frac{dp(Y)}{dY} - \frac{w_i}{a_i}, \end{aligned}$$

where we have used (17) in the last equality. The first-order condition with the total derivative then leads to:

$$p(Y) = \frac{w_i}{a_i} - m y_i \frac{dp(Y)}{dY}.$$

Whenever $p(\cdot)$ is a decreasing function, this means that the “perfectly competitive” price will typically be larger than the marginal cost of each firm. Actually it resembles the price that would be fixed by an oligopoly of price-maker firms. These considerations can be easily accounted for in our dynamics, for instance, by replacing (7) with:

$$\frac{\partial Y}{\partial L} = \frac{w}{\sigma},$$

where $0 < \sigma < 1$ measures the oligopolistic bias of the labor market, as in [Dossetto et al. 2022](#). That real wages have been lower than labor productivity for some decades is amply confirmed by empirical data in many industrialized countries (see [OECD 2018](#)). How much lower than unity should σ be depends again upon the power relationships among private companies on a given market and the institutional rules that possibly aim at preventing collusion between them.

4.4 Climate Change

It is now time to return to the topic alluded to in the introduction: the relationships between macroeconomics and climate change.

4.4.1 Damage Functions

Production and services, as well as consumption, generate greenhouse gases (GHG) which accumulate in the atmosphere. This additional concentration in GHG fuels an increase in radiative forcing, which acts as the feedback loop to the economy, by changing the surface temperature, therefore impacting resources, infrastructures, and behaviors. The way in which temperature increases impact economic production is quantified through what is called a damage function, that often takes the form of a mapping from temperature anomaly in °C to production losses in % of real output (and sometimes capital). Figure ?? exhibits three damage functions that have been discussed in the literature. While they more or less coincide below the +2 °C threshold, they diverge on a range of +6°C: whereas [Nordhaus 2014](#) estimates that only 10% of the world real GDP would be lost under a +6°C global warming, [Dietz and Stern 2015](#) finds that GDP would be cut by 90% for the same level of temperature anomaly.

Identifying the “right” damage function at the world level is certainly a formidable task, and perhaps an impossible one. For instance, [Wuillez et al. 2020](#) challenges yet another such function introduced in the literature by showing that, when applied to a -4 °C cooling (as opposed to warming), it would predict a triple of the world GDP even though half of today’s industrialised world would be under the ice. However, whatever being the modeler’s choice of the damage function, the underlying macroeconomic set-up matters a great deal for our understanding of the climate resilience of an economy, or the lack thereof. [Keen 2021](#), for instance, has shown, using the neo-classical DICE equilibrium model with rational expectations of [Nordhaus 2013](#), that, even after having lost about 90% of its real GDP, the world economy should return to its equilibrium state—a quite unappealing feature which is interpreted by these authors as being the artifact of the built-in postulate, in [Nordhaus 2013](#) that a market economy should always be at equilibrium, no matter what.

INSERT FIGURE 10

Now that we have a hopefully more satisfactory understanding of the far-from-equilibrium dynamics of an economy, we can scrutinize its interplay with climate change. A first tentative pass has been taken by ([Bovari et al. 2018a](#), [Bovari et al. 2018b](#) and [Bovari et al. 2020](#)) where Nordhaus’s climate back-loop is coupled with a world-calibrated version of the Leontief model with corporate debt, inflation, an endogenous interest rate (given by some Taylor rule ([Taylor \(1993\)](#))) and possibly credit restrictions. It turns out that, absent climate change, the world economy would converge at a slow rate towards some balanced growth path around the 23rd century. Now, when global warming enters the picture, the economy may be pulled out of the basin of attraction of the “good” equilibrium. Figure ?? shows the trajectory of two of the three key variables in Goodwin-Keen: wage share and employment rate.

INSERT FIGURE 11

(i) With Nordhaus’s optimistic damage function, climate change does tend to drive the world economy away from the good equilibrium, but market forces act as stabilizing forces, and after a transitory period, the economy locks onto a stabilizing trajectory.

(ii) But when incorporating Dietz-Stern’s more severe damage function instead, the impact of climate change overwhelms market forces, so that the whole economy experiences forced degrowth with employment rate and wage shares in free fall before the end of the 21st century.²³

INSERT FIGURE 12

These insights can be used to answer questions such as: in which basin of attraction are we currently locked onto? Given estimates from climate science, which policies will ensure that we converge towards a “good” equilibrium in the long run? Among such policies and trajectories, which ones are more desirable? The task is complexified by the fact that, not only is global warming counteracting the convergence of the economy towards a desirable long-term steady state, it is also reshaping the geometry of its basins of attraction, as shown by figure ???. What is more, the “good” equilibrium itself is not fixed but moves (and actually, deteriorates) as temperature increases, as shown by figure ??. The warmer the planet, the less desirable becomes the “good” steady state. In particular, with the Dietz-Stern damage function, the ratio of corporate debt to output becomes close to 300% around the “good” steady state for a 2°C warming, and reaches 500% for a 3°C warming. The wage share also shrinks significantly, so that inequalities widen with climate change even along the interior equilibrium. This is an instance of path-dependency (or hysteresis) due to the interplay between the biosphere and our anthroposphere.

INSERT FIGURE 13

4.4.2 Integrated Dynamic Environment-Economy (IDEE) Models

The previous models couple climate to the economy only through a simple back-loop, the one used by Nordhaus 2013. A more ambitious approach consists in coupling a full-fledged climate model to the economy, that can generate spatial dynamics. Martin et al. 2023 and Giraud and Valcke 2023 couple iLoveclim —a medium-size climate model, (Goosse et al. 2010, Bouttes et al. 2015)— with a macroeconomic SFC predator-prey model and studies various scenarios where GHG emissions are endogenized. In this specific macroeconomic model, two components have been added. The first (i) is endogenous growth *à la Kaldor-Verdoorn*, as represented in Kaldor 1957 and Romer 1983:

$$\frac{\dot{a}}{a} = \alpha + \gamma_g g \quad (18)$$

These dynamics also introduce path-dependence from growth to technical progress (and *vice versa*). One consequence is that, in general, the interior steady state need no longer be asymptotically stable.

The second element (ii) consists in improving the unrealistic feature of corporate debt being able to explode to infinity around a debt-deflationary steady state. One way to do so consists in adding a collateral requirement and the possibility of default in the indebtedness process of firms in the spirit of (Geanakoplos and Zame 2014 and Fostel and Geanakoplos 2014). As much lending in modern economies, indeed, corporate debt is secured by some collateral, which, here, is the

²³Of course, some important phenomena are uncertain, such as labor productivity, climate sensitivity or the absorption capacity of oceans. Monte-Carlo simulations then permit to incorporate this uncertainty in the analysis, as shown in Bovari et al. 2018a. Unfortunately, it is not enough to challenge the gloomy prospects of deterministic scenarios.

current stock of capital valued at its current market price. Default is accompanied by a transfer of ownership over the collateral from borrowers to lenders. As a consequence, the debt-to-output ratio, d , can no longer exceed the capital-to-output ratio, ν . Moreover, the authors assume that, as d gets close to its upper bound, ν , firms refrain from investing—a way to answer the legitimate criticism expressed in [Pottier and Nguyen-Huu 2017](#) about the unrealistic feature of the Keen model, where corporates keep investing in the neighborhood of the “bad” equilibrium while their profit rate vanishes.

The coupling of this SFC predator-prey macrodynamics with iLoveclim is dubbed an IDEE (Integrated Dynamic Environment-Economy) model. Its conclusions are striking:

(i) Due to global warming, and even with the “optimistic” Nordhaus’s damage function, there is a risk for firms to reach an irreversible financial tipping point in business-as-usual scenarios above which the world private sector would be trapped into a cascade of defaults. This would lead to a “financial Minsky moment” (Carney) which, with the Dietz-Stern damage function, would be reached as early as in the last quarter of the 21st century. One consequence is that, in the very long run, the choice of the damage function becomes irrelevant: action is needed however “optimistic” our views might be about the impact of climate change.

(ii) The temperature associated with this financial tipping point ranges from $+2.3^{\circ}\text{C}$ to $+3.8^{\circ}\text{C}$, depending upon the damage function. This provides a new —financial— justification for the Paris Agreement target of $+2^{\circ}\text{C}$.

(iii) A carbon tax policy cannot be dispensed with if a catastrophic endgame is to be avoided. Moreover, it must be quickly implemented and ambitious: about 2015US\$ 400/t by 2050.

(iv) Moreover, it must be accompanied by an equally ambitious public expenditure policy in order to enable the private sector to afford both the cost of mitigation and that of adaptation: public spending of at least 50% of the abatement costs are needed to avoid the financial tipping point. This expense, however, never exceeds 10% of the current GDP. It enables to reach net-zero emissions before 2075, even with the Dietz-Stern damage function, and to stay broadly around a $+2^{\circ}\text{C}$ temperature deviation by 2100. The estimated global cost of the low-carbon transition —approximately 90 US\$ trillions up to 2035— is in good agreement with [The Global Commission on the Economy and Climate 2018](#).

INSERT FIGURE 14

Absent such a voluntaristic action from both the public and the private sectors, ?? describes the state of the planet in the last third of the 21st century and the first third of the next one, suggesting that most of the Tropical belt would become uninhabitable. [Mora et al. 2017](#) showed, indeed, that high temperature and humidity combinations —called “deadly heatwaves”— might generate situations that exceed human thermoregulatory capacity: sweating becomes ineffective at high relative humidity, potentially leading to death. Empirical data suggests the existence of a *lethal temperature threshold* ([Mora et al. 2017](#)), which depends on relative humidity. What figure ?? shows is that a business-as-usual scenario would make a large part of the tropics uninhabitable before the last quarter of this century, even though the average global temperature increase would be around $+2.3^{\circ}\text{C}$ and would reach $+3.8^{\circ}\text{C}$ “only” around 2134.

Climate science, today is essentially driven by thermodynamics and fluid mechanics. The first two laws of thermodynamics play a decisive role in it. One way to go further in combining macroeconomics with environmental sciences is to rewrite macro-dynamics as a thermodynamic flow. This is far from being an easy task as most conventional economic models violate both laws of thermodynamics.

5 Dynamics of Material Flows

5.1 Why Energy Matters

The process of providing equations describing the (nonlinear) dynamics of the world economy was used as early as in 1972 in [Meadows et al. 1972](#): there, back-loops between material quantities (e.g., population, industrial capital, pollution, agricultural systems, non-renewable resources, wastes, etc.) and delays in the response of various subsystems (e.g., the effects of increasing pollution levels on human life expectancy may not be recognized for some decades after the pollution is emitted) are shaping a complex dynamics simulated by the computer model World3. Their work remained essentially overlooked by economists, until the physicist Graham Turner back-tested the projections of the Meadows report, comparing them with historical data. He finds that thirty years of historical data compared very well with the “standard run” scenarios (to be interpreted as the “business-as-usual” prolongation of historical trends) of the famous report to the Club of Rome ([Turner 2008](#), [Turner 2012](#) and [Turner 2014](#)). This path-breaking work suggests that taking into account planetary limits other than climate change is essential if we are to understand the macroeconomic evolution of a country, let alone the planet. After all, global warming is but one of the various planetary feedbacks experienced by the anthroposphere as a consequence of its own pollution. On the other hand, the fact that the conventional peak oil has already been reached at the planetary level around 2006 ([IEA 2010](#)) while the world economy still heavily depends upon oil (especially for mobility purposes) might soon reveal the crucial nature of energy for economic prosperity.

Such a viewpoint stands in sharp contrast with the custom, popular in macroeconomics until recently, to consider that energy and matter play a minor role in growth accounting and, therefore, in macroeconomics as such. While matter has long been nearly entirely neglected, the habit had been taken to calibrate output elasticity with respect to energy according to the cost share of energy. In most countries, this leads to the statement that energy elasticity should have been close to 8% on average in the last 4 decades—an affirmation that deeply contradicts findings by several economists and historians ([Csereklyei et al. 2016](#)).

A textbook result (known as the cost-share theorem) underlies this habit. It argues that, in perfectly competitive markets, under constant returns to scale and absent any externality of omitted variables, output elasticity of any production factor should equal its cost share. Denoting $\mathbf{p} = (p_i)_i \in \mathbb{R}_+^n$, the price of inputs, the profit maximization program of the representative producer reads:

$$\max_x pY(x) - \mathbf{p} \cdot x \tag{19}$$

where $x = (x_i)_i$ is the input vector and, p , the output price. The standard first-order condition leads to

$$\varepsilon_i = \frac{x_i}{Y(x)} \times \frac{\partial Y(x)}{\partial x_i} = \frac{p_i x_i}{\mathbf{p} \cdot x}, \tag{20}$$

where the left-hand side, ε_i , is the elasticity of output with respect to x_i and the right-hand-side, the cost share of x_i in the overall production cost, $\mathbf{p} \cdot x$. Besides the critique already formulated in

section 4.3 supra about the treatment of (19), there are a number of good reasons to believe that (20) is violated in most countries and for nearly all inputs (Kümmel et al. 2010, Kümmel 2011). Suppose, indeed, that the producer actually faces constraints —think of geological depletion for oil or wind intermittence for wind turbines—, so that (19) actually reads:

$$\begin{aligned} \max_x pY(x) - \mathbf{p} \cdot x \\ \text{s.t. } f(x) = 0. \end{aligned}$$

Shadow prices, ρ , then enter the cost-share equality (20):

$$\varepsilon_i = \frac{x_i(p_i - \rho \frac{\partial f(x)}{\partial x_i})}{\mathbf{p} \cdot x - \rho \frac{\partial f(x)}{\partial x_i}}. \quad (21)$$

Whenever the constraint embodied by $f(x) = 0$ becomes binding, ρ may grow to infinity, so that a large output elasticity with respect to x_i (say, energy) becomes compatible with a small cost-share.

Together with the imperative (arising from the conversation with climate sciences) to make macroeconomics compliant with the first two laws of thermodynamics, this suggests that embedding macroeconomics into a genuinely thermodynamic setting, where the role of energy (and matter) can be rigorously scrutinized, is more than an epistemological exercise but might be a must in order to improve the relevance of economics as a social science (Hall et al. 2001).

5.2 The Economy as a Dissipative Structure

Nicholas Georgescu-Roegen had anticipated the importance of the second law of thermodynamics (Georgescu-Roegen 2013) but was still working with the nineteenth-century stage of thermodynamics. Today, the key concept that comes to mind is Prigogine’s notion of a “dissipative structure” (Prigogine 1978), that is, an out-of-equilibrium metabolism which can be “born” and “die,” and which can only be maintained by some input of energy and matter and under the condition of being able to exude wastes. The human body is a dissipative structure, along with all living bodies.²⁴ An economy can be viewed as “social-ecological metabolism” in much the same way as an individual organism.

During the process of metabolising its “food,” a dissipative structure can essentially perform three tasks: (i) produce some work; (ii) expand its structure; (iii) self-organize its internal structure. In either case, it functions as an energy-matter converter which turns low-entropy energy (e.g. fossil fuels) and high-entropy matter (e.g., a pile of soil) into high-entropy energy (heat) and low-entropy matter (a manufactured product or productive infrastructure made of copper extracted from the soil) —while exuding high entropy energy and matter wastes.

Following Ayres and Warr 2005 and Ayres and Warr 2010, the work (i) performed by an economic metabolism can be understood as “useful work” —a generalization of mechanical work grouped into the following four categories: Muscle Work, Electricity, Mechanical Drive, and Heat (low, mid, and high temperatures). Useful work should definitely not be confused with real GDP: the former is a physical output, the latter is a statistical construct based on a monetary evaluation of market transactions and some price-deflator. While it has been argued that GDP is nowadays strongly influenced by the financial sphere, useful work should not be much impacted by financial transactions, except *via* the material grids of information and the data center on which globalized finance is based.

Operation (ii) concerns the material expansion of an economy. In recent years, material flow analysis has provided a wealth of data and analysis of the different ways this has happened in the

²⁴A star or a typhoon are examples of non-living dissipative structures.

past and my happen in the near future. According to the 2016 report of the International Resource Panel, “annual global extraction grew from 22 billion metric tons in 1970 to around 70 billion metric tons in 2010”. Whether this trend can be continued is doubtful given the thermodynamical limits imposed on the quantity of energy and water needed for this extraction process. The pace of its likely deceleration depends upon the growing scarcity of some key minerals such as copper (Vidal et al. 2017, Vidal et al. 2019) and the speed at which our economies will find substitutes for these natural resources. The fact remains that, courtesy of the thermo-economic approach, economic stock-flow consistency and the first law of thermodynamics enable to treat the conservation of energy, matter and money on the same foot (Noel et al. 2022).

Finally, as shown by Noel et al. 2022 and Herbert et al. 2022, the possible complexification (iii) of an economic metabolism can be measured through its internal entropy: Provided it gets enough energy (and matter) for that purpose, a dissipative structure can reduce its internal entropy at the cost of increasing the entropy of its environment. This is the crucial step where the second law eventually enters the picture.

The interplay between these three metabolic operations (i), (ii) and (iii), the intensity at which they are performed by a given economy, and their impact on its environment then become the main focus of an ecologically informed economic analysis (see Noel et al. 2022 and Herbert et al. 2022 for details). Needless to say, changes in the internal structure of an economic metabolism may have a significant impact on the political and social rules and habits prevailing in this polity (Haberl et al. 2019). Today, the literature dedicated to the political economy of the type of energy policy that prevails in an economy is burgeoning and very promising (e.g., Mitchell 2013 and Seow 2021). It is certainly part of the kind of complexity economics that is required to take seriously the role of thermodynamics in our polities.

From an analytical viewpoint, taking into consideration the materiality of an economy forces us to free ourselves from the convenient toy-model of a single-good economy and, in a way, to open the periodic table with its 118 elements... This requires taking up the challenge of a multisectoral macrodynamics whose notorious difficulty has plagued economic analysis for many decades.

6 Multisectoral Dynamics

6.1 From One-Commodity Economies to Multisectorality

Admittedly, one-commodity economies are such abstract constructions that they seem to loose connection with the reality of an industrial world —where economic actors deal every day with literally millions of commodities and services, most of which are not substitutable.²⁵ How come we, economists, still rely on one-commodity aggregate production functions, as we have done so far in this essay? The main justification for postulating that the production sector of, say, a country can be captured by such a simplistic function is the good empirical fit of such aggregate production functions with empirical time series. Consider the Cobb-Douglas function

$$Y = AK^\alpha L^\beta, \tag{22}$$

which obtains from the CES production function considered earlier when $\eta \rightarrow 0^+$. One easily shows that profit maximization entails that the share of wages, $\omega = wL/Y$, should always satisfy $\omega = \beta$. Similarly, $rK/Y = \alpha$, where the rate of return, $r := \partial Y/\partial K$, conflates both the short run interest

²⁵As Eric Beinhocker puts it: “The Wal-Mart near JFK Airport has over 100,000 different items in stock, there are over 200 television channels offered on cable TV, Barnes & Noble lists over 8 million titles, the local supermarket has 275 varieties of breakfast cereals, the typical department store offers 150 types of lipstick, and there are over 50,000 restaurants in New-York City alone.” (Beinhocker 2007, pp. 8-9.)

rate and the marginal productivity of capital. Taking the logarithm of (22) leads to the linear equation

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L. \quad (23)$$

As a matter of fact, empirical tests suggests that the linearity of the (log-linearized) production function appears to be a remarkably consistent finding across countries and across time. Moreover, the sum of the estimated coefficients, $\alpha + \beta$, usually approximates closely to unity —which presumably explains why we should assume that returns to scale are constant. Next, there is a quite satisfactory agreement between the estimated labor exponent, β , and the share, ω , observed in the data. This is usually interpreted as amply confirming that the marginal productivity of labor is equal to the wage share, and that the production sector of an economy can indeed be captured adequately through (22), or some variant of it.

Unfortunately, it has been proven by [Shaikh 1974](#) and [McCombie 2001](#), that this remarkable agreement of data with (22) simply arises from the fact that, provided that factor shares remain constant, (23) is but an accounting identity. Since it happens that, during the first 40 years after World War II, wages and profits have grown more or less at the same pace in most industrialized countries, their respective shares have remained approximately constant. The theory of production functions *seemed* therefore to be confirmed while we were actually only dealing with the artifact of accounting methods. In the past three decades, the rise of the profit share (at the expense of the wage share) in most industrialized countries —not mentioning the globalized South— has lead to the failure of the reassuring empirical agreement. But we are so deeply convinced by our traditional interpretation of (23) that we may be tempted to blame a time of economic turbulences rather than considering the possibility that the previously established agreement was, in fact, an *accident* (the constancy of factor shares across time) coupled with a *tautology* (an accounting identity).

This observation can lead to two complementary attitudes: the first is to acknowledge that the Cobb-Douglas (or, more generally, the CES) specification is not pertinent, and to replace it with another aggregate production function. This is the approach followed, e.g., by [Kümmel and Lindenberger 2020](#), who introduce a LinEx function, which not only depends upon capital and labor but also upon energy use. It is based on the remark made above that energy plays a much more decisive role in our economies than the textbook cost-share theorem suggests.²⁶ The second reaction goes a step further and aims at reconsidering the theory of aggregate production functions *as such*. As we shall now see, in this section and in the next, this is not an easy task, as it requires to reconsider some of the most basic tenets of conventional economic analysis.

[Samuelson 1962](#) has indeed characterized the core of the conventional theory through three key “parables,” which are all challenged when questioning the formalism of aggregate production functions.

- (i) The real return on capital, r (often identified with the short run rate of interest) is determined by the diminishing marginal productivity of capital. It should therefore be a purely technical parameter.
- (ii) A greater quantity of capital leads to a lower marginal product of additional capital and thus to a lower rate of interest, and the same inverse, monotonic relation with the rate of interest also holds for the capital/output ratio and sustainable levels of consumption per head.

²⁶Cf. section 5.1.

(iii) The distribution of income between laborers and investors is explained by relative factor scarcities/supplies and marginal products. The price of labor services, w , is determined by the relative scarcity and marginal productivity of labor, $\partial Y/\partial L$.

The three parables of this one-commodity narrative depend on a physical understanding of capital and labor: changes in factor quantities cause inverse changes in factor prices, allowing powerful, unambiguous predictions like parable (ii). We already know, however, that the textbook first-order condition of profit maximization leads to a wrong specification of prices: r will therefore be presumably different from $\partial Y/\partial K$.²⁷ How much different? It depends, among other things, upon the number of firms producing capital—a number which is typically fluctuating across time, so that the neat conclusion of “parable” (ii) need no longer hold. Moreover, we also know that, in a multi-commodity world, “everything is possible” at the aggregate level so that there might be multiple rates of return, r , compatible with profit maximization.²⁸ Whence the three parables altogether are imperiled. This suggests one escape road: to give up the restriction to a single commodity. After all, this is already what the LinEx production function performs by adding energy to capital and labor. The need to shift away from the simplistic one-commodity world was actually at the heart of the celebrated Cambridge dispute.

6.2 The Cambridge Controversy

Considering a world populated by multiple commodities raises a number of difficult questions.²⁹ Heterogeneous capital goods cannot be measured and aggregated in physical units (even in kg or joules). Instead, capital monetary valuation must be used. The monetary value of capital can be measured either as the cost of production, which takes time, or the present value of the future flow of incomes it produces. In either case, since the measure involves time, it presumes a discount factor which, quite often, is identified with the rate of interest. The latter being allegedly determined by the quantity, i.e. the value, of capital, there is a circularity which again undermines the whole theoretical building—to say the least. Moreover, even if there was a way to find a fixed point that puts an end to the vicious circle between K and r , this interdependence causes Wicksell effects, i.e., changes in the value of the capital stock associated with different interest rates, arising from either inventory revaluations of the same physical stock due to new capital goods prices (price Wicksell effects) or differences in the physical stock of capital goods (real Wicksell effects).

In the 1950s’ and 1960s’, these effects were termed *reswitching* and *capital-reversing*, and received much attention. Reswitching occurs when the same capital-to-labor ratio is preferred at two or more rates of interest while other techniques are preferred at intermediate rates.³⁰ The same technique is associated with two different rates r , violating parables (i) and (ii). With capital-reversing, “a lower capital/labor ratio is associated with a lower interest rate. In comparing two steady state equilibrium positions, it is as though capital services have a lower price when capital is “more scarce.” Capital-reversing implies that the demand curve for capital is not always downward sloping, violating parables (ii) and (iii).”

These pathologies have been at the heart of the famous Cambridge controversy, opposing economists from Cambridge UK (Piero Sraffa, Joan Robinson, Luigi Pasinetti and Pierangelo Garegnani) to Cambridge US (Paul Samuelson, Robert Solow, Frank Hahn and Christopher Bliss).

²⁷See section 4.3 *supra*.

²⁸See section 2.3 *supra*.

²⁹We loosely follow [Harcourt 2012](#) in this short wrap up of the Cambridge controversy.

³⁰As [Harcourt 2012](#) put it: “At lower values of the interest rate $[r]$, the cost-minimizing technique “switches” from a to b and then (“reswitches”) back to a .”

Contrary to what is sometimes claimed in textbooks, the issue was not the possible instability of “capitalism” —the latter refers to the indirect debate between Harrod-Domar and Solow— but the very meaning of aggregate production functions. Joan [Robinson 1953](#) wrote:

“The production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $Q = f(L, K)$ where L is a quantity of labor, K a quantity of capital and Y a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labor; he is told something about the index-number problem in choosing a unit of output; and then he is hurried on to the next question, in the hope that he will forget to ask in what units K is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next.”

As acknowledged by Paul [Samuelson 1966](#), the “English corner” won the analytical discussion: apart in a one-commodity world, none of the three “parables” can be postulated. A first step towards solving this conundrum therefore consists in extending our SFC dynamics to a multisectoral economy.

6.3 Intersectoral dynamics

To address the issues raised by the Cambridge controversy, as well as to be able to properly incorporate material flows in our analysis, we now extend the family of SFC macrodynamics presented above to a multisectoral economy. Following [Valcke and Giraud 2022](#), consider S distinct production sectors indexed by i . For the sake of concreteness, let us assume that Say’s law is not in force (as in [4.1](#)) and that there is some substitutability between labor and capital in the production function of each sector. Thus, each sector i is equipped with a CES production function. The main differences with the one-commodity set-up are as follows.

- (i) Sector-related fields and parameters are now S -dimensional vectors: $(K_i)_i, (p_i)_i$, etc. In particular, aggregate consumption in commodity i is given by $C_i = C_i Y_i$ and investment by $\kappa_i = \frac{\kappa}{\sum_j \Xi_{ij} p_j}$, where κY designates, as before, global, real investment.
- (ii) Besides capital and labor, the output of sector i can be used as an intermediate consumption by sector j . These intersectoral relations are taken care of courtesy of a non-negative $S \times S$ matrix $(\Gamma_{ij})_{i,j}$. Entry $\Gamma_{ji} \geq 0$ is the quantity of commodity i needed for the production of one unit of good j .
- (iii) The capital of sector i is built using outputs from sectors $j = 1, \dots, S$. These relationships are quantified *via* a second, non-negative $S \times S$ matrix $(\Xi_{ij})_{i,j}$. Entry $\Xi_{ji} \geq 0$ is the quantity of commodity i needed for the production of one unit of capital j .
- (iv) For simplicity, the following variables remain sector-independent (i.e. scalar): the total population of workers N , the population growth rate n , the short run interest rate r , the wage level w —which is driven by a short run Philips curve $\dot{w}/w = \phi(\sum_i \lambda_i)$ —, the degree of money illusion γ , the Harrod-neutral labor-enhancing technological progress α .

The dynamics of inventories [\(13\)](#) becomes:

$$\dot{V}_i = Y_i - C_i - \sum_j (\Gamma_{ji} Y_j + \Xi_{ji} I_j),$$

where I_j is the real investment of sector j , and C_i denotes aggregate consumption of commodity i . The unitary production cost becomes:

$$c_i/p_i = \omega_i + \sum_j \Gamma_{ij} \frac{p_j}{p_i}$$

Corporate debt in sector i obeys the following ODE:³¹

$$\underbrace{\dot{D}_i := rD_i}_{\text{banking sector}} + \underbrace{wL_i - C_i p_i}_{\text{households}} + \underbrace{I_i \sum_j \Xi_{ij} p_j - p_i \sum_j \Xi_{ji} I_j}_{\text{intermediate consumption}} + \underbrace{Y_i \sum_j p_j \Gamma_{ij} - p_i \sum_j \Gamma_{ji} Y_j}_{\text{investment}} \quad (24)$$

The dynamics of the multisectoral economy boils down to a $3S$ -dimensional system. Since, in the monosectoral case, the reduced system is 3-dimensional, $3S$ is the minimal dimension we could hope for in the S -sector case. It appears, as before, that the dynamical system admits different types of steady states: a set of non-inflationary Solovian equilibria (with a balanced growth path), a debt-deflationary equilibrium, and trivial equilibria that are economically meaningless. Asymptotic local stability of these steady states, however, obtains under more stringent conditions than in the one-sector case, so that instability seems rather to be the rule. On the other hand, multisectorality opens the road for a very rich intersectoral dynamics, as shown by the following example.

Consider a two-sector economy where sector 1 only produces the consumption good and sector 2, the capital good.

- (i) $\mathcal{C} = (C_1, C_2) = (1, 0)$: households spend all their revenues on the consumption commodity.
- (ii) $\Xi = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$: sector 1 purchases its investment from sector 2.
- (iii) $\forall i, j, \Gamma_{ij} = 0$ no intermediate consumption.
- (iv) Suppose that all the other parameters ($A_i, \eta_i, b_i \dots$) are identical in both sectors, and that initial conditions are far from the interior equilibrium

As shown by (??), the economy exhibits endogenous business cycles quite similar to that of a one-sector Van der Ploeg model. In this particular example, they are damped as the economy approaches the balanced growth path of Solovian long run steady state. However, the sectoral level looks quite different: it is only their aggregation that portrays the familiar dynamics — another instance of a phenomenon of emergence.

INSERT FIGURE 15

7 Conclusion

To a certain extent, this entire chapter has revolved around reshaping our understanding of capital. Starting with Samuelson's idea that "capital is time," we have moved to the need to take account of the hysteresis due to the spillover of investment on the path-dependent dynamics of an economy. We then saw that capital is fragile: accounting for the impact of global warming on capital increases

³¹We have indicated the recipients of each flow.

the long-lasting effect of climate change on our well-being. Embedding macroeconomics into thermodynamics even reveals that capital is essentially a way to extract and metabolize energy and matter. Next, we reported that capital cannot be understood as a one-dimensional monetary catch-all: the Cambridge controversy made it clear that capital must be multidimensional and does have a material component. Finally, capital should not even be considered as a stock but rather as a flow within the broader perspective of joint production. Obviously, these changes of perspective are demanding. They are the price to be paid if one wants to account for complexity in economics.

Two domains have not been covered in this chapter, both in order to keep it within reasonable dimensions and because these domains deserve more research: a) money and b) the public sector. About a), as pointed out by [Pottier and Nguyen-Huu 2017](#), the basic Keen model can still be interpreted as adhering to the loanable funds theory of money. Therefore, we still lack an analytical framework where endogenous money creation and money nonneutrality can be demonstrated, along the empirical lines described by [Werner 2016](#). [Dossetto et al. 2022](#) is a first tentative pass in this direction. About b), an avenue has been opened by [Costa Lima et al. 2014](#) by adding a public sector to the basic Keen model. In the absence of government intervention, as already said, this system admits, among others, two locally stable equilibria: a “good” one with a finite level of debt and nonzero wages and employment rate, and a “bad” one characterized by infinite debt and vanishing wages and employment. The authors show how the addition of public spending and taxation prevents the “bad” steady state from being locally stable. More generally, the dynamic approach adopted in this chapter and in the literature quoted above opens the door for a renewed understanding of public policy, which consists in driving the economy toward a desirable locus of the phase space while avoiding threatening crossings. Not that we should return to a cockpit-like conception of economy policy as if the state could control the whole economy, we have seen enough emergence phenomena above to know, by now, that an economy cannot be piloted like a spaceship. But, as exemplified by [Costa Lima et al. 2014](#), the recognition of the complexity of far-from-equilibrium economic dynamics does not leave us powerless, quite on the contrary: it is undoubtedly the sine qua non condition to regain a form (even if weakened) of agency on the world. Especially in a time of multidimensional and planetary crisis. As the French socio-philosopher Edgar Morin puts it:

Intelligence that is fragmented, compartmentalised, mechanistic, disjunctive, and reductionistic breaks the complexity of the world into disjointed pieces, splits up problems, separates that which is linked together, and renders unidimensional the multidimensional. It is an intelligence that is at once myopic, colour blind, and without perspective; more often than not it ends up blind. It nips in the bud all opportunities for comprehension and reflection, eliminating at the same time all chances for corrective judgement or a long-term view. Thus, the more problems become multidimensional, the less chance there is to grasp their multidimensionality. The more things reach crisis proportions, the less chance there is to grasp the crisis. The more problems become planetary, the more unthinkable they become. Incapable of seeing the planetary context in all its complexity, blind intelligence fosters unconsciousness and irresponsibility. ([Morin and Le Moigne 1999](#), chapter 7)

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