



ELECTROMECHANICAL COUPLING COEFFICIENT: NEW APPROACH TO STUDY AUXETIC PIEZOELECTRIC HARVESTERS

PowerMEMS+ 2024

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¹Université Savoie Mont-Blanc, **SYMME**, F-74000 Annecy, France and

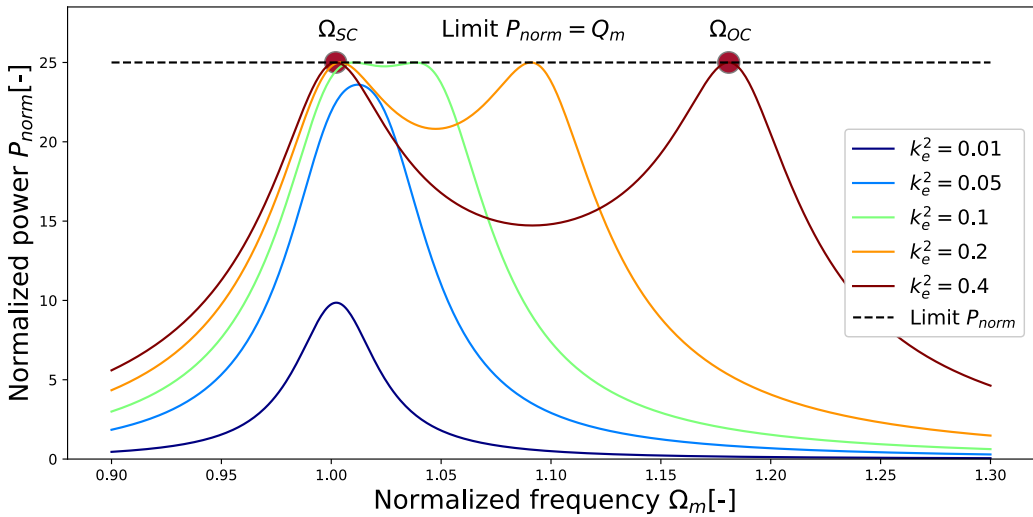
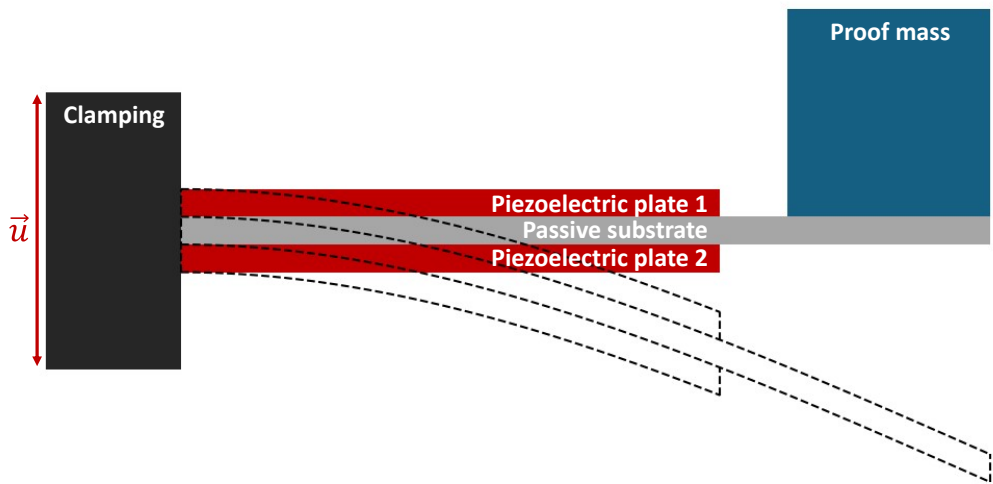
²Université de Bordeaux, CNRS, Bordeaux INP, **IMS**, UMR 5218, 33400 Talence, France

Tønsberg 11.19.2024

- I. Introduction
- II. Performance assessment method
- III. Model Implementation
- IV. Conclusion

I. Introduction

1. Piezoelectric energy harvesting

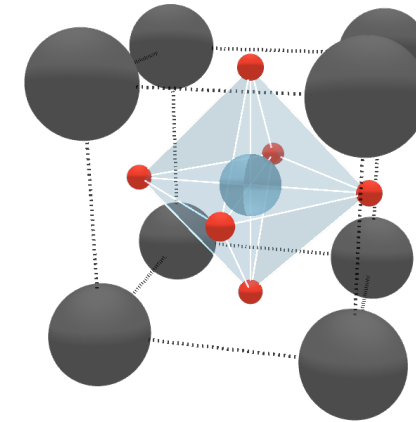
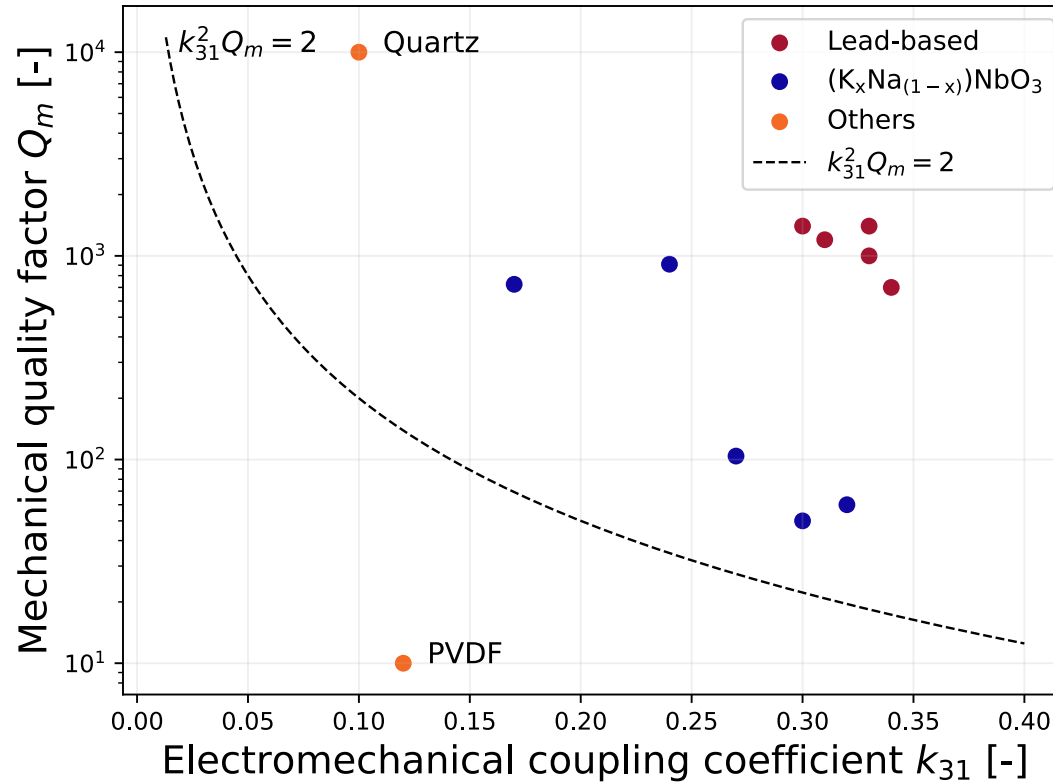


[1] Morel *et al.*, Journal of Intelligent Material Systems and Structures (2019)

- Two important factors on normalized power :
 - The mechanical quality factor Q_m
 - The global electromechanical coupling coefficient k_e^2

2. Lead-free piezoelectric material

Lead-Free $(K_xNa_{1-x})NbO_3$ (KNN)

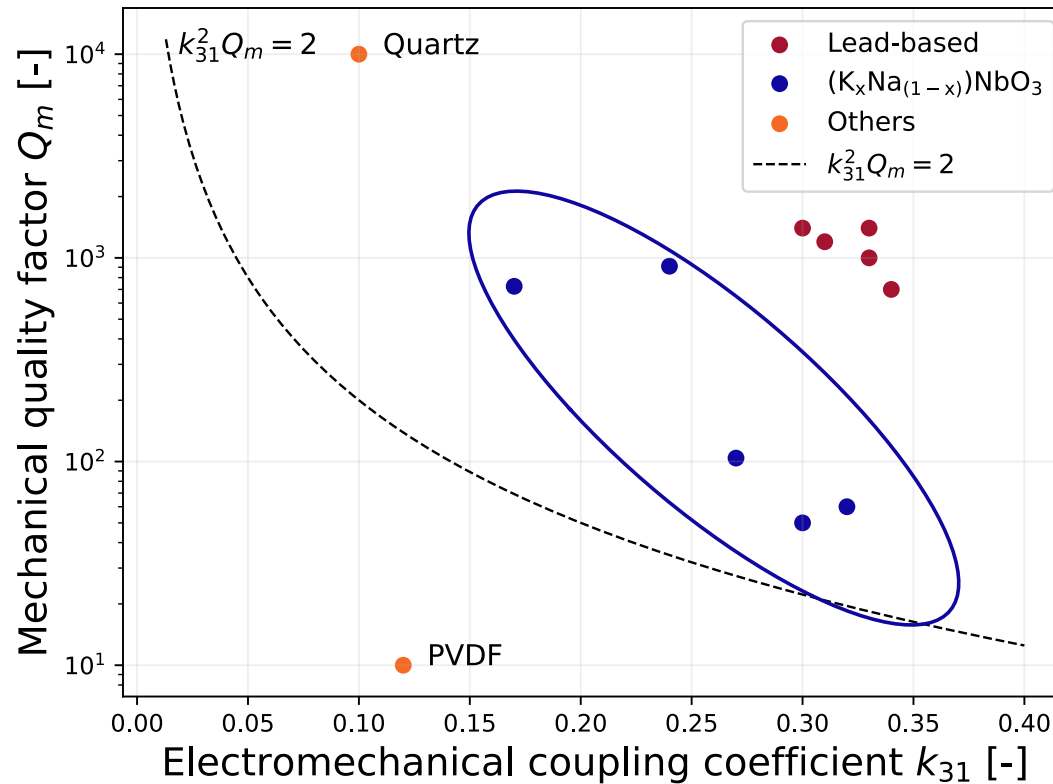


- : Potassium / Sodium ($K_xNa_{(1-x)}$)
- : Oxygen (O)
- : Niobium (Nb)

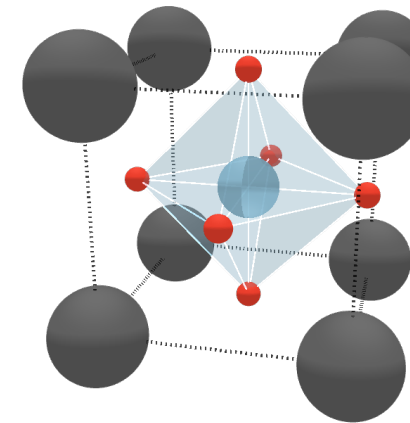
$$k_e^2 \propto k_{31}^2$$

- [2] Piezoelectric characteristics datasheet from Noliac
- [3] Piezoelectric characteristics datasheet from Niterra

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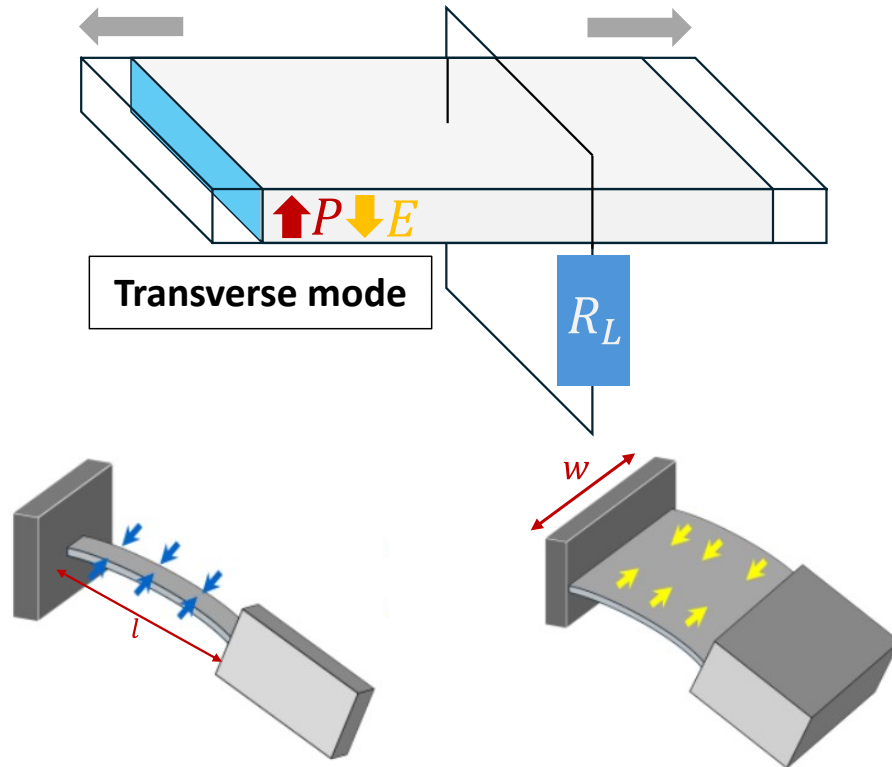
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- Ensure a high material coupling coefficient and a well design to achieve an optimal global coupling coefficient

3. Material electromechanical coupling

$k_{e, piezo}^2$ - different exposures

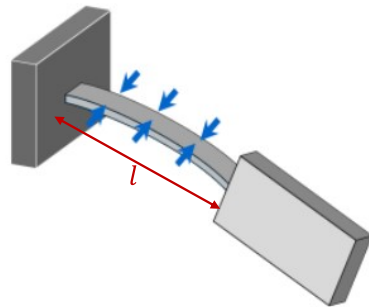
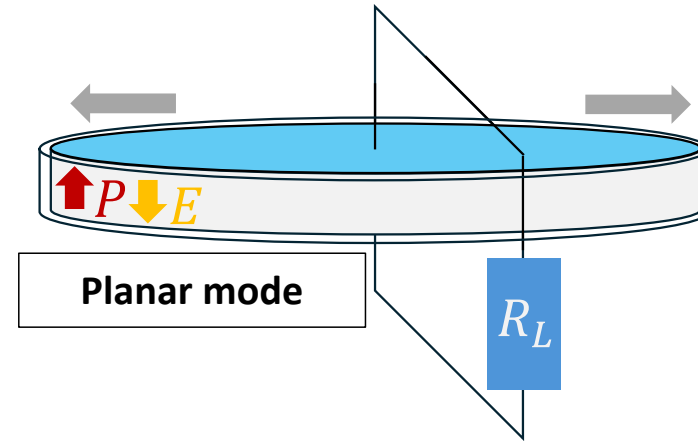
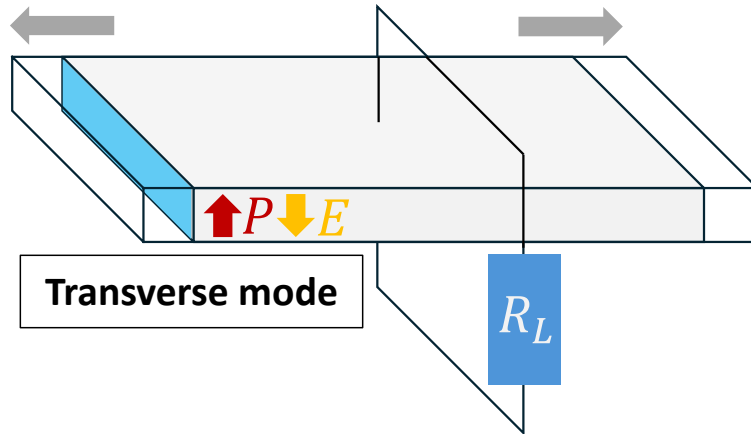


$$(k_{31}^2)^l = \frac{d_{31}^2}{s_{11}^E \epsilon_{33}^T}$$

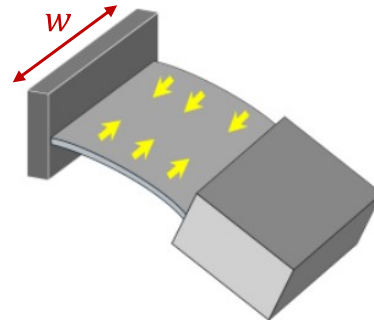
$$(k_{e,31}^2)^w = \frac{(e_{31}^{ef})^2}{c_{11}^{ef} \epsilon_{33}^{ef}}$$

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$$(k_{e,31}^2)^w = \frac{(e_{31}^{ef})^2}{c_{11}^{ef} \epsilon_{33}^{ef}}$$

$$k_p^2 = \frac{2}{1 + s_{12}/s_{11}} \frac{d_{31}^2}{s_{11}^E \epsilon_{33}^T}$$

$$(k_{31}^2)^l < (k_{31}^2)^w < k_p^2$$

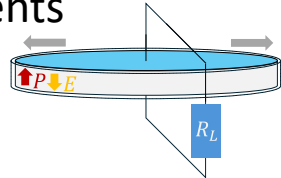
- d_{31} and e_{31} : Piezoelectric constants
- s_{11}^E and c_{11} : Mechanical stiffness and compliance
- ϵ_{33} : Electrical permittivity

[4] Gibus *et al.*, Applied Energy (2020)

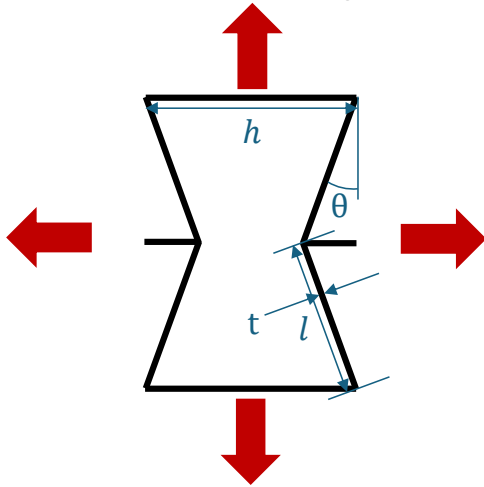
- Expose the piezoelectric material radially to enhance the coupling coefficient

4. Auxetic structures

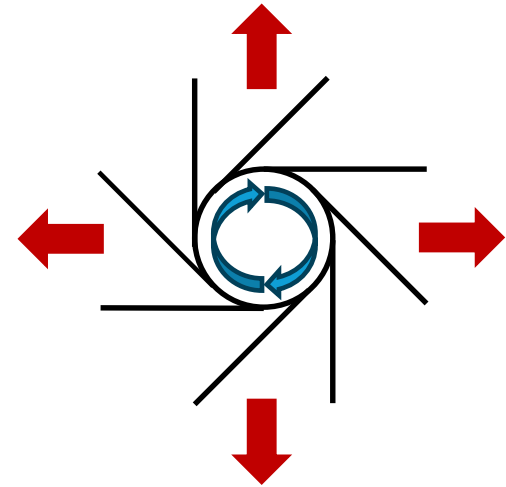
Objective: Invert lateral strain sign and get planar mode in piezoelectric elements



Re-entrant honeycomb

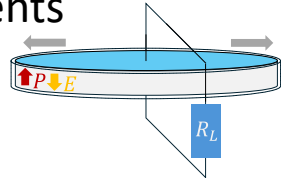


Chiral lattice

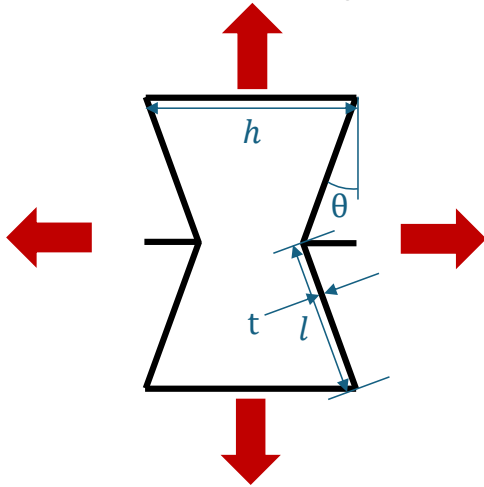


4. Auxetic structures

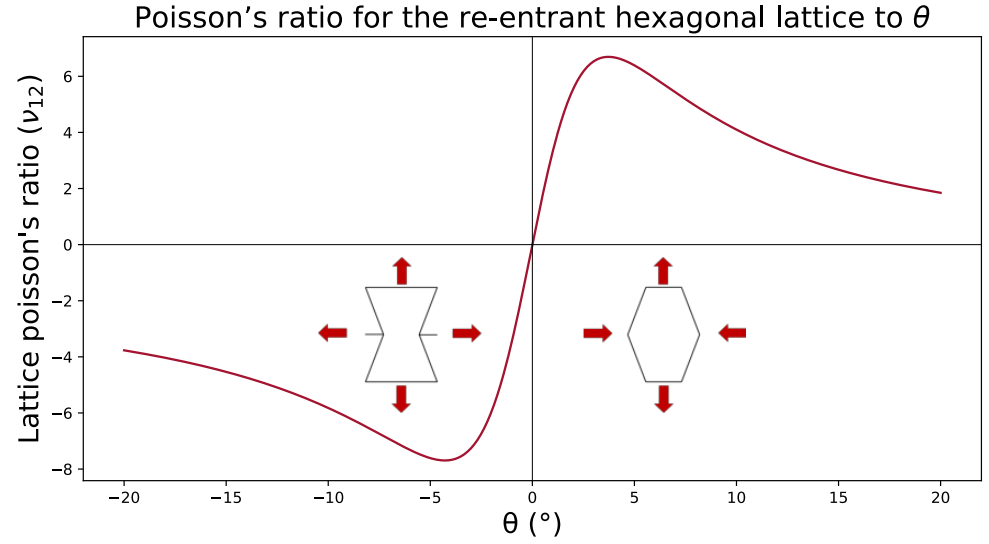
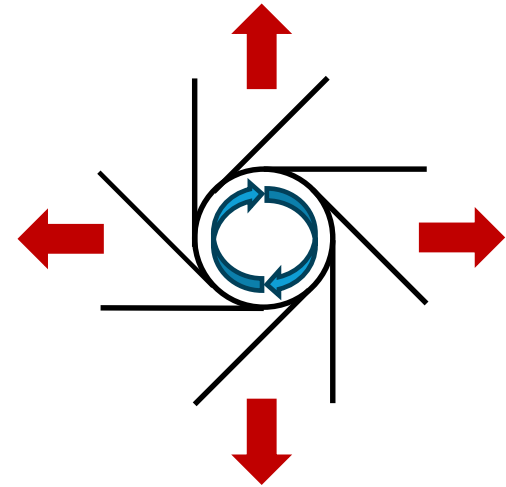
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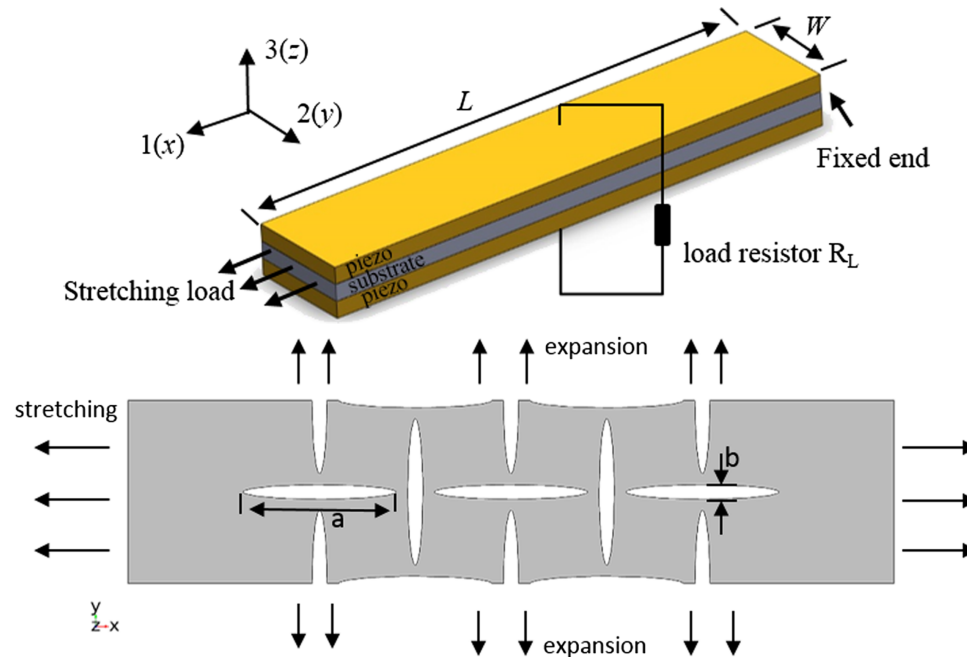
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[5] Gibson and Ashby, Press Syndicate of the University of Cambridge (1997)

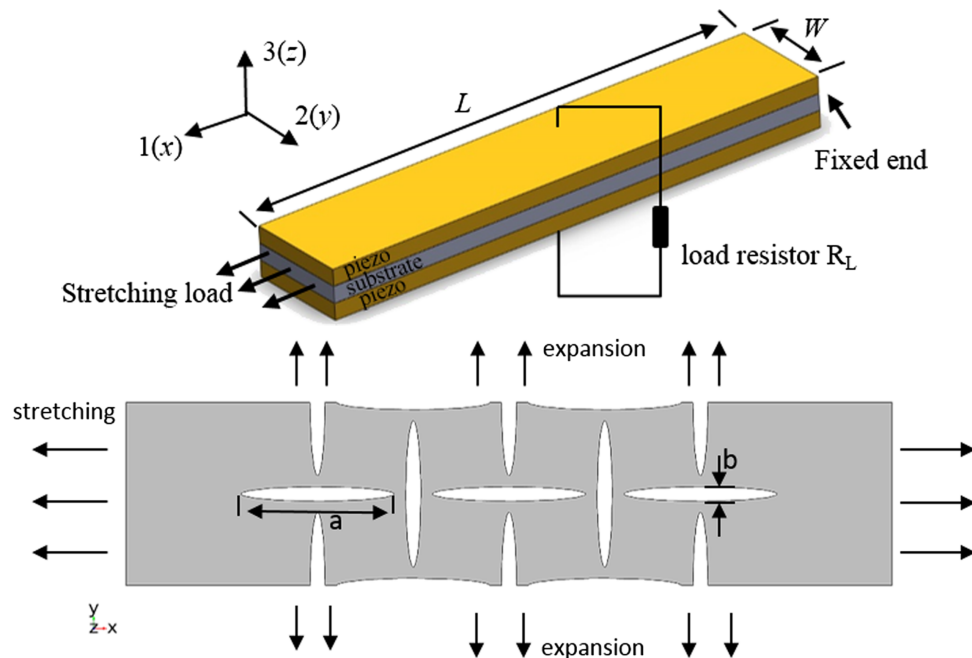
- Design re-entrant honeycomb lattice based substrates to reach planar mode exposure

5. Energy harvesters with auxetic substrates (out of resonance)

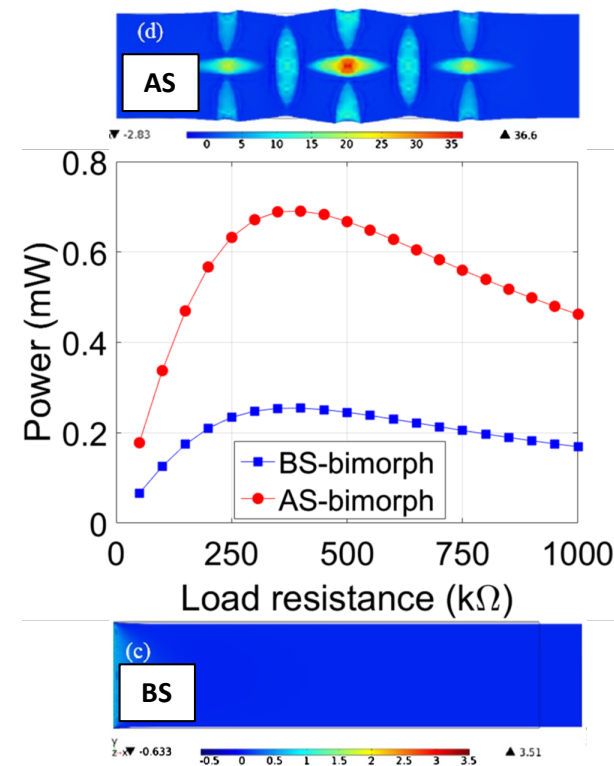


[6] Li *et al.* Aip Advances (2017)

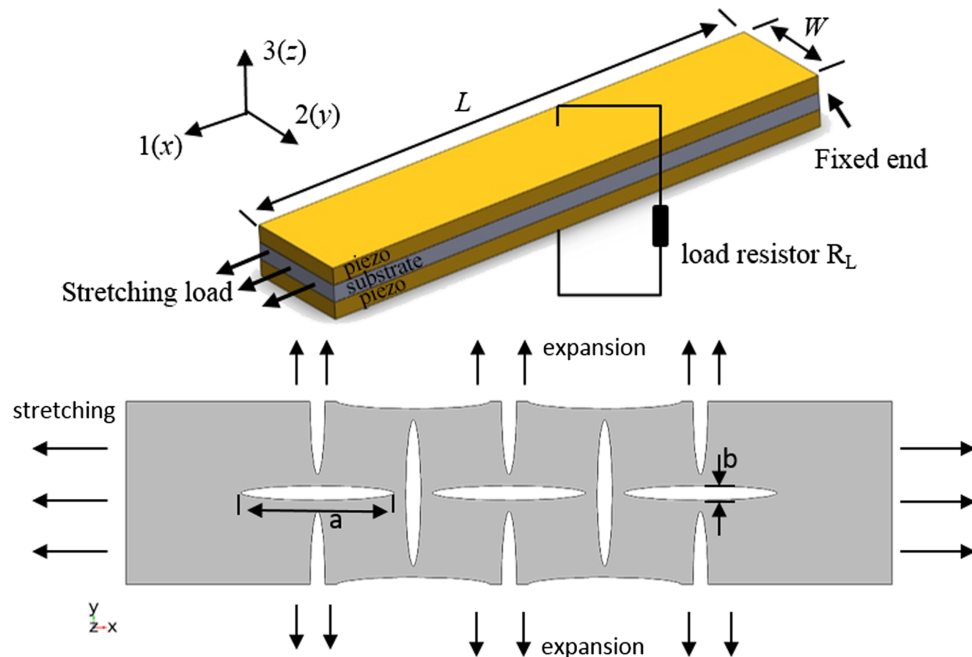
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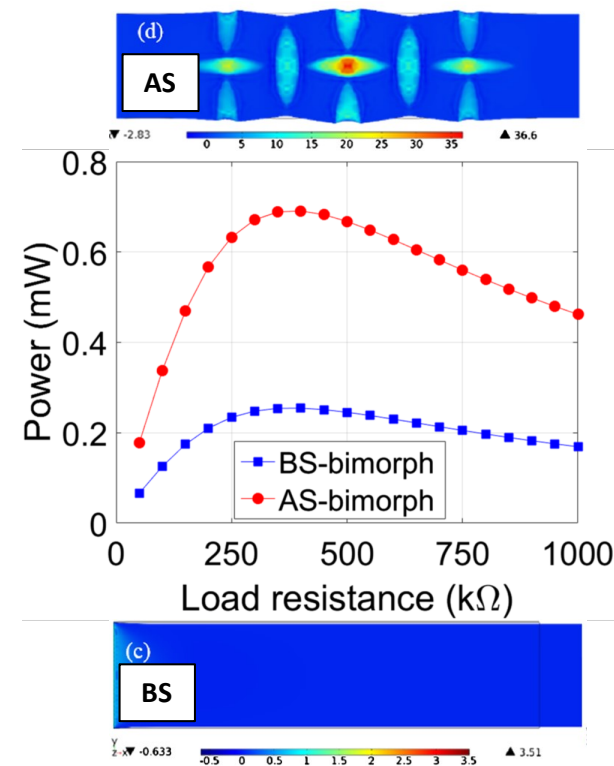
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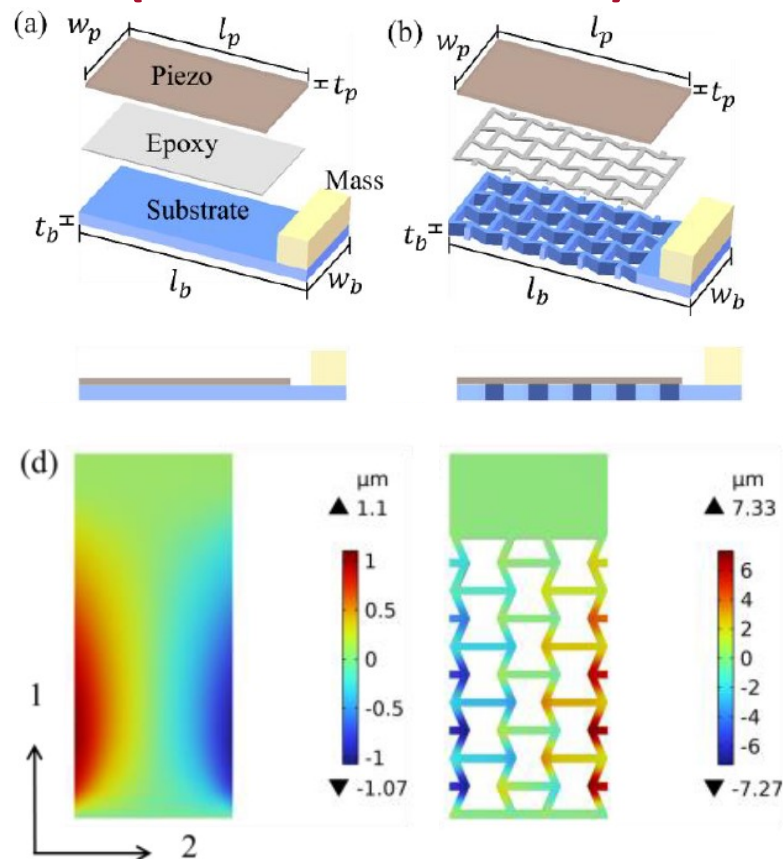
$$P_{\max} = \frac{\pi f t_p A d_{31}^2}{\epsilon_{33}} \cdot (\bar{\sigma}_{11} + \bar{\sigma}_{22})^2$$

- Better output power performance with higher average stress out of resonance



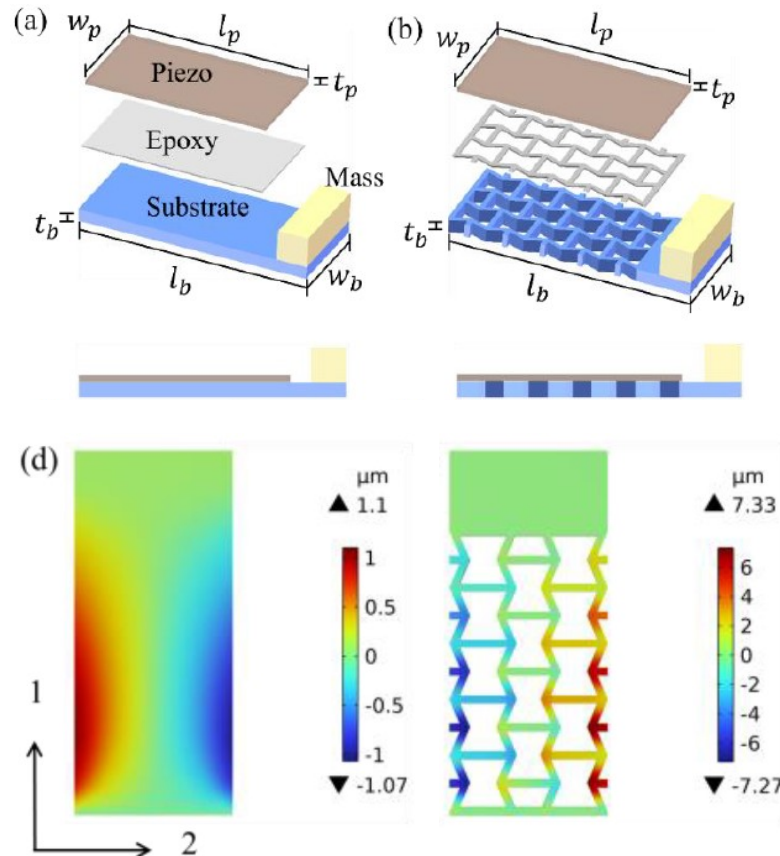
- A : electrode area
- $\bar{\sigma}_{11}$: average longitudinal stress
- $\bar{\sigma}_{22}$: average lateral stress

6. Energy harvesters with auxetic substrates (under resonance)



[7] Chen *et al.*, IEEE Internet of Things Journal (2024)

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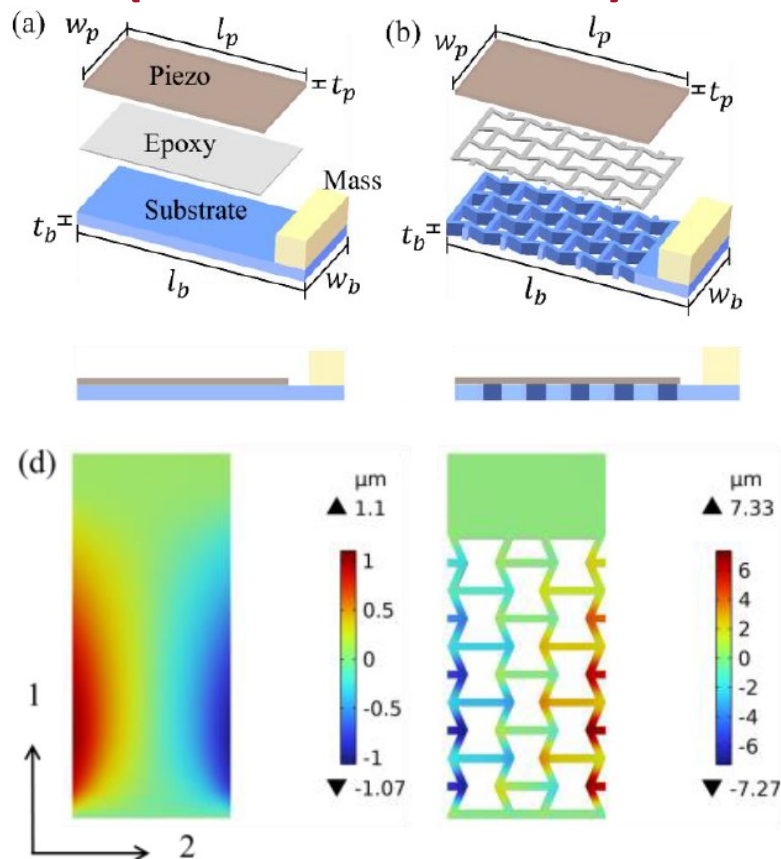


Harvester	Auxetic A	Auxetic B
P_{\max} (mW)	4.34	5.16
$(\sigma_{11}^- + \sigma_{22}^-)^2$ (MPa ²)	44.89	70.90
f_0 (Hz)	43.33	36.48

$$P \propto (\sigma_{11}^- + \sigma_{22}^-)^2$$

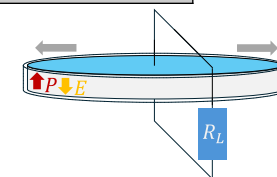
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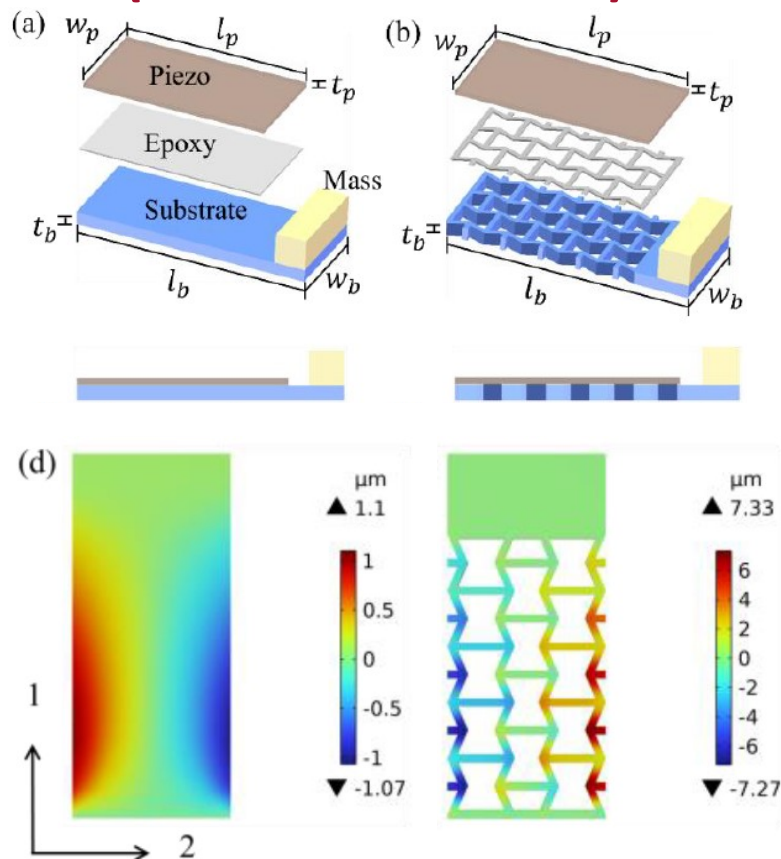
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Auxetic harvesters shouldn't be assessed with power output

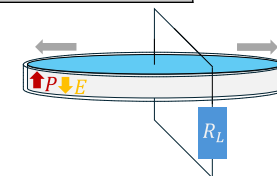
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The power limit is given by:

$$P_{lim} = \frac{M\gamma^2 Q_m}{8\omega_0}$$

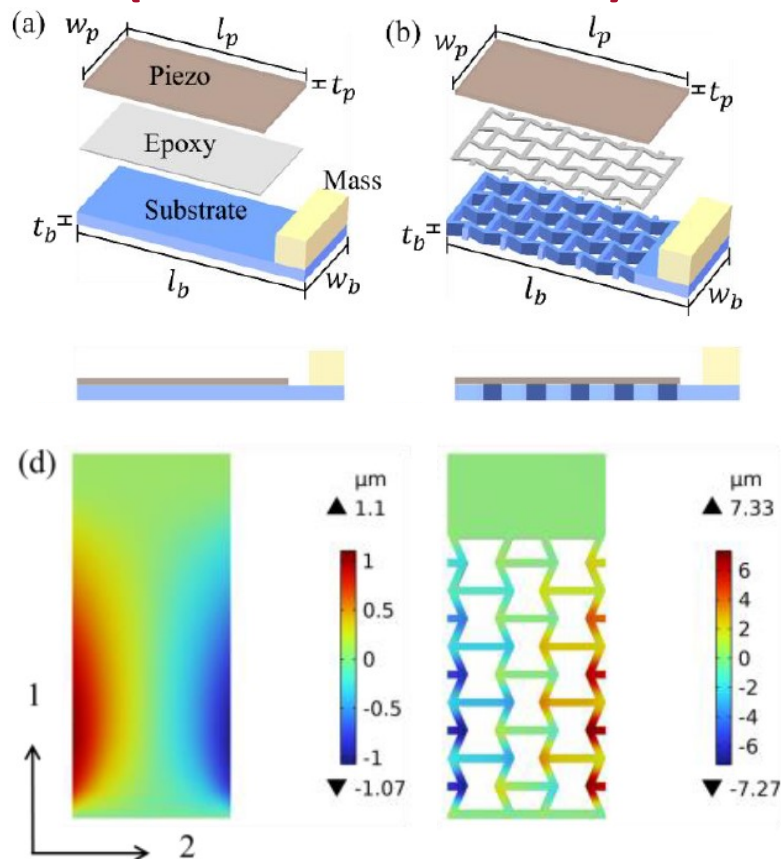
Validated by Chen *et al.* values as:

$$\frac{P_{\max,B}}{P_{\max,A}} = \frac{f_{0,A}}{f_{0,B}} = 1.19$$

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- How can the performance of auxetic substrates be properly assessed ?

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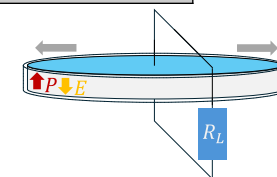


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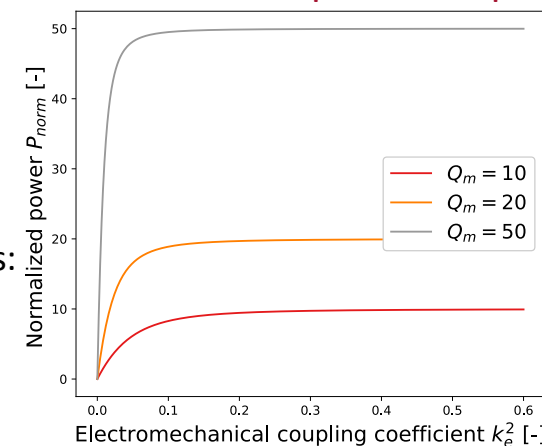
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II. Performance assessment method

1. Elastic energy ratio and coupling coefficient

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System elastic energy is:

$$U = \frac{1}{2} K_{piezo} R(t)^2 + \frac{1}{2} K_{sub} R(t)^2$$

U_{piezo} ↗ ↘ U_{sub}

- U_{piezo} : Piezo elements' elastic energy
- U_{sub} : Substrate's elastic energy
- K_{piezo} : Piezo elements' stiffness
- K_{sub} : Substrate's elastic stiffness

[8] Sodano *et al.*, Strain (2004)

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Global electromechanical coupling coefficient k_e^2 :

$$k_e^2 = k_{e,piezo}^2 \frac{U_{piezo}}{U}$$

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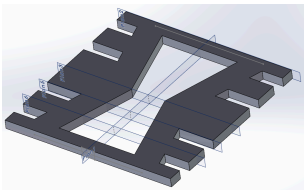
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Integration of energy density from finite element simulation

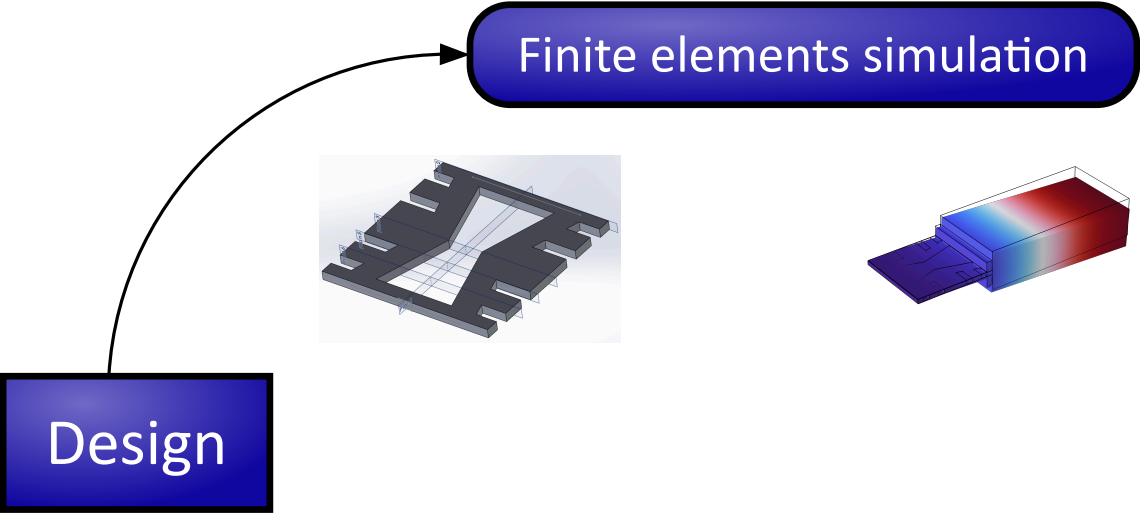
- The material coupling coefficient can be computed with finite element simulation method

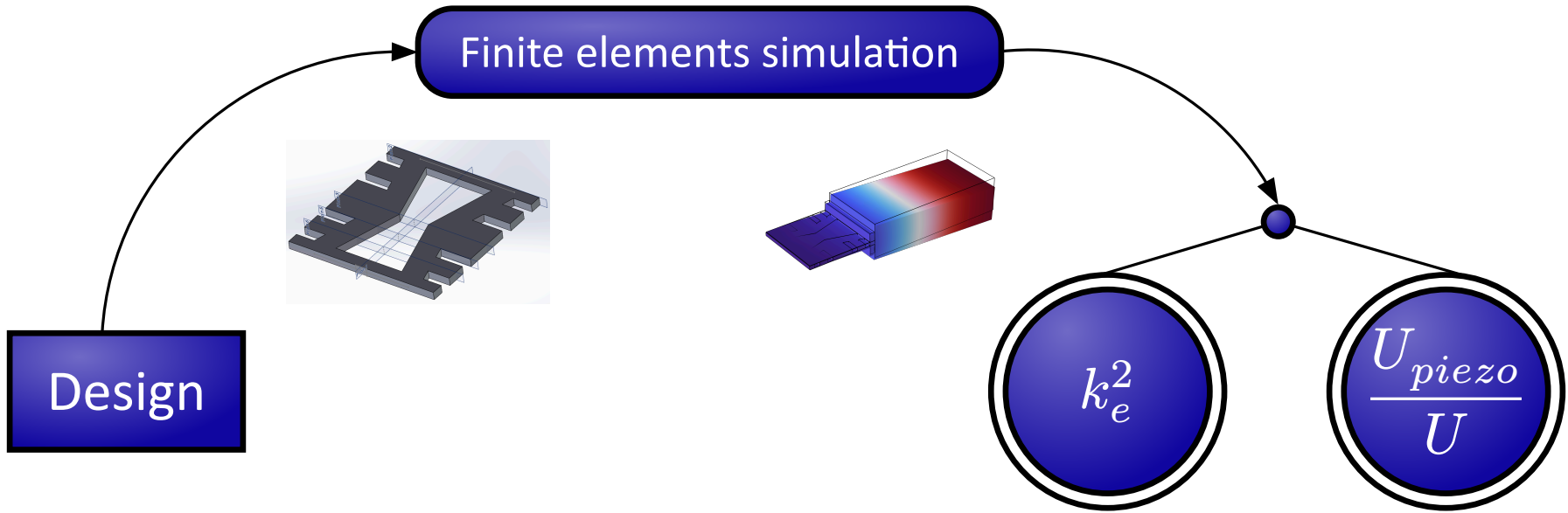
2. System designing

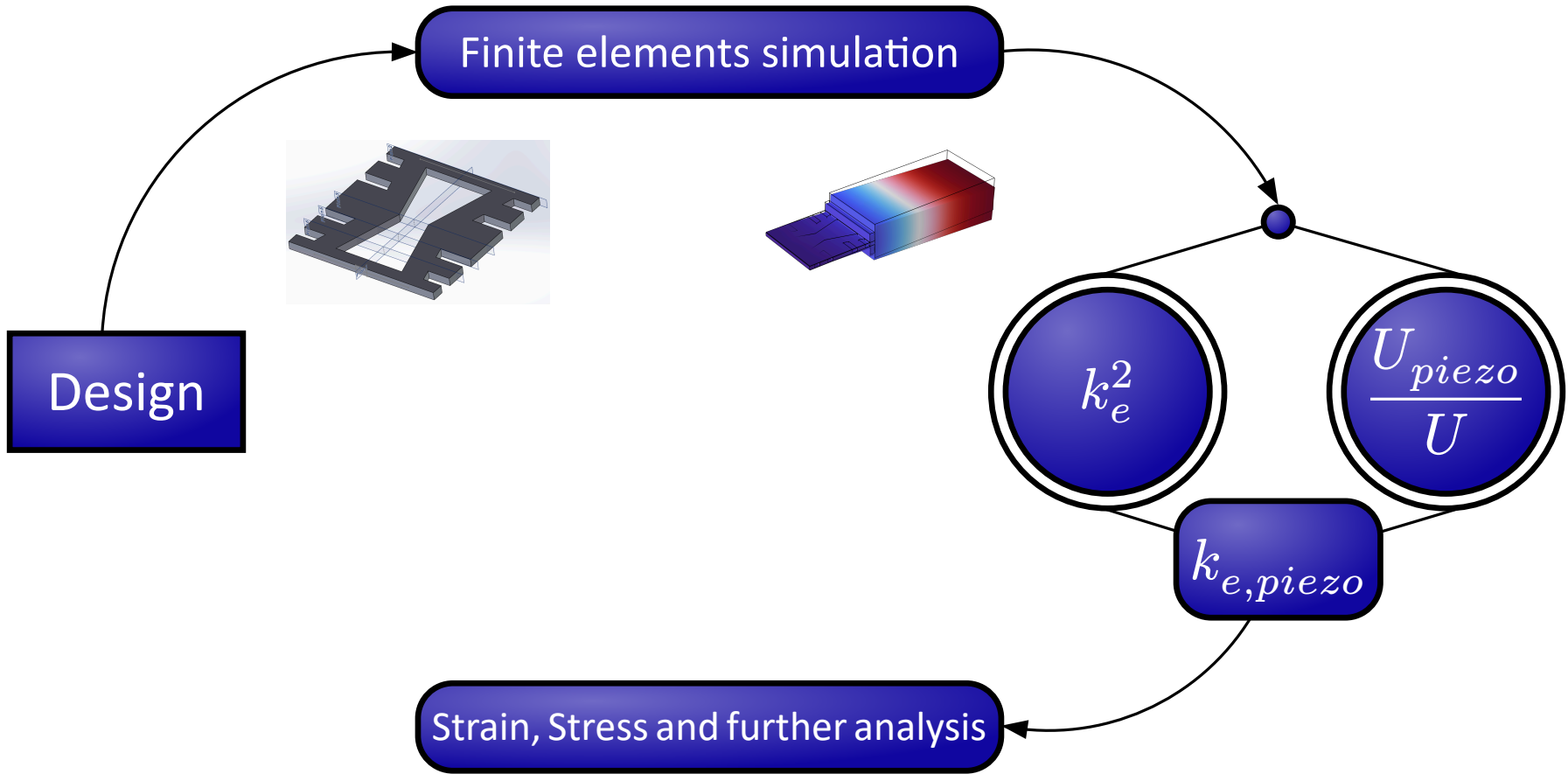
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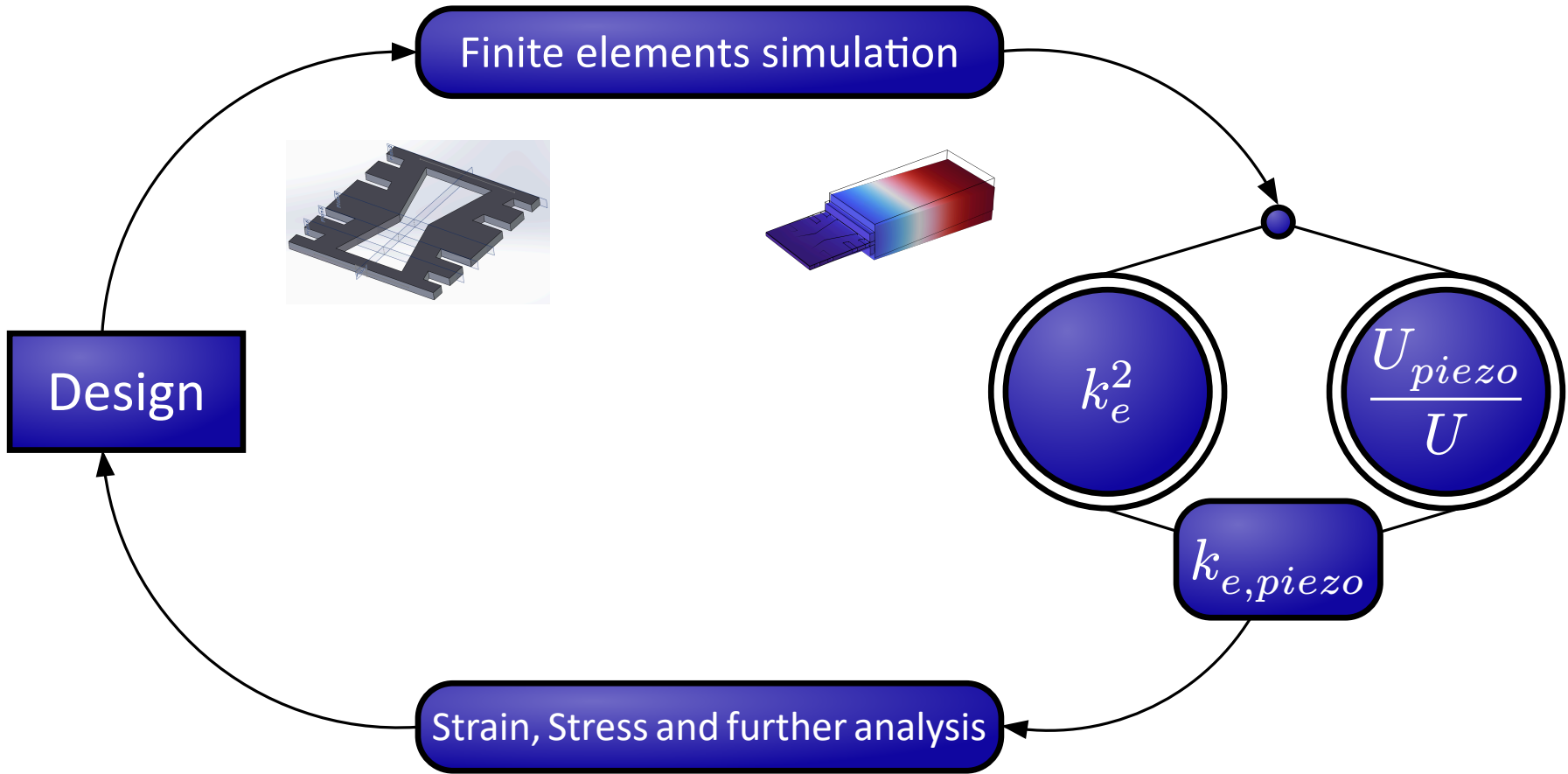


Design







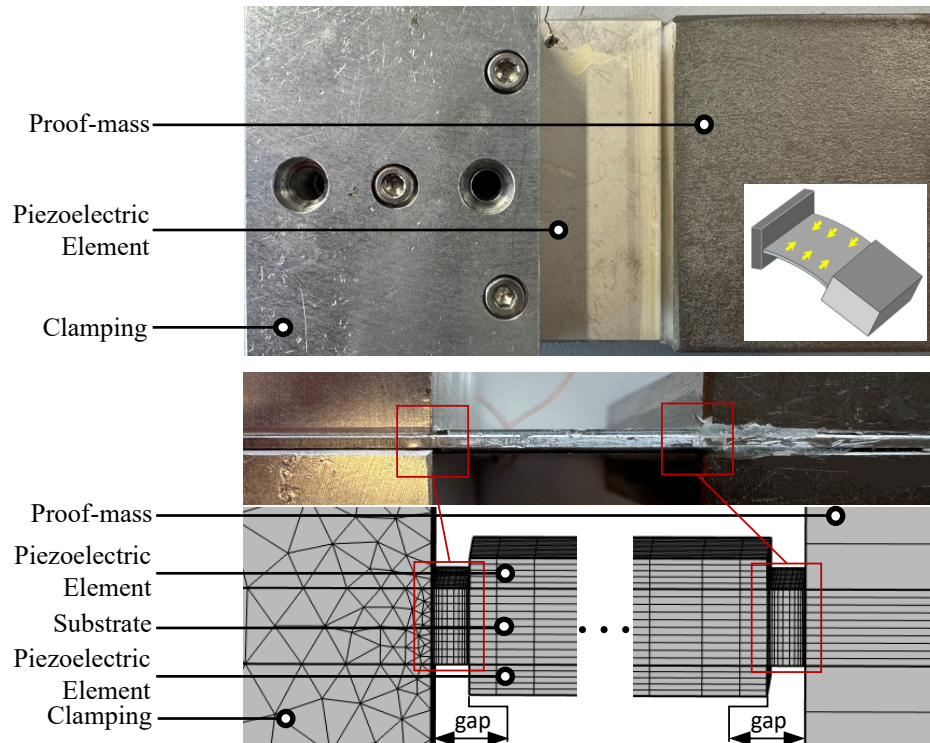


III. Model Implementation

- Perovskite ceramic
- Industrial $K_{0.5}Na_{0.5}NbO_3$
- KNN LF02B (commercial name)
 - $d_{31} = 95 \cdot 10^{-12} \text{ C/N}$
 - $Q_m = 104$
 - $T_c = 563.15 \text{ K}$
 - $\tan(\delta) = 1.3 \%$
- Lead-free materials with promising performance

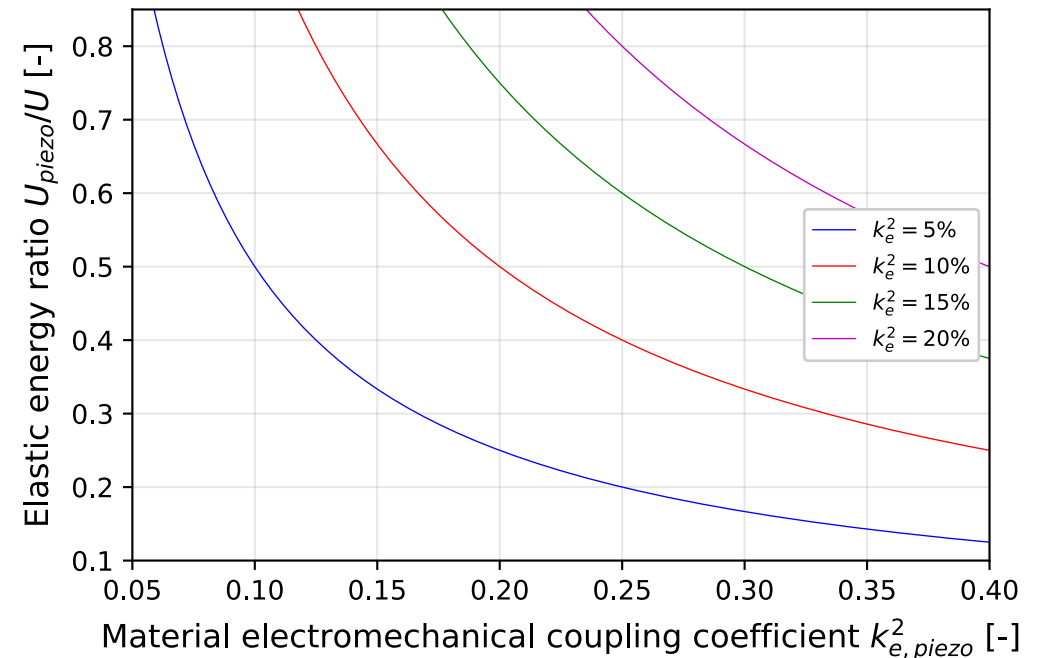
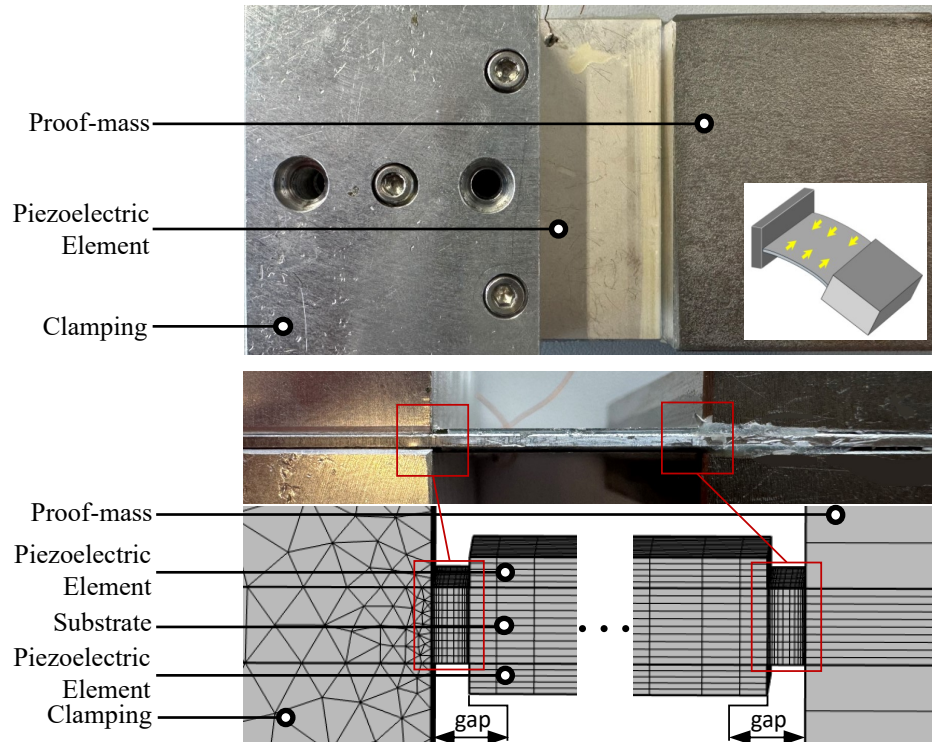


2. Application to a bimorph wide cantilever



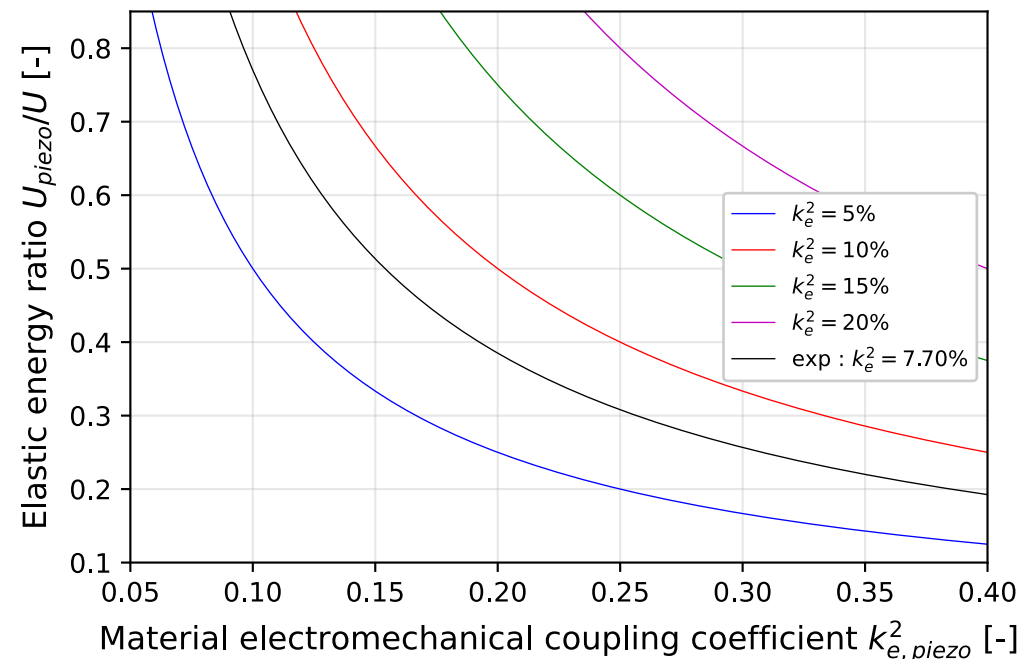
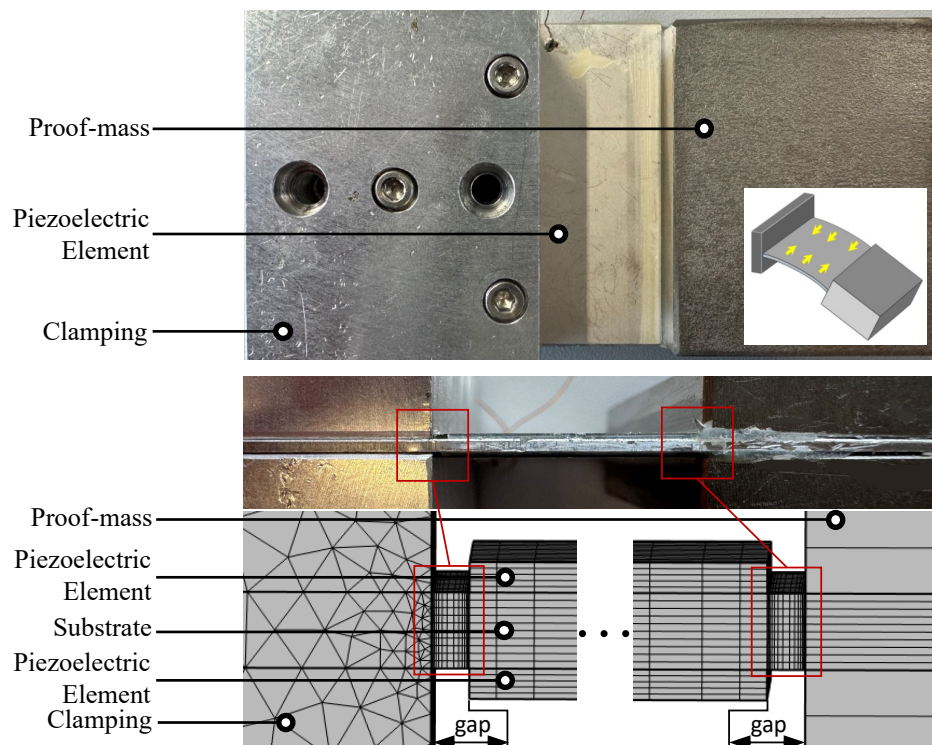
- Only decrease of elastic energy ratio but no change on piezoelectric layers' exposure
- Could auxetic substrate permit to obtain planar mode ?

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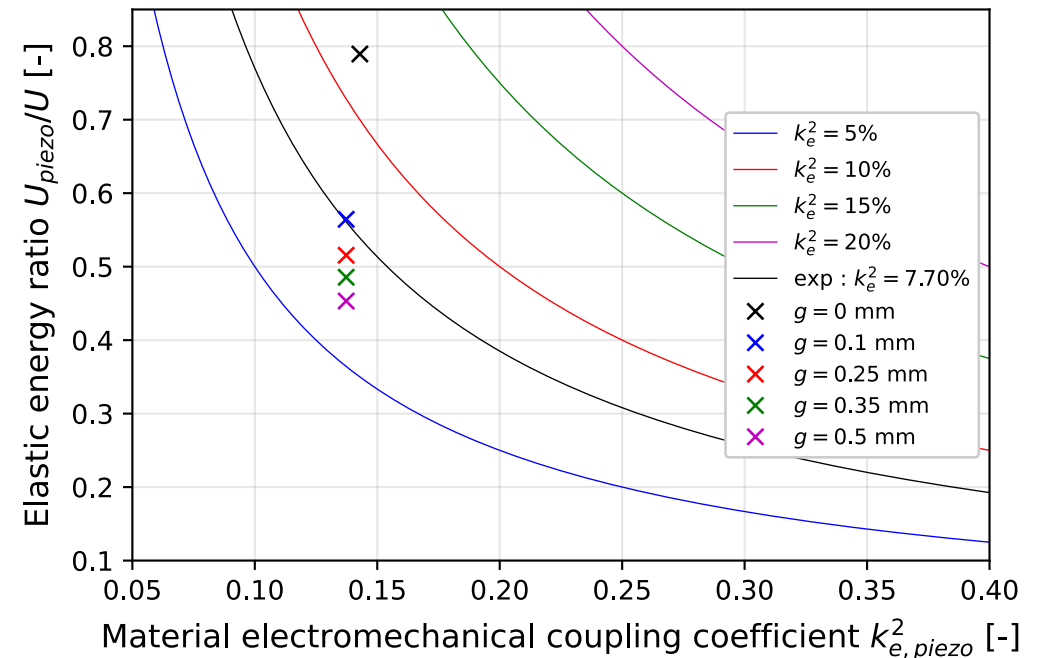
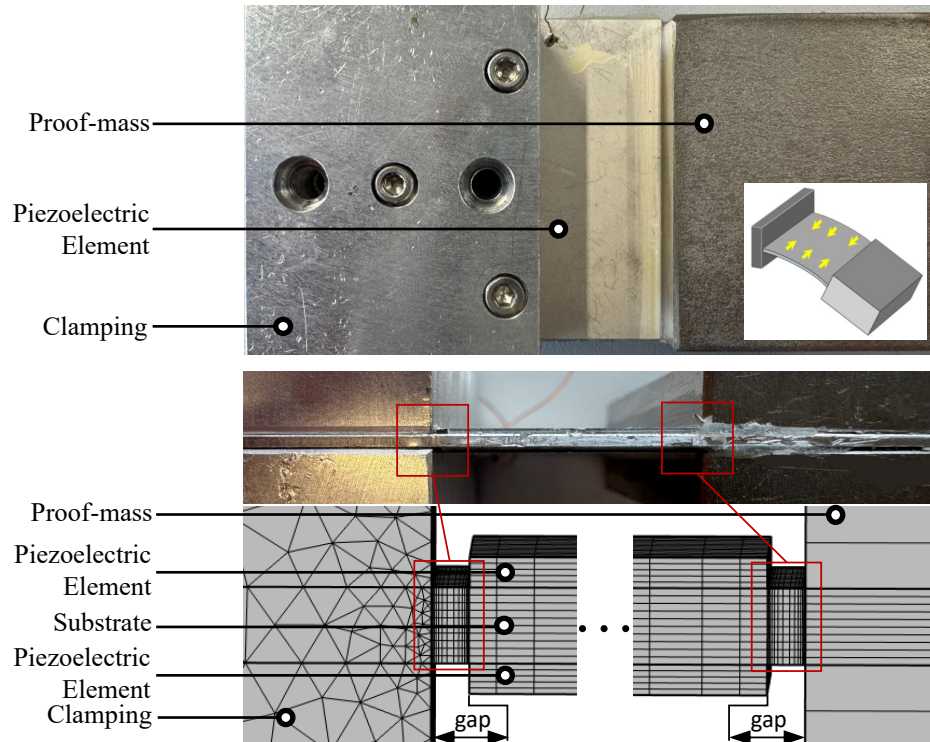
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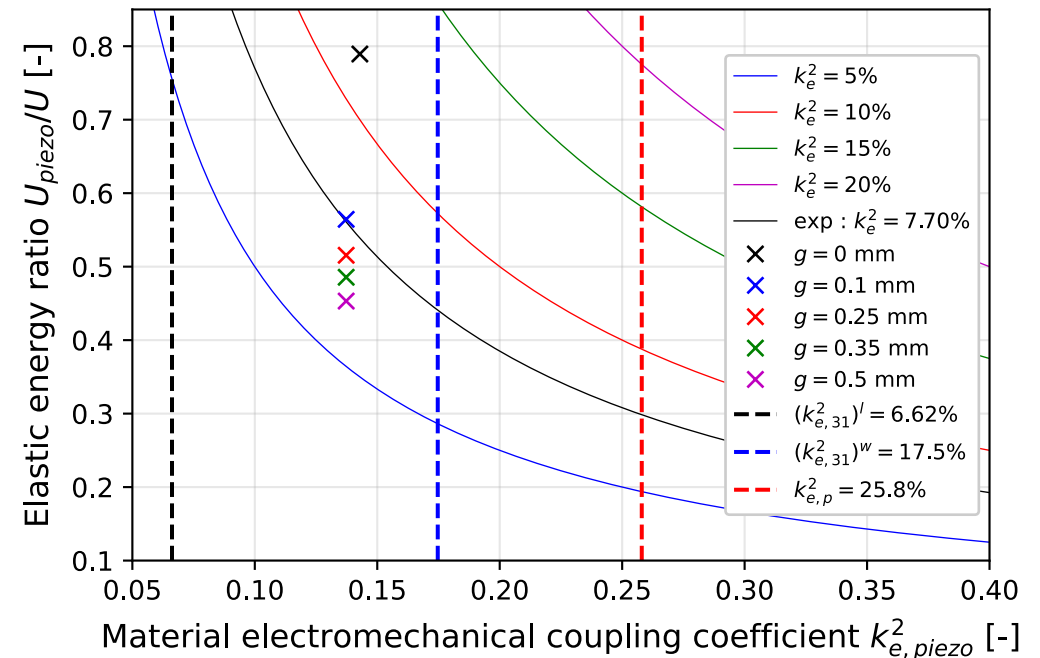
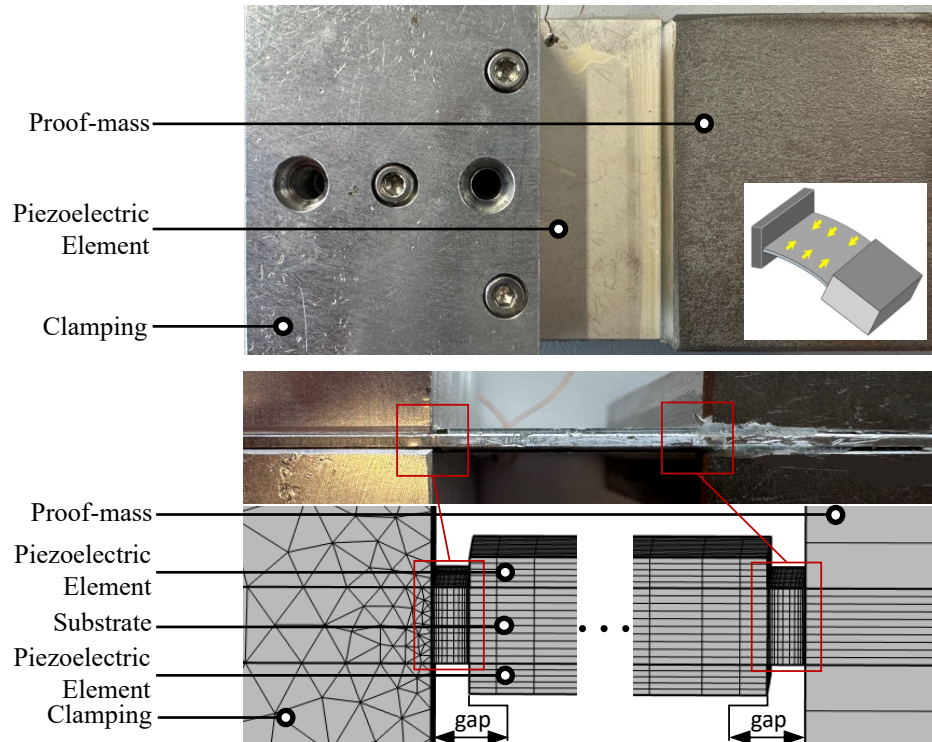
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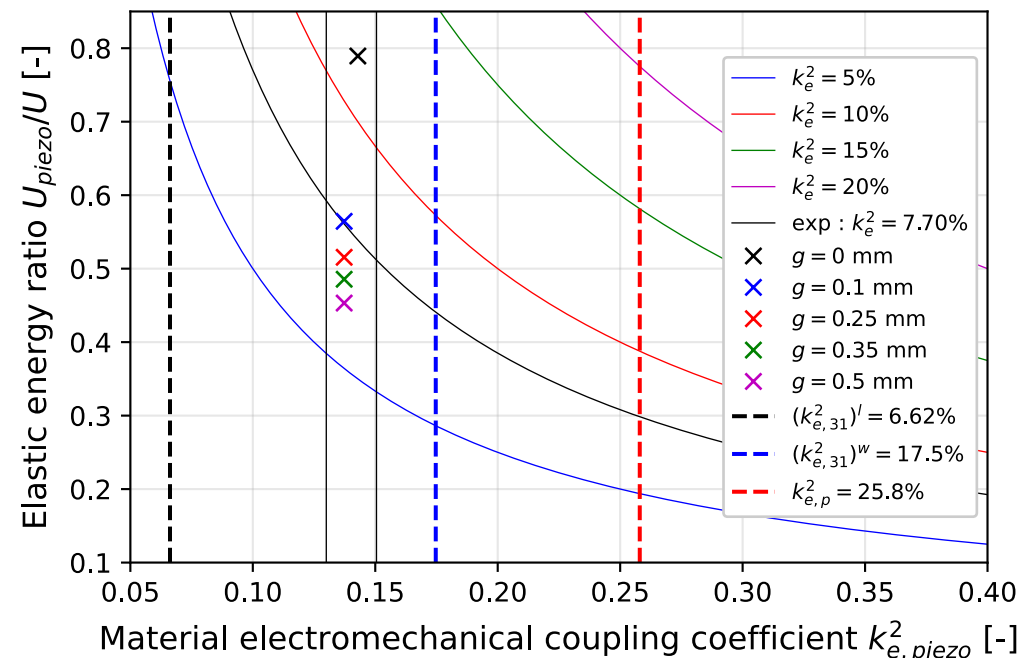
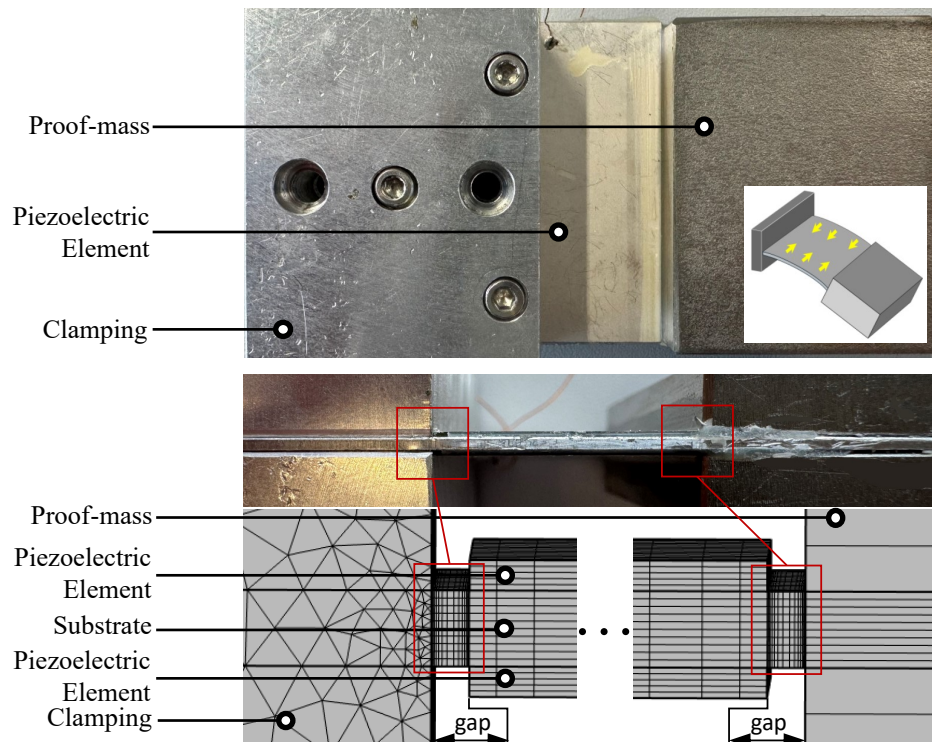
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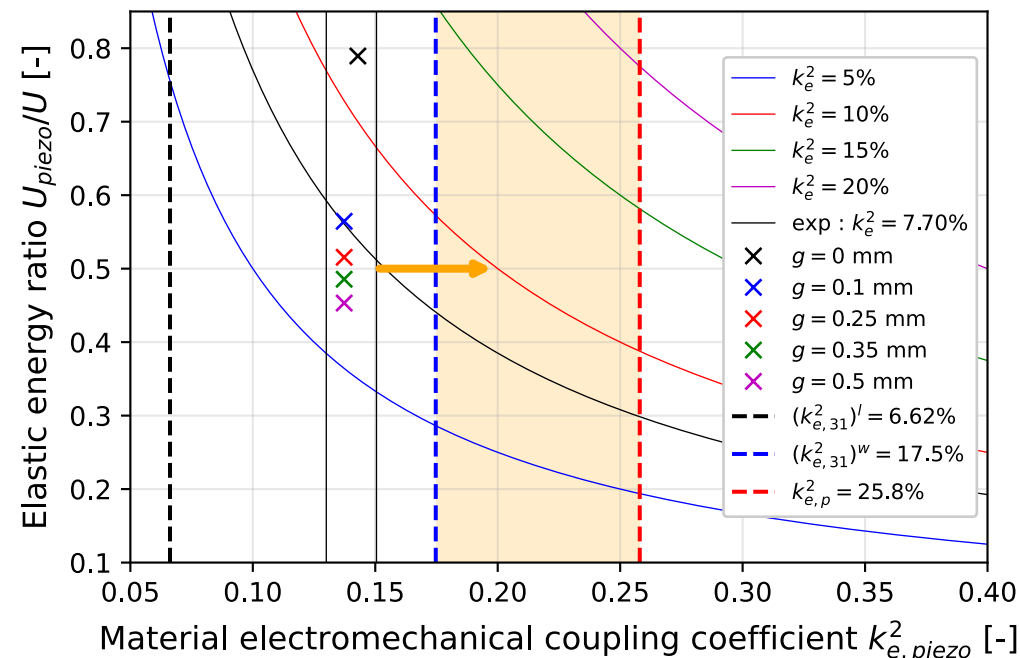
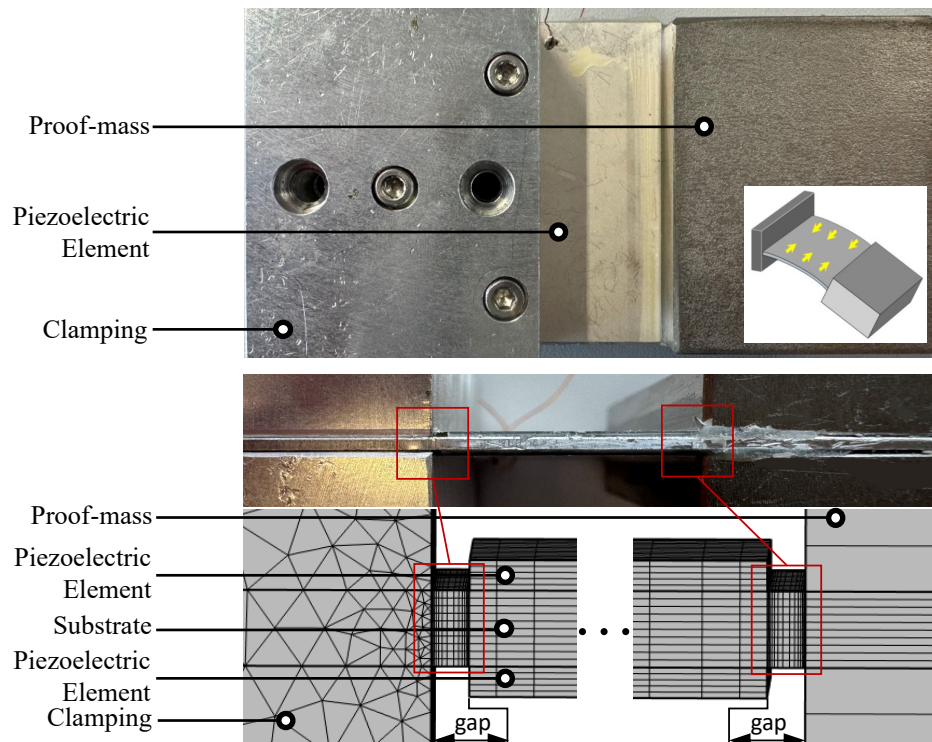
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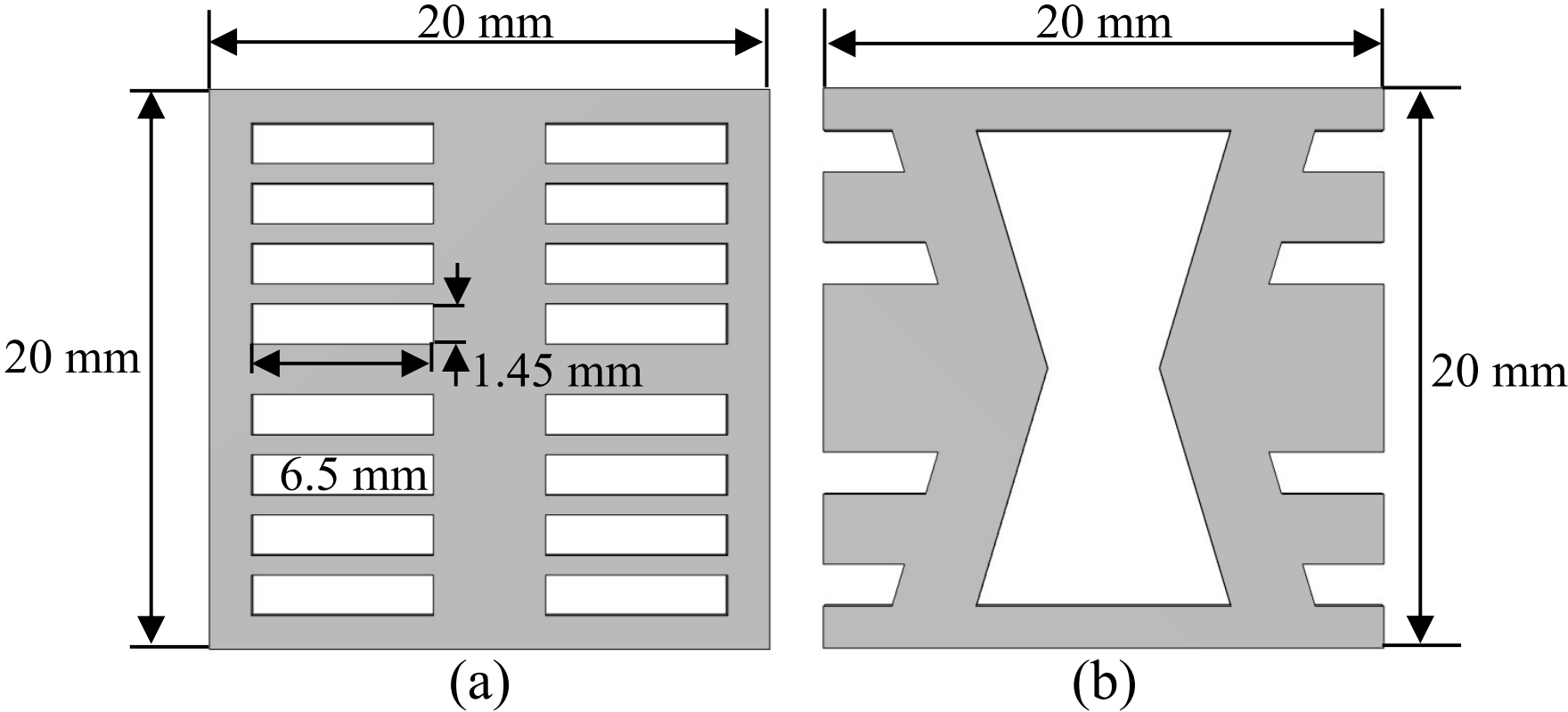
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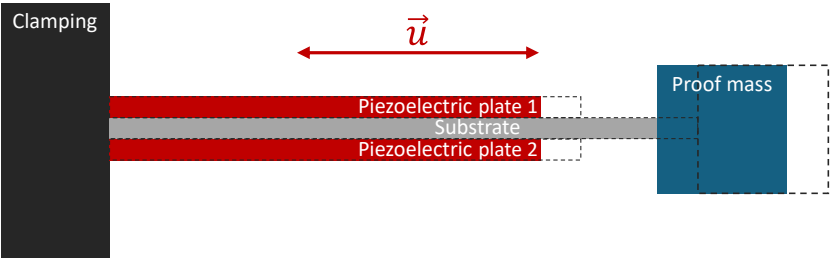
3. Comparison of different substrates



- Does hollowing the substrate have an impact on the material coupling coefficient or is the stiffness decrease the only relevant factor ?

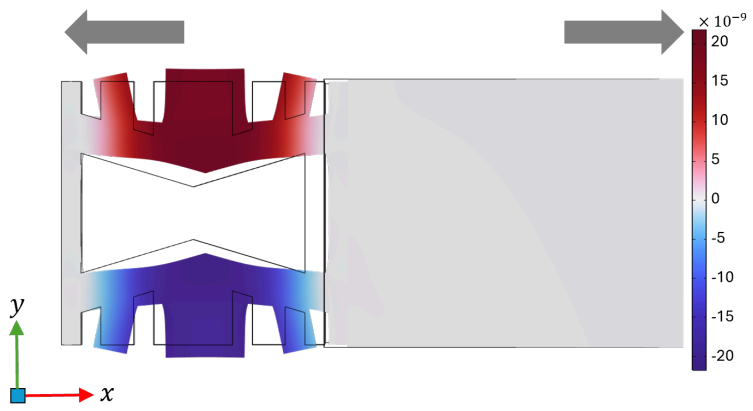
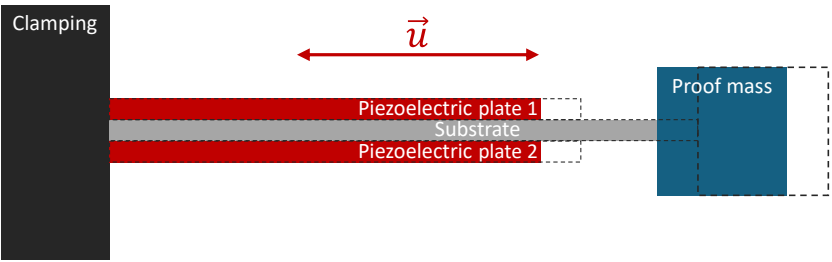
4. Results on axial resonance mode

III. Model Implementation

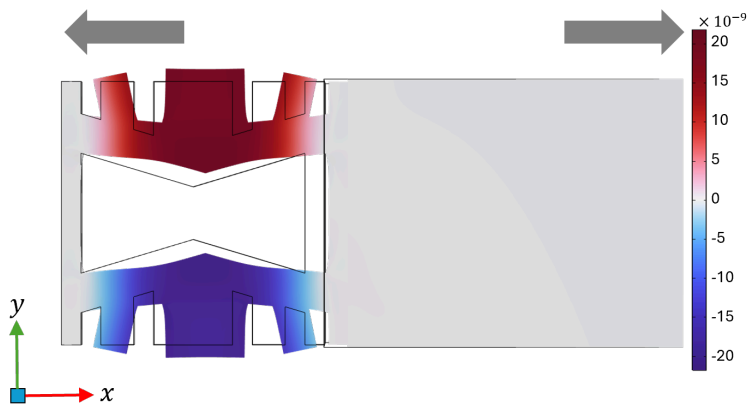
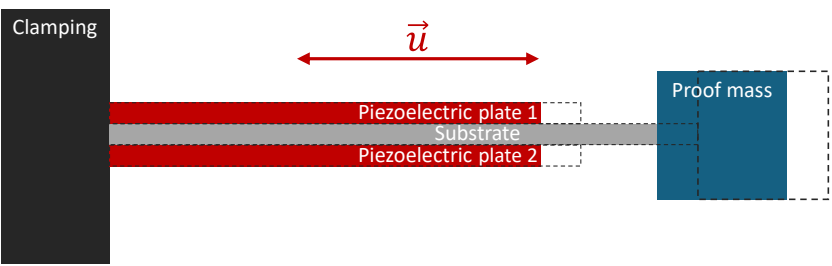


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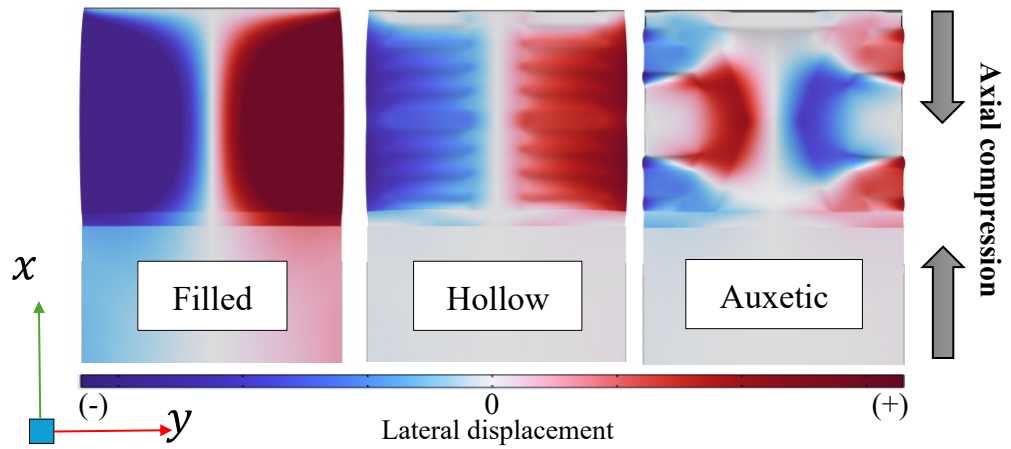
III. Model Implementation



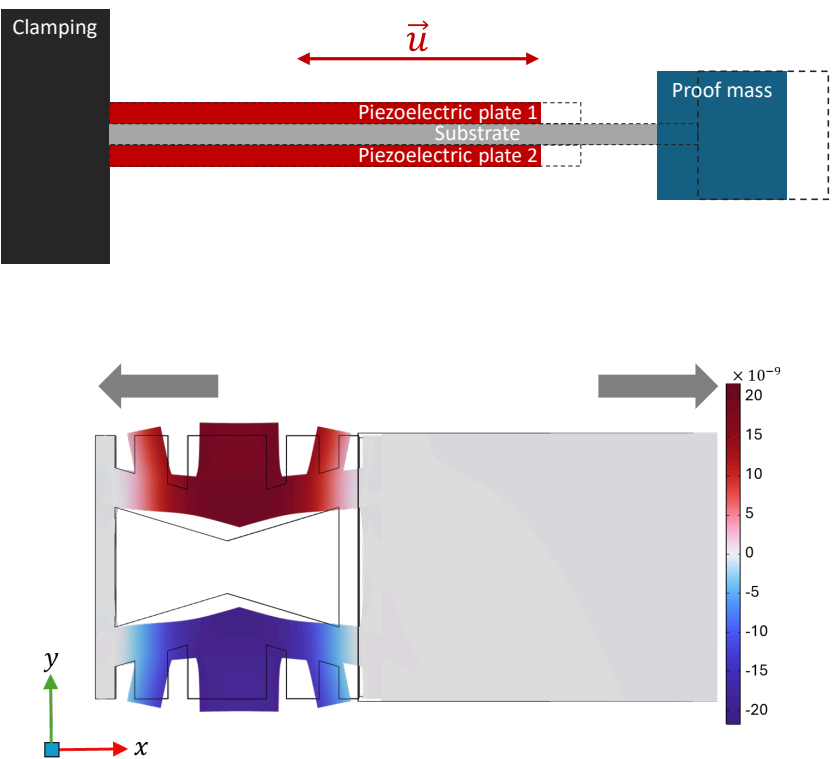
4. Results on axial resonance mode



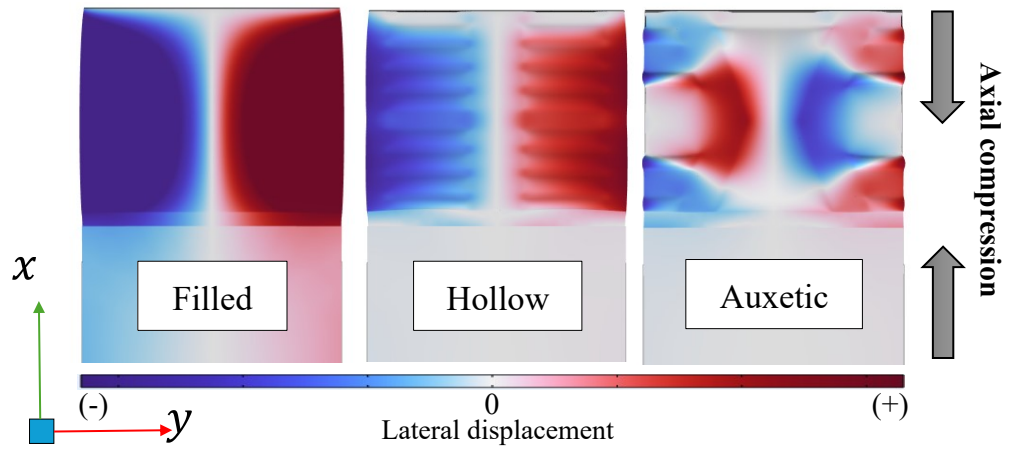
III. Model Implementation



4. Results on axial resonance mode



III. Model Implementation

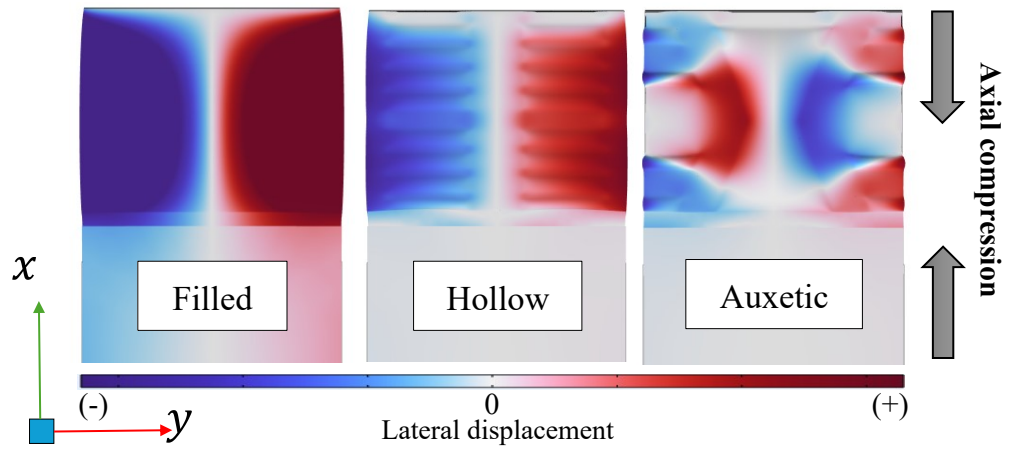
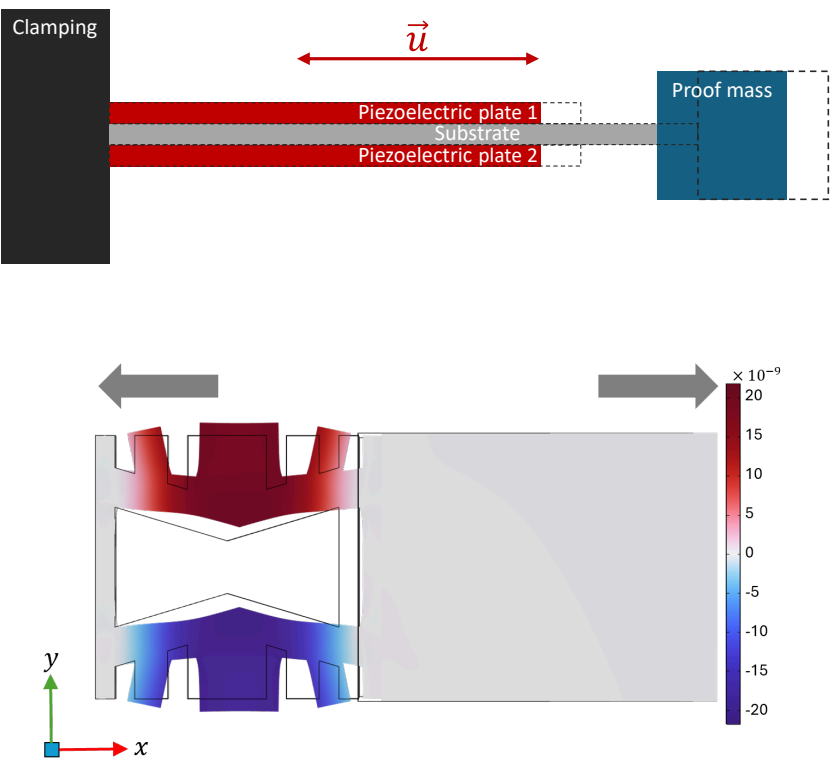


Substrate types :	Filled	Hollow	Auxetic
k_e^2	0.55 %	1.54 %	2.45 %
Energy Ratio	5.13 %	16.73 %	24.96 %
$k_{e,piezo}^2$	10.72 %	9.21 %	9.82 %

- The global coupling coefficient increases but only thanks to the Energy ratio
- There is no improvement on $k_{e,piezo}^2$

4. Results on axial resonance mode

III. Model Implementation

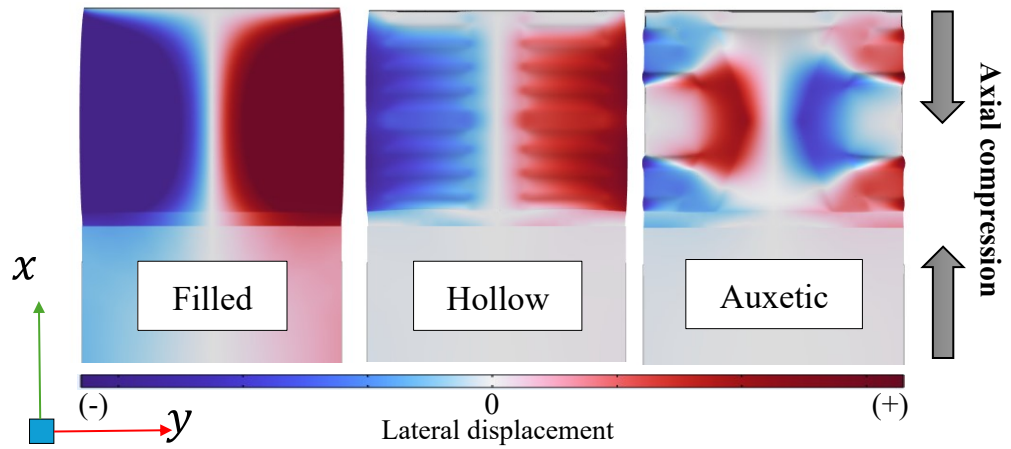
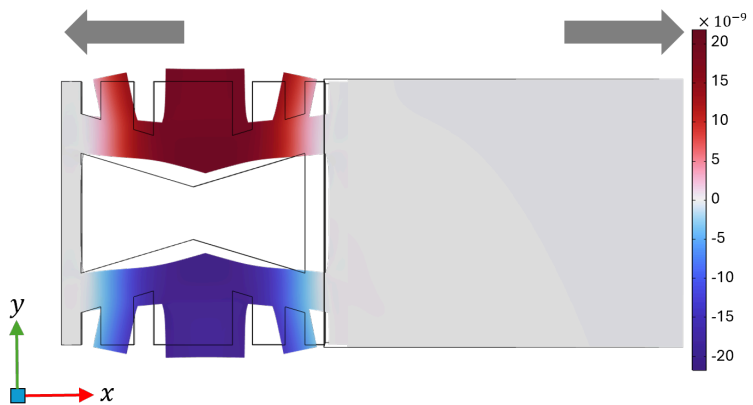
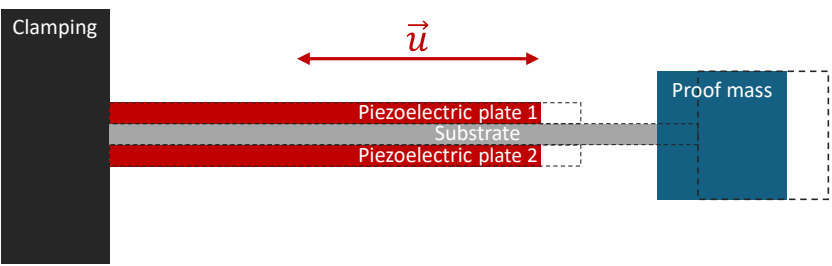


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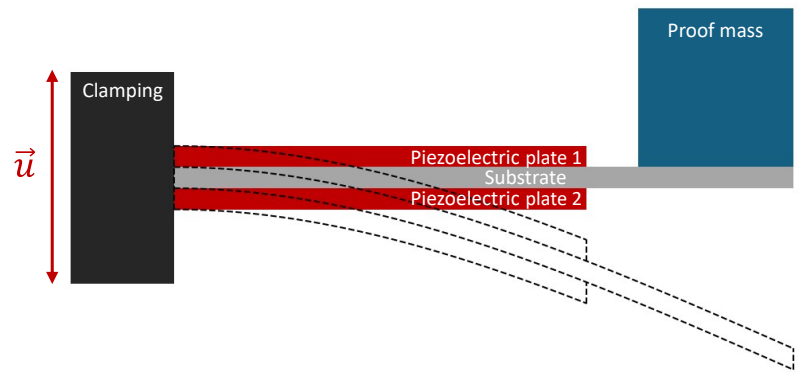


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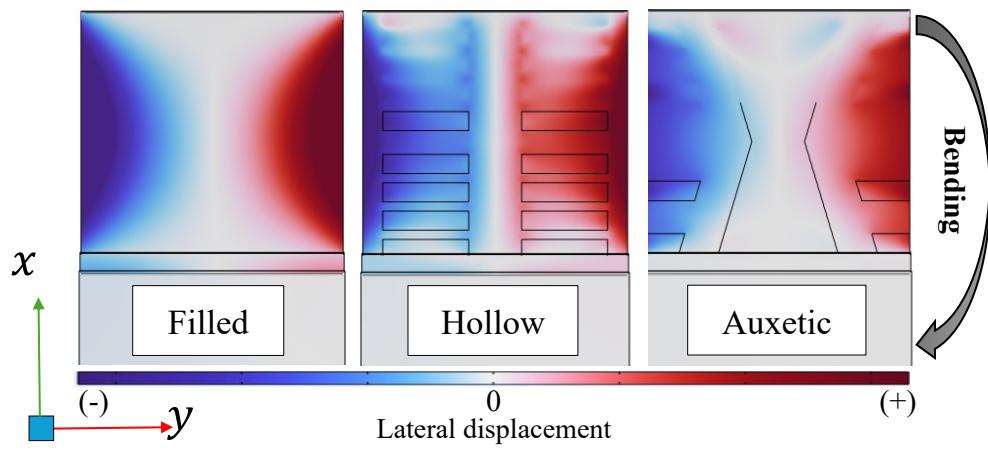
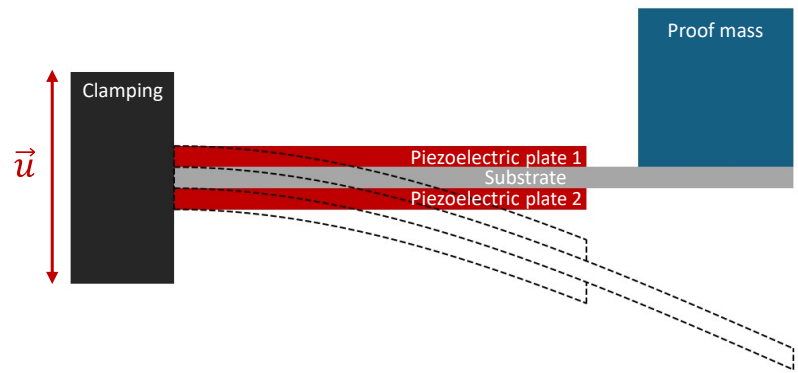
5. Results on bending resonance mode

III. Model Implementation



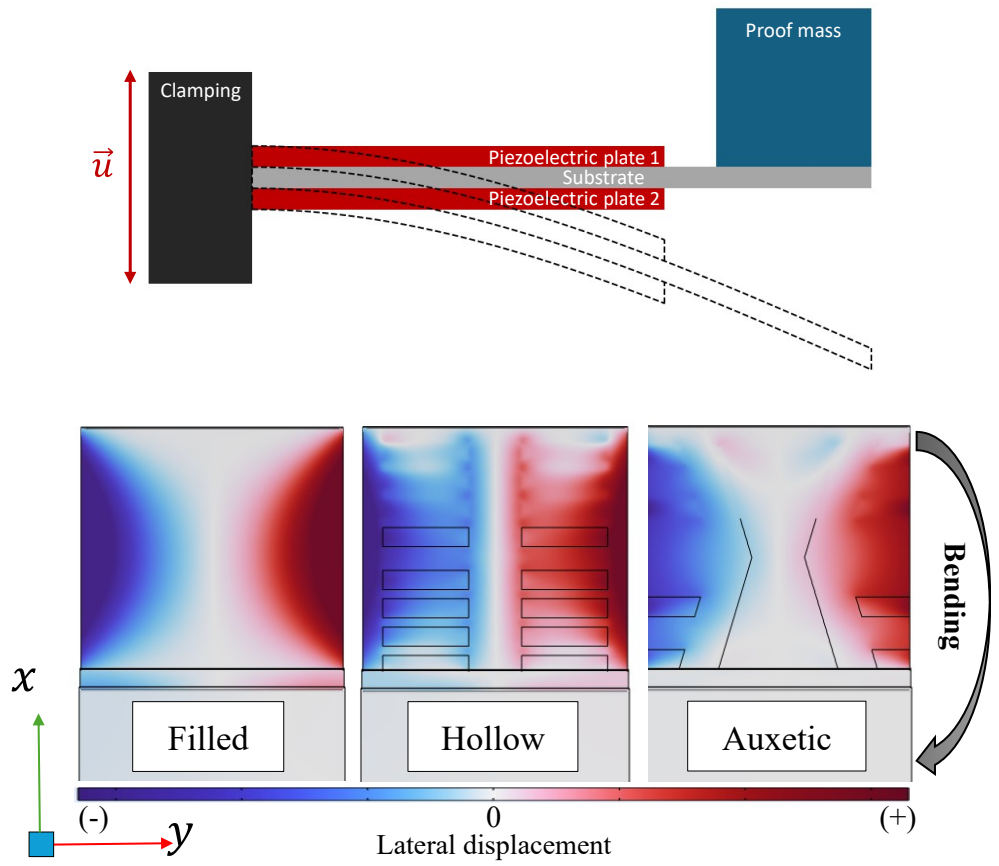
5. Results on bending resonance mode

III. Model Implementation



5. Results on bending resonance mode

III. Model Implementation

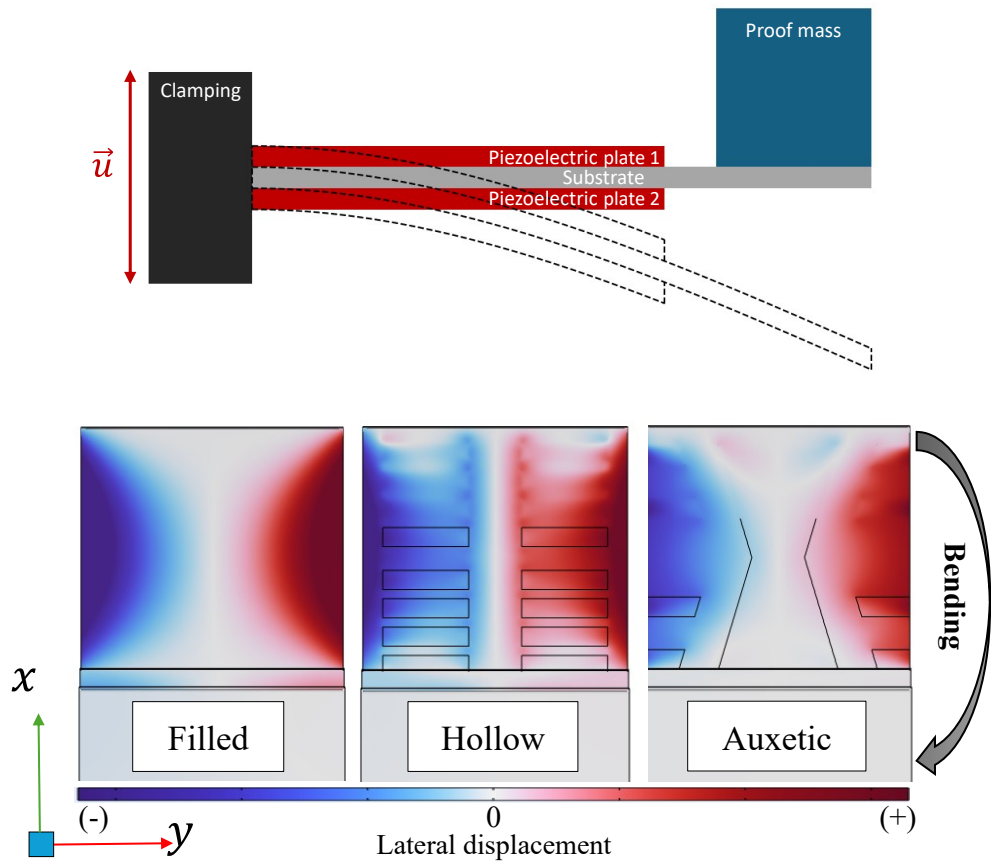


Substrate types :	Filled	Hollow	Auxetic
k_e^2	2.38 %	5.02 %	4.32 %
Energy Ratio	18.00 %	40.62 %	42.09 %
$k_{e.piezo}^2$	13.20 %	12.36 %	10.26 %

- The displacement does not change for bending resonance mode
- The global coupling decreases for auxetic substrate such as the material coupling

5. Results on bending resonance mode

III. Model Implementation

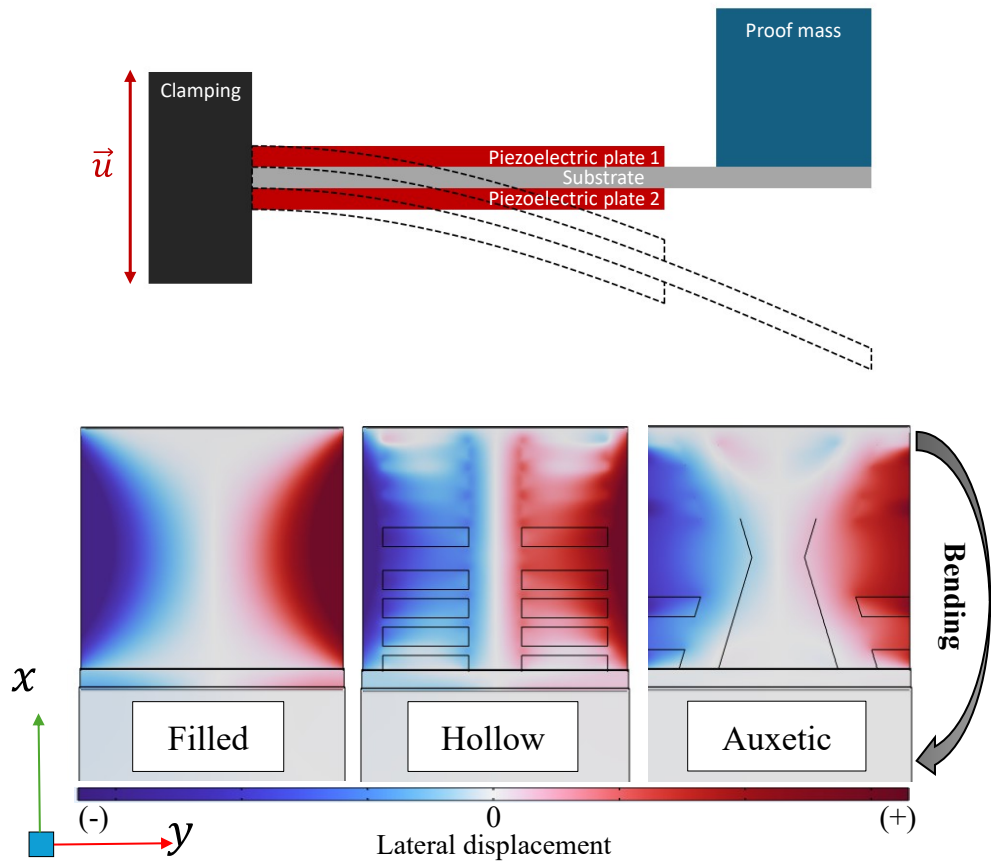


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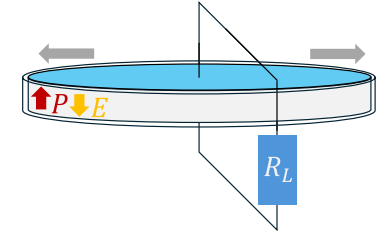


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IV. Conclusion

1. Piezoelectric energy harvesters assessment



Results :

- The material coupling coefficient could be improved by exposing the material to planar mode
- The material coupling coefficient can be computed using finite element simulation
- There are difficulties in designing working solutions with auxetic substrates, and more investigation is needed

Future work

- Work on auxetic design to achieve the planar mode and expand possibilities for bandwidth and/or power output.

Tusen takk !



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