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# ***Optimal Combination of Requirement and Reward in Financial Incentive Programs for Weight Loss***

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## **Abstract**

The paper analyzes how programs offering financial rewards for losing weight should be adapted to participants' preferences. Specifically, we highlight the conditions under which demanding and highly remunerative programs or – conversely – easily achievable but less remunerative programs should be proposed. We show that financial incentives are more likely to encourage efforts to lose weight if the level of difficulty of the objective and the reward in case of success increase with: 1) the degree of risk seeking; 2) the degree of cross-prudence in quality of life. This suggests that participants should be proposed a weight loss program tailored to their preferences towards risk and correlation.

**JEL:** I12, D90

**Keywords:** financial incentives, weight loss, preferences towards risk, preferences towards correlation

## 1. Introduction

Prevention actions implemented during the last decades to curb unhealthy behaviors have not fully achieved their objectives. Although they have been declining for several years, the smoking rate (29% of individuals aged 15+ smoke) and the average alcohol consumption (the equivalent of 8.6 liters of pure alcohol per individuals aged 15+ per year) are higher in Europe than in the other regions of the world. When it comes to obesity, which has nearly tripled worldwide since 1975, both the trend and level are alarming: 39% of individuals aged 18+ were overweight in 2016, and 13% were obese (WHO European Health Report 2018). Promoting healthy lifestyle thus presents major challenges for European countries.

Taxation, information campaigns, and measures targeting specifically the consumption of tobacco and/or alcohol (ban on sale to minors, ban on advertising, ban on smoking in public places, *etc.*) and unhealthy food (obligation to mention the composition of products on labels, warnings in advertising messages, *etc.*) are the main instruments used by public authorities to influence individual behaviours. In addition to having produced disappointing results, especially with regard to obesity, some of them present major drawbacks. Taxation of products harmful to health can for instance have detrimental effects if it does not cause a reduction in their consumption, thereby leading to a potential reduction in the budget available for the purchase of essential goods and services<sup>1</sup>. Questioning the relevance of new prevention instruments likely to overcome the weaknesses of the existing ones could therefore prove to be useful.

The instruments typically used by public authorities are characterized by the contrast existing between the costs and the benefits of the prevention efforts they are supposed to encourage: while the difficulties related to smoking cessation or to weight loss are undergone immediately, the benefits of such actions are uncertain, intangible, and delayed. Financial rewards granted upon the achievement of predefined short-term health objectives (quit smoking for 6 months, lose 8kg in 16 weeks, *etc.*) mitigate this gap by associating a certain, tangible and immediate return with healthy behaviors. These incentives create some extrinsic motivation to undertake the first efforts to quit smoking, exercise or eat healthy. The assumption is that once these initial efforts have been made, subsequent efforts will seem relatively less difficult. Their continuation during post-incentive periods would thus be facilitated, potentially leading to lasting behavioral changes.

Our paper deals specifically with financial incentive programs for weight loss. Experiments assessing their effectiveness have so far provided mixed results. Volpp et al. (2008) compared the weight losses of unincentivized subjects with those of subjects who knew that they would receive either the earnings of a lottery (lottery group) or a fixed reward which required the prior payment of a deposit by the subject (deposit contract group) if they managed to lose 0.45 kg per week for 16 weeks. Volpp et al. (2008) find that weight loss at 16 weeks was on average significantly higher among participants from both incentivized groups (5.89 kg in the lottery group and 6.3 kg in the deposit contract group) than among those in the control group (1.75 kg). Financial incentives also significantly increased the share

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<sup>1</sup> This effect being more likely to occur among the poorest populations (see Gospodinov and Irvine, 2009; Farrelly, Nonnemaker and Watson, 2012).

of participants reaching the weight-loss objective: 10.5% in the control group compared to about one-half in the incentive groups (52.6% in the lottery group, 47.4% in the deposit contract group).

Paloyo et al. (2012) randomized obese patients from a rehabilitation clinic into three experimental arms: a control group and two treatment groups receiving respectively a reward of 150€ and 300€ for achieving a personalized weight loss objective fixed by the medical staff (corresponding to about 6 to 8% of the patient's initial weight). The authors address two questions: 1) the effect of reward size on weight loss; 2) whether the effect of the reward differs across subgroups of patients. They show that receiving any financial incentive has a significant effect on weight loss among obese patients, but that increasing the financial incentive from 150€ to 300€ has significant effects only among two subgroups of patients: women and migrants.

The experiment conducted by Cawley and Price (2013) displays more contrasting results<sup>2</sup>. In their case-study of a company program, voluntary employees were enrolled over a year in three treatment groups offering financial rewards for achieving weight loss objectives, and a quasi-control group offering a flat 20\$ participation fee. The three financial incentives took the following form: a reward proportional to the weight loss, without deposit (program A); two deposit programs<sup>3</sup>, with lump-sum or proportional rewards (programs B and C, respectively). Surprisingly, Cawley and Price (2013) indicate that dropout rates were higher in the incentivized groups than in the quasi-control group (76% of participants quitted program A by 12 months, compared to merely 60% in programs B and C and to 48% in the control group). Weight losses at 12 months were on average significantly higher in the deposit contract groups B and C than in the control group, even after controlling for attrition. However, the proportional reward without deposit (program A) did not conduct to weight loss significantly different from that of the control group.

Other studies indicate that financial incentives have not significant impact on weight loss<sup>4</sup>. Patel et al. (2016) evaluated increases in employer contributions to the health insurance premium as incentives to encourage weight loss. Participants of the study were randomly assigned to a control group or to one of three treatment groups in which they received a financial incentive to lose 5% of their baseline weight. The reward in case of success took the form of: 1) an immediate health insurance premium reduction; 2) a delayed health insurance premium reduction; 3) the earnings of a lottery. The authors show that there were no statistically significant differences in mean weight reduction between the four groups at 6 months and 12 months.

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<sup>2</sup> Note, however, that compared to Volpp et al. (2008), the financial incentives granted in the experiment of Cawley and Price (2013) were more modest.

<sup>3</sup> In the group without deposit (A), the reward is granted quarterly: employees receive 3\$ per-percentage-point of weight loss for a decrease of 15% – or less – of initial weight, and up to 5\$ per-percentage-point when they lose 20% to 30% of their weight. In the deposit group with lump-sum payment (B), participants make a 109.45\$ deposit which is reimbursed after 12 months if a 5% weight loss target is achieved, and were rewarded a 100\$ bonus if they lost 10% or more. The deposit is lost if participants failed to reach the target of 5%. The deposit program with quarterly payments (C) is similar to B, but repayments and subjects weighting occur on a quarterly basis: participants received 15\$ for 1% weight loss, 21\$ for 2%, 30\$ for 3%,..., 90\$ for 20% or more.

<sup>4</sup> In addition, it must also be noted that the efficiency of the financial incentives in the long-term - *i.e.* after the end of the program - has been questioned (see for instance Paul-Ebbohimen and Avenell, 2008 and Spahn et al. 2010).

The relevance of financial incentives to promote weight loss remains an open question. However, the effectiveness of these programs is likely to be underestimated if they are not optimally designed. The literature is quite silent on this issue. First, the objective of the experiments conducted so far has been to determine whether financial incentives produced behavioral changes. Detecting differences between unincentivized and incentivized groups was therefore the focus of these studies. In the latter, the financial incentive programs combined multiple features: lump-sum payment when a weight loss threshold is reached vs. payment proportional to the weight loss, certain vs. uncertain reward in case of success, with or without an initial deposit paid by the subject (and not recovered in case of failure in achieving the objective), individual-oriented vs. group-oriented programs, etc. Isolating the effectiveness of some features of incentive programs is thus difficult. Second, to the best of our knowledge<sup>5</sup>, there are no theoretical contributions evaluating the relative effectiveness of the different types of incentive programs.

In this paper, we use a theoretical framework to determine how participants' preferences should influence the design of financial incentive programs for weight loss. More precisely, we determine the conditions under which participants entering a weight loss program should be proposed more demanding objectives associated with higher rewards or – conversely – easier objectives associated with lower rewards. In the literature, the objectives differ widely from one experiment to another without explicit justification<sup>6</sup>. For instance, in Volpp et al. (2008), subjects obtained a financial reward if they lost at least 7.2kg within 16 weeks – independently of their baseline weight –, whereas rewards were proportional to the weight loss achieved in other studies (Cawley and Price, 2013; Patel et al. 2016). We address this issue on the assumption that the optimal combination of objective and reward might depend on the participants' attitude towards risk and correlation.

In that sense, our article ties into the recent literature that promotes health interventions adapted to individual – rather than broad – preferences. Andreoni et al. (2021) identified time preferences of Lady Health Workers (LWH) in Pakistan and designed personalized bonus contracts to achieve a specific objective: equalizing the vaccination efforts provided by LHW on two different days. Their analysis shows that contracts tailored to time preferences perform better than non-personalized contracts in that respect. Aggarwal et al. (2020) also use information about time preferences with another objective: designing financial incentive contracts to encourage walking among diabetics in India. They compare a time-bundled contract likely to work particularly well for subjects with higher discount rate over effort (*i.e.*, rewarding participants for complying with a daily target of 10,000 steps a minimum number of days per week) with another incentive scheme (providing a payment that increases linearly with the number of days the participant has reached the 10,000-step objective). The result of their experiment matches with their theoretical predictions: the relative efficacy of time bundled over linear contract depends on the participants' level of impatience.

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<sup>5</sup> Crainich (2019) provides a theoretical analysis of the effects of financial incentives on the efforts to stop smoking. However, the paper does not compare the effectiveness of different incentive programs, but instead focuses on subjects' behavior during and after the incentive period.

<sup>6</sup> The weight loss objectives are justified in some studies. For instance, Paloyo et al. (2015) specify that the target assigned to each patient was established by the medical staff of a rehabilitation clinic.

While these two papers focus on time preference, we analyze how individuals' preferences towards risk and towards correlation should shape the personalized financial incentives program for weight loss proposed to each participant. In particular, if public authorities' objective is to maximize the efforts subjects make to lose weight, our theoretical model shows that the weight loss target of the program and the reward in case of success should increase both with the participants' degree of risk seeking and with their degree of cross prudence in quality of life, *i.e.*, a particular form of correlation aversion (see Eeckhoudt, Rey and Schlesinger, 2007, and Appendix C for a definition and an explanation of the concept). The explanations for these two results being not straightforward, we therefore also provide their underlying intuition.

The article is organized as follows. The second section introduces the assumptions and notations of the model. Section 3 describes how individuals make efforts to lose weight when they participate to a financial incentive program. The optimal combination of weight loss objective and reward in case of success is characterized in Section 4. Section 5 concludes.

## 2. Assumptions and notations

An overweight subject participates in a weight loss program implemented by public authorities. Registration for the program is free of charge for the subject. Two periods are considered:  $t = 0$  (during the program) and  $t = 1$  (after the program). A weight loss objective – which level of difficulty is denoted by  $q$  – is assigned to the subject. In case he/she fulfills this objective, the subject receives a financial reward  $s$  at  $t = 1$ . Given a closed budget  $B$ , the regulator's objective is to select the combination of reward and difficulty that maximizes the subject's effort to lose weight.

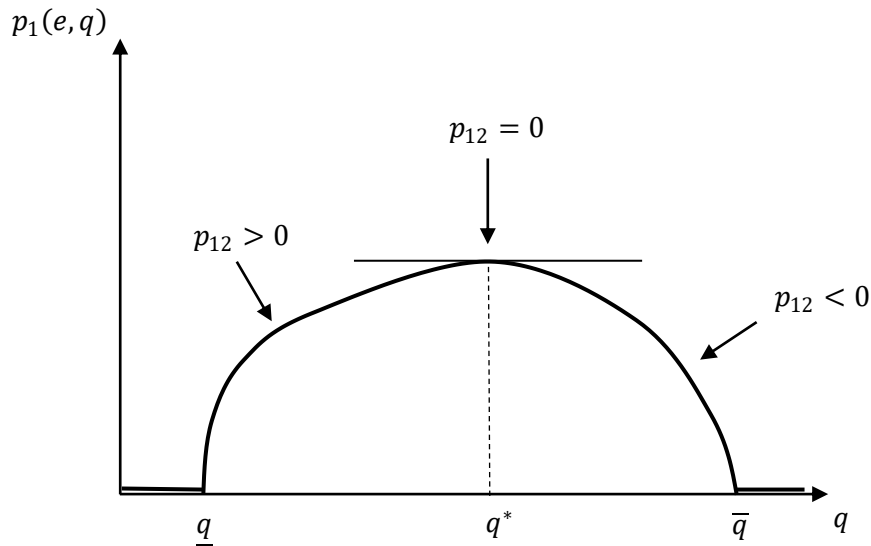
The representative subject is assumed to maximize expected utility. His/her preferences are represented by a bi-dimensional utility function  $u(w, h)$  that depends on wealth ( $w$ ) and on quality of life ( $h$ ). Higher levels of wealth and quality of life are preferred by the subject ( $u_1 > 0$  and  $u_2 > 0$  respectively), who is risk averse in each of these two dimensions ( $u_{11} < 0$  and  $u_{22} < 0$  respectively)<sup>7</sup>. No assumption is made about the sign of  $u_{12}$ . Without loss of generality, the subject's utility function is the same across periods. The preference rate for  $t = 1$  is normalized and  $\tau \geq 1$  captures the preference for  $t = 0$ .

During the program ( $t = 0$ ), the subject makes efforts such as eating less salt and sugar, drinking less alcohol, practicing more physical exercise, *etc.* to lose weight. These efforts – which intensity is denoted by  $e$  – have no monetary costs but are demanding and thus reduce the subject's quality of life from  $h$  to  $h - e$ . At the end of the program ( $t = 1$ ), efforts have a sure benefit and a random benefit. First, they improve the subject's health state, and consequently his/her quality of life by  $h(e)$  (with  $h'(e) > 0$  and  $h''(e) < 0$ ), whether the objective of the program is fulfilled or not. Second, they increase the probability – denoted by  $p(e, q)$  – to reach the objective of the program, and therefore to obtain the financial reward. The effect of efforts on weight loss is thus assumed to be random, since it depends on several factors such as the quality of sleeping, hormonal cycles, *etc.* The following

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<sup>7</sup> First and second derivatives of the utility function with respect to the first argument (wealth) are respectively denoted by  $u_1(w, h)$  and  $u_{11}(w, h)$ . First and second derivatives of the utility function with respect to the second argument (quality of life) are respectively denoted by  $u_2(w, h)$  and  $u_{22}(w, h)$ . The cross derivative is denoted by  $u_{12}(w, h)$ .

assumptions are made about the probability of reaching the objective: 1)  $p_1(e, q) \geq 0, \forall e, q$  (additional efforts increase or leave unchanged the probability of reaching the objective); 2)  $p_2(e, q) \leq 0, \forall e, q$  (higher levels of difficulty reduce or leave unchanged the probability of reaching the objective); 3)  $p_{12}(e, q) \leq 0 \Leftrightarrow q \geq q^*$  where  $q^* \in [\underline{q}, \bar{q}]$ ,  $\forall e, q$ . The rationale behind the third assumption is depicted in Figure 1. Efforts have no effect on the probability of success either when the objective is so easy that no efforts are required to achieve it ( $p_1(e, q) = 0$  if  $q < \underline{q}$ ) or when the objective is unachievable whatever the efforts undertaken ( $p_1(e, q) = 0$  if  $q > \bar{q}$ ). When the level of difficulty of the objective is intermediate, *i.e.*, when  $q \in [\underline{q}, \bar{q}]$ , efforts have positive returns ( $p_1(e, q) > 0$ ). The third assumption thus implies that there is a unique level of difficulty that maximizes the effect of effort on the probability of reaching the objective, and that any increase in the level of difficulty raises (respectively decreases) the marginal effect of effort when the level of difficulty is below (resp. above) that threshold. In addition, we also assume – as depicted in Figure 1 – that increases in the level of difficulty of the objective modify the effect of effort on the probability of success at the decreasing rate, *i.e.*,  $p_{122}(e, q) \leq 0$ .



*Figure 1: marginal effect of effort on the probability of success*

Finally, note that the relationship shown in Figure 1 between the marginal effect of effort on the probability of success and the level of difficulty of the objective assigned to the participant holds for a given level of effort.

### 3. Subjects' efforts

Given these assumptions, the participant's intertemporal maximization problem is given by:

$$\text{Max}_e Z = \tau u(w, h - e) + [(1 - p(e, q))u(w, h + h(e)) + p(e, q)u(w + s, h + h(e))]$$

where the first term captures the welfare loss resulting from the effort made during the program ( $t = 0$ ) to lose weight, and the second term the expected utility at the end of the program ( $t = 1$ ).

We use the following notations for the two states at  $t = 1$ , *i.e.*, failure and success in reaching the weight loss objective of the program:  $F = (w, h + h(e))$ ;  $S = (w + s, h + h(e))$ .

The first order condition associated with this maximization problem is given by Eq. (1):

$$\frac{\partial Z}{\partial e} = -\tau u_2(w, h - e) + p_1(e, q)[u(S) - u(F)] + h'(e)[(1 - p(e, q))u_2(F) + p(e, q)u_2(S)] = 0 \quad (1)$$

We assume that the second order condition ( $SOC = \frac{\partial^2 Z}{\partial e^2} < 0$ ) is met<sup>8</sup>.

The first order condition (1) is the sum of three terms. The first term ( $-\tau u_2(w, h - e)$ ) is the marginal cost of the efforts made to lose weight during the program ( $t = 0$ ). The second and third terms correspond to the marginal benefits of these efforts. They respectively result from the higher probability of obtaining the financial reward ( $p_1(e, q)[u(S) - u(F)]$ ), *i.e.*, the extrinsic motivation to undertake the effort, and from the health improvement resulting from the subject's behavioral change ( $h'(e)[(1 - p(e, q))u_2(F) + p(e, q)u_2(S)]$ ), *i.e.*, the intrinsic counterpart.

Applying the implicit function theorem to the first order condition, we determine the effects of changes in the reward granted in case of success ( $s$ ) and in the level of difficulty of the weight loss objective ( $q$ ) on the intensity of effort ( $e$ ) provided during the program.

$$\frac{de}{ds} = -\frac{\frac{\partial^2 Z}{\partial e \partial s}}{SOC} = -\frac{[p_1(e, q)u_1(S) + p(e, q)h'(e)u_{12}(S)]}{SOC} > 0 \quad (2)$$

$$\frac{de}{dq} = -\frac{\frac{\partial^2 Z}{\partial e \partial q}}{SOC} = -\frac{[p_{12}(e, q)[u(S) - u(F)] + p_2(e, q)h'(e)[u_2(S) - u_2(F)]}{SOC} \geq 0 \quad (3)$$

Eq. (2) indicates that any increase in the reward  $s$  has two effects on the effort to lose weight: 1) it increases the motivation to obtain the payment insofar as the financial gap between reaching and not reaching the objective increases; 2) when the marginal benefit of quality of life rises (resp. falls) with wealth, *i.e.*, when  $u_{12} > 0$  (resp.  $u_{12} < 0$ ), any increase in  $s$  increases (resp. decreases) the marginal utility of the quality of life in case the objective is reached, and therefore the benefits of the efforts to lose weight. As a result, increasing  $s$  may have a negative effect on the effort made to lose weight when  $u_{12} < 0$ . However, if that were the case, public authorities would not implement this type of program. We thus assume that  $\frac{de}{ds} > 0$ .

Eq. (3) measures the effect of making the weight loss objective more demanding on the effort made by the subject. The first term ( $p_{12}(e, q)[u(S) - u(F)]$ ) is related to the incentives to obtain the payment: when  $p_{12}(e, q) > 0$  (resp.  $p_{12}(e, q) = 0$ ; resp.  $p_{12}(e, q) < 0$ ), increasing the degree of difficulty increases (resp. does not modify; resp. decreases) the effects of additional weight loss efforts

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<sup>8</sup> A sufficient condition for this program to define a maximum is  $u_{12} \leq 0$ .

on the probability of reaching the objective (see figure 1). The second term captures the fact that when the difficulty of reaching the objective increases, the probability of obtaining the payment decreases: whether this raises or reduces the incentives to lose weight depends on the interaction between wealth and quality of life in the subject's utility (*i.e.*, on the sign of  $u_2(S) - u_2(F)$ ), which is dictated by the sign of  $u_{12}$  as  $S$  and  $F$  only differ in terms of wealth). As a result, the overall effect of changes in the level of difficulty of the objective on the subject's propensity to make efforts to lose weight cannot be identified:  $\frac{de}{dq} \gtrless 0$

#### 4. Public authorities' decision

We adopt an intuitive approach to explaining how the optimal level of difficulty of the weight loss objective ( $q$ ) and the reward in case of success ( $s$ ) are jointly determined<sup>9</sup>. We assume that public authorities design programs with the aim of maximizing the level of efforts ( $e$ ) a given individual makes to lose weight. To capture the way individuals' preferences influence the optimal joint values of  $q$  and  $s$ , we consider two subjects. We suppose in sub-section 4.1. that a subject is risk averse and correlation neutral, so that the effect of risk aversion is isolated. In sub-section 4.2., we examine the effect of correlation preferences by assuming that a subject is risk averse but not correlation neutral (*i.e.*, either correlation averse or correlation seeker).

##### 4.1. Risk preferences

When defining the optimal levels of  $s$  and  $q$ , public authorities take the subject's behavior, *i.e.*,  $\frac{de}{ds}$  and  $\frac{de}{dq}$ , into account. Furthermore, given that, on the one hand, increasing the level of difficulty of the weight loss objective reduces the probability of success and, on the other hand, the total amount spent must be equal to a fixed budget  $B$ , public authorities can rise the financial reward they offer when they set more demanding objectives. Formally,  $B - p(e, q)s = 0$  implies  $\frac{ds}{dq} = -\frac{p_2(e, q)s}{p(e, q)} > 0$ , which indicates that changes in  $s$  and  $q$  respecting the budget constraint necessarily go in the same direction.

When a subject is risk averse ( $u_{11} < 0$ ) and correlation neutral ( $u_{12} = 0$ ), Eqs. (2) and (3) become:

$$\frac{de}{ds} = -\frac{p_1(e, q)u_1(S)}{SOC} > 0 \quad (2')$$

$$\frac{de}{dq} = -\frac{p_{12}(e, q)[u(S) - u(F)]}{SOC} \gtrless 0 \quad (3')$$

Suppose first that  $s$  and  $q$  and the resulting efforts  $e$  are such that  $p_{12}(e, q) > 0$ . In that case, increasing  $q$  has a direct positive effect on the efforts this subject makes to lose weight (Eq. (3')). In addition, given that  $\frac{ds}{dq} > 0$  along the budget line, it also indirectly increases the efforts made via the improved reward in case of success (Eq. (2')). Therefore, if  $p_{12}(e, q) > 0$ , any increase in  $q$  (direct effect) combined with the resulting mechanical increase in  $s$  (indirect effect) increase the efforts the subject makes to lose weight. As a result,  $p_{12}(e, q)$  cannot be positive at the optimum. For higher

<sup>9</sup> A more formal approach leading to the same analytical results is provided in Appendix A.

values of  $q$  and  $e$  such that  $p_{12}(e, q) < 0$ , the direct effect of increasing  $q$  is negative. However, increases in  $q$  also imply a positive indirect effect on the effort through the increase of the financial reward  $s$  in case of success. As long as the indirect effect dominates the direct effect, a more demanding weight loss objective combined with a higher reward should be offered to the subject. There should be no further increases in  $q$  and  $s$  when the direct effect is just equal to the indirect one, i.e., when  $\left| \frac{de}{dq} \right| = \frac{de}{ds}$ .

Formally, these two effects are expressed in Eq. (4), which holds at the optimum:

$$de = \underbrace{p_{12}(e, q)[u(S) - u(F)]dq}_{\text{direct effect}} + \underbrace{p_1(e, q)u_1(S)ds}_{\text{indirect effect}} = 0 \quad (4)$$

Using  $\frac{ds}{dq} = -\frac{p_2(e, q)s}{p(e, q)}$  and the mean-value theorem to replace  $[u(S) - u(F)]$  by  $su_1(\alpha, h + h(e))$  with  $\alpha \in [w, w + s]$ , Eq. (4) becomes:

$$-p_{12}(e, q)u_1(\alpha, h + h(e)) = -\frac{p_1(e, q)p_2(e, q)}{p(e, q)}u_1(w + s, h + h(e)) \quad (5)$$

The right-hand side of Eq. (5) corresponds to the marginal benefit of increasing  $s$  (indirect effect). It is always positive and can be either increasing or decreasing with joint increases in  $q$  and  $s$ . The left-hand side of Eq. (5) corresponds to the marginal cost of increasing  $q$  (direct effect). It is necessarily positive at the optimum (which implies  $p_{12}(e, q) < 0$ ) and is increasing with joint increases in  $q$  and  $s$ .

Suppose that the subject is becoming more risk averse, so that the difference between  $u_1(\alpha, h + h(e))$  and  $u_1(w + s, h + h(e))$  increases. Eq. (5) indicates that, in such a case, lower values of  $q$  and  $s$  are required for the (negative) direct effect of the increase in the level of difficulty to be equal to its (positive) indirect effect due to the increase in the financial reward it allows. As a result, the level of difficulty of the weight loss objective and the reward in case of success decrease with the subject's degree of risk aversion at the optimum.

Formally, rearranging Eq. (5), we obtain<sup>10</sup>:

$$-\frac{p_{12}(e, q)}{p_2(e, q)} + \frac{p_1(e, q)}{p(e, q)} \frac{u_1(w + s, h + h(e))}{u_1(\alpha, h + h(e))} = 0 \quad (6)$$

where the ratio  $\frac{u_1(w + s, h + h(e))}{u_1(\alpha, h + h(e))}$  – which is lower than 1 (resp. is higher than 1; resp. is equal to 1) when subjects are risk averse (resp. seekers; resp. neutral) – measures their degree of risk aversion (see Appendix B). This leads to Proposition 1.

**Proposition 1:** *When subjects are correlation neutral, the level of difficulty of the weight loss objective of the program and the reward in case of success must decrease with their degree of risk aversion.*

<sup>10</sup> Note that Eq. (5) is a particular case of Eq. (A1) (see Appendix A) taking account of the subject's correlation neutrality (which implies  $u_{112} = 0$ ).

## 4.2. Correlation preferences

The optimal combination of the level of difficulty of the objective and the reward is obtained when increases in  $q$  and  $s$  have no more effect on the effort made by the subject to lose weight. When the subject is risk averse ( $u_{11} < 0$ ) and not correlation neutral ( $u_{12} \neq 0$ ), we obtain the following equilibrium condition by using Eqs. (2) and (3):

$$de = \underbrace{[p_{12}(e, q)[u(S) - u(F)] + p_2(e, q)h'(e)[u_2(S) - u_2(F)]}_{\text{direct effect}} dq + \underbrace{[p_1(e, q)u_1(S) + p(e, q)h'(e)u_{12}(S)]}_{\text{indirect effect}} ds = 0 \quad (7)$$

Using  $\frac{ds}{dq} = -\frac{p_2(e, q)s}{p(e, q)} > 0$  and the mean-value theorem to express  $u(S) - u(F)$  and  $u_2(S) - u_2(F)$ ,

Eq. (7) can be rewritten:

$$\begin{aligned} & -p_{12}(e, q)u_1(\alpha, h + h(e)) - p_2(e, q)h'(e)u_{12}(\beta, h + h(e)) \\ & = -\frac{p_1(e, q)p_2(e, q)}{p(e, q)}u_1(w + s, h + h(e)) - p_2(e, q)h'(e)u_{12}(w + s, h + h(e)) \end{aligned} \quad (8)$$

where  $\alpha \in [w, w + s]$  and  $\beta \in [w, w + s]$ .

The comparison of Eqs. (5) and (8) indicates that when individuals are not correlation neutral,  $-p_2(e, q)h'(e)u_{12}(\beta, h + h(e))$  and  $-p_2(e, q)h'(e)u_{12}(w + s, h + h(e))$  are respectively added to the direct and indirect effects of changes in  $q$  and  $s$  (*i.e.*, to the marginal cost of increasing  $q$  and to the marginal benefit of increasing  $s$ ). The overall effect thus depends on whether  $u_{12}(S, h + h(e)) \gtrless u_{12}(\beta, h + h(e))$ , *i.e.*, on whether  $u_{112} \gtrless 0$ . In Appendix C, we show that the sign of  $u_{112}$  indicates whether individuals are cross-prudent or cross imprudent in quality of life, while we show in Appendix D that  $[u_{112}(w + s, h + h(e))/u_1(\alpha, h + h(e))]$  provides one way to measure the intensity of cross-prudence/imprudence in the sense of Ross (1981). Therefore, the higher the intensity of cross-prudence in quality of life in the sense of Ross, the more subjects should be offered demanding weight loss objectives combined with high rewards in case of success.

Rearranging the terms of Eq. (8), we obtain Eq. (9), which corresponds to the equilibrium condition formally established in Appendix A (see Eq. (A1)). The comparison of Eqs. (6) and (9) shows how the intensity of cross-prudence/imprudence comes on top of that of risk aversion to determine the optimal values of  $q$  and  $s$ .

$$-\frac{p_{12}(e, q)}{p_2(e, q)} + \frac{p_1(e, q)}{p(e, q)} \frac{u_1(w + s, h + h(e))}{u_1(\alpha, h + h(e))} + h'(e)(w + s - \beta) \frac{u_{112}(\gamma, h + h(e))}{u_1(\alpha, h + h(e))} = 0 \quad (9)$$

where  $\gamma \in [\beta, w + s]$ .

To understand the intuition behind this result, note first that if public authorities increase simultaneously the level of difficulty of the objective and the payment in case of success in such a way that the budget constraint is respected (*i.e.*, so that  $\Delta p(e, q)s = 0$ ), they impose a mean-preserving

spread in the distribution of wealth – *i.e.*, a mean-preserving increase in risk – to subjects. To demonstrate this, compare the incentive programs A and B such that  $p_A > p_B$ ,  $s_A < s_B$  and  $p_A s_A = p_B s_B$ . Formally, the two distributions associated with these programs correspond to  $\tilde{A} = (s_A, 0; p_A, 1 - p_A)$  and  $\tilde{B} = (s_B, 0; p_B, 1 - p_B)$ . Consider the distribution  $\tilde{A}' = (s_A + \tilde{\varepsilon}, 0; p_A, 1 - p_A)$  where  $\tilde{\varepsilon} = (s_B - s_A, -s_A; \frac{p_B}{p_A}, 1 - \frac{p_B}{p_A})$  is a zero-mean risk ( $\mathbb{E}[\tilde{\varepsilon}] = 0$ ). The new distribution  $\tilde{A}'$  is equivalent to  $\tilde{B}$ . As a result,  $\tilde{B}$  is constituted by the addition of a zero-mean risk to the success branch of  $\tilde{A}$ . Therefore, any mean-preserving shift from a less demanding program yielding a lower reward (such as program A) to a more demanding program yielding a higher reward (such as program B) corresponds to a mean-preserving increase in risk (see Rothschild and Stiglitz, 1970) or, equivalently, to a 2<sup>nd</sup> order increase in risk *à la* Ekern (see Ekern, 1980). When agents are cross-prudent in quality of life, any increase in the financial risk they face increases the marginal utility of their quality of life (see Appendix C). As indicated by the third term of Eq. (1), this reinforces their incentives to make efforts to lose weight. This leads to Proposition 2.

**Proposition 2:** *Higher degrees of cross-prudence in quality of life enhance the incentive to make efforts to lose weight when more demanding weight loss objectives and higher rewards for success are offered.*

Note the difference between Proposition 1 – which states that the higher the degree of subjects' risk aversion, the lower the optimal combination difficulty/reward – and Proposition 2. In the latter, we do not claim that higher levels of  $q$  and  $s$  should necessarily be associated with higher degrees of cross-prudence in quality of life. This is due the fact that changes in cross-prudence and in risk aversion are not independents. As a result, if higher degrees of cross-prudence in quality of life go along with higher degrees of risk aversion, the effect on the optimal levels of difficulty of the objective and reward in case of success is ambiguous. However, if subject A is more risk-seeker and more cross-prudent than subject B, a more demanding and more rewarding program should be offered to subject A. Likewise, if subject C is more risk-averse and more cross-imprudent than subject D, a less demanding and less rewarding program should be offered to subject C.

#### 4. Conclusion

The cost of overweight and obesity is high: for instance, Cawley and Meyerhoefer (2012) estimated that each case of adult obesity raised health insurance expenditures by 2346\$ annually. Financial incentives associated with the achievement of weight loss objectives are instruments public authorities could combine with more standard ones - such as information campaigns to consumers, banning the advertising of unhealthy foods, taxation, *etc.* – to help individuals in their efforts to eat healthy foods and to exercise.

Other experiments should however be carried out before determining whether financial incentives are effective tools to promote healthy behavior. The evaluations implemented so far have offered financial incentives which are not adapted to the characteristics of the subjects. In this paper, we show that knowing and using these characteristics is crucial to properly designing financial incentive programs for weight loss. Specifically, we show how individuals' preferences towards risk and correlation

influence the level of difficulty of the objective and the reward that should be proposed to participants. According to our theoretical model, the best way to support the efforts that risk seeker and cross-prudent subjects make to lose weight is to offer a program combining difficult weight loss targets which high rewards in case this objective is reached. Symmetrically, risk averse and cross-imprudent subjects will engage more efforts to lose weight if they enroll in less demanding programs granting lower rewards in case of success.

Finally, note that if subjects were offered several combinations of rewards and weight loss targets meeting the public authority budget constraint, they would not self-select the one defined in section 4. While we assume that financial incentive programs are implemented to encourage subjects' efforts to lose weight, subjects would consider their expected utility if they could self-select the one that suit them best. Imposing the target and the reward is therefore the only way for public authorities to maximize weight loss. This presupposes, however, that the relevant preferences can be properly measured. Fortunately, the literature offers solutions in that respect: measuring the degree of risk aversion in experiments is now quite established, and public authorities could also exploit the most recent contributions on the elicitation of cross-prudence (see Eeckhoudt, Rey and Schlesinger, 2007, for the general principle and Attema, l'Haridon and van de Kuilen, 2019, for an application).

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## 6. Appendices

### Appendix A

Public authorities maximize the efforts subjects make to lose weight while respecting the budget constraint. The Lagrangian of this program is given by:

$$\text{Max}_{s,q,\lambda} L = e + \lambda[B - p(e, q)s]$$

The first order conditions of this constrained maximization program are given by:

$$\frac{\partial L}{\partial s} = \frac{de}{ds} - \lambda \left[ p_1(e, q) \frac{de}{ds} s + p(e, q) \right] \leq 0; s \geq 0; s \frac{\partial L}{\partial s} = 0 \quad (\text{A1})$$

$$\frac{\partial L}{\partial q} = \frac{de}{dq} - \lambda \left[ p_1(e, q) s \frac{de}{dq} + p_2(e, q) s \right] \leq 0; q \geq 0; q \frac{\partial L}{\partial q} = 0 \quad (\text{A2})$$

$$\frac{\partial L}{\partial \lambda} = B - p(e, q)s \geq 0; \lambda \geq 0; \lambda \frac{\partial L}{\partial \lambda} = 0 \quad (\text{A3})$$

Note first that the payment granted to participants in case they achieve their objective must be positive at the optimum ( $s > 0$ ). If  $s = 0$ , Eq. (A3) implies  $\lambda = 0$ , so that  $\frac{\partial L}{\partial s} > 0$ , which contradicts Eq. (A1) since  $\frac{de}{ds} > 0$  (see Eq. (2)). The degree of difficulty of the weight loss objective must also be positive:  $q = 0$  implies  $p_1(e, q) = p_2(e, q) = p_{12}(e, q) = 0$ , which in turn lead to  $\frac{de}{dq} = 0$  and to  $\lambda = 0$  (so that Eq. (A2) is respected). However, the last equality contradicts Eq. (A1), implying that the corner solution  $q = 0$  can also be ruled out. As a result, interior solutions prevail, implying that the first order conditions of the constrained maximization program are met if  $\frac{\partial L}{\partial s} = 0$  and  $\frac{\partial L}{\partial q} = 0$ . These two equalities imply that the optimum is reached when:

$$p_2(e, q) s \frac{de}{ds} = p(e, q) \frac{de}{dq}$$

Inserting Eqs. (2) and (3) – which define  $\frac{de}{ds}$  and  $\frac{de}{dq}$ , respectively – in the last condition, we obtain:

$$\begin{aligned} & p_1(e, q) p_2(e, q) s u_1(S) + p(e, q) p_2(e, q) h'(e) s u_{12}(S) = \\ & p(e, q) p_{12}(e, q) [u(S) - u(F)] + p(e, q) p_2(e, q) h'(e) [u_2(S) - u_2(F)] \end{aligned}$$

We use the mean value theorem twice to re-express the right-hand side of the equality sign as follows:

$$u(S) - u(F) = u(w + s, h + h(e)) - u(w, h + h(e)) = s u_1(\alpha, h + h(e)) \text{ with } \alpha \in [w, w + s]$$

$$u_2(S) - u_2(F) = u_2(w + s, h + h(e)) - u_2(w, h + h(e)) = s u_{12}(\beta, h + h(e)) \text{ with } \beta \in [w, w + s]$$

So that we obtain:

$$\begin{aligned} & p_1(e, q) p_2(e, q) s u_1(S) + p(e, q) p_2(e, q) h'(e) s u_{12}(S) = \\ & p(e, q) p_{12}(e, q) s u_1(\alpha, h + h(e)) + p(e, q) p_2(e, q) h'(e) s u_{12}(\beta, h + h(e)) \end{aligned}$$

Which, after rearranging the terms, gives:

$$p_{12}(e, q)p(e, q)u_1(\alpha, h + h(e)) \\ = p_2(e, q)p_1(e, q)u_1(S) + p(e, q)p_2(e, q)h'(e)[u_{12}(S) - u_{12}(\beta, h + h(e))]$$

We use once again the mean value theorem to re-express the right-hand side of the equality sign as follows:

$$u_{12}(w + s, h + h(e)) - u_{12}(\beta, h + h(e)) = (w + s - \beta)u_{112}(\gamma, h + h(e)) \text{ with } \gamma \in [\beta, w + s]$$

The optimal values of  $s$  and  $q$  are then defined by the following expression:

$$-\frac{p_{12}(e, q)}{p_2(e, q)} + \frac{p_1(e, q)}{p(e, q)} \frac{u_1(w + s, h(e))}{u_1(\alpha, h(e))} + h'(e)(w + s - \beta) \frac{u_{112}(\gamma, h + h(e))}{u_1(\alpha, h + h(e))} = 0 \quad (\text{A4})$$

where  $\alpha \in [w, w + s]$ ,  $\beta \in [w, w + s]$  and  $\gamma \in [\beta, w + s]$ .

## Appendix B

An individual  $v$  is more risk averse than another individual  $u$  if  $v(x) = \varphi(u(x))$  with  $\varphi' > 0$  and  $\varphi'' < 0$ , i.e. if  $v$  is an increasing and concave transformation of  $u$  (see Pratt, 1964). Therefore:

$$v'(x) = \varphi'(u(x))u'(x)$$

$$v'(y) = \varphi'(u(y))u'(y)$$

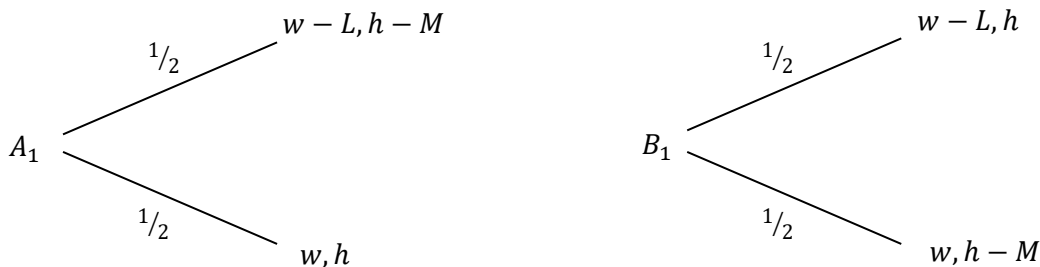
Assume that  $x > y$ . We have:

$$\frac{v'(x)}{v'(y)} = \frac{\varphi'(u(x))u'(x)}{\varphi'(u(y))u'(y)} > \frac{u'(x)}{u'(y)}$$

As a result,  $\frac{v'(x)}{v'(y)} > \frac{u'(x)}{u'(y)}$  when individual  $v$  is more risk averse than individual  $u$ , so that the ratio of marginal utilities evaluated at two separate points provides a measure of the degree of risk aversion.

## Appendix C

Richard (1975) proposed an interpretation of the sign  $u_{12}$  based on the choice that an individual makes between the two lotteries  $A_1$  and  $B_1$ :



In each of the above lotteries, two losses, denoted by  $L$  and  $M$ , are respectively added to the first argument (wealth) and to the second argument (quality of life) of the individual's wellbeing. However, these two losses are not allocated in the same way in  $A_1$  and in  $B_1$ : while they are perfectly positively

correlated in  $A_1$ , they are perfectly negatively correlated in  $B_1$ . Individuals who are averse to this correlation prefer  $B_1$  over  $A_1$ , while the reverse preference hold for those who are correlation seekers.

Richard (1975) showed that in the expected utility model, the relative preference for  $A_1$  or  $B_1$  depends on the sign of  $u_{12}$ :

$$\begin{aligned} R = EU(B_1) - EU(A_1) &= \frac{1}{2}[u(w - L, h) + u(w, h - M)] - \frac{1}{2}[u(w - L, h - M) + u(w, h)] \\ &= \frac{1}{2}[u(w - L, h) - u(w, h)] - \frac{1}{2}[u(w - L, h - M) + u(w, h - M)] \end{aligned}$$

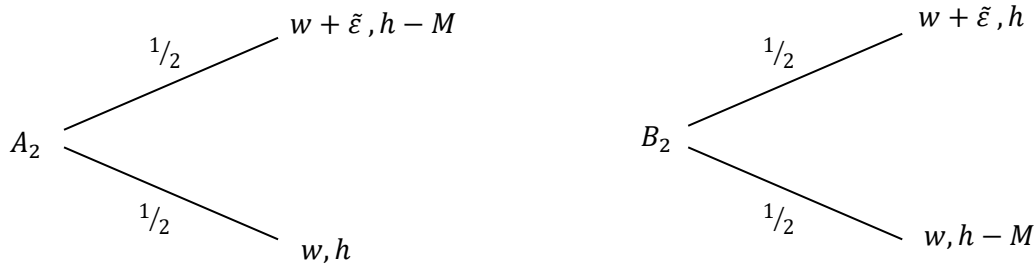
From the last expression, it is straightforward to show that:

$$R \geq 0 \Leftrightarrow u_{12} \leq 0$$

As a result,  $u_{12} < 0$  indicates a preference for disaggregating harms (which take the form of sure losses in this case) related to health and quality of life. Conversely,  $u_{12} > 0$  corresponds to a preference for the aggregation of these two harms.

Eeckhoudt, Rey et Schlesinger (2007) generalized this result by showing that the same kind of preference can be established when the harm added to one argument of the utility function takes the form of a zero-mean risks while the harm added to the other argument of the utility function is a sure loss.

Let us consider the distributions  $A_2$  et  $B_2$ . Compared to distributions  $A_1$  et  $B_1$ , we notice that the harm added to wealth is not the sure loss  $L$  but the zero-mean risk  $\tilde{\varepsilon}$  ( $E(\tilde{\varepsilon}) = 0$  is assumed).



In the expected utility model, Eeckhoudt, Rey et Schlesinger (2007) show that the sign of  $u_{112}$  drives the preference that individuals have for  $A_2$  or  $B_2$ :

$$\begin{aligned} S = EU(B_2) - EU(A_2) &= \frac{1}{2}[Eu(w + \tilde{\varepsilon}, h) + u(w, h - M)] - \frac{1}{2}[Eu(w + \tilde{\varepsilon}, h - M) + u(w, h)] \\ &= \frac{1}{2}[Eu(w + \tilde{\varepsilon}, h) + u(w, h)] - \frac{1}{2}[Eu(w + \tilde{\varepsilon}, h - M) + u(w, h - M)] \end{aligned}$$

Using Jensen's inequality, it can be shown that:

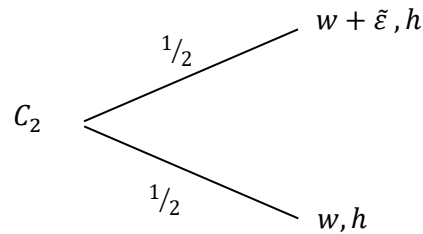
$$S \geq 0 \Leftrightarrow u_{112} \geq 0$$

Therefore,  $u_{112} > 0$  ( $u_{112} < 0$ ) indicates a preference for disaggregating (resp. aggregating) two harms which take the form of a sure loss added to quality of life and of a zero-mean risk added to

wealth. Individuals are said to be cross-prudent (resp. cross-imprudent) in quality of life when  $u_{112} > 0$  (resp.  $u_{112} < 0$ ).

Using the same technique, we can show that if the harm added to wealth is a sure loss while that added to quality of life is a zero-mean risk, the preference for the allocation of these two harms is driven by the sign of  $u_{122}$ . More precisely,  $u_{122} > 0$  (resp.  $u_{122} < 0$ ) indicates a preference for disaggregating (resp. aggregating) two harms that take the form of a zero-mean risk added to quality of life and of a sure loss added to wealth. Individuals are said to be cross-prudent (resp. cross-imprudent) in wealth when  $u_{122} > 0$  (resp.  $u_{122} < 0$ ).

Let us show now that increases in financial risk increase (resp. decrease) the marginal utility of quality of life if individuals are cross-prudent (resp. cross-imprudent) in quality of life. Consider the lottery  $C_2$  defined as follows:



Suppose that a sure loss  $M$  must be added to one of the two branches of  $C_2$ . To which branch does an individual prefer the sure loss  $M$  to be added? The answer depends on whether the individual is cross-prudent or cross-imprudent in the quality of life, since the sure loss  $M$  attached to the upper branch of  $C_2$  correspond to  $A_2$ , while  $B_2$  is obtained if the sure loss  $M$  is attached to the lower branch of  $C_2$ . However, regardless of the individual's preferences, he always prefers the sure loss  $M$  to be associated with the lowest marginal utility of quality of life. Therefore, increasing the financial risk has the consequence of increasing (resp. decreasing) the marginal utility of quality of life if the individual's preferences are such that  $u_{112} > 0$  (resp.  $u_{112} < 0$ ).

#### Appendix D

Ross (1981) extended the standard definition of risk aversion provided by Arrow (1965) and Pratt (1964) to deal with situations where the numerator and the denominator of the coefficient of absolute risk aversion are not evaluated at the same level of wealth. Crainich, Eeckhoudt and Le Courtois (2020) generalized it by providing several potential ways to define higher order risk aversion applied in the bivariate context (see their definition 2.2). Among these definitions, the one adapted to the case presented in section 4.3 is the following. An individual  $v$  is more cross prudent in health in the sense of Ross than another individual  $u$  if:

$$\frac{v_{112}(x, h)}{v_1(y, h)} \geq \frac{u_{112}(x, h)}{u_1(y, h)} \quad \forall x, y, h$$

Graphically, this corresponds to Figure 2, where  $x$  and  $y$  are any potential values that  $w$  may take.

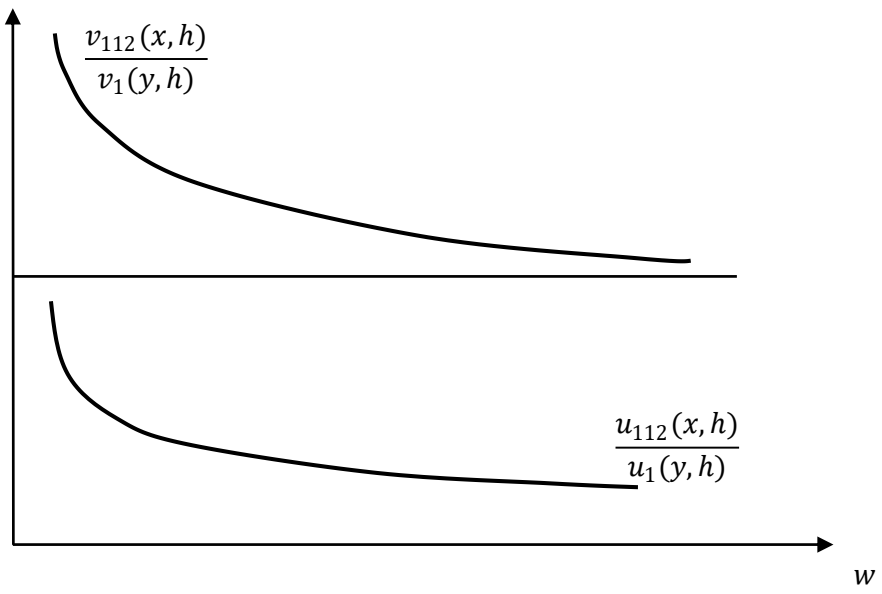


Figure 2