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# Generalizing from examples: an epistemological contribution

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In mathematics courses, the use of examples is often reduced to illustrating concepts (Bills et al., 2006). Since more than four decades, the international didactic community has investigated the potential of examples for expressing one's proof (Balacheff, 1988; Zaslavsky, 2018). Considering the difficulties regarding proof and proving experienced by students at the transition to university mathematics (Selden, 2012), such use of examples is promising insofar as it “reduces the level of abstraction and suspends or even eliminates the need to deal with formalism and symbolism” (Zaslavsky, 2018, p. 290). Although flourishing, the research has not stabilized the vocabulary (interchangeable use of the terms *generic example*, *generic proof*, *generic argument* and even *proof by generic example*) and it is often unclear what the authors speak about (Dogan and Williams-Pierce, 2021). We have contributed to clarify those terms by defining different levels of genericity in (Trouvé, 2023). An acceptance within this polysemy echoes to Steiner's stance that “it is not, then, the *general* proof which explains; it is the *generalizable* proof” (1978, p. 144, emphasis in the original). As part of our research on the use of examples for teaching and learning proof, we focus on generalizing from examples.

To get insight into this use of examples, we choose to work in the first-order predicate calculus to be able to take over the instantiation processes (Barrier, 2016). Given a deductive theory  $T$ , the statement  $\forall y \in Y, Q(y)$  will be said more general than the statement  $\forall x \in X, P(x)$  if and only if  $\{x \in X, P(x)\} \subset \{y \in Y, Q(y)\}$ . We describe and illustrate three proof processes for obtaining  $\forall y \in Y, Q(y)$  by generalization:

1. by using (and possibly changing) a proof of  $\forall x \in X, P(x)$ , for elements of  $Y$ ,
2. by using the statement  $\forall x \in X, P(x)$  in a proof of  $\forall y \in Y, Q(y)$ , and
3. by identifying the generic character (Trouvé, 2023) of (part of) a proof of the statement  $\forall x \in X, P(x)$ .

As exploring the experts' practices is important to think the didactical transposition, we wonder if and how mathematicians generalize from examples. To address these research questions, we analyze the answers of 13 mathematicians to a questionnaire designed to tackle more generally the question of genericity. It leads to the following results. The second process is not mentioned in any answer. In contrast, the first and the third processes are respectively cited in 3 and 5 answers. The corresponding answers illustrate the complexity of these processes, both in terms of the diversity of their shape and their possible interaction. 2 of the 14 mathematicians speak about generalizing from examples without involving them in a proving process. For 3 others,

the description is not precise enough to decide whether the examples are used for generalizing, and if they are involved in a proof process or not.

This study enhances our understanding of proof processes in mathematics and stresses the relevance of incorporating them in the learning and teaching of proof. In a didactic perspective, we wonder if such processes live in the curriculum and if teachers recognize them as goals of the mathematics class. Besides, we question the possibility to devolve situations involving such processes. If so, what are their specificities? How can (or should) they be implemented? We plan to study these issues in our future research.

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