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Drop Medusa: Direct numerical simulations of high-frequency Faraday waves on spherical drops

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The dynamics of a drop under vibration is a classical and long-standing fluid dynamics problem. Lamb [1] studied the free oscillations of a viscous drop, and Lundgren et al. [2] carried out the first numerical study of the flow. Although several numerical techniques were then utilised to elucidate the oscillatory phenomena [3–5], these studies were limited to axisymmetric, moderate amplitude, low capillary mode oscillations, and were unable to attain long-time, finite-

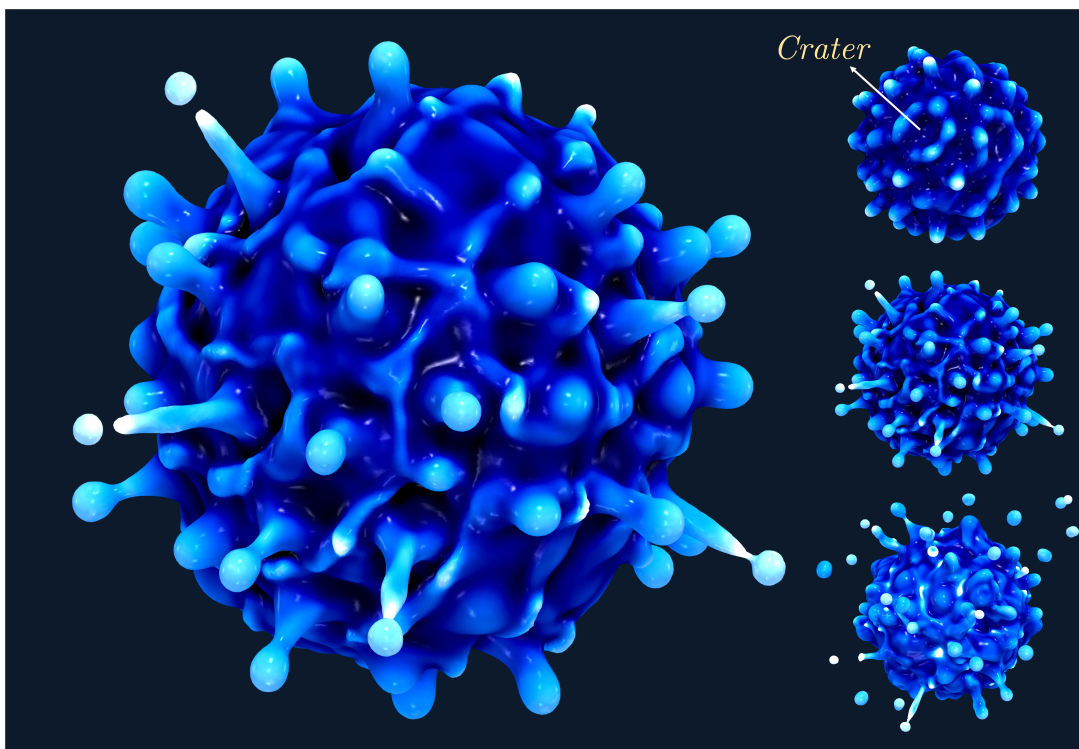


FIG. 1: Drop Medusa: (left) prominent threads of jets forming on the interface that resemble the living snake-like hair of the Greek mythological figure, Medusa; (right) interfacial evolution at $t = 380T$, $385T$, and $387T$, top to bottom, respectively. (<https://doi.org/10.1103/APS.DFD.2023.GFM.P0030>)

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amplitude motion. Recently, three-dimensional numerical simulations of moderate amplitude and low-frequency oscillations were carried out by Ebo-Adou et al. [6], covering spherical harmonics ranging from 1 to 6, referred to as the spherical analogue of Faraday waves. These authors observed regular and simple patterns. At higher amplitudes and frequencies, it is inevitable that many spherical harmonics will be excited and interact nonlinearly. Here, we extend the numerical study of Ebo-Adou et al. [6] to high frequency and amplitude vibration of a spherical drop. This is similar to the subject of our previous Gallery of Fluid Motion award presentation [7] where we discussed similar dynamics for a vertically-vibrated sessile drop.

We performed three-dimensional direct numerical simulations of a drop of volume $\mathcal{V} = 100\mu\text{L}$ using our in-house Navier-Stokes multiphase solver, *BLUE* [8]. A triply-periodic cubic computational domain, encompassing water and air, is decomposed into $8 \times 8 \times 8$ subdomains each of resolution 64^3 meshgrids. The corresponding global mesh structure is 512^3 grid cells of uniform size $22.5\ \mu\text{m}$. The density of water and air are set to $998\ \text{kg/m}^3$ and $1.205\ \text{kg/m}^3$, and their dynamic viscosities to $10^{-3}\ \text{kg/m.s}$ and $1.82 \times 10^{-5}\ \text{kg/m.s}$, respectively, while the surface tension is set equal to $0.0714\ \text{N/m}$. The volumetric force on a sphere analogous to the modulated gravitational forces on a planar interface is $\mathbf{F}_v = \tilde{\rho} A \sin(2\pi ft)(\mathcal{H}(r, t) r/R + (1 - \mathcal{H}(r, t)) R^2/r^2)\mathbf{e}_r$, where $\mathcal{H}(r, t)$ is the Heaviside function (that is 1 for water and 0 for air) at the radial coordinate r and R is the radius of the drop; $\tilde{\rho} = \rho_w \mathcal{H}(r, t) + \rho_a (1 - \mathcal{H}(r, t))$ is the single-fluid formulated density, where ρ_w and ρ_a are the density of water and air, A is the acceleration amplitude, and f the frequency of vibration. The frequency is set to $f = 1040\ \text{Hz}$, as per Vukasinovic et al. [9]. We initialize the location of the interface at $\zeta(\theta, \phi) = R(1 + \epsilon Y_{20}^0(\theta, \phi))$, where $\epsilon = 0.005$ and Y_{20}^0 is the 20th axisymmetric spherical harmonic. As in Panda et al. [7], we ramped up the acceleration to $A = 300g$ at a rate of $50g$ per 60 forcing time periods $T (= 1/f)$ up to $t = 360T$ to set up an irregularly perturbed interface. The kinetic energy decreases significantly before increasing again, suggesting that the initial perturbation has been damped and reorganized, so that the subsequent appearance and dynamics are independent of the initialisation.

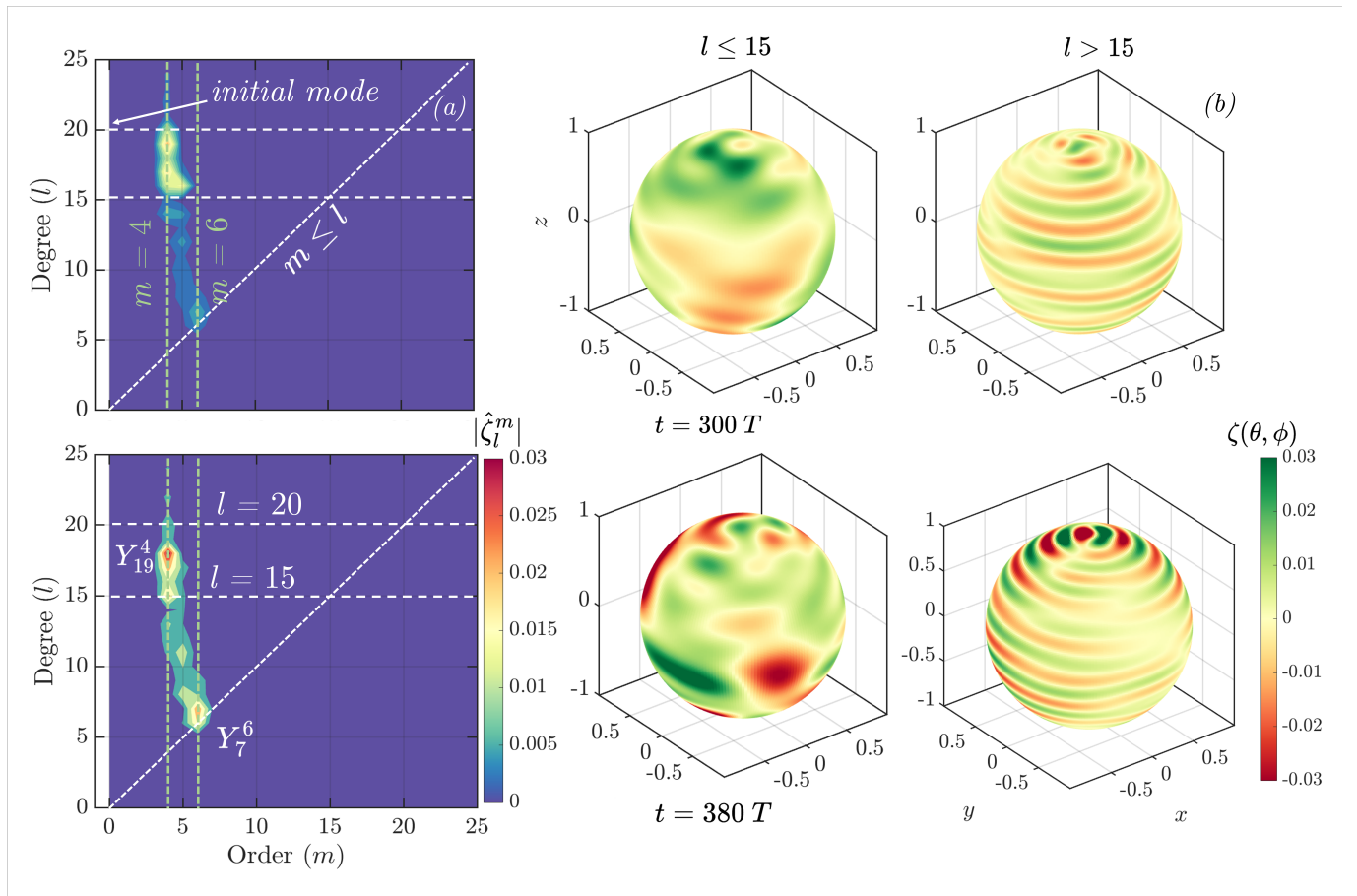


FIG. 2: (a) Spherical harmonic spectrum: modulus of the spherical harmonic coefficients $|\hat{\zeta}_{l,m}|$ at $t = 300T$ (top) and $380T$ (bottom); (b) interface reconstruction at $t = 300T$ (top) and $380T$ (bottom) from only the low-degree (left, $l < 15$) or high-degree (right, $l > 15$) spherical harmonics.

Figure 1 shows the behaviour of the Faraday waves on the drop at $t = 380T$, $385T$, and $387T$, colored according

to the magnitude of the interfacial pressure. The craters (shown at $t = 380T$) collapse to form jets that eject droplets [9, 10]. In order to better understand this phenomenon, we compute the spectral coefficients obtained from the spherical harmonic decomposition $\zeta(\theta, \phi, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \widehat{\zeta}_l^m(t) Y_l^m(\theta, \phi)$, where Y_l^m is the spherical harmonic function of degree $l \in [0, \infty)$ and order $m \in [-l, l]$. In fig. 2(a), we plot the square modulus $|\widehat{\zeta}_l^m(t)|$ of the complex coefficients at $t = 300T$ and $380T$. Initially at $t = 0$, the only non-zero coefficient is that of Y_{20}^0 . At $t = 300T$, the excited spectral coefficients are mainly in the narrow band $m \in [4, 6]$ and $l \in [15, 20]$; for this reason, we use $l = 15$ as the boundary between high- and low- degree modes. We then reconstruct the surface to examine the role of higher and lower modes. For $l > 15$, we observe low- m modes that are centered with respect to the z -axis that is distinguished at $t = 0$ via the axisymmetric initial condition Y_{20}^0 . It is somewhat surprising that the orientation of the initial condition persists, despite the damping out of the initial surface perturbation and the emergence of new modes. By $t = 380T$, the amplitudes of lower- l modes have greatly increased in the system, as shown in the lower part of fig. 2(a). More specifically, a range of $l \in [6, 20]$ is excited in the $m \in [4, 6]$. The dominant modes are found to be Y_{19}^4 and Y_7^6 . The reconstructed interface for $l \leq 15$ shows large patches of increased ζ with no apparent order. The amplitude of $|\widehat{\zeta}_l^m(t)|$ for $l > 15$ also increases, and remains oriented around the z -axis.

The superposition of all of these modes results in a highly deformed interface, as shown in fig. 1. James et al. [10] described the way in which an erratic surface ζ leads to negative curvature zones on the drop surface, called craters. We associate these craters with the prominent red patches in fig. 2(b) and with spherical wavenumbers of order $l \in [5, 15]$, $m \in [4, 6]$ in fig. 2(a). The craters undergo capillary-driven collapse, which in turn leads to jet formation at $t = 385T$. Following their pinch-off and ejection, the jets retract and fall back into the vibrating drop. These violent dynamics create a resemblance with the Greek mythological figure, Medusa, captured by the title of this paper.

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- [1] H. Lamb, Proceedings of the London Mathematical Society **1**, 51 (1881).
 - [2] T. Lundgren and N. Mansour, J. Fluid Mech. **194**, 479 (1988).
 - [3] T. Patzek, R. Benner Jr, O. Basaran, and L. Scriven, J. Comput. Phys. **97**, 489 (1991).
 - [4] O. A. Basaran, J. Fluid Mech. **241**, 169 (1992).
 - [5] S. Meradji, T. Lyubimova, D. Lyubimov, and B. Roux, Cryst. Res. Technol. **36**, 729 (2001).
 - [6] A.-H. Ebo-Adou, L. S. Tuckerman, S. Shin, J. Chergui, and D. Juric, J. Fluid Mech. **870**, 433 (2019).
 - [7] D. Panda, L. Kahouadji, L. Tuckerman, S. Shin, J. Chergui, D. Juric, and O. Matar, Phys. Rev. Fluids **8**, 110510 (2023).
 - [8] S. Shin, J. Chergui, and D. Juric, J. Mech. Sci. Tech. **31**, 1739 (2017).
 - [9] B. Vukasinovic, M. K. Smith, and A. Glezer, J. Fluid Mech. **587**, 395 (2007).
 - [10] A. James, B. Vukasinovic, M. K. Smith, and A. Glezer, J. Fluid Mech. **476**, 1 (2003).