



HAL
open science

Integration of evolutionary prejudices in Dempster-Shafer theory

Florence Dupin de Saint-Cyr, Francis Faux

► **To cite this version:**

Florence Dupin de Saint-Cyr, Francis Faux. Integration of evolutionary prejudices in Dempster-Shafer theory. *International Journal of Approximate Reasoning*, In press, 178, pp.109351. 10.1016/j.ijar.2024.109351 . hal-04859615

HAL Id: hal-04859615

<https://hal.science/hal-04859615v1>

Submitted on 30 Dec 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Integration of evolutionary prejudices in Dempster-Shafer theory*

Florence Dupin de Saint-Cyr Francis Faux
bannay@irit.fr francis.faux@univ-jfc.fr

December 2024

Abstract

This paper deals with belief change in the framework of Dempster-Shafer theory in the context where an agent has a prejudice, i.e., a priori knowledge about a situation. Our study is based on a review of the literature in the social sciences and humanities. Our framework relies on the claim that prejudices and evidences should be dealt with separately because of their very different natures (prejudices being at the meta level, governing the evolution of beliefs). Hence, the cognitive state of an agent is modeled as a pair whose components reflect its prejudices and uncertain beliefs. We propose a general formalism for encoding the evolution of this pair when new information arrives, this is why the study is related to Dempster's revision. Several cases of prejudice are described: the strong persistent prejudice (which never evolves and forbids beliefs to change), the prejudice that is slightly decreasing each time a belief contradicts it, etc. A general example with several prejudices and complex masses illustrates our approach.

Keyword: Theory of belief functions, Dempster-Shafer Theory, Belief change, Prejudices, Stereotypes, Dempster rule, Evidences, Uncertainty

*This is an accepted manuscript of an article published in the International Journal of Approximate Reasoning, Volume 178, 2025, 109351, ISSN 0888-613X, available at <https://doi.org/10.1016/j.ijar.2024.109351>

1 Introduction

When one is interested in formalizing human reasoning the subject of prejudices cannot be avoided. Indeed, this subject is fascinating for two reasons: on the one hand, prejudices seem to be necessary shortcuts that accelerate reasoning and decision¹, on the other hand, they are often unconscious mechanisms that should at least be detected in order to rationalize them or to cope with them with awareness. These two aspects may explain why it seems important as a human being to better understand prejudice mechanisms in order to try to be aware of our own prejudices and even to control them. To the best of our knowledge, in computer science, there is not yet any explicit formalization of this phenomenon, while there are a lot of studies about it in human sciences. This is why we need to base our own study on the large state of the art in this domain. In the remaining of this introduction we first review the studies about cognitive biases, dogmatism and stereotypes before exploring how Dempster-Shafer theory may help to handle prejudices.

1.1 Modest overview of human sciences perception of prejudices, dogmatisms and stereotypes

A prejudice can be defined as an a priori favorable or unfavorable ² “opinion adopted without examination, imposed by an environment, an education” (Montaigne, Essais, II, 12, ed. P. Villey and V.-L. Saulnier, p.506). A prejudice can be disproved on the basis of facts. Prejudices can be more or less strong, the strongest they are the more difficult it is to disprove them and the more they will influence the reasoning. Therefore, the way people consider information is influenced not only by the type of information they receive, but also by the purpose for which they process it. For example Hilton and al. [14] show that stereotyped thinking generally serves multiple purposes as it can appear as a means of simplifying the demands placed on the perceiver or as a means of justifying the status quo.

Prejudice is a major theme in psychology, and the subject of an abundant literature: Nelson [27] observes that between 1977 and 2004, around 2000 articles were published about prejudices with theoretical and empirical studies. The author explains the growing attraction of this subject by the fact that it is a clear topic

¹In his famous book *The Nature of prejudice* the American psychologist Gordon Allport [2] claims that “prejudice is essentially a by product of the necessary mental shortcuts the human brain uses to process the vast amount of information it takes in”.

²“(good or bad) opinion that one forms in advance” (Lanoue, Discours pol. et milit., 436 in Littré,1587).

characterizing human nature and its tendency to behave as a member of various groups with important implications for one's own life and that of others. Moreover, the revolutionary work of Tversky and Kahneman [39] gave birth to a new way to analyze human being behavior by highlighting the fact that human beings have reasoning biases and develop heuristics to speed up decision-making. It is worth noticing that these biases are not necessarily considered as bad behavior by the authors themselves, since they provide crucial help and can be very effective in many cases. The main biases and heuristics revealed by [39] are the *availability heuristic* which is the tendency to not looking for new information, but to consider only what "comes to mind", the *confirmation bias* which consists of giving priority to information that corroborates previous ideas or hypotheses and that is in line with one's own beliefs, the *anchoring bias* which consists in building a line of reasoning from a given point provided by the problem statement, the situation or previous practice, it sets the initial framework for the analysis and tends to limit the possibilities for subsequent adjustment. Among these three well defined tendencies, the confirmation and anchoring biases seem to be the basis for human beings' use of prejudice.

An extreme version of prejudice is the concept of *dogmatism*, it refers to closed-minded people, i.e., individuals that can be rigid, inflexible and not open to new information, new experiences, or new environments. In 1954, Rokeach [32] analyzed the nature and meaning of dogmatism which he defines as a relatively closed organization of beliefs and disbeliefs about reality, organized around a central set of beliefs with absolute authority. This organization provides a framework for intolerance and tolerance patterns and is a more complex form of resistance to change than mere rigidity since several axioms govern dogmatism: perception that arguments pointing similarities between beliefs and disbeliefs are irrelevant, denial of events contradicting the belief system but coexistence of contradictions in this system. The level of dogmatism was assessed by Rokeach [33] using a complex 89-item rating scale to measure individual differences between open and closed belief systems. A simplified conceptualization was proposed by Shearman and Levine in [35] with an assessment of the level of closed-mindedness on a simplified 10-item scale. The theory of dogmatism may be a good explanation for *echo chambers*, which are increasingly mentioned these days when talking about closed communities in social networks. It seems very important for democracy or simply free thinking defense to struggle against the extremism generated by these echo chambers which is reinforced by algorithms that only provide arguments in line with what the members of a community already believe.

While dogmatism can be viewed as an extreme attitude, *stereotypes* are very

common. From a psychological point of view, they refer to the images we construct about social groups and beliefs, and are defined by Leyens et al. in [20] as “a set of beliefs about personal attributes shared by a group of people”. According to Lippmann [21], stereotypes are indispensable for coping with the complexity of our social environment. They allow us to simplify reality and adapt to it more easily. However, these representations are not without negative consequences, giving rise to discriminatory behavior such as racism or sexism, or interfering with the coding and restitution of information. In [24], Martin et al. studied “how societal stereotypes might form and evolve via cumulative cultural evolution”. They use the view of Allport [2] who sees stereotypes as “characterizations of social categories whereby group membership is associated with the possession of certain attributes (e.g., scientists are geeky, Scottish people are miserly, men like the color blue)”. Martin et al. acknowledge that stereotypes lead to prejudice and discrimination against minority groups, biased thinking and behavior, but they also underline that they are playing a vital cognitive role in efficiently structuring social information. As regards the formation and evolution of stereotypes, their main idea is that, as in evolution theory but in a communicative aspect, in order to survive, a stereotype should be easy to memorize and transmit to others. Moreover they argue that the reason why stereotypes exist and persist is that they are perfectly adapted to human cognition because they are the cumulative product of human cognition in a cycle: observation, cognitive representation, social transmission, observation. Stereotypes are forming because of the limits of human capacities at each step of the cycle. They validate their ideas with an experiment described in [25], where they show that “a structural plateau” in which the stereotype no more evolves, is reached after several cycles, typically around the fourth transmission of information.

As we have seen, prejudices and stereotypes are concise ways of reasoning despite the lack of precise information. This is why a formalism able to handle epistemic uncertainty is suitable for our purpose. In the following, we propose to extend Dempster-Shafer theory, that is well suited to combine evidences from different sources, to handle this new type of (meta) information.

1.2 An informal introduction to Dempster-Shafer theory

The aim of this article is to formalize some cognitive mechanisms (essentially stereotypes) described above within the framework of Dempster-Shafer theory. Dempster-Shafer theory was first introduced by Arthur P. Dempster in the context

of statistical inference, then developed by Glenn Shafer into a formal framework for representing and reasoning with uncertain information [34]. G. Shafer viewed belief functions as the result of the conjunctive combination of pieces of evidence such as (more or less unreliable) testimonies from different sources, in order to form a representation of beliefs about certain aspects of the world. This theory is well adapted to perform epistemic analysis in situations where there is few information on which to evaluate a probability or when the information is non-specific, ambiguous, or conflicting. This framework makes possible to express that there is some evidence on a set of events A without specifying how uncertain are each item of A whereas in a probabilistic setting the probability of each event of A should be known.

This theory has been well studied and developed in order to reason with several sources of information, and there were attempts to encode the retraction of information: the operation of removal (or *retraction*) was proposed by many authors [12, 18, 36, 37] in order to decrease the degree of belief by retracting some piece of evidence. In particular, the model called “Latent Belief Structure” introduced by Smets [37] and then studied in [30], considers a pair of belief functions: one representing the confidence part, and the other the diffidence part by playing the role of a moderator that can annihilate, via retraction, some information supplied by the former. Recently, this model was reinterpreted by Dubois et al. [8] in terms of *prejudice* on the part of the information recipient, and retraction was considered as a special kind of belief change. Its role is to weaken the support of some focal sets of a belief function, possibly stemming from the fusion of the incoming information. The authors suggest that prejudices are due to some prior knowledge that is more entrenched than incoming new pieces of uncertain evidence. They explain how this prior information can affect a belief function and code a prejudice by a negative mass function that must be combined with a positive mass function (representing a testimony). However, as the result of this combination must be a positive mass (a negative mass cannot be interpreted), this approach is highly constrained. Consequently, only particular forms of prejudices and beliefs can be modeled, for example when the prejudice is directed against a focal set of a separable belief function. This paper is an attempt to extend their approach.

1.3 Prejudices in a social choice example

In order to illustrate the main goal of our paper, the following example presents a situation of an election campaign with 6 candidates in which an agent has preju-

ances about some candidates and wants to reason in presence of new information.

Example 1 (Opinions and prejudices for election) *An agent has a strong a priori prejudice against members of the International Flat Earth Society and knows that candidates 3 and 5 belongs to it. The agent has also a weak prejudice against having not handsome people elected, and find that candidates 1, 3, 4, 6 are not handsome. This amounts to take into account a prejudice p_1 of strength 0.8 against $A_1 = \{3, 5\}$, and a prejudice p_2 of strength 0.25 against $A_2 = \{1, 3, 4, 6\}$.*

Before voting the agent meets two friends 1 and 2 in whom he has a respective trust of 0.6 and 0.05 (friend 2's argumentation is often far-fetched, this is why the agent has a low trust in him). Friend 1 says that for him the two candidates 2 and 3 are the most competent for governing the country. Friend 2 says that all the candidates 2, 3 and 5 have a PhD degree, hence will be good presidents. Following these testimonies, the agent updates his belief state.

Later, another friend, called Friend 3, who is trusted at 0.55 said that candidates 3, 4 and 6 have all signed a manifest for the climate (and the agent believes that this manifest is good for the country). Figure 1 illustrates this example where prejudices are represented by rectangles and beliefs with ovals.

In this paper we study what will be the new state of beliefs of the agent and will the agent prejudices change.

As precised at the end of the psychological approaches summary, we do not address the case of the integration of a new prejudice or the reinforcement of an existing one. More precisely the incoming information is only a testimony which may reduce a prejudice (or not affect it at all) but cannot increase nor create a new one. The *creation* of prejudice is left for further studies. Moreover, we are going to use the word “prejudice” as a meta-information representing a preconceived idea based on stereotypes or dogmatism.

This paper is an attempt³ to encode prejudices (and more precisely stereotypes) in the formal setting of Dempster-Shafer theory. Note that in this article, we consider that stereotypes are given and cannot increase (taking into account the idea of structural plateau of Martin et al. [25]), but since we are interested in information acquisition, we consider that agents' awareness of their own prejudices can evolve, and we therefore code a decreasing (not increasing) effect. The

³More precisely, it is our second attempt since a first version of this study was presented at the 17th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2023) held in Arras (France).

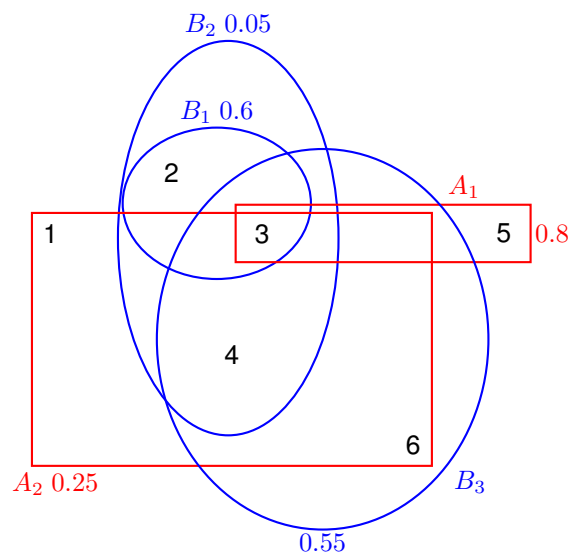


Figure 1: The a priori prejudices of an agent against some candidates in a election campaign (rectangles), the beliefs from three witnesses are represented by ellipses.

main result of our article is a new explicit formalization of the revision mechanism at stakes when dealing with prejudices. Our framework relies on the claim that prejudices and evidences should be dealt with separately because of their very different natures (prejudices being more at the meta level that governs the evolution of beliefs). However, one particularity of this proposal (due to the use of Dempster-Shafer theory) is that commensurability between evidence and prejudice strengths is imposed losing a little of the meta character of the phenomenon.

Some necessary background about Dempster-Shafer theory are introduced in Section 2. This section also provides a reminder of defiance functions, the retraction operation and latent structures, and is completed by a presentation of the essential properties of revision either by a sure information or by a mass function. Section 3 formalizes the evolution of the agent’s belief state in presence of prejudice and uncertain information, five axioms are given to govern this evolution. Section 4 defines how to handle this evolution in the framework of Dempster-Shafer theory. In Section 5, our approach is illustrated on the general example presented above (“Opinions and prejudices for an election”) and some other applications are evoked. Finally, Section 6 concludes the paper by a comparison with the literature and some perspectives.

2 Background

2.1 Basics about belief functions

Let us consider a finite set $\Omega = \{\omega_1, \dots, \omega_N\}$, called the frame of discernment, whose elements represent descriptions of possible situations, states of the world, one of which corresponding to the truth. In Dempster-Shafer theory [34], the uncertainty concerning an agent’s state of belief on the real situation is represented by a *mass function* defined as a mapping $m : 2^\Omega \rightarrow [0, 1]$ such that $m(\emptyset) = 0$ and verifying $\sum_{A \subseteq \Omega} m(A) = 1$. $m(A)$ expresses the proportion of evidence that the current state is in A . Each subset $A \subseteq \Omega$ such as $m(A) > 0$ is called a *focal set* of m .

An *elementary testimony* T with weight $(1 - \beta)$ in favor of a non-contradictory and non-universal proposal $A \in 2^\Omega \setminus \{\Omega, \emptyset\}$ is represented by the *simple mass function* $m : 2^\Omega \rightarrow [0, 1]$ denoted by $m = A^\beta$ in [37, 6] such that

$$m = A^\beta \quad \text{denotes} \quad \begin{cases} m(A) = 1 - \beta \\ m(\Omega) = \beta \end{cases} \quad (1)$$

Here, β evaluates the lack of confidence in the testimony T also called mistrust.

In presence of multiple sources of information or multiple uncertain testimonies, the result of the *conjunctive combination* of two mass functions m_1 and m_2 , noted $m_1 \odot_2$, is defined as follows:

$$m_1 \odot_2(A) = \sum_{A_1 \cap A_2 = A} (m_1(A_1) \cdot m_2(A_2)) \quad (2)$$

In particular, $m_1 \odot_2(\emptyset)$ represents the conflict between the mass functions. We will denote \oplus the normalized conjunctive combination rule, called Dempster's rule of combination [5] (which assigns a zero mass to the empty set and divides all the masses of the focal elements by $1 - m_1 \odot_2(\emptyset)$).

A *belief function* $Bel(A)$ is a non-additive set function which represents the total quantity of pieces of evidence supporting the proposition $A \subseteq \Omega$ and is defined by

$$Bel(A) = \sum_{\emptyset \neq E \subseteq A} m(E) \quad (3)$$

The *plausibility* $Pl(A)$ is the dual set-function of $Bel(A)$ where

$$Pl(A) = 1 - Bel(\bar{A}) \quad i.e. \quad Pl(A) = \sum_{E \cap A \neq \emptyset} m(E) \quad (4)$$

A mass function m can be equivalently represented by its associated *commonality function* defined for all $A \subseteq \Omega$ by

$$Q(A) = \sum_{B \supseteq A} m(B) \quad (5)$$

The commonality function $Q(A)$ represents the total quantity of incomplete evidence that makes *all* elements of A possible. [34] calls a *separable support function*, a belief function $m = \oplus_{i=1}^k A_i^{\beta_i}$ resulting from Dempster rule combination of the simple mass functions $A_i^{\beta_i}$ with $A_i \neq \Omega$ and $0 < \beta_i < 1$, in which each simple mass represents an independent testimony.

Separability for belief functions is a useful property that enables calculations to be simplified by dealing with each simple mass independently, however not all belief functions are separable. To overcome this difficulty, Smets [37] has discovered that by extending simple mass functions A^β with β initially over the unit interval to the interval $(1, +\infty)$, any standard mass function can be decomposed into several simple (generalized) mass functions.

2.2 Defiance, retraction and latent structures

Thanks to the extension to generalized mass functions, Smets introduced the *retraction* of a simple mass function B^y (with $y \in (0, 1)$) supporting B from a simple support function A^x (with $x \in (0, 1)$), denoted by $\textcircled{\otimes}$. It allows to represent the fact that an agent believing in A with strength $1 - x$, do not want (with strength $1 - y$) to believe B . Formally,

$$A^x \textcircled{\otimes} B^y = A^x \textcircled{\ominus} B^{1/y} \quad (6)$$

To illustrate the notion of retraction in an intuitive way, let us consider the following example.

Example 1 *Let us suppose that a person named John meets a friend in whom he has a trust level of $1 - x_1$ who claims that a certain person, say Linda, is a banker b and then meets another friend (trusted with level $1 - x_2$) who claims that Linda is a philanthropist p . The process of merging these two testimonies (typically Dempster's combination rule) leads us to assign $(b^{x_1} \textcircled{\ominus} p^{x_2})$, i.e., a degree of belief $(1 - x_1)(1 - x_2)$ to the fact $b \cap p$ that she is a philanthropic banker.*

However in the past, John had a conflict with his banker over a credit problem. John therefore finds it hard to believe that a banker could be a philanthropist. In other words, he has anchored prejudicial information with a strength $1 - y$. On this basis, we would like to erode, or even remove, the detrimental information $(b \cap p)^y$ from the merging of information b^{x_1} and p^{x_2} . Thus we compute the retraction operation $m = (b^{x_1} \textcircled{\ominus} p^{x_2}) \textcircled{\otimes} (b \cap p)^y$ which leads to a mass on $b \cap p$ equal to $1 - \frac{(x_1 + x_2 - x_1 x_2)}{y}$. When $y = (x_1 + x_2 - x_1 x_2)$ the information on $b \cap p$ disappears completely. When $y > (x_1 + x_2 - x_1 x_2)$, the result is still a mass (always positive) and the information on $b \cap p$ is attenuated.

Note however that in Equation (6), $A^x \textcircled{\otimes} B^y$ is a belief function if and only if $A = B$ and $x/y < 1$. Indeed the result $A^x \textcircled{\otimes} B^y$ is NOT a belief function in general as this mass function may fail to be positive. Retraction also fails if the set to retract is not focal or when the focal set B to be retracted intersects some other focal set A without being included in it (i.e., $B \cap A \neq B$). Despite these limitations, retraction can also be applied in the following two special cases [8]: case 1) when the set of focal sets of m is closed under conjunction and B is focal, case 2) when the mass is separable.

In case 1), we can compute $m' = m \textcircled{\ominus} B^y$. Now, $\textcircled{\ominus}$ attributes values to intersections of the focal sets of m with B , since m is closed under intersection then

m' has the same focal elements as m . By examining the different cases of intersection between a focal set A and B , we get three cases either $A = B$, or $A \subset B$ or ($A \cap B \neq A$ and $A \cap B \neq B$, named “otherwise”). The reader can check that $m \circledast B^y$ gives the following results in this three cases:

$$\forall A \in \mathcal{F}, m'(A) = \begin{cases} y.m(B) + (1-y)Q(B) & \text{if } A = B, \\ m(A) + (1-y) \cdot \sum_{\emptyset \neq D \subseteq \bar{B}} m(A \cup D) & \text{if } A \subset B, \\ y.m(A) & \text{otherwise.} \end{cases} \quad (7)$$

Finally, in order to obtain a mass function for m' , the values of $m'(A)$ should be positive in three situations yielding the following complex condition given in inequation (8) [8]:

$$1 < y < \min\left(\frac{Q(B)}{Q(B) - m(B)}, \min_{E \subset B} \frac{\sum_{D \subseteq \bar{B}} m(E \cup D)}{\sum_{\emptyset \neq D \subseteq \bar{B}} m(E \cup D)}, \min_{E \not\subseteq B} \frac{1}{m(E)}\right) \quad (8)$$

In case 2), retracting a focal set E_J from a separable mass function m affects and may also delete all focal sets $E_I \subset E_J$, namely all combinations between the merging of information E_J from sources indexed in J , with information from other sources.

Based on the canonical decomposition of belief functions and the retraction operation, the concept of *latent belief structure (LBS)* [37, 30] was defined as a pair of separable non dogmatic⁴ masses m^c and m^d called respectively the *confidence and diffidence components* such that $m = m^c \circledast m^d$ with $m^c = \bigcircledast_{A \in C} A^{w(A)}$ and $m^d = \bigcircledast_{A \in D} A^{\frac{1}{w(A)}}$. The disjoint subsets C and D come from a partition of 2^Ω such that $C = \{A | A \subset \Omega, w(A) \in (0, 1]\}$ and $D = \{A | A \subset \Omega, w(A) \in (1, \infty)\}$. The diffidence component may be interpreted as a prejudice against the subset D . However, only a few particular cases of prejudice can be modeled by LBS because the constraints linked to the retraction operation detailed above are very restrictive.

2.3 Revision in Dempster-Shafer theory

In the following we describe the revision by a sure information while in the next subsection a more general view is taken for revising by any mass function.

⁴A mass is dogmatic when $m(\Omega) = 0$.

2.3.1 Revising by a sure observation C

When dealing with statistical data, Dubois and Denoeux [7] differentiate revision and prediction with respect to the new piece of information. The initial information corresponds to a belief function encoded by a mass function (the assignment of non-negative weights $m(E)$ to subsets E of Ω). This mass m is modified in order to take into account the new piece of information saying that the states in C are observed. When this new piece of information is totally certain then it is a *revision* and Dempster conditioning is used for revising m by the new piece of information (represented by C) considered totally certain.

$$m(B||C) = \sum_{E: B=C \cap E \neq \emptyset} m(E) \quad (9)$$

In other words $m(B||C) = Pl(E \cap C)$. Moreover they define $Pl(B||C) = Pl(B \cap C)/Pl(C)$ and $Bel(B||C) = 1 - Pl(\overline{B}||C)$. The following classical example illustrates the notion of revision.

Example 2 (Ellsberg’s paradox) *We consider an urn with three kinds of balls: white, black and red. We know that 1/3 of balls are reds, the universe is all the possible outputs obtained after the event to draw a ball from the urn: $\Omega = \{\omega_1, \omega_2, \omega_3\}$ where ω_1 (resp. ω_2, ω_3) represents the fact that the ball is white (resp. black and red). The mass representing the initial information is named m in Table 1. We don’t know the proportions of balls of each kind, let us call β the proportion of white among the white and black balls. We learn that the ball that is extracted is not black: $C = \{\omega_1, \omega_3\}$. It gives the results presented in Table 1 column 4.*

In the case called “revision” by Dubois and Denoeux [7], Dempster conditioning transfers the full mass of each focal set E to $E \cap C \neq \emptyset$ (followed by a renormalization). This means that the new information C modifies the initial belief function in such a way that $Pl(\overline{C}) = 0$, i.e., situations where C is false are considered as impossible. In the “prediction” case, only a proportion of the mass of E is transferred to $E \cap C$, but after normalization the rest is distributed over the new focal elements (the ones in $E \cap C$).

E	m	Pl	$m_{ C}$	m_I	$m \circ m_I$	$m_{I'}$	$m \circ m_{I'}$
\emptyset	0	0	0	0	0	0	0
$\{1\}$	0	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{2}{3}$	0.5	$0.5 + \frac{0.8}{3}$
$\{2\}$	0	$\frac{2}{3}$	0	0	0	0.1	0.1
$\{3\}$	$\frac{1}{3}$	1	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{0.4}{3}$
$\{1, 2\}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0	0	0	0
$\{1, 3\}$	0	1	0	1	0	0.4	0
$\{2, 3\}, \Omega$	0	1	0	0	0	0	0

$\sigma(A, B)$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 2\}$	$\{1, 3\}$	$\{2, 3\}$	Ω
$\{1\}, \{2\}$	0	0	0	0	0	0	0
$\{3\}$	0	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\{1, 2\}$	1	1	0	1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\{1, 3\}, \{2, 3\}, \Omega$	0	0	0	0	0	0	0

Table 1: Computing the revision $m_{||C}$ by $C = \{\omega_1, \omega_3\}$ of the mass functions m in the Ellsberg paradox example, where ω_1 (resp. ω_2 and ω_3) means white (resp. black and red). The top table shows the revision $m \circ m_I$ (resp. $m \circ m_{I'}$) of a given mass m by the simple mass m_I (resp. with the more complex mass $m_{I'}$). The bottom table gives the specialization matrix for m . In this table ω_i is abbreviated i for any $i \in \{1, 2, 3\}$.

2.3.2 Revising by a new mass function m_I :

Ma et al. [23] have introduced a “revision operator” \circ that applies on two mass functions m and m_I on Ω and defined by:

$$\text{For any } E \neq \emptyset, \quad m \circ m_I(E) = \sum_{A \cap B = E} \sigma(A, B) \cdot m_I(B) \quad (10)$$

where $\sigma(A, B)$, called *specialization matrix*, is such that $\sigma(A, B) = 0$ when $A \cap B = \emptyset$ and otherwise:

$$\sigma(A, B) = \begin{cases} \frac{m(A)}{Pl(B)} & \text{for } Pl(B) > 0 \\ 0 & \text{for } Pl(B) = 0 \text{ and } A \neq B \\ 1 & \text{for } Pl(B) = 0 \text{ and } A = B \end{cases} \quad (11)$$

In other words, it flows down a portion of $m_I(B)$ to $A \cap B$, making the revision result a “specialization”⁵.

⁵Specialization was introduced in [10], m specializes m' iff there exists a square matrix Σ with general term $\sigma(A, B)$ being a proportion (i.e., verifying $\sum_A \sigma(A, B) = 1$, for any B . $\sigma(A, B) >$

Example 2 (continued): An example of specialization matrix is depicted at the bottom of Table 1, it is the one of the mass m representing the Ellsberg's paradox (Example 2) which is used to revise m by the certain information C giving $m_{||C}$. Table 1 also shows the result of the revision of the mass m by a simple mass function m_I and by a more complex one $m_{I'}$, in column 6 and 8 respectively.

2.4 Synthesis: the three ways to represent negative beliefs

There are several ways to express a negative belief about an information represented by a set of worlds B ,

- either the agent has some beliefs about the world and also believes that the real world is not in B , it means that his beliefs are combined with the complementary set of B (denoted \bar{B}).
- either the agent learns that he should believe that the real world is not in B hence that he should *revise* his beliefs by \bar{B}
- either the agent does not want to believe in B anymore, he wants to delete any belief in B , it is called a *retraction* by B

Note that revision is a stronger change than retraction since it can be viewed as a retraction of B to which \bar{B} is added while in retraction the agent may remain ignorant about \bar{B} . Indeed believing in the complementary of B is different from not believing in B .

Table 2 presents an abstract example with the three different approaches about negative information: conjunctive combination with the complementary (column 4), revision by the complementary (column 5) and retraction (last column). In this example, we have information about the beliefs of an agent on three focal sets A , B and $A \cap B$ materialized by a consistent mass function m . In the three cases we assume that there is a level of uncertainty u (or doubt) about B , translated into:

- \bar{B} is believed with $1 - u$ for the complementary approach
- \bar{B} should be believed with $1 - u$ for the revision approach,
- B should be deleted with a strength u , i.e., believed with $1/u$ with the retraction approach.

0 implies $A \subseteq B$ for any A, B such that $m(A) = \sum_B \sigma(A, B)m'(B)$ for all A . In [23], the definition of specialization matrix is taken in a broader sense: only imposing that $\sigma(A, B) > 0$ implies $A \cap B \neq \emptyset$ for any A, B .

It is important to note that retraction of the focal set B differs (last column) from conjunctive combination with the complementary of this focal set \bar{B} (fourth column). Indeed retraction allows us to focus and reduce or delete the mass on B (it is possible to find values for x, y, z and u such that $m \circledast B^u(B) = 0$ with $y, u \neq 0$) while negation never allows us to cancel beliefs on B (since $yu \neq 0$ as soon as $y, u \neq 0$). In other words, integrating a piece of evidence on the complementary of B (\bar{B}^u) is different from canceling an evidence for B (by integrating $B^{1/u}$). Revision gives priority to the new piece of information, hence revising by \bar{B} amounts to transfer pieces of evidence from A, B and $A \cap B$ to $A \setminus B$ (since $A \setminus B \subset \bar{B}$).

	m	\bar{B}^u	Negation $m \circledcirc \bar{B}^u$	Revision $m \circ \bar{B}^u$	Retraction $m \circledast B^u = m \circledcirc B^{1/u}$
\emptyset	0	0	$(y+z)(1-u)$	0	0
A	x	0	xu	xu	x/u
B	y	0	yu	yu	$1-x-z-$ $(1-x-y-z)/u$
$A \cap B$	z	0	zu	zu	$x+z-x/u$
$A \setminus B$	0	0	$x(1-u)$	$1-u$	0
\bar{B}	0	$1-u$	$(1-x-y-z)(1-u)$	0	0
Ω	$1-x$ $-y-z$	u	$(1-x-y-z)u$	$(1-x-y-z)u$	$(1-x-y-z)/u$

Table 2: Three different views of “negative information”: negation, revision and retraction

In order to formalize prejudices, our approach takes a complementary point of view relatively to revision and retraction since it allows us to make evolve the beliefs either by transfer or by attenuation. The important difference is the introduction of a new dimension for prejudices, enabling us to distinguish them from negative evidence and treat their evolution independently.

3 Formalizing prejudices

In this section, we propose to model prejudices against a piece of evidence. We propose to define what happens in the situation where the receiver already has some prejudices and some beliefs, the prejudice being characterized by a strength (which both influences its tenacity and how new information is integrated). We

study how new incoming information can modify both the levels of prejudice and beliefs.

3.1 Desirable properties for prejudices wrt incoming information

We do not address the case of the integration of a new prejudice or the reinforcement of an existing one. More precisely the incoming information is only a testimony that can reduce a prejudice (or not affect it at all) but cannot increase nor create one. The *creation* of prejudice is left for further studies. It leads to enunciate the first axiom about prejudices:

Axiom p1 *The prejudice against a piece of information cannot increase with the arrival of a new evidence.*

Now, it seems natural to conserve the prejudice against a piece of information when there is no new information about it, which leads to the second axiom about prejudices:

Axiom p2 *When there is no evidence in favor of a piece of information, the prejudice against it is not disturbed.*

Incoming information cannot be more trusted by the fact that there is a prejudice against it (since confidence can either decrease or remain at this level), which leads to a first axiom about beliefs.

Axiom m1 *The level of confidence in an incoming piece of information can only decrease if there is already a prejudice against it.*

A similar axiom as **Axiom p2** can be stated concerning beliefs:

Axiom m2 *When there is no prejudice against a piece of information, the arrival of new evidence about it should maintain its confidence level unchanged.*

For beliefs and prejudices of same strengths about the same piece of information, *we take the position* that a prejudice prevails over a belief. Hence the prejudice annihilates the confidence, as expressed by the following axiom:

Axiom m3 *When the strength of a prejudice against a piece of information is equal to the confidence level about the same incoming information, the confidence on it is cancelled.*

3.2 A framework for encoding a belief state with prejudices and beliefs

In order to both encode prejudice and beliefs, we propose to consider the pair (p, m) where m is a mass function and p is a prejudice against some set A . Intuitively, for all set A , with $\emptyset \subset A \subset \Omega$, $p(A)$ represents the threshold of evidence required to change one's mind about A :

- $p(A) = 0$ indicates the absence of prejudice against A
- $p(A) = 1$ means an unshakable prejudice against A .

Definition 1 (belief state) A belief state is a pair (p, m) where p , representing a prejudice against some piece(s) of information, is a prejudice function $p : 2^\Omega \setminus \{\Omega, \emptyset\} \rightarrow [0, 1]$ and m is a mass function on the same universe Ω . p is extended to Ω and \emptyset by setting $p(\Omega) = 1 - \sum_{X \subset \Omega} p(X)$ and $p(\emptyset) = 0$ (normalization).

Remark 1 Note that we impose that there is no prejudice on the empty set (because we do not see any intuitive meaning for this case). Moreover, we impose that the sum of the prejudices is 1 in order to have a commensurable scale with masses as evoked at the end of introduction.

In contrast with the Latent Belief Structure introduced by Smets [38] where a mass is decomposed into a pair of separable mass functions called respectively the confidence and diffidence, m can be any mass function (i.e., not necessarily separable) and p has not the same semantic as a mass. The aim of our study is to propose a framework in which the evolution of prejudices and beliefs are described through the arrival of new evidences.

We are now in position to express more formally the five axioms governing the evolution of a belief state (p, m_0) , where we suppose that p is a combination of prejudices against several subsets $A_i \subseteq \Omega$, m_0 being any mass function on Ω representing the initial beliefs of an agent, and we assume that m_1 is an incoming evidence. This evidence is taken into account by setting $m = m_0 \odot m_1$. m being a mass on Ω , it admits at least one focal set B_j on which the agent has a confidence, say β_j (i.e., $m(B_j) = \beta_j$). In the following formal version of the six axioms, we denote by p' and m' , the prejudice and the mass of the new belief state (p', m') resulting from the evolution of the belief state (p, m_0) with the arrival of m_1 . The axioms should hold for any focal set B_j and any subset A_i concerned by the prejudice p .

Axiom p1 $p'(A_i \cap B_j) \leq p(A_i \cap B_j)$: the prejudice against $A_i \cap B_j$ cannot increase.

Axiom p2 If $\beta_j = 0$ and $A_i \cap B_j \neq \emptyset$ then $p'(A_i) = p(A_i)$ (the prejudice against A_i is not disturbed)

Axiom m1 $m'(A_i \cap B_j) \leq m(A_i \cap B_j)$ (the confidence on $A_i \cap B_j$ decreases or remains)

Axiom m2 When $p(A_i \cap B_j) = 0$ then $m'(B_j) = m(B_j)$ (the beliefs are not disturbed)

Axiom m3 When $p(A_i \cap B_j) = m(A_i \cap B_j)$, then $m'(A_i \cap B_j) = 0$ (the confidence on $A_i \cap B_j$ is annihilated)

Remark 1 *From axioms p2 and m2, we get that if, for any focal set B_j and any subset A_i concerned by the prejudice p , $A_i \cap B_j = \emptyset$ then $p' = p$ and $m' = m$: the prejudice and the confidence remain the same.*

In the following section (Section 4.1), we are going to study the case where there is only one focal set for the beliefs (called A) which is also the set on which the prejudice is focusing, while in Section 4.2 there is evidence on a set called B , and one prejudice that focuses on a set (called A). Section 4.3 deals with complex masses and prejudices.

4 Handling the evolution of a belief state

In this paper, we propose to use a global tenacity function, denoted f , that describes how prejudice thresholds evolve when a new discordant information arrives. This tenacity function is decreasing or remaining constant (Axiom m1), because in this paper we restrict our study to the case where no piece of evidence can increase a prejudice. We also propose to use a function g that governs the evolution of the confidence in the incoming information.

4.1 Simple belief state with Beliefs and Prejudice on the same focal set

The simplest situation occurs when the prejudice and the evidence (testimony) are concerning the same information.

Definition 2 (evolving belief state) A simple belief state about $A \in 2^\Omega \setminus \{\emptyset, \Omega\}$ is denoted by $(p = {}_f^\alpha A, m = A_g^{1-\beta})$ with $\alpha, \beta \in [0, 1]$, where p is called simple prejudice against A of strength α , m is a simple mass function on A with confidence β , and f and g are functions from $[0, 1] \times [0, 1]$ to $[0, 1]$ that governs respectively the evolution of the prejudice and mass according to the following rules:

- $\alpha < \beta$: the prejudice is canceled or reduced and the confidence in A decreases according to g , denoted $({}_{f^{(\alpha, \beta)}} A, A_g^{g(\alpha, \beta)})$,
- $\alpha \geq \beta$ the prejudice decreases or remains but the informative mass $m(A)$ is deleted : the pair becomes $({}_{f^{(\alpha, \beta)}} A, A_g^1)$.

with f and g being functions that are conform to axioms **p1**, **p2**, **m1**, **m2** and **m3**.

In other words, in the first case of Definition 2, when the evidence is sufficiently convincing, the prejudice is deleted, i.e., the threshold of persuasiveness required to change one's mind is overtaken, the prejudice of strength α was low compared to the strength of the evidence β . In the second case of this definition, we consider that a strong prejudice cancels all information about the focal set: the prejudice remains but is possibly attenuated while the evidence is rejected. More generally, the attenuation of the prejudice depends on its evolving nature, represented by the function f dependent of α and β . Note that f the characteristic function for the prejudice evolution is not dual to the function g that governs the evolution of the confidence since it concerns a different kind of information, f being more at the meta level.

Let us come back to the conformity (required by Definition 2) of the functions f and g with the axioms. First, in order to define the function f that governs the evolution of the prejudice against A from ${}_f^x A$ towards ${}_{f^{(x, y)}} A$, where x and y are variables in $[0, 1]$ that will represent the respective strengths of the prejudice and the evidence, we are looking for a function f s.t. $\forall x, y \in [0, 1]$,

$$\begin{cases} f(x, y) \leq x & \text{(Axiom p1)} \\ f(x, 0) = x & \text{(Axiom p2)} \end{cases}$$

It means that f is not much constrained, several functions can be used. Here are some examples of special cases for f :

- $f_1(x, y) = \max(0, x - \varepsilon)$ decreases by $\varepsilon \in [0, 1]$ (independently of y value) after receiving each new evidence.

- $f_2(x, y) = \max(0, x - y)$ decreases in function of the strength of the certainty on the piece of information, at most the prejudice is removed.
- $f_3(x, y) = x$ leads to the conservation of the prejudice whatever the strength of the piece of information in the case of a narrow-minded person.

In order to define the function g that governs the evolution of the confidence in A from A^{1-y} towards $A^{g(x,y)}$, we are looking for a function g such that:

$$\forall x, y \in [0, 1] \text{ with } x \leq y, \begin{cases} g(x, y) \geq 1 - y & \text{(Axiom m1)} \\ g(0, y) = 1 - y & \text{(Axiom m2)} \\ g(x, x) = 1 & \text{(Axiom m3)} \end{cases}$$

In the following we propose to always use the function g defined by:

$$g(x, y) = \begin{cases} \frac{1-y}{1-x} & \text{for any } x, y \text{ with } x \neq 1 \\ 1 - y & \text{when } x = 1 \end{cases}$$

which is a solution to the three constraints induced by **Axioms m1, m2 and m3**. This means that g will be omitted in the following.

Due to the assumption taken for this paper (that prejudices cannot be created or be increased by the arrival of new evidences): if there is no prejudice against A then whatever the new piece of evidence that may arrive about A , it cannot create a new prejudice on A . This amounts to having a simple mass function as shown in the following proposition:

Proposition 1 *The pair $({}_f^0A, A^{1-\beta})$ is equivalent to the simple mass function $m = A^{1-\beta}$ with no prejudice.*

Proof: Here $\alpha = 0$, hence $\beta \geq \alpha$, according to Definition 2, the pair becomes $({}_f^0A, A^{1-\beta})$, since either $\beta \neq 0$ and we use the first item, or $\beta = 0$ then we use the second item and the fact that $f(0, 0) = 0$. \square

Remark 2 *When the listener has a great capacity of reconsideration, e.g., $f(x, y) = \max(0, x - \varepsilon)$, with $\varepsilon = 1$, it means that with a high confidence β on the focal set A and $\beta > \alpha$, the new incoming information will delete the prejudice and will lower the confidence leading to a belief state reduced to a simple mass (the one obtained by revising the initial mass $1 - \beta$ with $1/(1 - \alpha)$): the pair $({}_{f^{(\alpha, \beta)}}^fA, A^{1-\beta})$ is equivalent to a revision of the simple mass function $m = A^{1-\beta}$*

E	$p = {}_{f_2}^{0.2}A$	$p_n = {}_{f_3}^1A$	m_a	m_b	(p', m'_a) $\alpha < \beta$	(p', m'_b) $\alpha \geq \beta$	(p'_n, m'_{na}) $\alpha \geq \beta$
$E \in 2^\Omega \setminus \{\emptyset, A, \Omega\}$	0	0	0	0	(0, 0)	(0,0)	(0,0)
$A=\{1, 3\}$	0.2	1	0.4	0.1	(0,0.25)	(0.1,0)	(1,0)
Ω	0.8	0	0.6	0.9	(1,0.75)	(0.9,1)	(0,1)

Table 3: Three different evolutions of belief states

by the simple mass function $A^{\frac{1}{1-\alpha}}$ yielding a simple mass m' such that $m' = A^{1-\beta} \circ A^{\frac{1}{1-\alpha}} = A^{\frac{1-\beta}{1-\alpha}}$. This is due to Equation (10), $A^{1-\beta} \circ A^{\frac{1}{1-\alpha}}(\Omega) = \sigma(\Omega, \Omega) \frac{1}{1-\alpha}$ and $\sigma(\Omega, \Omega) = \frac{m(\Omega)}{Pl(\Omega)} = 1 - \beta$.

In the following example we illustrate Definition 2 with a universe of three worlds and two prejudices of different strengths applied on two simple mass functions. They all focus on the same subsets of worlds.

Example 3 Let us consider a universe $\Omega = \{\omega_1, \omega_2, \omega_3\}$ where three situations are possible. The columns p and p_n of Table 3 show two prejudices: p is a prejudice against $\{\omega_1, \omega_3\}$: $p = {}_{f_2}^{0.2}\{\omega_1, \omega_3\}$, p_n represents the prejudice of a narrow-minded agent against these two same situations: $p_n = {}_{f_3}^1\{\omega_1, \omega_3\}$. Three belief states are considered (p, m_a) , (p, m_b) and (p_n, m_a) where the beliefs are $m_a = \{\omega_1, \omega_3\}^{0.6}$ and $m_b = \{\omega_1, \omega_3\}^{0.9}$.

The column (p', m'_a) shows the results of the conjunctive combination of m_a and p as described in Definition 2, i.e., the prejudice disappear and the mass on A decreases: $p'(\{\omega_1, \omega_3\}) = 0$ and $m'_a(\{\omega_1, \omega_3\}) = 1 - \frac{1-\beta}{1-\alpha} = 1 - 0.4/0.8 = 0.25$. In the case where the beliefs are described by $m_b = \{\omega_1, \omega_3\}^{0.9}$ then the prejudice decreases with f_2 and the evidences are canceled (column (p', m'_b)). The last column (p'_n, m'_{na}) shows the resulting belief state when receiving information m_a with the prejudice p_n .

4.2 Simple belief state with beliefs and prejudice on two different sets

Let us consider two sets A and B that are not included in each other (more precisely such that $A \neq \Omega$, $B \neq \Omega$ with $(A \setminus B) \neq \emptyset$ and $(B \setminus A) \neq \emptyset$). Assume that there is a prejudice ${}_{f_2}^\alpha A$ (prejudice against A of strength α with evolving function f) and an evidence $m = B^{1-\beta}$ (in favor of B with strength β).

Definition 3 (simplification with no transfer) Given a belief state (p, m) such that $p = \frac{\alpha}{f}A$ and $m = B^{1-\beta}$ with $A, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$, this state is simplified into (p', m') such that:

If $A \cap B = \emptyset$ then no change: $(p', m') = (p, m)$

Else first compute (p^0, m^0) and then renormalize:

- $\alpha < \beta$ (weak prejudice): the prejudice and the mass decreases
 - $p^0(A) = f(\alpha, \beta)$, $m^0(B) = 1 - \frac{1-\beta}{1-\alpha}$
- $\alpha \geq \beta$ (strong prejudice): the prejudice decreases and the mass is canceled
 - $p^0(A) = f(\alpha, \beta)$, $m^0(B) = 0$
- Normalization: $p'(\Omega) = 1 - p^0(A)$, $m'(\Omega) = 1 - m^0(B)$

The above definition is rather drastic, as soon as there is an intersection between prejudice and belief, the prejudice decreases or remains on the whole set A initially concerned by the prejudice and the beliefs are canceled or decreased on the whole initial focal set B . However, when $A \neq B$ provided that $B \setminus A \neq \emptyset$ and $A \setminus B \neq \emptyset$, there is no frontal contradiction between the beliefs and the prejudice, it means that information can be transferred to $B \setminus A$, and a prejudice can remain against $A \setminus B$. Concerning the part $A \cap B$, we consider that it should behave as seen in Definition 2. The following definition propose to transfer beliefs and prejudices towards the subsets that are not conflicting (i.e., not overlapping).

Definition 4 (simplification with transfer) Given a belief state (p, m) such that $p = \frac{\alpha}{f}A$ and $m = B^{1-\beta}$ with $A, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$, this state is simplified into (p', m') such that:

If $A \cap B = \emptyset$ then no change: $(p', m') = (p, m)$

Else:

- concerning prejudices: α remains or decrease to $f(\alpha, \beta)$. $f(\alpha, \beta)$ is then transferred to $A \setminus B$ and $A \cap B$ s.t.: $p'(A \setminus B) + p'(A \cap B) = f(\alpha, \beta)$ and $p'(\Omega) = 1 - f(\alpha, \beta)$ and $p'(\emptyset) = 0$
- concerning masses: we first define m^0 then we normalize
 - [only when $B \setminus A \neq \emptyset$] $m^0 = \text{transfer}(B, B \setminus A, m)$
 - $\alpha < \beta$ (weak prejudice): the mass decreases or remains

- * [only when $A \cap B \neq \emptyset$] $m^0(A \cap B) = 1 - \frac{1-\beta}{1-\alpha}$
- $\alpha \geq \beta$ (strong prejudice): the mass is canceled
- * [only when $A \cap B \neq \emptyset$] $m^0(A \cap B) = 0$
- [only when $B \subseteq A$] $m^0(\Omega) = 1 - m^0(A \cap B)$
- Renormalization: for all $E \subseteq \Omega$, $m'(E) = \frac{m^0(E)}{\sum_{F \subseteq \Omega} m^0(F)}$

Where $\text{transfer}(E, F, t)$ denotes the function t' from Ω to $[0, 1]$ obtained from t by setting first $t' = t$ then $t'(E) = 0$ and $t'(F) = t(E) + t(F)$.

Note that, in the above definition, we only impose that $p'(A \setminus B) + p'(A \cap B) = f(\alpha, \beta)$. Hence the precise distribution of the (potentially attenuated) strength of the prejudice, $f(\alpha, \beta)$, on the two subsets $A \cap B$ and $A \setminus B$ is not given. In order to be consistent with Definition 2, in the case $A = B$, it would be desirable to impose $f(\alpha, \beta)$ on $A \cap B$, and to transfer α to $A \setminus B$, but this would give a total amount greater than $f(\alpha, \beta)$. Now, to conserve the same total amount $f(\alpha, \beta)$ of attenuated prejudice about A , several decompositions could be used. The simplest one, that do not take into account the size of the sets $A \cap B$ and $A \setminus B$, is to set $p'(A \cap B) = \frac{f(\alpha, \beta)^2}{\alpha + f(\alpha, \beta)}$ and $p'(A \setminus B) = \frac{\alpha \cdot f(\alpha, \beta)}{\alpha + f(\alpha, \beta)}$. Indeed, the reader can check that $p'(A \setminus B) + p'(A \cap B) = f(\alpha, \beta)$ and that this corresponds to set $k = \frac{f(\alpha, \beta)}{\alpha + f(\alpha, \beta)}$ in the equation $k \cdot \alpha + k \cdot f(\alpha, \beta) = f(\alpha, \beta)$. This decomposition will be used in the following example.

As expected, prejudices and masses do not behave the same: in presence of a prejudice, evidences decrease or remain (and in parallel doubt increases or remain), the prejudice can be seen as a kind of meta-information that allows us to reduce the size of the subsets concerned by the evidence. Indeed with a prejudice against A and an evidence about B , evidence remains only on the smaller sets $B \setminus A$ and maybe on $A \cap B$. Concerning prejudices they can only decrease or remain (as assumed in all this article), hence the ‘‘prejudice about Ω ’’ can only increase.

Example 3 (continued): Let us consider a prejudice of strength α against a new overlapping subset $\{\omega_2, \omega_3\}$, encoded by $p = \frac{\alpha}{f}\{\omega_2, \omega_3\}$. The fourth columns in Table 4 shows the results of the simplification, given by Definition 4, of m_a and p with $\alpha = 0.2$, i.e., the prejudice disappears on ω_3 but is transferred on ω_2 ($p'(\{\omega_2\}) = \frac{\alpha \cdot f(\alpha, \beta)}{\alpha + f(\alpha, \beta)} = (0.2 \times 0.1) / (0.2 + 0.1) = 0.067$). The mass $m_a(\{\omega_1, \omega_3\} = 0.4$ is transferred to $\{\omega_1\}$ ($m^0(\{\omega_1\}) = 0.4$). Concerning $\{\omega_3\}$ (the intersection), $p'(\{\omega_3\}) = \frac{f(\alpha, \beta)^2}{\alpha + f(\alpha, \beta)} = 0.1^2 / 0.3 = 0.033$ and $m^0(\{\omega_3\}) = 1 - g(\alpha, \beta) =$

E	$p = \frac{\alpha}{f}A$	m_a	m_b	(p', m'_a) $\alpha = 0.2 < \beta = 0.4$ $f = \max(0, x - 0.1)$	(p', m'_b) $\alpha = 0.2 \geq \beta = 0.1$ $f = \max(0, x - 0.1)$	(p', m'_b) $\alpha = 1 \geq \beta$ $f = x$
$\{\emptyset, \{1, 2\}\}$	0	0	0	(0,0)	(0, 0)	(0,0)
$\{1\}$	0	0	0	(0,0.32)	(0,0.1)	(0,0.1)
$\{2\}$	0	0	0	(0.067,0)	(0.067,0)	(0.5,0)
$\{3\}$	0	0	0	(0.033,0.2)	(0.033,0)	(0.5,0)
$B=\{1, 3\}$	0	0.4	0.1	(0,0)	(0,0)	(0,0)
$A=\{2, 3\}$	α	0	0	(0,0)	(0,0)	(0,0)
Ω	$1 - \alpha$	0.6	0.9	(0.9,0.48)	(0.9,0.9)	(0,0.9)

Table 4: Simplification with transfer of prejudice and beliefs from A and B to $\{2\}$ and $\{1\}$ respectively and with attenuation or cancelation on $\{3\}$.

$1 - 0.6/0.8 = 0.25$). Then m' is obtained by renormalization, masses are divided by $0.4 + 0.25 + 0.6 = 1.25$. In the case where the belief is described by m_b , the prejudice is stronger than the beliefs, hence, depending on f , the prejudice against $\{\omega_3\}$ decreases (in column 5) or remains (in column 6) and is transferred to $\{\omega_2\}$ as in column 4. The evidence for $\{\omega_3\}$ is canceled but transferred to $\{\omega_1\}$ ($m'_b(\{\omega_1\}) = 0.1$). The last column presents the case of a narrow minded agent with $f(\alpha, \beta) = \alpha = 1$, hence $p'(\{\omega_2\}) = p'(\{\omega_3\}) = 1/2$. This may seem counter-intuitive since it is different from $p'(\{\omega_2, \omega_3\}) = 1$ and there is no reason why the strong prejudice should disappear on $\{\omega_2, \omega_3\}$.

As seen in the previous example, transfer may not be desirable in case of a strong prejudice, because in that case the intuition would lead to maintain the prejudice on the whole initial set, this leads to a more refined definition of the simplification process, as follows:

Definition 5 (simplification with weak prejudice transfer) Given a belief state (p, m) such that $p = \frac{\alpha}{f}A$ and $m = B^{1-\beta}$ with $A, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$, this state is simplified into (p', m') such that:

If $A \cap B = \emptyset$ then no change $(p', m') = (p, m)$

Else:

- concerning prejudices: the prejudice remains or decrease to $f(\alpha, \beta)$.
 - $\alpha < \beta$ (weak prejudice): $f(\alpha, \beta)$ is transferred to $A \setminus B$ and $A \cap B$:
 $p'(A \setminus B) + p'(A \cap B) = f(\alpha, \beta)$ and $p'(\Omega) = 1 - f(\alpha, \beta)$ and $p'(\emptyset) = 0$
 - $\alpha \geq \beta$ (strong prejudice): $p'(A) = f(\alpha, \beta)$, $p'(\Omega) = 1 - p'(A)$

- concerning masses: for all $E \subseteq \Omega$, $m'(E)$ is defined as in Definition 4

The previous definition is an example where there is a hybrid handling of the transfer of prejudices. Other solutions could be adopted namely sharing the prejudice $f(\alpha, \beta)$ on A and $A \setminus B$ and $A \cap B$ or no mass transfer in case of strong prejudice.

Example 3 (continued): Table 5 shows the results of the simplification with transfer of weak prejudice only, given by Definition 5. Column 4 is unchanged with respect to Table 4, Columns 5 and 6: the prejudice being stronger than beliefs, it remains or decreases on the whole set A according to f . In column 6, the prejudice behaves well (remaining unchanged) as what we expect from a narrow minded person.

E	$p = \frac{\alpha}{f}A$	m_a	m_b	(p', m'_a) $\alpha = 0.2 < \beta = 0.4$ $f = \max(0, x - 0.1)$	(p', m'_b) $\alpha = 0.2 \geq \beta = 0.1$ $f = \max(0, x - 0.1)$	(p', m'_b) $\alpha = 1 \geq \beta$ $f = x$
$\{\emptyset, \{1, 2\}\}$	0	0	0	(0,0)	(0, 0)	(0,0)
$\{1\}$	0	0	0	(0,0.32)	(0,0.1)	(0,0.1)
$\{2\}$	0	0	0	(0.0670,0)	(0,0)	(0,0)
$\{3\}$	0	0	0	(0.033,0.2)	(0,0)	(0,0)
$B=\{1, 3\}$	0	0.4	0.1	(0,0)	(0,0)	(0,0)
$A=\{2, 3\}$	α	0	0	(0,0)	(0.1,0)	(1,0)
Ω	$1 - \alpha$	0.6	0.9	(0.9,0.48)	(0.9,0.9)	(0,0.9)

Table 5: Simplification with transfer of beliefs and weak prejudice only.

From now on, we consider that prejudices and evidences are governed by Definition 5. In the two following propositions, we study the extreme cases where there is no prejudice or no evidence (Proposition 2). Note that in the case where there are some initial beliefs about a set B , Definition 5 imposes that even with a prejudice of strength 0 (i.e., no prejudice) against a set A , the masses on B are divided between two subsets of B : namely $A \cap B$ and $B \setminus A$. This could seem strange, one could prefer to have no change at all concerning B . However, we could consider that saying that the user has no prejudice against a set A is different from saying nothing. Indeed it means that the two subsets $A \cap B$ and $B \setminus A$ have some reason to be differentiated.

Proposition 2 When $\alpha = \beta = 0$ there is no change in the belief state after simplification. When there is no prejudice but a mass: $\alpha = 0$, we get that the initial belief state $(\frac{0}{f}A, B^{1-\beta})$, with $A, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$, is equivalent to the combination

of two simple masses $(A \cap B)^{1-\beta}$ and $(B \setminus A)^{1-\beta}$ (hence amounts to have the same global mass β on B). When there is no evidence: $\beta = 0$, we get that the initial belief state $(\overset{\alpha}{f}A, B^1)$ with $A, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$ is equivalent to the simple prejudice $\overset{\alpha}{f}A$.

Proof: When $\alpha = \beta = 0$, due to the constraint $f(x, 0) = x$, we get $f(\alpha, \beta) = 0$, hence for any set $E \subset \Omega$, $p'(E) = 0$ and $m'(E) = 0$: no change.

When $\alpha = 0 < \beta$, for any set $E \subset \Omega$, $p'(E) = 0$ and the mass is both transferred to $B \setminus A$ and decreased on $A \cap B$ resulting in $m'(B \setminus A) = \beta$ and $m'(A \cap B) = \beta$.

When $\beta = 0$ then $\alpha \geq \beta$, $m^0(A \cap B) = 0$ and $m(B \setminus A) = \beta = 0$, then for any non-empty set $E \subset \Omega$ we get $m^0(E) = 0$, hence $m'(E) = 0$. Concerning prejudices: $p'(A) = f(\alpha, 0) = \alpha$, hence we get the simple prejudice $\overset{\alpha}{f}A$. \square

The following proposition enables us to recover the simple case where prejudice and beliefs focus on the same subset ($A = B$).

Proposition 3 *Definition 5 is conform to Definition 2 when $A = B$.*

Proof: In that case $A \cap B = A$ and $A \setminus B = \emptyset$ hence $p'(A) = f(\alpha, \beta)$ and $p'(\Omega) = 1 - f(\alpha, \beta)$ in both cases (weak or strong prejudice). $m^0(B)$ is either set to 0 (strong prejudice) or to $1 - g(\alpha, \beta)$. Moreover, $A \setminus B = B \setminus A = \emptyset$, hence there is no transfer, the mass on Ω becomes $1 - m^0(A \cap B) = 1 - m^0(B)$. At the end, the renormalization takes place with no effect. Hence we get either $(\overset{f(\alpha, \beta)}{f}A, A_g^{g(\alpha, \beta)})$ when the prejudice is weak or $(\overset{f(\alpha, \beta)}{f}A, A_g^1)$ when the prejudice is strong. \square

Note that when $B \subseteq A$, the prejudice remains or decrease on the whole set A (when it is strong) and is distributed on B and $A \setminus B$ (when it is weak), the evidences either disappear or decrease on B according to the strength of the prejudice.

4.3 Discussion on beliefs and prejudice about several sets

In the previous subsections, we have demonstrated how to manage prejudices and evidences when considering only two sets. In this subsection, we discuss the more complex case i.e., in presence of many prejudices that may intersect, and evidence modeled by a mass with several focal sets.

4.3.1 Preliminary: Intersection of focal sets and decomposability in the theory of belief functions (without prejudice)

In this paper we propose two ways to enforce decomposability and thus to estimate the mass/prejudice on an intersection, namely the “veto approach” and the “cumulative approach”. More precisely, the veto approach takes the max of the values while the cumulative one combines them.

Definition 6 (veto/cumulative approach) *Let us consider a function v that attributes to two overlapping non-empty focal sets X_1, X_2 the respective values $v(X_1) = x_1, v(X_2) = x_2$ with $v(X_1 \cap X_2) = 0$*

- *The veto approach defines v_V on three disjoint focal sets $X_1 \setminus X_2, X_2 \setminus X_1, X_1 \cap X_2$:*
 - *if $X_1 \setminus X_2 \neq \emptyset$ then $v_V(X_1 \setminus X_2) = x_1$*
 - *if $X_2 \setminus X_1 \neq \emptyset$ then $v_V(X_2 \setminus X_1) = x_2$*
 - *$v_V(X_1 \cap X_2) = \max(x_1, x_2)$*
- *The cumulative approach defines v_C such that:*
 - *if $X_1 \setminus X_2 \neq \emptyset$ then $v_C(X_1 \setminus X_2) = x_1$*
 - *if $X_2 \setminus X_1 \neq \emptyset$ then $v_C(X_2 \setminus X_1) = x_2$*
 - *$v_C(X_1 \cap X_2) = \min(1, x_1 + x_2)$*

This definition will help us to handle the case where several prejudices or several masses should be taken into account simultaneously. This is a simple example of combination functions, other more or less complex approaches could be used.

4.3.2 Beliefs on two sets and prejudice against one set

Given a belief state (p, m_0) and an incoming evidence m_1 such that p is a prejudice against A with strength α and $m = m_0 \oplus m_1$ is a mass function attributing β_1 and β_2 to B_1 and B_2 with $A, B_1, B_2 \in 2^\Omega \setminus \{\emptyset, \Omega\}$: this state is simplified into (p', m') such that:

- When B_1 and B_2 are disjoint and the prejudice is intersecting B_1 : (Figure 2.a shows this situation where the prejudice is strong)

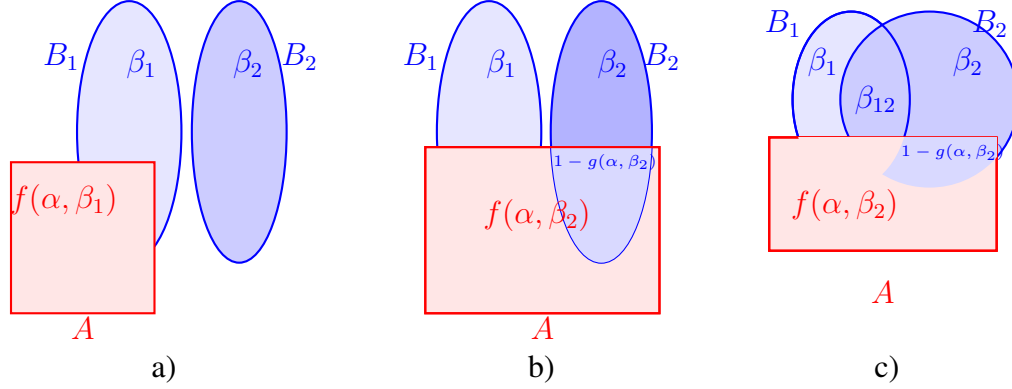


Figure 2: Two evidences on B_1 and B_2 with masses β_1 and β_2 , a simple prejudice $f^\alpha A$ with $\beta_1 < \alpha < \beta_2$. a) shows the case with disjoint masses and a prejudice only overlapping one, b) shows the prejudice overlapping both sets with a veto approach for masses. c) overlapping masses with $\beta_{12} < \alpha$ on $B_1 \cap B_2$.

- Concerning the prejudice, according to Definition 5, if $\alpha < \beta_1$ (weak prejudice), the strength $f(\alpha, \beta_1)$ is distributed to $A \cap B_1$ and $A \setminus B_1$ else it remains on the whole set A . Finally, $p'(\Omega)$ is set to $1 - f(\alpha, \beta_1)$.
- Concerning the masses, β_1 is transferred from B_1 to $B_1 \setminus A$, then $A \cap B_1$ either receives 0 or $1 - g(\alpha, \beta_1)$ according to the relative values of the prejudice and the mass. m^0 is then normalized in order to obtain a mass m' .
- When B_1 and B_2 are disjoint and the prejudice intersects both: (Figure 2.b shows this situation with the particular case $\beta_1 < \alpha < \beta_2$)
 - Concerning the prejudice, a veto approach would lead to select $\beta = \max(\beta_1, \beta_2)$ while a cumulative approach would lead to select $\beta = \min(1, \beta_1 + \beta_2)$, then if $\alpha < \beta$ (weak prejudice), the strength $f(\alpha, \beta)$ is distributed to $A \cap B_1$ and $A \cap B_2$ and $A \setminus (B_1 \cup B_2)$ else it remains on the whole set A . Finally, $p'(\Omega)$ is set to $1 - f(\alpha, \beta)$.
 - Concerning the masses, β_1 is transferred from B_1 to $B_1 \setminus A$, then $A \cap B_1$ either receives 0 or $1 - g(\alpha, \beta_1)$ according to the relative values of the prejudice and the mass, similarly for B_2 . m^0 is then normalized in order to obtain a mass m' .

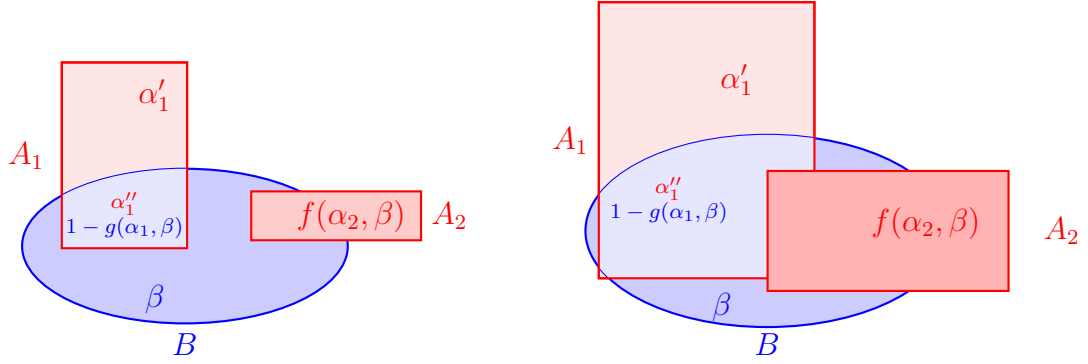


Figure 3: Two prejudices on A_1 and A_2 with strengths α_1 and α_2 , a simple mass $B^{1-\beta}$ with $\alpha_1 < \beta < \alpha_2$. The left side of the figure shows the case with disjoint prejudices, the right side shows overlapping prejudices with a veto approach. α_1' and α_1'' represent the distribution of $f(\alpha_1, \beta)$, i.e., such that $\alpha_1' + \alpha_1'' = f(\alpha_1, \beta)$.

- When B_1 and B_2 intersect, we assume that the mass on $B_1 \cap B_2$ is known and equal to β_{12} then the prejudice is decomposed on at most four disjoint focal sets $A \cap (B_1 \cap B_2)$, $A \cap (B_1 \setminus B_2)$, $A \cap (B_2 \setminus B_1)$ and $A \setminus (B_1 \cup B_2)$ by using either the Veto or the Cumulative approach, this is similar to the previous case where each disjoint focal set is dealt with independently (see Figure 3.c where a veto approach is used).
- When neither B_1 nor B_2 intersects with A then the prejudice and the masses keep their values (according to **Axioms p2** and **m2**).

4.3.3 Beliefs about one focal set and prejudices against two sets

Given a belief state (p, m_0) such that p is a prejudice against A_1 and A_2 with respective strength α_1 and α_2 and m_1 is an incoming evidence such that $m = m_0 \odot m_1 = B^{1-\beta}$ is a simple mass function with $A_1, A_2, B \in 2^\Omega \setminus \{\emptyset, \Omega\}$: this state is simplified into (p', m') such that:

- When A_1 and A_2 are disjoint: let us assume without loss of generality that $\alpha_1 \leq \alpha_2$, the left side of Figure 3 shows how prejudices (in red) and masses (in blue) are distributed on the different sets
 - Concerning prejudices, according to Definition 5, if $\alpha_1 < \beta$ (weak prejudice), the strength $f(\alpha_1, \beta)$ is distributed to $A_1 \cap B$ and $A_1 \setminus B$

else it remains on the whole set A_1 and similarly for A_2 . If α_2 is weak then $f(\alpha_2, \beta)$ is distributed to $A_2 \cap B$ and $A_2 \setminus B$ else it remains on A_2 . Finally, $p'(\Omega)$ is set to $1 - f(\alpha_1, \beta) - f(\alpha_2, \beta)$.

- Concerning the masses, β is transferred from B to $B \setminus (A_1 \cup A_2)$, then $A_1 \cap B$ (and $A_2 \cap B$ respectively) either receives 0 or $1 - g(\alpha_1, \beta)$ (respectively $1 - g(\alpha_2, \beta)$) according to the relative strengths of the prejudices and masses. The mass is then normalized in order to obtain m' .
- When A_1 and A_2 intersect: in a cumulative approach the prejudice is decomposed on at most three disjoint focal sets $A_1 \cap A_2$, $A_1 \setminus A_2$ and $A_2 \setminus A_1$ where the intersection receives an aggregation of the strength on A_1 and A_2 while the two remaining subsets keep their value, the three values are then normalized. The Veto approach results in at most two disjoint focal sets A_1 (the one with the highest strength) and $A_2 \setminus A_1$ (the second prejudice truncated but with its original strength). Then it amounts to come back to the previous case where each prejudice is dealt with independently (see the right side of Figure 3 where a veto approach is used).

In the general case, several prejudices can affect several subsets and the same subsets could be affected by several of them. The idea is to deal with each subset separately and to evaluate the aggregation of prejudices on each subset by using either a veto or cumulative approach and then reason with masses that are affected to them, see the following general example.

5 Applications

In this section, the introductory example is treated in its entirety, and three other practical applications are discussed.

5.1 The “Opinions and prejudices for an election” example

Let us come back to the example presented in the introduction. Tabular 6 presents the result obtained successively. The prejudices of the agent are represented in columns p_1 and p_2 , the first meeting of the two friends of the agent is given on Figure 4.a and column m_{12} . The belief state of the agent is described in column (p', m'_{12}) , a veto approach is first used to determine the strength of the

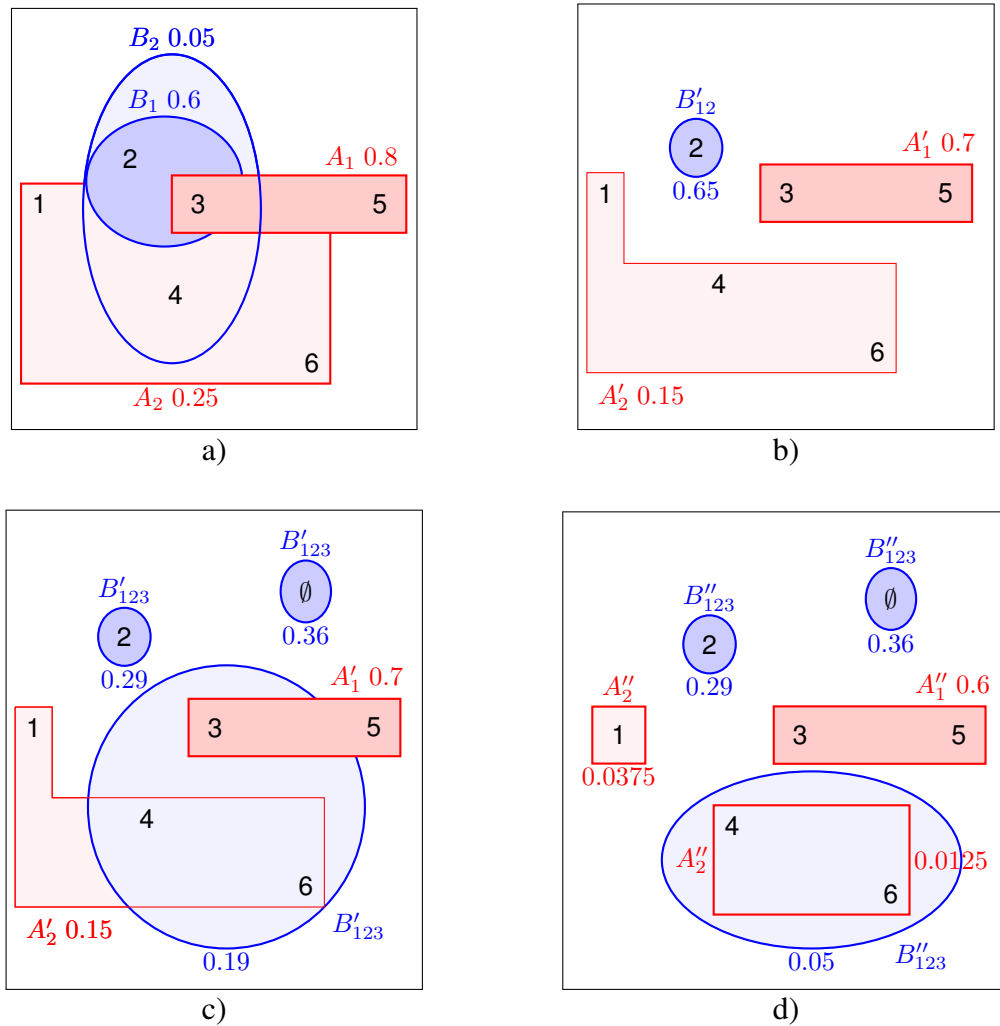


Figure 4: a) The agent with prejudices against A_1 and A_2 meeting his two friends with confidence in favor of B_1 and B_2 , b) the resulting belief state with a veto approach, c) the meeting of the third friend with a confidence of 0.55 in favor of B_3 which is aggregated to B'_{12} giving B'_{123} , d) final belief state

prejudice against $\{3\}$. The prejudice against A_1 is decreased (to 0.7 by using $f(x, y) = \max(0, x - 0.1)$) because of the evidences on the sets $\{2, 3\}$ and $\{2, 3, 4\}$ that intersect it. The prejudice on A_2 is transferred to $A'_2 = \{1, 4, 6\}$ then decreased to 0.15 since there are evidences on the set $\{2, 3, 4\}$ that intersects A'_2 . The belief state is then simplified into (p', m'_{12}) (fifth column of the table), the

E	p_1 $= \frac{0.8}{f} A_1$	p_2 $= \frac{0.3}{f} A_2$	m_{12}	(p', m'_{12}) $f = \max(0, x - 0.1)$	m_3	m_{123} $= m'_{12} \odot m_3$	(p'', m'_{123}) $f = \max(0, x - 0.1)$
other sets	0	0	0	(0,0)	0	0	(0,0)
$\{\emptyset\}$	0	0	0	(0,0)	0	0.36	(0,0.36)
$\{1\}$	0	0	0	(0,0)	0	0	(0.0375,0)
$\{2\}$	0	0	0	(0,0.65)	0	0.29	(0,0.29)
$\{3\}$	0	0	0	(0,0)	0	0	(0,0)
$\{4, 6\}$	0	0	0	(0,0)	0	0	(0.0125,0.05)
$B_1=\{2, 3\}$	0	0	0.6	(0,0)	0	0	(0,0)
$B_2=\{2, 3, 4\}$	0	0	0.05	(0,0)	0	0	(0,0)
$B_3=\{3, 4, 6\}$	0	0	0	(0,0)	0.55	0.19	(0,0)
$A_1=\{3, 5\}$	0.8	0	0	(0.7,0)	0	0	(0.6,0)
$A_2=\{1, 3, 4, 6\}$	0	0.25	0	(0,0)	0	0	(0,0)
$A'_2=\{1, 4, 6\}$	0	0	0	(0.15,0)	0	0	(0,0)
Ω	0.2	0.75	0.35	(0.15,0.35)	0.45	0.16	(0.35,0.3)

Table 6: Study of evolution of masses and prejudices in the example 1 described in introduction

masses on B_1 and B_2 are transferred to $\{2\}$ since the prejudices are stronger than evidences on 3 and 4, this is summarized in Figure 4.b. Then the new evidence m_3 arrives with a mass of 0.55 on $B_3 = \{3, 4, 6\}$, first the evidences m'_{12} and m_3 are aggregated into m_{123} described in the seventh column of the table and on Figure 4.c, we propose to use the conjunctive combination \odot . As the masses are inconsistent (no intersection of the focal sets B'_{12} and B_3), this combination creates a mass on \emptyset , which expresses a conflict of level 0.36. We could have chosen to use Demster-Rule in order to combine these masses but due to its normalization effect, the conflict would have been removed by this operation. Once the masses are aggregated, the simplification step of the belief state can take place in the last column of the table and in Figure 4.d. In this step, the prejudice on A'_1 decreased (to 0.6 by using $f(x, y) = \max(0, x - 0.1)$) because of the evidences on the set $\{3, 4, 6\}$ that intersect it. The prejudice A'_2 is both transferred to $\{1\}$ then decreased to $p''\{1\} = \frac{0.15f(0.15,0.19)}{0.15+f(0.15,0.19)} = \frac{0.15 \max(0,0.15-0.1)}{0.15+\max(0,0.15-0.1)} = \frac{0.15 \times 0.05}{0.15+0.05} = 0.0375$ and transferred to $B'_{123} \cap A'_2 = \{4, 6\}$ (as the evidence on $\{3, 4, 6\}$ intersects A'_2) then attenuated to $p''\{4, 6\} = \frac{f(0.15,0.19)^2}{0.15+f(0.15,0.19)} = \frac{0.05^2}{0.15+0.05} = 0.0125$. Therefore $f(0.15, 0.19) = 0.5$ is distributed in order to conserve the same total amount of attenuated prejudice A''_2 ($p''\{1\} + p''\{4, 6\} = 0.05$). Finally, the evidence on B'_{123} is transferred to $\{4, 6\}$ then decreased to $1 - \frac{1-0.19}{1-0.15} = 0.05$. The prejudice A'_3 that intersects B'_{123} with more strength ($0.7 > 0.19$) delete the evidence on $\{3\}$. The evidence on the subsets $\{2\}$ and \emptyset that intersects with no prejudice are not changed. The final belief state indicates to vote for candidate 2.

The constant flow of information leads to a lessening of the strength of preju-

dices, but also to their possible fragmentation. It is worth noticing that prejudices are not only lowering the beliefs about some subsets but also are entailing that evidence are gradually more focused, hence augmenting the informative content of the belief state. Concerning the emptyset, by definition there is no prejudice against it, however as seen in Figure 4.c) when information is contradictory, a mass can be attributed to it. This explains why the weight of the conflict (the mass on the emptyset) remains when prejudices are taken into account.

This example shows how the theory we propose can be put to practical use. More engineering or practical applications can be considered, in particular the ones that are already dealt with in the framework of Dempster-Shafer theory, this is evocated in the next section.

5.2 A look at other practical applications

A first application is about the fusion of sensor information in autonomous vehicles where evidential-based approaches are used [19, 15]. In the context of semi-autonomous vehicles, it would be interesting to combine these approaches with a model of the human agent that uses the vehicle, in order to take into account high-level information (like her preferences and prejudices) and to understand how the driver of the vehicle uses both sensor information and meta-information. For this purpose, our approach could enhance the expressivity of the existing works.

Another practical application using the Dempster-Shafer theory concerns image processing where the different color channels are considered as distinct sources of information that should be merged [11, 26, 4]. In this kind of works, images are labeled by humans, these humans may have prejudices that could be made explicit. Then our approach could be used to study how humans integrate the evidences coming from the different channels, into their own beliefs and prejudices, in order to ultimately know if the labels are correct or should be suspected.

A last practical application is in the domain of social choice (vote, social networks). In this domain, the classic objective is to merge the uncertainty of different agents in order to make a global decision, or to merge agents preferences in order to propose recommendations. Several works are using evidential theory for this purpose, e.g. [28] which models the preferences as mass functions that are combined with Dempster's rule of combination. Incorporating prejudices could be useful to better represent the cognitive states of agents and then better study the evolution of their social interactions.

6 Summary, related work, perspectives

In the framework of the Dempster-Shafer theory, this paper presents a way to formalize some cognitive mechanisms (essentially stereotypes). The main idea of our proposal is to deal with prejudices and evidences separately by using a pair representing a belief state. Prejudices are seen as a meta information that both evolves and governs the integration of new beliefs. We proposed five axioms governing the evolution of beliefs and prejudices in the presence of new evidence. In this study, an agent's prejudices are characterized by the targeted information and their strength against it (which also determines their tenacity). This work generalizes [8] and is consistent with the definition of simple mass evolution in presence of a prejudice. This generalization allows us to overcome the problem of negative mass handling due to the retraction process.

The revision operation is a classical topic broadly studied in knowledge representation literature [16, 1], and also in the particular context of belief functions by [7]. In the next paragraph we recall the approaches that deal with information deletion, namely retraction and updating. Indeed, Dempster-Shafer theory is made to add new pieces of evidence through Dempster combination rule, nevertheless the issue of deleting or modifying the agent's belief state when some evidence is invalidated or modified has deserved some attention.

In probability and possibility theory, the retraction operation consists of a division followed by a normalization. Since 1984, Ginsberg [12] proposed a special case of retraction applied to belief functions in the simple case of a frame of discernment with only two elements. In the valuation-based system framework, Shenoy [36] defined removal as point-wise division followed by normalization (if normalization is possible). Kramosil [17] generalized the notion of belief functions with basic signed measure assignment (BSMA) and proposed an operation inverse to Dempster's rule. He introduced the notion of q -invertibility that may be seen as generalizing non-dogmaticism. Pichon [29] pursued Kramosil's seminal work by defining the so-called *conjunctive signed weight function*. But the absence of a semantic, the lack of intuitive interpretation of such generalized belief functions and the fact that only the conjunctive rule is used to combine BSMA's (normalization cannot be applied) are obstacles to the potential use of this approach. Smets [37] generalized the concept of simple support function, allowing the diffidence values to range on the positive reals and introduced the retraction operation defined by the division of commonality functions. Smets defined then the concept of *latent belief structure* for non dogmatic mass functions. This concept is studied in more

details by Pichon and Denoeux [30]. Lukaszewski [22] proposes an algorithm for what he calls *updating* which consists in removing or changing some pieces of evidence without carrying out all the combinations again except for the ones that have been deleted or modified. Dubois, Faux and Prade [8] consider retraction as a special symmetric belief change operation that avoids the explicit use of negative mass functions.

Our approach takes a complementary point of view relatively to revision and retraction since it allows us to make evolve the beliefs either by transfer or by attenuation. The important difference is the introduction of a new dimension for prejudices allowing us to distinguish them from negative evidence and to handle their evolution independently. Indeed prejudices involve a meta-information about the anchoring of the beliefs which is different from the certainty level of these beliefs. Note that in order to be able to combine new information with anchored beliefs (i.e., “prejudices”), we make the strong assumption of comparability (which is debatable but allowed us to handle simply practical cases). Nevertheless at the end, when the new information has been taken into account, the new belief state still differentiates prejudices and beliefs.

An extension of this work would be to translate our study with a qualitative point of view, by considering qualitative prejudices and evidences (as it is done for qualitative masses in [9]). An aspect that has not been mentioned much is that: prejudice could be viewed as a kind of defeasible information, indeed a prejudice is a deliberate concealment of exceptions for reasons of (self) persuasion while a default is a temporary concealment through non-monotonic reasoning (see e.g. default logic [31] or systems with default rules [13] and their possibilistic counterpart [3]). Moreover, an important application of our study could be its use in the context of decision theory, where in addition to uncertainty and utilities we could add prejudices. A more long term perspective of this paper would be to study the combination of two belief states that have both prejudices and beliefs and that influence each other. For instance, we could imagine to combine the belief state of a social group (treated as a meta-agent) with the one of an individual agent that is confronted with the group’s opinion. This kind of combination was considered as out of the scope of the current study where we reason only from the point of view of one agent who receives an information, indeed the transmission of prejudice seems for the moment a much more complex subject to formalize.

Let us conclude that prejudices deserve more studies because they are both *essential* for reasoning quickly and *dangerous* for jumping too blindly to conclusions, so incorporating them into the modeling of an agent’s reasoning can provide

tools to make people aware of this type of bias and help gain better control over them.

References

- [1] Carlos E Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *The journal of symbolic logic*, 50(2):510–530, 1985.
- [2] Gordon Willard Allport, Kenneth Clark, and Thomas Pettigrew. *The nature of prejudice*. Addison-wesley Reading, MA, 1954.
- [3] Salem Benferhat, Didier Dubois, and Henri Prade. Nonmonotonic reasoning, conditional objects and possibility theory. *Artificial Intelligence*, 92(1-2):259–276, 1997.
- [4] Isabelle Bloch. Defining belief functions using mathematical morphology—application to image fusion under imprecision. *International journal of approximate reasoning*, 48(2):437–465, 2008.
- [5] Arthur P Dempster. A generalization of bayesian inference. *Journal of the Royal Statistical Society: Series B (Methodological)*, 30(2):205–232, 1968.
- [6] Thierry Denœux. Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence. *Artificial Intelligence*, 172(2-3):234–264, 2008.
- [7] Didier Dubois and Thierry Denœux. Conditioning in Dempster-Shafer theory: prediction vs. revision. In *Belief Functions: Theory and Applications: Proceedings of the 2nd International Conference on Belief Functions, Compiègne, France 9-11 May 2012*, pages 385–392. Springer, 2012.
- [8] Didier Dubois, Francis Faux, and Henri Prade. Prejudice in uncertain information merging: Pushing the fusion paradigm of evidence theory further. *International Journal of Approximate Reasoning*, 121:1–22, 2020.
- [9] Didier Dubois, Francis Faux, Henri Prade, and Agnès Rico. Qualitative capacities: Basic notions and potential applications. *International Journal of Approximate Reasoning*, 148:253–290, 2022.

- [10] Didier Dubois and Henri Prade. On the unicity of Dempster rule of combination. *International Journal of Intelligent Systems*, 1(2):133–142, 1986.
- [11] Francis Faux and Franck Luthon. Theory of evidence for face detection and tracking. *International Journal of Approximate Reasoning*, 53(5):728–746, 2012.
- [12] Matthew L. Ginsberg. Non-monotonic reasoning using Dempster’s rule. In *Proc. Nat. Conf. on Artificial Intelligence. Austin, TX, August 6-10, 1984.*, pages 126–129, 1984.
- [13] Moisés Goldszmidt and Judea Pearl. Qualitative probabilities for default reasoning, belief revision, and causal modeling. *Artificial Intelligence*, 84(1-2):57–112, 1996.
- [14] James L Hilton and William Von Hippel. Stereotypes. *Annual review of psychology*, 47(1):237–271, 1996.
- [15] Shriram C Jugade and Alessandro C Victorino. Grid based estimation of decision uncertainty of autonomous driving systems using belief function theory. *IFAC-PapersOnLine*, 51(9):261–266, 2018.
- [16] Hirofumi Katsuno and Alberto O Mendelzon. Propositional knowledge base revision and minimal change. *Artificial Intelligence*, 52(3):263–294, 1991.
- [17] Ivan Kramosil. Measure-theoretic approach to the inversion problem for belief functions. *Fuzzy Sets and Systems*, 102(3):363 – 369, 1999.
- [18] Ivan Kramosil. *Probabilistic Analysis of Belief Functions*. Kluwer, New York, 2001.
- [19] Marek Kurdej, Julien Moras, Véronique Cherfaoui, and Philippe Bonnifait. Controlling remanence in evidential grids using geodata for dynamic scene perception. *International Journal of Approximate Reasoning*, 55(1, Part 3):355–375, 2014.
- [20] Jacques-Philippe Leyens, Vincent Yzerbyt, and Georges Schadrin. *Stereotypes and social cognition*. Sage Publications, Inc, 1994.
- [21] Walter Lippmann. Public opinion. URL: http://infomotions.com/etexts/gutenberg/dirs/etext04/pbp_nn10.htm, 1921.

- [22] Tomasz Lukaszewski. Updating the evidence in the Dempster-Shafer theory. *Informatica, Lith. Acad. Sci.*, 10:127–141, 01 1999.
- [23] Jianbing Ma, Weiru Liu, Didier Dubois, and Henri Prade. Revision rules in the theory of evidence. In *2010 22nd IEEE International Conference on Tools with Artificial Intelligence*, volume 1, pages 295–302. IEEE, 2010.
- [24] Douglas Martin, Sheila J Cunningham, Jacqui Hutchison, Gillian Slessor, and Kenny Smith. How societal stereotypes might form and evolve via cumulative cultural evolution. *Social and Personality Psychology Compass*, 11(9):e12338, 2017.
- [25] Douglas Martin, Jacqui Hutchison, Gillian Slessor, James Urquhart, Sheila J Cunningham, and Kenny Smith. The spontaneous formation of stereotypes via cumulative cultural evolution. *Psychological Science*, 25(9):1777–1786, 2014.
- [26] Pauline Minary, Frédéric Pichon, David Mercier, Eric Lefevre, and Benjamin Droit. Face pixel detection using evidential calibration and fusion. *Int. J. Approx. Reason.*, 91:202–215, 2017.
- [27] Todd D Nelson. *The psychology of prejudice*. Pearson Allyn and Bacon Boston, MA, 2006.
- [28] Van-Doan Nguyen and Van-Nam Huynh. Integrating with social network to enhance recommender system based-on dempster-shafer theory. In *Computational Social Networks: 5th International Conference, CSoNet 2016, Ho Chi Minh City, Vietnam, August 2-4, 2016, Proceedings 5*, pages 170–181. Springer, 2016.
- [29] Frédéric Pichon. *Belief functions: canonical decompositions and combination rules*. PhD thesis, Université de Technologie de Compiègne, 2009.
- [30] Frédéric Pichon and Thierry Denoeux. On latent belief structures. In *Symbolic and Quantitative Approaches to Reasoning with Uncertainty: 9th European Conference, ECSQARU 2007, Hammamet, Tunisia, October 31-November 2, 2007. Proceedings 9*, pages 368–380. Springer, 2007.
- [31] Raymond Reiter. A logic for default reasoning. *Artificial intelligence*, 13(1-2):81–132, 1980.

- [32] Milton Rokeach. The nature and meaning of dogmatism. 1954.
- [33] Milton Rokeach. The open and closed mind: Investigations into the nature of belief systems and personality systems. 1960.
- [34] Glenn Shafer. *A mathematical theory of evidence*. Princeton university press (reedition 2021), 1976.
- [35] Sachiyo M Shearman and Timothy R Levine. Dogmatism updated: A scale revision and validation. *Communication Quarterly*, 54(3):275–291, 2006.
- [36] Prakash P. Shenoy. Conditional independence in valuation-based systems. *International Journal of Approximate Reasoning*, 10(3):203–234, 1994.
- [37] Philippe Smets. The canonical decomposition of a weighted belief. In *Proc. 14th Int. Joint Conf. on Artificial Intelligence (IJCAI), Montreal, Aug. 20-25*, volume 2, pages 1896–1901, 1995.
- [38] Philippe Smets. The transferable belief model for quantified belief representation. In *Quantified Representation of Uncertainty and Imprecision*, pages 267–301. Springer, 1998.
- [39] Amos Tversky and Daniel Kahneman. Judgment under uncertainty: Heuristics and biases: Biases in judgments reveal some heuristics of thinking under uncertainty. *science*, 185(4157):1124–1131, 1974.