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A model for French voters

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Research about opinion dynamics is a thriving domain, that can help us understand pressing contemporary issues such as polarization. The theoretical behavior of the models is well understood, but confrontations with real data are scarce. We evaluate the *empirical reliability* of the well-known Voter Model. Its macroscopical properties were previously shown to match with vote-shares in U.S. elections. We take a fine-grained approach and investigate the reliability of the model on a microscopical level. We uncover a tight correspondence between the equilibrium state of the model and individual ground-truth opinions of users in a highly heterogeneous French Twitter dataset.

Introduction. Models of opinion dynamics provide potent tools to unveil mechanisms driving phenomena such as polarization [1–7], echo chambers [8, 9] or filter bubbles [10]. These models usually comprise a network of agents endowed with opinions that evolve through interactions with one another. Social, psychological and technological features may govern these interactions, *e.g.* negative influence [3, 6, 7, 11], stubbornness [12–14], bounded confidence [6, 15], or recommendation algorithms [5, 10, 16]. Exploring the effect of such features and combinations thereof, can help us understand how ideological landscapes are shaped in various circumstances. This is particularly useful in the current context of raising concerns about the role of online social platforms in political debates and democratic processes [17–19].

While classical models of opinion dynamics have been extensively studied from a theoretical point of view, it is not clear to what extent they are *empirically reliable*, that is, how faithfully they can describe the formation and evolution of opinions in real-life. Studies addressing this question are scarce, although they exhibit an increasing trend. Recurrent sources of data employed towards this goal include surveys [2, 20–22], election results [23–27], and more recently sets of interactions extracted from online social platforms [1, 4, 15, 28, 29]. The celebrated Voter Model has been the subject of several studies pertaining to empirical reliability. The seminal paper of Fernández-Gracia et al. [27] demonstrated the model’s ability to capture statistical features of votes shares in U.S. elections. These results were confirmed and enriched in subsequent works [23–26, 30]. We refer the interested reader to [31, 32] for in-depth reviews of the research about the empirical applications of opinion dynamics models.

The models are built to emulate global phenomena—*e.g.* polarization, echo chambers—and most do not pretend to truthfully describe individual processes of opinion formation. Naturally therefore, research about the empirical reliability of the models usually take a macroscopical point of view. For example, Valensise et al. [29] investigate the capability of four

different opinion dynamics models to reproduce distributions of opinions observed in various online social platforms. We take a step outside the intended scope of use for opinion dynamics models, as we evaluate the empirical reliability of the multi-dimensional Voter Model with zealots (VMZ) [12] on the microscopical level. That is, we investigate its ability to capture *individual* opinions in real-life cases.

We leverage a directed, weighted network of retweets between Twitter users, collected during the campaign of the 2017 French Presidential Election [33]. We uncover a strong correspondence between individual opinions in the equilibrium state of the VMZ and their real values. Users are clearly separated along party lines in the opinion space of the model, and the opinion identified as most likely by the model matches the ground-truth in 92.6% of cases. Discord probabilities let us distinguish with high accuracy between user pairs with the same ground-truth opinion and those who hold different views. The robustness of these results is confirmed through comparison with a baseline that considers solely direct connections with accounts of political entities. Moreover, neither an undirected, unweighted version of the retweet network, nor the follow and mention networks produce comparable results. Our findings highlight the necessity for a fine-grained modelling approach, and contribute to the growing literature on the empirical reliability of opinion dynamics models.

The #Elysée2017fr dataset. Fraisier et al. [33] collected retweets and mentions between $\sim 23,000$ accounts in the last six months leading to the 2017 French Presidential Election. Follow relationships between the accounts were collected in another study [34]. The accounts were selected based on the presence of political keywords in their tweets, and include about 2,000 non-individual political entities, *e.g.* official party accounts, activist groups, etc. On the basis of the content of their tweets and profile description, accounts were manually labelled by the creators of the dataset to indicate the party they supported in the election. Are considered the five main competing parties: FI (France Insoumise, far-left), PS (Parti Socialiste, traditional left), EM (En Marche, center), LR (Les Républicains, traditional right), FN (Front National, far-right). We call *ground-truth opinion* and denote by y_i the label of user i . There are a few unlabelled accounts and accounts with multiple labels, which we discard for the sake of simplicity

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(resp. $\sim 3,400$, ~ 800 accounts).

Retweet interactions induce a directed, weighted network such that the weight w_{ij} of the edge from j to i is defined as the number of times that i has retweeted j . If this number is positive, we say that j is a leader of i . To apply the VMZ, users are initially endowed with random opinions chosen uniformly in the discrete opinion space $\mathcal{S} = \{FI, PS, EM, LR, FN\}$. We treat accounts of political entities as *zealots*, as we fix their opinions to the ground-truth and never update it. We reserve the term *user* to refer to all the other accounts. The dynamics of the VMZ unfold as follows: each step, a user i is chosen at random and adopts the opinion of one of their leaders, chosen at random with probability w_{ij} for leader j [35].

A crucial aspect of our methodology is that the opinions of political entities are static, and set to match their ground-truth labels. Hence the term *zealots*, often used in the literature to refer to agents with immutable opinions [12, 14, 24, 36]. It seems natural to assume that party affiliations of political entities are immutable: the Parisian branch of FI for example, is and will always be—by definition—in support of FI. As these affiliations are public knowledge, it is not such a restrictive hypothesis to assume that they are known. This choice is also motivated by a technical reason: zealots guarantee the existence of a unique equilibrium state, at least for the regions of the network that they can reach [13, Thm. 2.1]. Otherwise, there may exist a multitude of different equilibrium states, the potential diversity of which entails a difficulty in their interpretation. Zealots also act as reference points, ensuring a unique correspondence between opinions in the model and ground-truth opinions.

We call supporters of party s all users with ground-truth opinion $y_i = s$. We let \mathcal{L}_i denote the set of leaders of i . We let z_i^s denote the total weight of edges from zealots with opinion s towards user i . The vector $z_i \in [0, 1]^5$ thus describes the distributions of direct connections between zealots and user i .

Equilibrium opinions. The system is driven towards a unique state of equilibrium, wherein the opinion of i fluctuates according to an individual probability distribution

$$x_i = (x_i^{FI}, x_i^{PS}, x_i^{EM}, x_i^{LR}, x_i^{FN}). \quad (1)$$

The entry x_i^s can be interpreted as the likeliness attributed by the model to the possibility that i supports party s . We proved in [12] that

$$x_i^s = \sum_{j \in \mathcal{L}_i} w_{ij} x_j^s + z_i^s. \quad (2)$$

The existence of a unique solution requires that every user can be reached by at least one zealot via a path in the network [13, Thm. 2.1], and we remove all the users who cannot be. We restrict ourselves to the largest weakly connected component of the user network [37]—all other connected components are of size one or two. After these filtering steps, the system is composed of 15,607 users and 1,842 zealots. We then compute the x_i^s values for the users. Remark that the ground-truth opinion y_i of i does not bear involvement in the computation of x_i^s —only the ground-truth opinions of zealots are used. This

justifies the following analysis, where we compare x_i and y_i in several fashions to assess the reliability of the VMZ.

Via the equilibrium opinion distributions x_i , the VMZ is able to identify supporters of each party with very high accuracy. The most likely opinion of i according to the model, i.e. $\operatorname{argmax}_{s \in \mathcal{S}} x_i^s$, matches the ground-truth y_i for 92.6% of users. Party-wise accuracy values are shown in Fig. 1a (plain bars). FN supporters are the easiest to identify (accuracy 0.95), while PS supporters are the hardest (accuracy 0.81). The worse performance of the VMZ for PS is a pattern that will repeat throughout our analysis. It might be due to the dire situation of the PS at the time, which was considerably weakened after five years of difficult Hollande presidency, and its voting base torn between the rise of FI on the left and EM on the right.

Not only does the VMZ attributes a higher probability to ground-truth opinions than to others, but the difference between the probabilities is quite large. For the supporters of each party s we confront, in Fig. 1b, the distributions of

$$x^s = \{x_i^s : i \in \mathcal{N}_s\}, \quad (3)$$

$$x^{-s} = \{x_i^t : i \in \mathcal{N}_s, t \neq s\}. \quad (4)$$

The two are concentrated as opposite sides of the unit interval. The former is located towards 1, with means ranging from $\langle x^s \rangle = 0.71$ for PS to $\langle x^s \rangle = 0.86$ for EM, while the latter is strongly skewed towards 0, with means ranging from $\langle x^{-s} \rangle = 0.03$ for EM to $\langle x^{-s} \rangle = 0.07$ for PS. Note that we find consistently low probabilities for x^{-s} (standard deviation ranging from 0.09 for FN supporters to 0.15 for PS), while the magnitude of x^s is much more varied (standard deviation between 0.16 for FN supporters and 0.31 for PS).

An interpretation of the striking difference between the distributions of x^s and x^{-s} , is that the supporters of each party occupy a specific region in the five-dimensional space that contains the x_i . Figures 1c shows that in this space, users are significantly closer to those who support the same party than to others. Furthest away are the supporters of EM and FN, and closest to one another are those of EM and PS. As an illustrative example, we show in Fig. 2 the bidimensional distributions of (x_i^{EM}, x_i^{PS}) for EM and PS supporters, and (x_i^{EM}, x_i^{FN}) for EM and FN supporters. In the latter case, the densities are concentrated at opposite sides of the unit square. The former case however presents a different picture, as supporters of PS populate a larger and more central area of the subspace (Fig. 2, top plots). To confirm these results, we find the best separating hyperplane between supporters of the different parties, by fitting a basic Support Vector Classifier (SVC) to the (x_i, y_i) pairs. The result is very accurate, as the SVC correctly estimates the opinion of 93% of the users. Party-wise accuracy scores are shown in Fig. 1a (hatched bars). In summary, through the individual equilibrium distributions x_i , the VMZ embeds users in a space where supporters of each parties are located in a specific region, separated from the others, and therefore clearly identifiable.

Discord. Individual equilibrium opinion distributions x_i describe the stabilization of the system at the user level. It is often useful to also describe stabilization at the level of edges, or more generally user pairs, via the notion of active links or

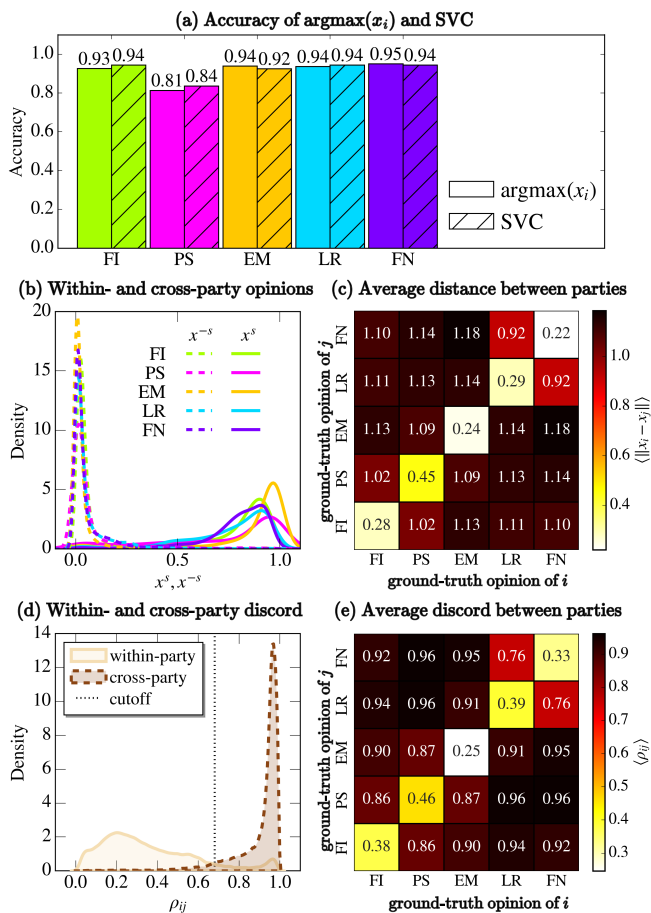


FIG. 1: Correspondence between theoretical opinion distribution x_i and ground-truth political leanings y_i . **(a)** Accuracy of $\text{argmax}(x_i)$ (plain bars) and SVC (hatched bars) for the retrieval of ground-truth opinions y_i from x_i . **(b)** Distribution of within-party (x^s) and cross-party (x^{-s}) opinions among supporters of each party s , distinguished by color. **(c)** Average euclidean distance between supporters of the same party (diagonal cells) and supporters of different parties (off-diagonal cells). **(d)** Distribution of discord probabilities ρ_{ij} between supporters of the same party (blue) and supporters of different parties (red, dashed). The vertical dotted line indicates the cutoff of the logistic regression model: if ρ_{ij} is on the left of it, the logistic regression predicts that i and j support the same party, otherwise it predicts that they support different parties. **(e)** Average discord probability between supporters of the same party (diagonal cells) and supporters of different parties (off-diagonal cells). Smoothed distributions are computed via kernel density estimation.

discord [12, 36, 38]. The discord probability ρ_{ij} is the probability to find i and j holding two different opinions. Under the same conditions as for x_i , this quantity converges towards

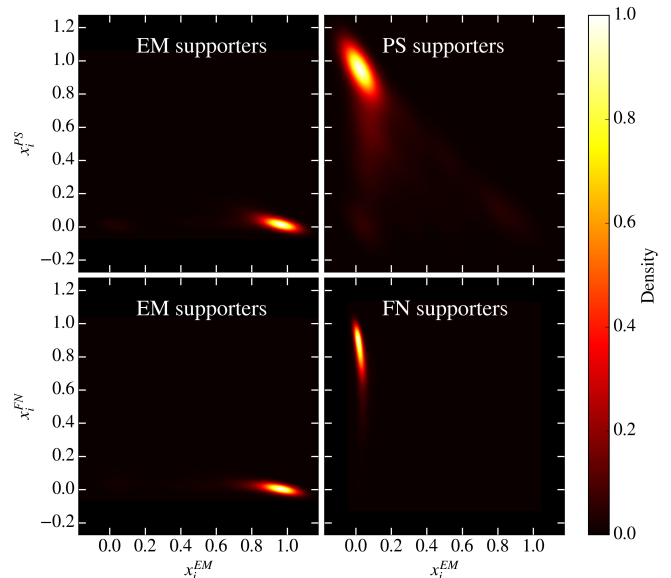


FIG. 2: **(Top)** Distributions of theoretical opinions towards EM and PS, among EM supporters (left) and PS supporters (right). **(Bottom)** Distributions of theoretical opinions towards EM and FN, among EM supporters (left) and FN supporters (right).

an equilibrium state, defined by

$$\rho_{ij} = \frac{1}{2} \left[\sum_{k \in \mathcal{L}_i} w_{ik} \rho_{jk} + \sum_{k \in \mathcal{L}_j} w_{jk} \rho_{ik} + \sum_{s \in \mathcal{S}} z_i^s (1 - x_j^s) + \sum_{s \in \mathcal{S}} z_j^s (1 - x_i^s) \right]. \quad (5)$$

Certain conditions of independence between the opinions of i and j allow to use simply $\rho_{ij} = \sum_{s \in \mathcal{S}} x_i^s (1 - x_j^s)$. The derivation of these formulas is detailed in ref. [12]. As long as the existence and unicity of the x_i vectors are guaranteed, the same goes for the ρ_{ij} values. The value ρ_{ij} can be interpreted as the likeliness attributed by the VMZ to the possibility that i and j do not support the same party.

Averages $\langle \rho_{ij} \rangle$ for each party pair are shown in Fig. 1e. EM supporters exhibit the lowest within-party discord, while PS exhibits the highest. The lowest cross-party discord is observed between LR and FN, the highest between PS and both LR, FN. In Fig. 1d, we plot separately the within-party and cross-party distributions of ρ_{ij} . The two differ greatly, the latter being located towards 1 and the former towards 0. Users with the same ground-truth opinion are much more likely to hold the same opinion in the VMZ than those who do not. In fact, a simple logistic regression model is able to discriminate between within-party and cross-party pairs, based on the values ρ_{ij} , with accuracy 0.93. This highlights the ability of the VMZ to reliably distinguish friends from foes. Because discord spans a narrower range cross-party than within-party, foes are easier to identify than friends (accuracy 0.934 versus 0.902).

Estimating opinions directly from zealots. The value of x_i^s corresponds to the probability that a backward random walk initiated in i hits a zealot with opinion s before a zealot with any other opinion [13, Prop. 3.2]. Therefore zealots—here political entities—play an crucial role in the equilibrium state of the VMZ. One may then wonder, are the encouraging results we just exposed a mere consequence of the fact that people are preferentially connected with political entities of the party they support? The question is particularly relevant, given that retweet networks are known to exhibit political homophily [39–41]. Thus, we expect the vectors z_i to provide good indicators of ground-truth opinions; but if they were to be as good or even better than x_i , one may question the relevance of using Eq. 2. Because 27% of users do not have direct connections with zealots, one cannot infer anything from z_i for about a quarter of the considered population. Therefore the use of the vectors x_i is already beneficial in that it extends the realm of the analysis. Compared to x_i , the z_i vectors position users closer to those with the same ground-truth opinion y_i and further away from those with a different y_i (respective average Euclidean distances 0.190 and 1.339, versus 0.219 and 1.146 for x_i). Surprisingly however, we find that x_i is more accurate than z_i to identify ground-truth opinions. Indeed, $\text{argmax}(x_i)$ exhibits a slightly higher accuracy than $\text{argmax}(z_i)$, and so does a Support Vector Classifier fitted on the (x_i, y_i) pairs (respective accuracies 0.962 versus 0.951, and 0.964 versus 0.948). Therefore, the VMZ improves on a simple analysis of connections between users and political entities.

Comparison with other networks. User networks can be extracted from online social interactions in different ways. To put our results into perspective, we investigate whether different data collection and processing methods impact the empirical reliability of the VMZ. We consider three other networks. The *UU* network, an undirected and unweighted version of the retweet network. The *follow* network, where $w_{ij} = 1$ if i follows j and 0 otherwise. The *mention* network, where w_{ij} is the number of times that i mentioned j in a tweet. In Table I we show that the retweet network performs much better than the others according to all the metrics we have used in the above analysis. The fact that the UU network gives poorer results is an especially relevant finding, given that most studies on the Voter Model focus on undirected unweighted networks. The poorer performances of the follow and mention networks are not too surprising. Indeed, while retweets are often considered to be markers of endorsement, people may follow a broader range of the political spectrum [], and mentions are often used to express hostility towards outgroup members [42?].

Discussion. The VMZ can accurately estimate individual opinions in a large, fine-grained, heterogeneous online population. The separation between users with different ground-truth opinions is clear in the opinion space of the model, and the model correctly identifies the ground-truth for 92.6% of the users. In addition, discord probabilities act as very good proxies to distinguish user pairs with the same ground-truth opinion from others. This is particularly surprising, given that it emerges as the result of a microscopical analysis while the model was originally built for macroscopical analyses. Our

| | Retweet | UU | Follow | Mention |
|---|--------------|-------|--------|--------------|
| Accuracy $\text{argmax}(x_i)$ | 0.926 | 0.749 | 0.842 | 0.644 |
| Accuracy SVC | 0.928 | 0.928 | 0.890 | 0.840 |
| $\langle x^s \rangle$ | 0.926 | 0.507 | 0.471 | 0.337 |
| $\langle x^{-s} \rangle$ | 0.044 | 0.123 | 0.132 | 0.166 |
| $\langle \ x_i - x_j\ \rangle_{\text{within}}$ | 0.296 | 0.188 | 0.205 | 0.164 |
| $\langle \ x_i - x_j\ \rangle_{\text{cross}}$ | 1.097 | 0.617 | 0.545 | 0.301 |
| $\langle \rho_{ij} \rangle_{\text{within}}$ | 0.354 | 0.591 | 0.638 | 0.682 |
| $\langle \rho_{ij} \rangle_{\text{cross}}$ | 0.902 | 0.779 | 0.795 | 0.730 |
| Accuracy logistic(ρ_{ij}) | 0.929 | 0.861 | 0.853 | 0.778 |

TABLE I: Comparison between the results of the VMZ applied on the directed weighted retweet network, the undirected unweighted retweet network (UU), the follow network and the mention network. Subscripts precise whether the averages are within- or cross-party. $\langle \|x_i - x_j\| \rangle$ is the average Euclidean distance in the opinion space of the model for the considered user pairs, and $\langle \rho_{ij} \rangle$ the average discord probability. logistic(ρ_{ij}) denotes a logistic regression model trained to distinguish between supporters of the same party and supporters of different parties on the basis of the ρ_{ij} values.

analysis advocates for the pursuit of more theoretical works dedicated to directed, weighted, heterogeneous networks with multi-dimensional opinions.

Empirical applications of opinion dynamics models are often restricted to sweeps of the parameter space in order to find parameter values for which the model best reproduces the observed data. This is a natural and essential step, given that most opinion dynamics models rely on social and psychological parameters, for which there exist no universal inference methods. To demonstrate the empirical reliability of the models in a robust way however, these analyses should ideally be complemented with evaluations of the models thus calibrated on different datasets. This is no easy task, given the difficulty to obtain fine-grained data, the heterogeneity of datasets, and the absence of universal inference methods for model parameters. Our methodology is parameter-free and thus circumvents this thorny issue, but future research should strive to address it.

Our results add to the existing body of evidence for the empirical reliability of the Voter Model. Other datasets, and extensions such as the partisan Voter Model [43] or the nonlinear Voter Model [44] among others, could be the ground of further testing. Because we analyzed the equilibrium state of the model, our findings pertain to the long-term behavior or the model. Future works should strive to integrate temporal aspects in the study of the empirical reliability of the models. Overall, we call for more research dedicated to the evaluation of opinion dynamics models on the microscopical level.

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