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### **To cite this version:**

Frederic Mazenc, Alessio Iovine. A bounded controller for a class of bidirectional boost DC/DC converters: Application to storage devices. 2024. hal-04840365

## **HAL Id: hal-04840365 <https://hal.science/hal-04840365v1>**

Preprint submitted on 16 Dec 2024

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# A bounded controller for a class of bidirectional boost DC/DC converters: Application to storage devices

Frédéric Mazenc<sup>1</sup>, Alessio Iovine<sup>2</sup>

*Index Terms*—Bounded control, Nonlinear control, Control for Power Converters, boost converters

*Abstract*—This paper lies in the research field of advanced control for power converters. We propose a bounded controller for a DC/DC boost converter, which ensures boundedness and stability of the closed loop systems. Guidelines on how to select the controller's parameters are provided.Simulations confirm that the developed control strategy effectively regulates the converter.

#### I. INTRODUCTION

The development and application of sophisticated control methods to regulate power converters in Alternate Current (AC) or Direct Current (DC) grids in general and DC/DC converters in particular has become a standard practice [1]–[3], both in case of single converter situations [4]–[6] or when interacting to share a common task [7], e.g., voltage regulation in DC grids [8], [9]. Indeed, the advancement of technology and the increasing complexity of tasks, coupled with a growing demand for enhanced performance, necessitate more advanced control techniques beyond the conventional Proportional Integral Derivative (PID) controllers.

However, despite the plethora of advanced control approaches, only a few consider the physical limitations of the systems and their impact on performance and operational range, e.g., boundedness of the control inputs impacting the convergence rate [10], [11] and the feasible working region [1], [12], [13] and/or boundedness of the dynamics [14]–[16]. Developing an unbounded control law for systems where large control values are restricted

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is common in the literature due to the complexity of constructing bounded feedbacks. As a result, bounds are empirically considered when selecting control law gains to estimate the system's operating region.

The present paper focuses on the development of bounded controllers for boost DC/DC converters [7], [11]–[13], [17], which are known for their inherent complexity due to their bilinear nature and non-minimum phase behavior. Unlike other approaches that involve time derivatives, which may lead to undesirable oscillations in the control law or significant control errors in the presence of small measurement errors [10], [11], [13], or complex computations involving Linear Matrix Inequalities (LMI) [12], [17] or receding horizon techniques [6], the approach presented in this paper introduces a straightforward controller that relies solely on desired reference values, measured dynamics, and bounded functions. We focus on the possibility to control the power output of a voltage source, e.g., a storage device, by selecting its output current or the voltage of the input capacitance of the converter. The controller is designed for a class of DC/DC boost power converters that satisfy a specific condition on the parameters' values. The goal of the control is to improve the speed of convergence of the solutions. Through the tuning of some parameters, we can achieve locally an arbitrarily fast speed of convergence. Even if restrictive, the conditions considered in our study allow for the consideration of a broad range of boost converters.

The paper is organized as follows. Section II introduces the considered model, while Section III provides the proposed bounded controller together with a stability analysis and practical elements on how to select the controller's parameters to respect the bounds on the duty cycle. Section IV outlines simulations to validate the approach. Conclusions are provided in Section V.

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Research supported in part by the Agence Nationale de la Recherche (ANR) via grant "Power Systems as Cyber-Physical Systems for enhancing the Energy Transition " (ESTHER), number ANR-22-CE05-0016.



Fig. 1. The adopted electrical scheme. The boost DC/DC converter is introduced and is connected to a output voltage  $V_O$  and to an input one  $V_I$ .

#### II. MODELING

The considered model for a DC/DC bidirectional boost converter connected to a voltage source  $V_I$ , e.g., a storage device, and an output voltage  $V_O$ , e.g., a strong DC bus, is shown in Fig. 1. The equivalent circuit representation can be expressed using a state space average model, under the hypothesis to then use a Pulse Width Modulation (PWM) technique for control purposes [18], [19]. Three state variables are needed for the system model: the capacitor voltages  $V_{C_1} : \mathbb{R} \to$  $\mathbb{R}^+$  and  $V_{C_3}$  :  $\mathbb{R} \to \mathbb{R}^+$  ( $x_1$  and  $x_3$  respectively) and the inductor current  $I_{L_2}$  :  $\mathbb{R} \to \mathbb{R}$   $(x_2)$ . Let  $X(t) = (x_1(t), x_2(t), x_3(t)) = (V_{C_1}(t), I_{L_2}(t), V_{C_3}(t)).$ In the remaining part of the paper, time dependence will be omitted due to lack of space. The values of the parameters  $C_1$ ,  $C_3$ ,  $R_1$ ,  $R_3$ ,  $L_2$ ,  $R_{01}$  are known and positive, and represent the values of the capacitors, resistances and the inductor, respectively. The voltages  $V_I \in \mathbb{R}^+$  and  $V_O \in \mathbb{R}^+$  are constant or piece-wise constant. The measured signals are the states  $x_1, x_2, x_3$ and the external constant voltages  $d_1 = V_I$ ,  $d_2 = V_O$ . By definition of boost (step-up) converter, the disturbances are such that  $d_1 > 0$ ,  $d_2 > 0$ ,  $d_2 > d_1$ . The control input is denoted as  $v = 1 - u$ , which is defined as the duty cycle of the circuit. Therefore, it results in  $v \in [0, 1]$ . According to average modeling, the electrical scheme in Fig. 1 is represented by the following dynamical equations:

$$
\begin{cases}\n\dot{V}_{C_1} = \frac{1}{R_1 C_1} (V_I - V_{C_1}) - \frac{1}{C_1} I_{L_2}, \\
\dot{I}_{L_2} = \frac{1}{L_2} V_{C_1} - \frac{R_{01}}{L_2} I_{L_2} - \frac{1}{L_2} V_{C_3} v, \\
\dot{V}_{C_3} = \frac{1}{R_3 C_3} (V_O - V_{C_3}) + \frac{1}{C_3} I_{L_2} v\n\end{cases}
$$
\n(1)

which are rewritten in state-space with  $X(t)$  previously defined as

$$
\begin{cases}\n\dot{x}_1 = -a_1x_1 - a_2x_2 + a_1d_1, \\
\dot{x}_2 = a_3x_1 - a_3x_3v - a_4x_2, \\
\dot{x}_3 = -a_5x_3 + a_6x_2v + a_5d_2\n\end{cases}
$$
\n(2)

where the values of the parameters are shown in Table I.

In addition let us observe that

$$
a_2 \neq 0 \tag{3}
$$

and

$$
a_3 > 0.\t\t(4)
$$

These properties will be used in the mathematical developments of the next sections.

#### *A. Equilibrium point*

Let us determine the equilibrium point of the system (2) with  $v \equiv v_*$  where  $v_* > 0$  is a constant. Let  $x<sub>*</sub> = (x<sub>1*</sub>, x<sub>2*</sub>, x<sub>3*</sub>)$  denote an equilibrium point. One can prove easily that

$$
x_{1*} = \left(\frac{a_6 v_*^2}{a_5} + \frac{a_4}{a_3}\right) a_1 \frac{d_1 - v_* d_2}{a_1 \left(\frac{a_6 v_*^2}{a_5} + \frac{a_4}{a_3}\right) + a_2} + v_* d_2,
$$
  
\n
$$
x_{2*} = a_1 \frac{d_1 - v_* d_2}{a_1 \left(\frac{a_6 v_*^2}{a_5} + \frac{a_4}{a_3}\right) + a_2},
$$
  
\n
$$
x_{3*} = \frac{a_6 v_*}{a_5} a_1 \frac{d_1 - v_* d_2}{a_1 \left(\frac{a_6 v_*^2}{a_5} + \frac{a_4}{a_3}\right) + a_2} + d_2.
$$
\n(5)

Clearly, in order to have that  $v \in [0, 1]$ , the controller at the equilibrium, i.e.,  $v_*,$  must satisfy this constraint as well, i.e.,  $v_* \in [0, 1]$ . Actually, several constraints arise on the dynamical evolution of the system and on its desired equilibrium point due to the bounds on the control input. The exact computation of the set of admissible initial conditions such that the bounds are respected is out of the scope of the present paper. We will consider the values of initial conditions, for which  $v(t) \in [0,1]$  for all  $t \geq 0$ .

The equilibrium point in  $(5)$ ,  $x_{*}$ , is function of the value of the control at the equilibrium,  $v_*$ , i.e.,  $x_* = f(v_*)$ . In practice, the available degree of freedom given by the control inputs is utilized to establish a desired equilibrium, leading other dynamics to align accordingly. Consequently, we can define a desired equilibrium relative to a specific variable's reference. The possibilities are:

• a value  $x_1^e$  is imposed;

$$
x_{1*} = x_1^e
$$
  
\n
$$
x_{2*} = \frac{a_1}{a_2}(d_1 - x_1^e)
$$
  
\n
$$
x_{3*} = \frac{1}{2} \left[ d_2 + \sqrt{d_2^2 + 4\beta_1 \phi_1 (x_1^e - \gamma_1 \phi_1)} \right],
$$
  
\n(6)

where  $\beta_1 = \frac{a_6 a_1}{a_5 a_2}$  $\frac{a_6a_1}{a_5a_2}$ ,  $\gamma_1 = \frac{a_4a_1}{a_3a_2}$  $\frac{a_4a_1}{a_3a_2}$ ,  $\phi_1 = d_1 - x_1^e$ . • a value  $x_2^e$  is imposed;

$$
x_{1*} = d_1 - \frac{a_2}{a_1} x_2^e
$$
  
\n
$$
x_{2*} = x_2^e
$$
  
\n
$$
x_{3*} = \frac{1}{2} \left[ d_2 + \sqrt{d_2^2 + 4 \frac{a_6}{a_5} x_2^e \left[ d_1 - \gamma_2 x_2^e \right]} \right],
$$
\n(7)

TABLE I THE POSITIVE VALUES OF THE PARAMETERS.

Parameter	Value	Parameter	Value
$a_1$		$a_2$	
$a_3$	42	$a_4$	
$a_5$		a <sub>6</sub>	

where  $\gamma_2 = \frac{a_2}{a_1}$  $\frac{a_2}{a_1} + \frac{a_4}{a_3}$  $\frac{a_4}{a_3}$ .

Let us observe that similar calculations apply also in the case where  $x_3^e$  is imposed. However, the goal of the present paper is to impose a reference value either for the voltage of the input capacitance, i.e.,  $x_1$ , or for the current, i.e.,  $x_2$ .

#### *B. Problem statement*

We introduce an assumption which is often satisfied in practice and is vital when we establish our main result. The inequality

$$
\left(\frac{1}{R_1C_1} - \frac{R_{01}}{L_2}\right)^2 > 4\frac{1}{C_1L_2} \tag{8}
$$

is satisfied.

Condition (8) in Assumption II-B reads

$$
(a_1 - a_4)^2 > 4a_2 a_3. \t\t(9)
$$

Let us propose the following problem: For the class of DC/DC boost power converters described by (1), i.e., (2), and satisfying condition (8), i.e., (9), find a bounded controller  $v$  ensuring that solutions of the system  $(2)$ converge asymptotically to a desired equilibrium point.

In the sequel, a controller solving Problem II-B for solutions of interest together with a theoretical proof showing stability is presented. Subsequently, a discussion ensues regarding the parameter selection process to ensure that the controller's values remain within the specified bounds, i.e.,  $v(t) \in [0,1]$  for all  $t \geq 0$  for solutions of interest.

#### III. MAIN RESULT

#### *A. Preliminaries*

Given the parameters in Table I, let us introduce the following constants:

$$
\Delta = \sqrt{(a_1 - a_4)^2 - 4a_2 a_3} \tag{10}
$$

$$
\alpha_1 = \frac{a_1 - a_4 - \sqrt{\Delta}}{2a_3} \quad , \quad \alpha_2 = \frac{a_1 - a_4 + \sqrt{\Delta}}{2a_3} \quad (11)
$$

$$
g_1 = \frac{a_1 + a_4 + \sqrt{\Delta}}{2}, \qquad g_2 = \frac{a_1 + a_4 - \sqrt{\Delta}}{2} \tag{12}
$$

$$
\beta = \frac{\alpha_1}{\alpha_2} + p \tag{13}
$$

$$
p = \frac{\alpha_1 \lambda_2}{\alpha_2 (g_1 - g_2 - \lambda_2)}\tag{14}
$$

where  $\lambda_2 > 0$  is a constant parameter such that

$$
g_1 - g_2 - \lambda_2 \neq 0 \tag{15}
$$

representing a desired convergence rate. Moreover, let us consider the constant parameters  $\lambda_1 > 0$  (another desired convergence rate),  $\epsilon_1 > 0$ ,

$$
\epsilon_2 = \frac{4}{3}\epsilon_1\tag{16}
$$

and the function  $\sigma_{\epsilon}$  defined by

$$
\sigma_{\epsilon}(s) = \begin{cases}\n s & s \in [-\epsilon, \epsilon], \\
 \epsilon \frac{s}{|s|} & |s| > \epsilon\n\end{cases}
$$
\n(17)

where  $\epsilon \in (0, 1)$ . Moreover, let us define

$$
\tilde{x}_1(t) = x_1 - x_{1*}, \qquad \tilde{x}_2(t) = x_2 - x_{2*}
$$
 (18)

and perform a change of variables

$$
z_i = \tilde{x}_1 + \alpha_i \tilde{x}_2 \tag{19}
$$

with  $i = 1, 2$ . Finally, let us introduce the function  $\psi$ defined by:

$$
\psi(s) = \begin{cases}\n(1 - \epsilon)x_{3*} & s \le (1 - \epsilon)x_{3*}, \\
s & s \in [(1 - \epsilon)x_{3*}, (1 + \epsilon)x_{3*}], \\
(1 + \epsilon)x_{3*} & s \ge (1 + \epsilon)x_{3*}.\n\end{cases}
$$
\n(20)

where  $\epsilon$  is the constant involved in the definition of  $\sigma_{\epsilon}$ .

#### *B. Controller*

We are ready to state and prove the following result: Consider the controller

$$
v(\tilde{x}_1, \tilde{x}_2, x_3) = \frac{a_3 x_{1*} - a_4 x_{2*} - \omega(\tilde{x}_1, \tilde{x}_2)}{a_3 \psi(x_3)}
$$
(21)

where

$$
\omega = -\frac{1}{\alpha_2} \sigma_{\epsilon_2} (\lambda_2 \tilde{x}_1 + \lambda_2 \alpha_2 \tilde{x}_2)
$$
  
 
$$
-\frac{1}{\alpha_2} \frac{\beta}{|\beta|} \sigma_{\epsilon_1} (\lambda_1 (1+p) \tilde{x}_1 + \lambda_1 (\alpha_1 + p \alpha_2) \tilde{x}_2).
$$
 (22)

It solves Problem II-B for all the solutions such that there is  $T \geq 0$  such that  $x_3(t) \in [(1-\epsilon)x_{3*}, (1+\epsilon)x_{3*}]$  for all  $t \geq T$ , where  $\epsilon$  is the constant present in (17) and (20).

Remark. Theorem III-B is concerned only with the solutions such that  $x_3$  enters the interval  $[(1-\epsilon)x_{3*},(1+\epsilon)x_{3*}]$  $\epsilon$ ) $x_{3*}$ ] in finite time. This may seem a strong limitation of the proposed result. But this is not at all the case: the simulations we perform show that the solutions with initial conditions that are meaningful from a practical point of view satisfy this property when  $\epsilon = 0.1$  is chosen.

Consider a solution  $(x_1, x_2, x_3)$  of the system (2) in closed-loop with the control law (21) such that there is T > 0 such that  $x_3(t) \in [(1-\epsilon)x_{3*}, (1+\epsilon)x_{3*}]$  for all  $t > T$ .

Then, from the definition of  $\psi$  in (20), it follows that

$$
v(\tilde{x}_1, \tilde{x}_2, x_3) = \frac{a_3 x_{1*} - a_4 x_{2*} - \omega(\tilde{x}_1, \tilde{x}_2)}{a_3 x_3},
$$
 (23)

for all  $t \geq T$ . We deduce that the  $(x_1, x_2)$ -subsystem of the system (2) in closed-loop with the control law (21) can be rewritten as

$$
\begin{cases}\n\dot{x}_1 = -a_1 x_1 - a_2 x_2 + a_1 d_1, \\
\dot{x}_2 = a_3 x_1 - [a_3 x_{1*} - a_4 x_{2*} - \omega(\tilde{x}_1, \tilde{x}_2)] - a_4 x_2\n\end{cases}
$$
\n(24)

when  $t \geq T$ . As an immediate consequence,

$$
\begin{cases} \dot{\tilde{x}}_1 = -a_1 \tilde{x}_1 - a_2 \tilde{x}_2, \\ \dot{\tilde{x}}_2 = a_3 \tilde{x}_1 - a_4 \tilde{x}_2 + \omega(\tilde{x}_1, \tilde{x}_2) \end{cases} \tag{25}
$$

when  $t \geq T$ .

Next, in the remaining part of the proof we consider the case where  $t \geq T$ , without mentioning this. We have

$$
\dot{z}_1 = -a_1 \tilde{x}_1 - a_2 \tilde{x}_2 + \alpha_1 (a_3 \tilde{x}_1 - a_4 \tilde{x}_2 + \alpha_1 \omega = (-a_1 + \alpha_1 a_3) \tilde{x}_1 - (a_2 + \alpha_1 a_4) \tilde{x}_2 + \alpha_1 \omega = (-a_1 + \alpha_1 a_3) \left[ \tilde{x}_1 + \frac{a_2 + \alpha_1 a_4}{a_1 - \alpha_1 a_3} \tilde{x}_2 \right] + \alpha_1 \omega.
$$
 (26)

Now, observe that

$$
\frac{a_2 + \alpha_1 a_4}{a_1 - \alpha_1 a_3} = \alpha_1 \tag{27}
$$

because

$$
\alpha_1^2 a_3 + (a_4 - a_1)\alpha_1 + a_2 = 0. \tag{28}
$$

It follows that

$$
\dot{z}_1 = (-a_1 + \alpha_1 a_3) z_1 + \alpha_1 \omega. \tag{29}
$$

Similarly, one can prove that

$$
\dot{z}_2 = (-a_1 + \alpha_2 a_3) z_2 + \alpha_2 \omega.
$$
 (30)

Thus we have

$$
\begin{cases}\n\dot{z}_1 = -g_1 z_1 + \frac{\alpha_1}{\alpha_2} u, \\
\dot{z}_2 = -g_2 z_2 + u\n\end{cases} (31)
$$

with  $g_1$  and  $g_2$  as in (12) and

$$
u = \alpha_2 \omega. \tag{32}
$$

Now, let us analyze the stability properties of the system (31). Let us recall that (15) is satisfied and let us of observe that

$$
u = -\sigma_{\epsilon_2}(\lambda_2 z_2) + \rho \tag{33}
$$

where  $\rho$  is

$$
\rho = -\frac{\beta}{|\beta|}\sigma_{\epsilon_1}(\lambda_1(1+p)\tilde{x}_1 + \lambda_1(\alpha_1 + p\alpha_2)\tilde{x}_2). \quad (34)
$$

It follows from (16) that

$$
|\rho(\tilde{x}_1, \tilde{x}_2)| \le \overline{\rho} \tag{35}
$$

for all  $(\tilde{x}_1, \tilde{x}_2) \in \mathbb{R}^2$  with

$$
\overline{\rho} = \frac{3\epsilon_2}{4}.
$$
 (36)

Thus, we have

$$
\begin{cases}\n\dot{z}_1 = -g_1 z_1 - \frac{\alpha_1}{\alpha_2} \sigma_{\epsilon_2} (\lambda_2 z_2) + \frac{\alpha_1}{\alpha_2} \rho, \n\dot{z}_2 = -g_2 z_2 - \sigma_{\epsilon_2} (\lambda_2 z_2) + \rho.\n\end{cases}
$$
\n(37)

To analyze the stability properties of the  $z_2$ -subsystem of (37), let us introduce the positive definite function:

$$
U_2(z_2) = \frac{1}{2}z_2^2.
$$
 (38)

Its derivative along the trajectories of (37) satisfies

$$
\begin{array}{rcl}\n\dot{U}_2(t) & \leq & -g_2 z_2^2 - z_2 \sigma_{\epsilon_2}(\lambda_2 z_2) + |z_2|\overline{\rho} \\
& \leq & -g_2 z_2^2 - z_2 \sigma_{\epsilon_2}(\lambda_2 z_2) + |z_2|\frac{3\epsilon_2}{4}\n\end{array} \tag{39}
$$

where the last inequality is a consequence of (36). When  $|z_2(t)| \geq \frac{3\epsilon_2}{4\lambda_2}$ , then

$$
\dot{U}_2(t) \leq -g_2 z_2^2 - |z_2|\sigma_{\epsilon_2} \left(\lambda_2 \frac{3\epsilon_2}{4\lambda_2}\right) + |z_2| \frac{3\epsilon_2}{4}
$$
\n
$$
= -g_2 z_2^2 - |z_2| \frac{3\epsilon_2}{4} + |z_2| \frac{3\epsilon_2}{4}
$$
\n
$$
< 0.
$$
\n(40)

We deduce that there is  $t_1 \geq 0$  such that for all  $t \geq t_1$ ,

$$
|z_2(t)| \le \frac{3\epsilon_2}{4\lambda_2}.\tag{41}
$$

Then for all  $t \geq t_1$ ,

$$
\begin{cases}\n\dot{z}_1 = -g_1 z_1 - \frac{\alpha_1}{\alpha_2} \lambda_2 z_2 + \frac{\alpha_1}{\alpha_2} \rho, \\
\dot{z}_2 = -(g_2 + \lambda_2) z_2 + \rho.\n\end{cases} (42)
$$

Now, let us perform the change of variables

$$
z_3(t) = z_1(t) + pz_2(t). \tag{43}
$$

Then

$$
\dot{z}_3 = -g_1 z_1 - \frac{\alpha_1}{\alpha_2} \lambda_2 z_2 + \frac{\alpha_1}{\alpha_2} \rho
$$
  
\n
$$
-p(g_2 + \lambda_2) z_2 + p\rho
$$
  
\n
$$
= g_1[-z_3(t) + pz_2] - \left[\frac{\alpha_1}{\alpha_2} \lambda_2 + p(g_2 + \lambda_2)\right] z_2
$$
  
\n
$$
+ \left(\frac{\alpha_1}{\alpha_2} + p\right) \rho
$$
  
\n
$$
= -g_1 z_3 - \left[\frac{\alpha_1}{\alpha_2} \lambda_2 + p(g_2 - g_1 + \lambda_2)\right] z_2
$$
  
\n
$$
+ \left(\frac{\alpha_1}{\alpha_2} + p\right) \rho.
$$
  
\n(44)

From the definition of  $p$ , it follows that

$$
\dot{z}_3 = -g_1 z_3 + \beta \rho \tag{45}
$$

with

$$
\beta = \frac{\alpha_1}{\alpha_2} + p. \tag{46}
$$

Now, we observe that

$$
\beta = \frac{\alpha_1}{\alpha_2} + \frac{\alpha_1 \lambda_2}{\alpha_2 (g_1 - g_2 - \lambda_2)} = \frac{\alpha_1}{\alpha_2} \left[ 1 + \frac{\lambda_2}{g_1 - g_2 - \lambda_2} \right]
$$

$$
= \frac{\alpha_1}{\alpha_2} \frac{\sqrt{\Delta}}{\sqrt{\Delta - \lambda_2}}.
$$
(47)

Let us observe that Assumption 1 ensures that  $\alpha_1 \neq 0$ and  $\Delta > 0$ . We deduce that  $\beta \neq 0$ . Then through simple calculations, one can prove that

$$
\rho = -\frac{\beta}{|\beta|} \sigma_{\epsilon_1}(\lambda_1 z_3). \tag{48}
$$

Then, according to (42) and (45), for all  $t \ge t_1$ ,

$$
\begin{cases}\n\dot{z}_3 = -g_1 z_3 - |\beta|\sigma_{\epsilon_1}(\lambda_1 z_3), \\
\dot{z}_2 = -(g_2 + \lambda_2) z_2 - \frac{\beta}{|\beta|}\sigma_{\epsilon_1}(\lambda_1 z_3).\n\end{cases} (49)
$$

We can easily analyze the the stability properties of the z<sub>3</sub>-subsystem of (49). In particular there is  $t_2 \geq t_1$  such that for all  $t \geq t_2$ ,  $|z_3(t)| < \frac{\epsilon_1}{\lambda_1}$  $\frac{\epsilon_1}{\lambda_1}$ . It follows that for all  $t \geq t_2$ ,

$$
\begin{cases}\n\dot{z}_3 = -(g_1 + |\beta|\lambda_1)z_3, \\
\dot{z}_2 = -(g_2 + \lambda_2)z_2 - \frac{\beta}{|\beta|}\lambda_1 z_3.\n\end{cases} (50)
$$

Let us observe that an arbitrary large rate of convergence can be obtained by an appropriate choice of  $\lambda_1$  and  $\lambda_2$ . The controller  $v$  in (21) and (22) solves Problem II-B for the solutions we specified in Theorem 1. However, it is not guaranteed that  $v \in [0, 1]$ . To ensure this property, a proper choice of initial conditions and of the parameter  $\epsilon_1$  has to be made (recall that (16) holds). From physical

TABLE II THE VALUES OF THE PARAMETERS.

Parameter	Value	Parameter	Value
$R_{\rm 1}$	$100 \text{ m}\Omega$	R3	$100 \text{ m}\Omega$
Ch	$100 \text{ m}F$	$\mathcal{C}_{3}$	$10 \text{ m}F$
L۵.	$33 \text{ m}H$	$R_{01}$	$10 \text{ m}\Omega$

properties and the considered parameters in Table I, it is possible to say that, for realistic initial conditions, the dynamics of  $x_3$  will not move too far from the computed equilibrium in (5). Then we can consider an upper bound  $x_3^M$  and a lower bound  $x_3^m$  of  $x_3(t)$ . Therefore, a conservative selection of the parameter  $\epsilon_2$ (and then of  $\epsilon_1$ ) can be performed such that

$$
\omega \in [a_3x_{1*} - a_4x_{2*} - a_3\psi(x_3^m), a_3x_{1*} - a_4x_{2*}],
$$
 (51)

which ensures that  $v \in [0, 1]$ .

#### IV. SIMULATIONS

The simulations are developed in Matlab/Simulink2022b©. To demonstrate the effectiveness of the proposed control law, we use a piece-wise constant reference trajectory instead of a constant one. This approach, which is commonly used when testing the performance of power converters in closed-loop, relies on the controller's ability to ensure a satisfactory convergence before each change in the reference occurs. As described in Section II, we impose  $x_2^e$  and propose a comparison between two tuning parameters  $\varepsilon_2$  for the controller. Since no hypothesis is made for the voltage source of the converter, i.e., the device can act as providing or absorbing power, it is possible to consider that the reference  $x_2^e$  takes both positive and negative values. Parameters' values are in Table II. The considered simulation time is about 10 seconds.

#### *A. Case 1: choice of a value*  $\varepsilon_2 = 12000$

Fig. 2, 3 and 4 depict both the references  $x_{1*}$ ,  $x_{2*}$  =  $x_2^e$  and  $x_{3*}$  and the dynamical behaviour of the state variables  $x_1$ ,  $x_2$  and  $x_3$ , respectively, when the system is controlled by the control input introduced in equation (21) with target to impose a desired  $x_2^e$  and with the choice  $\varepsilon_2 = 12000$ . As shown, the controller perfectly meets its target, since the error dynamics converges to zero and a smooth behaviour with no overshoots results for the variables of main interest  $x_1$  and  $x_2$ . Furthermore, since the choice of the parameter  $\varepsilon_2$  and of the desired reference  $x_2^e$  have been properly done, the controller fits its boundaries, as it is depicted in Fig. 5.

Possible undesired behaviours can be the high overshoots taking place in the dynamics of  $x_3$  or the fact that the controller almost touches its boundaries at time 2s and 6s. To avoid these drawbacks, we focus now on a possible different choice of the parameter  $\varepsilon_2$ .

#### *B. Case 2: choice of a value*  $\varepsilon_2 = 4000$

Fig. 6, 7 and 8 are similar to Fig. 2, 3 and 4, as they show dynamics  $x_1$ ,  $x_2$  and  $x_3$  with respect to their references  $x_{1*}$ ,  $x_{2*} = x_2^e$  and  $x_{3*}$ . However, here the parameter  $\varepsilon_2$  in (21) is  $\varepsilon_2 = 4000$ . Similarly as in the previous case, the controller perfectly meets its target, since the error dynamics converges to zero for the variables of main interest  $x_1$  and  $x_2$ . However, we remark that the convergence rate is slower with respect to the case where  $\varepsilon_2 = 12000$  and also overshoots appears in the dynamics of  $x_1$  and  $x_2$ . This is not surprising, as the current choice is more limiting. As shown by Fig. 9, by saturating earlier the controller reduces its margin of maneuver, thereby affecting performance. However, it is worth noting that the controller remains further from its bounds, and the overshoots observed in the dynamics  $x_3$  are of lesser magnitude, as shown comparing Fig. 8 with Fig. 4.



Fig. 2. Case 1: The dynamical behaviour of  $x_1$  with respect to its reference  $x_1^*$  computed according to  $x_2^e$  and the value  $\varepsilon_2 = 12000$ .



Fig. 3. Case 1: The dynamical behaviour of  $x_2$  with respect to its reference  $x_2^*$  and with the value  $\varepsilon_2 = 12000$ .

From Fig. 5 and 9 it must be noted that the developed control action allows for a "smooth" curve in the



Fig. 4. Case 1: The dynamical behaviour of  $x_3$  with respect to its reference  $x_3^*$  and with the value  $\varepsilon_2 = 12000$ .



Fig. 5. Case 1: The controller  $u$  with respect to its physical bounds when  $\varepsilon_2 = 12000$ .



Fig. 6. Case 2: The dynamical behaviour of  $x_1$  with respect to its reference  $x_1^*$  computed according to  $x_2^e$  and the value  $\varepsilon_2 = 4000$ .

obtained duty cycle. It is important since it results in less stress for the components and the devices.

While in other approaches as in [11] this outcome is due to the proper choice of several parameters, here this outcome is a result of the dominant static part of the controller. Indeed, the dynamical part of the controller behaviour takes place only during transients due to a change in the desired equilibrium. Therefore, the choice of the parameters needs mainly to focus on the verification of the static bounds, thus to simplify the understanding and the implementation of the proposed



Fig. 7. Case 2: The dynamical behaviour of  $x_2$  with respect to its reference  $x_2^*$  and with the value  $\varepsilon_2 = 4000$ .



Fig. 8. Case 2: The dynamical behaviour of  $x_3$  with respect to its reference  $x_3^*$  and with the value  $\varepsilon_2 = 4000$ .



Fig. 9. Case 2: The controller  $u$  with respect to its physical bounds when  $\varepsilon_2 = 4000$ .

controller. The simulations demonstrate the efficacy of the proposed controller and highlight the advantages and disadvantages of choosing larger or smaller parameters, which influence the size of the controller.

#### V. CONCLUSIONS

This paper presents a novel bounded nonlinear controller designed for a specific class of DC/DC boost converters. The controller calculates the control input in real-time, taking into account physical constraints to ensure desired performance.

Simulation results validate the effectiveness of the proposed control strategy.

Future works will focus on output feedback stabilization of the systems when a delay in the measurements is present.

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