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# Highly Efficient Compact Acousto-Optic Modulator Based on a Hybrid-Lattice Hollow Core Fiber

Ricardo E. da Silva, Jonas H. Osório, David J. Webb, Frédéric Gérôme, Fetah Benabid, Marcos A. R. Franco and Cristiano M. B. Cordeiro

**Abstract**— We demonstrate the acousto-optic modulation of a hybrid-lattice hollow core fiber (HL-HCF) for the first time. For many years, optical fibers with reduced diameters have been the main solution to increase the interaction of acoustic and optical waves. However, the high drive voltages and large modulator components still employed drastically affect the efficiency and miniaturization of these devices. Here, we experimentally show that combining Kagomé and tubular lattices in HL-HCFs allows for enhancing the amplification of the acoustic waves and the modulation of the guided optical modes, thus providing high modulation efficiency even when using a fiber with a 240  $\mu\text{m}$  diameter. To the best of our knowledge, the measured HL-HCF's modulation efficiency (1.3 dB/V) is the highest compared to devices employing reduced diameter fibers. Additionally, we demonstrate a compact acousto-optic modulator with driver dimensions smaller than the HL-HCF diameter. Overall, our results show a promising alternative to solve the compromise of speed, efficiency, and compactness for integration with microscale all-fiber photonic devices.

**Index Terms**— All-fiber acousto-optic devices, modulation efficiency, hybrid-lattice hollow core fiber.

## I. INTRODUCTION

ALL-FIBER acousto-optic modulators (AOMs) enable remarkable applications in electrically tunable wavelength filters, frequency shifters, heterodyne vibration sensors, and pulsed fiber lasers [1], [2], [3], [4]. In these devices, the spectral and power properties of the optical fibers are dynamically tuned by the frequency and amplitude of the electrical signal, providing low loss and monolithic integration with fiber optic components.

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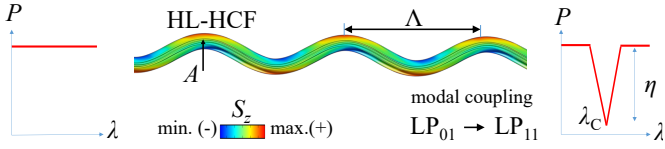
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AOMs based on flexural acoustic waves couple the power between the guided modes in the optical fiber. Previous studies employing distinct fiber types show that the fibers' mechanical and optical properties play an important role in the modulated transmission spectrum, defining the tuning wavelength range, notch bandwidth, modulation depth, and overall modulator efficiency. Thus, AOMs employing reduced diameter etched fibers, long interaction lengths and high voltages have been used to enhance the modulation depth [1], [2], [4], [5], [6]. However, the increasing consumed power might decrease the overall AOM efficiency, additionally requiring the use of power amplifiers which increase the setup complexity and cost. As alternatives, suspended core fibers [7] and hollow core fibers [8], [9], [10] achieve significantly high modulation depths and efficiencies with no need for tapering or etching techniques. The increased acoustic and optical interactions impact the size reduction of the AOM components, switching time, and consumed electric energy.

Recently, we have demonstrated that in solid silica fibers, the acoustic wave is mostly concentrated over the cladding, thus having a weak overlap with the optical modes in the fiber core [11]. Besides, we have shown that tubular-lattice hollow core fibers (TL-HCF) significantly increase the acousto-optic interaction consuming lower acoustic energy [10], [11] due to the silica reduction and the out-of-center position of the cladding tubes over the fiber cross section. Furthermore, TL-HCFs provide stronger spatial overlap of the coupled optical modes, hence contributing to increase the modulation depth of the spectral resonances. Here, we experimentally demonstrate the acousto-optic modulation of a hybrid-lattice hollow core fiber (HL-HCF), which displays a cladding formed by a Kagomé structure and a ring of tubes defining its air core. Optically, these two claddings decrease the confinement losses [12]. Additionally, due to its microstructure characteristics, the HL-HCF further reduces the silica content in the fiber cross section compared to other HCFs, while keeping the properties of tubular lattices that are beneficial for acousto-optic modulation. In this context, we show herein that combining Kagomé and tubular lattices in the HL-HCF allows for attaining a modulation efficiency of 1.3 dB/V, which is the highest among other devices, even when using fibers with considerable smaller diameters. Additionally, we report on a compact acousto-optic modulator with smaller dimensions than the HL-HCF itself. We hence understand that our results offer a promising solution for integrating high speed, efficiency, and compactness into microscale all-fiber photonic devices.



**Fig. 1.** Illustration of a hybrid-lattice hollow core fiber (HL-HCF) modulated by a flexural acoustic wave of amplitude  $A$  and period  $\Lambda$ . The acoustic wave induces a longitudinal strain  $S_z$ , coupling power between the fundamental mode  $LP_{01}$  and the higher-order mode  $LP_{11}$ , modulating a notch with depth  $\eta$  at the resonant wavelength  $\lambda_c$  in the power ( $P$ ) transmission spectrum.

## II. ACOUSTO-OPTIC MODULATION OF HCFs

When optical fibers are modulated by flexural acoustic waves, the refractive index profile over their cross section is altered. In HCFs, this variation is mainly caused by the fiber's geometric deformation that changes the optical path of the guided modes [11]. Fig. 1 illustrates a flexural acoustic wave with amplitude  $A$ , period  $\Lambda$ , and frequency  $f$ . The acoustically induced longitudinal strain  $S_z$  causes a net change in the refractive index, which is non-uniform over the fiber cross section. The acoustic wave couples the power from the fundamental mode  $LP_{01}$  to a higher-order mode  $LP_{1m}$  at the resonance wavelength  $\lambda_c$  when the optical beatlength  $L_B$  matches with the acoustic period  $\Lambda$ , *i.e.*,  $L_B = \Lambda$ , where [2], [5], [8],

$$L_B = \frac{\lambda_c}{n_{01} - n_{1m}}, \quad (1)$$

$n_{01}$  and  $n_{1m}$  are the effective refractive indices of the modes  $LP_{01}$  and  $LP_{1m}$ , and,

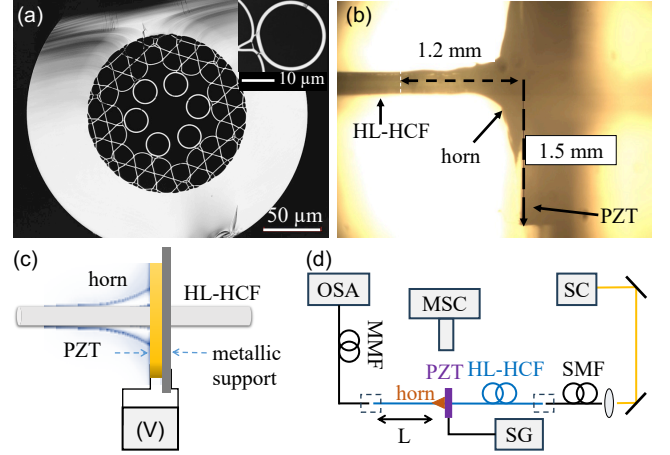
$$\Lambda = \left( \frac{\pi D c_{ext}}{2f} \right)^{\frac{1}{2}}, \quad (2)$$

where  $D$  is the fiber diameter and  $c_{ext} = 5740$  m/s is the extensional acoustic velocity. Thus, the modulated resonance's wavelength  $\lambda_c$  and depth  $\eta$  are tuned by the acoustic frequency and power, respectively. The effective indices  $n_{01}$  and  $n_{1m}$  can be calculated analytically as reported in [12]. We will use the latter equations to investigate the optical couplings in our acousto-optic configuration. The results and corresponding analyses can be found in the next section of the manuscript.

## III. EXPERIMENTAL SETUP

Fig. 2(a) shows the cross section of the HL-HCF, which is composed of a hybrid cladding formed by Kagomé-tubular lattices. The thickness of the struts in the Kagomé structure is 800 nm. The fiber core (34  $\mu\text{m}$  diameter) is defined by a ring of six tubes of 20  $\mu\text{m}$  in diameter and 1.25  $\mu\text{m}$  thickness. The fiber is made of pure silica and its outer diameter is 240  $\mu\text{m}$ . The inset in Fig. 2(a) shows a connecting tube of about 50 nm thickness between the Kagomé structure and a core tube, providing appropriate spacing between the two claddings and preventing leakage of the fundamental mode. Details about the HL-HCF design and fabrication are described in [12].

The acousto-optic modulator is composed of a HL-HCF segment, a piezoelectric transducer disc (PZT), and an axially connected acoustic horn, as shown in Fig. 2(b). The PZT is 200  $\mu\text{m}$  thick and has a diameter of 3 mm. The horn is made by molding a fixing adhesive during its fluid state using the thin tip of a needle and the help of a microscope (MSC). Fig. 2(b)



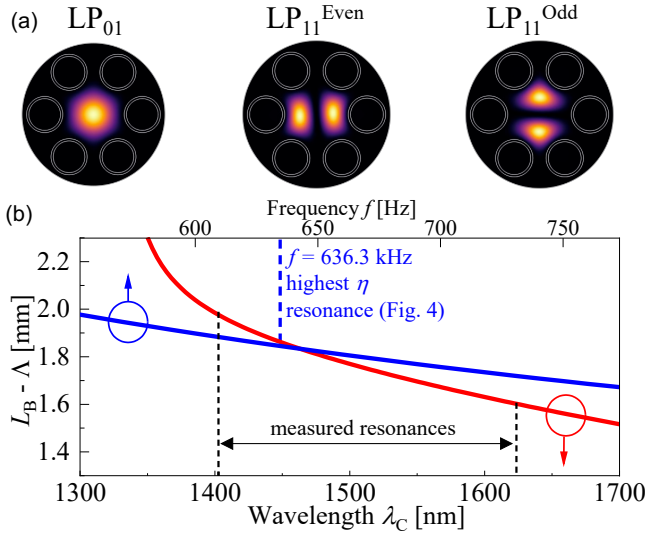
**Fig. 2.** (a) Cross section of the hybrid-lattice hollow core fiber (HL-HCF) showing in the inset a connecting tube between the Kagomé structure and a core tube. (b) Microscope view of the acousto-optic modulator (driver section) indicating the HL-HCF, acoustic horn and piezoelectric transducer (PZT). (c) Illustration of the modulator indicating the HL-HCF passing through the horn, PZT and metallic support. (d) Experimental setup.

indicates the horn length of  $\sim 1.2$  mm with a nearly exponential-like profile, decreasing from 2 mm to match the fiber diameter (240  $\mu\text{m}$ ). Fig. 2(c) illustrates additional details of the AOM. The HL-HCF passes through the horn, PZT and a thin metallic support (a 500  $\mu\text{m}$  hole is drilled in the PZT and support center). An electrode of 90  $\mu\text{m}$  in diameter is connected to the PZT by means of a conductive silver adhesive. The other PZT side is connected to the metallic support. The output fiber end is fixed forming a fiber interaction length of about  $L \sim 7.7$  cm (the whole modulator including horn and PZT is 7.8 cm long).

Fig. 2(d) illustrates the experimental setup. A sinusoidal electrical signal with a maximum peak-to-peak voltage of 10 V is applied to the PZT in the frequency range of  $f = 611$  to 733 kHz using a signal generator (SG). The HL-HCF's spectrum is measured by using a supercontinuum source (SC) and an optical spectrum analyzer (OSA). The HL-HCF input is butt-coupled to a single mode fiber (SMF), previously aligned with the SC beam. The output is butt-coupled to a multimode fiber (MMF) connected to the OSA. The coupling alignments are performed using micrometer stages, mirrors, and a lens.

## IV. RESULTS AND DISCUSSION

Fig. 3(a) shows the power distribution of the modes  $LP_{01}$  and  $LP_{11}$  (even and odd) simulated in the HL-HCF core with the finite element method as described in [10]. Fig. 3(b) shows the simulated beatlength  $L_B$  of the modes  $LP_{01}$  and  $LP_{11}$  between  $\lambda = 1300 - 1700$  nm calculated using (1). Here, the effective refractive indices  $n_{01}$  and  $n_{11}$  were computed employing the model reported in [13] and the fiber parameters indicated in the last section. Fig. 3(b) also shows the acoustic period  $\Lambda$  calculated from  $f = 550$  to 800 kHz using (2). We experimentally measured modulated resonances from  $\lambda = 1401$  to 1623 nm (222 nm range), indicating good agreement with the smooth central region of the curves in Fig. 3(b). Fig. 4 shows the measured modulated transmission spectrum of the HL-HCF, indicating the resonance with the highest achieved modulation depth (13.7 dB at  $\lambda_{c2} = 1523$  nm and  $f = 636.3$  kHz).

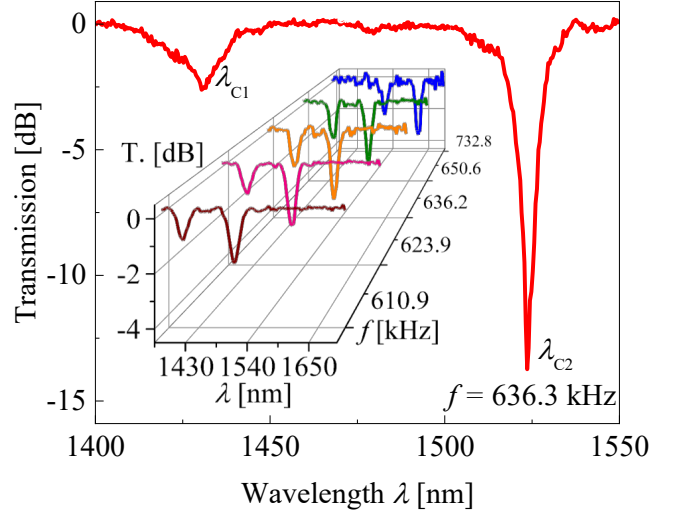


**Fig. 3.** (a) Simulated power distribution of the fundamental and higher order modes LP<sub>01</sub> and LP<sub>11</sub> in the HL-HCF core. (b) Analytical simulation of the optical beat length  $L_B$  of the modes LP<sub>01</sub> and LP<sub>11</sub> (red) and the variation of the acoustic period  $\Lambda$  with frequency (solid blue) indicating the range of the measured resonances. The dashed blue line indicates the frequency of the resonance with the highest modulation depth  $\eta$  in Fig. 4 (red).

The two notches in Fig. 4 indicate acoustically induced birefringence of the mode LP<sub>11</sub> into the modes LP<sub>11</sub><sup>Even</sup> and LP<sub>11</sub><sup>Odd</sup>. The average separation of the notches for the whole frequency range is 96 nm. This birefringence is caused by the non-uniform change of the refractive index over the fiber cross section. Having acoustically induced birefringence seems to be common in HCFs as seen in [9], [10] and explained in detail in [11]. The resonances' average half-height bandwidth is 20 nm.

Other resonances with modulation depths  $\eta$  changing from 2.1 to 3.5 dB (0.21 – 0.35 efficiencies) are shown in Fig. 4 (inset). The decreased  $\eta$  of these resonances might be caused by the phase deviation of the  $L_B$  and  $\Lambda$  curves in Fig. 3(b). We note in Fig. 3(b) that the resonance with the highest  $\eta$  ( $f$  indicated with a dashed blue line) nearly approaches an optimum matching of  $L_B$  and  $\Lambda$ . The differences between the simulated and measured values might be caused by variations of the HL-HCF's structure dimensions, material and induced birefringence. Also, distinct PZT or fiber resonances at  $f = 636.3$  kHz inducing higher amplitude acoustic waves might contribute for higher  $\eta$  [11].

The frequency tuning of the modes LP<sub>11</sub><sup>Even</sup> and LP<sub>11</sub><sup>Odd</sup> is shown in Fig. 5. The measured resonances cover tuning ranges of  $\lambda_{c1} = 1401 - 1521$  nm (LP<sub>11</sub><sup>Even</sup> - black) and  $\lambda_{c2} = 1495 - 1623$  nm (LP<sub>11</sub><sup>Odd</sup> - orange). The frequency tuning is almost linear for both modes ( $r^2 = 0.999$ ), with respective tuning rates of 0.98 and 1.04 nm/kHz. For  $\lambda_c = (n_{01} - n_{11})\Lambda$ , the slope  $\Delta\lambda_c/\Delta f$  depends on the modes' effective indices and fiber diameter as indicated in (1) and (2). Thus, a larger slope would provide a broader spectral tuning range ( $\Delta\lambda_c$ ) for a certain frequency interval ( $\Delta f$ ). However, limited tuning range is expected due to decreasing modulation efficiency with mismatching  $L_B$  and  $\Lambda$ . Additionally, the HL-HCF's nearly spaced high attenuation bands [12] might provide shorter tuning ranges compared to a TL-HCF [10] (here, limiting the shortest wavelength). At the longest wavelengths, tuning is usually limited by decreasing PZT's displacements with frequency.



**Fig. 4.** Acoustically modulated resonance with the highest achieved modulation depth in the transmission spectrum of the HL-HCF. The inset shows the modulated spectra tuned by the increasing acoustic frequency  $f$ .

Overall, the demonstrated 222 nm range is relevant compared to those achieved with other fiber types, as shown in Table 1. The spectral gaps in Fig. 5 are caused by the intrinsic frequency response of the small employed PZT providing fewer narrowband resonances compared to a large PZT [10] (larger PZTs might provide broader and stronger resonant bands at low frequencies). Additionally, the employed coaxial AOM design might excite longitudinal acoustic waves, which cannot couple the modes inducing tuning gaps. This does not affect further applicability of the demonstrated AOM since improved continuous tuning might be achieved by changing the PZT dimensions and material [4], [10], or employing shear modes PZTs, favoring the transversal excitation of flexural acoustic waves [9]. Overall, the HL-HCF provides continuous tuning for the entire effective frequency range, as seen in Fig. 3(b).

We compared the performance of our AOM with devices employing fibers with distinct diameters and geometries. The overall modulation efficiency (in dB/V) is estimated from the ratio of the maximum modulation depth and the respective consumed voltage. Table 1 summarizes the modulation efficiency, the fiber diameter, and the employed interaction length, which directly influence the overall modulation efficiency. The considered fibers include an etched SMF, an etched dispersion compensation fiber (DCF), a few-mode fiber (FMF), a photonic band gap (PBG-HCF), and a tubular-lattice (TL) HCF (references in Table 1). Note that the HL-HCF's modulation efficiency is 1.18x higher compared to the etched SMF (with  $\sim 60\%$  longer length) and 2.6x higher than the etched DCF (with  $\sim 60\%$  shorter length). Considering that the etched fibers are close in diameter, an average of  $\sim 2x$  higher efficiency is estimated for the HL-HCF, even employing a diameter 7x larger. Although the interaction length and the other modulator components can relevantly influence the overall efficiency, the dependence of the increasing efficiency with silica reduction is clearly noted for the distinct fibers in Table 1. As expected, the Kagomé structure provides extra silica reduction over the HL-HCF cross section even having thicker tubes compared to the TL-HCF [10], inducing higher strains in the nano struts than the

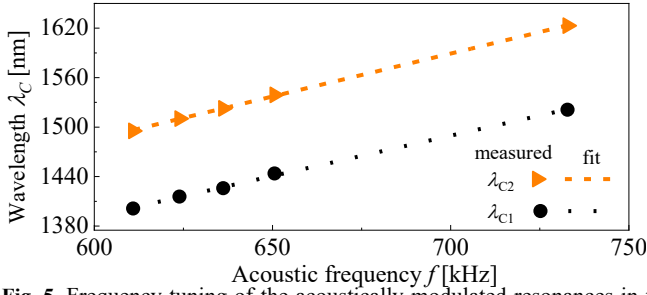


Fig. 5. Frequency tuning of the acoustically modulated resonances in the transmission spectrum of the HL-HCF.

induced strain in a solid silica jacket. We expect that increasing HL-HCF core, tubes and kagomé cladding diameters (and reducing tubes' and Kagomé struts thickness) might further enhance the modulation efficiency. Overall, the HL-HCF with suspended tubes provides an extra degree of freedom to amplify the strain compared to the TL and PBG HCFs.

Further advances might also come from HL-HCFs with different dimensions and cladding designs. In particular, having fibers with larger diameters is significantly beneficial to reducing the switching time  $\tau$  of AOMs. Considering the resonance parameters in Fig. 5, a switching time of  $\tau = 33 \mu\text{s}$  is estimated using  $\tau = L / (2\sqrt{0.5\pi D c_{ext} f})$  [5], which is almost half compared to that corresponding to the etched DCF ( $\tau = 57 \mu\text{s}$ ) employing similar frequency values [5]. Moreover, the compact 1.2 mm long acoustic horn considerably decreases the load on the thin PZT, also contributing to improving the AOM efficiency. This horn might still be reduced and optimized for matching the horn ends with the PZT and fiber diameters, while providing an exponentially tapered transition for proper acoustic wave amplification. Besides, using fixing composites with a material approaching the silica properties might also reduce acoustic reflection and absorption, improving the acoustic coupling to the optical fiber.

## V. CONCLUSION

We have experimentally demonstrated the highly efficient interaction of a HL-HCF and flexural acoustic waves. A good agreement between the analytical simulations and measured resonances is observed over a spectral range of 222 nm (1401 to 1623 nm). A maximum modulation depth of 13.7 dB is achieved at 10 V. The AOM modulation efficiency of 1.3 dB/V is higher than in other devices that use reduced diameter fibers. This efficiency is remarkably 2x higher than a 35  $\mu\text{m}$  etched fiber, indicating that the hybrid Kagomé-tubular lattice provides excellent amplification, hence increasing the induced strain in the fiber. The reduced acoustic horn (1.2 x 2 mm) and electrode (90  $\mu\text{m}$ ) contribute to relieving the load on the thin 200  $\mu\text{m}$  PZT, enabling the acoustic modulation of a 240  $\mu\text{m}$  fiber. The combined properties of the HL-HCF and modulator design provide the lowest consumed voltage (10 V), removing the need for power amplifiers. The short interaction length and large fiber diameter are promising to decrease the switching time of AOMs while providing better mechanical stability and protection. These features are highly attractive in applications requiring fast, compact, and efficient all-fiber integrated photonic devices.

TABLE I  
ACOUSTO-OPTIC DEVICES BASED ON DISTINCT FIBER TYPES

Fiber type	Diameter $D$ ( $\mu\text{m}$ )	Length (cm)	Tuning range (nm)	Efficiency (dB/V)	Reference
HL-HCF	240	7.7	1401–1623 (222 nm)	1.3	This work
SMF – ET	33	12.0	1080–1580 (500 nm)	1.1	[1]
PBG-HCF	120	12.0	1520–1580 (60 nm)	0.8	[9]
DCF – ET	35	4.8	1510–1590 (80 nm)	0.5	[6]
TL-HCF	200	7.7	743–1355 (612 nm)	0.4	[10]
FMF	125	N.A.	1500–1600 (100 nm)	0.2	[2]
SMF	125	N.A.	1543–1558 (15 nm)	0.2	[3]

Geometric parameters and efficiencies of AOMs employing distinct fiber geometries (ET = etched). N.A.: not available.

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