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Measuring Hierarchiness of Image Segmentations

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Abstract—Numerous segmentation methods are able to produce several partitions of the same image by tuning a scale parameter. In such a series of multilevel segmentations, if every region at a given level is included in a single region of the segmentation at the next level, then the series is called a hierarchy. Hierarchies are often desired for multiscale image representation and analysis due to their mathematical properties, leading to accurate and efficient solutions. Although certain effective strategies may not produce a hierarchy, it is uncertain whether their multiscale output is close to be one. This work explores several cases when analyzing two consecutive segmentations, as full inflation and full merge, for instance. From those, we provide three measures for evaluating the hierarchiness between two subsequent partitions: (i) nestedness; (ii) refinement error; and (iii) inflation ration. Using our proposals in a in-sequence pairwise comparison, as shown by the experimental results, it is possible to verify whether a multiscale segmentation is a hierarchy and, if not, to analyze the nature and extent of the hierarchical errors that prevent it from becoming hierarchical.

I. INTRODUCTION

In segmentation, an image is partitioned into several regions so that the object can be built by its comprising parts (ideally only one). However, in certain applications, the mere construction of one partition is insufficient for calculating, for instance, the object’s saliency. For that, one may generate several partitions so that the object information is presented in several and different granularities, from few to many regions produced (*i.e.*, coarser to finer segmentations). Consequently, finer regions describe the object’s characteristics with higher level of details, crucial for extracting local information, whilst coarser ones describe global information of the object, reducing the redundancy from finer partitions. Several works [1], [2] recur to such *multiscale segmentation*, where each segmentation is named as *scale* or *layer*.

Two strategies are often used for computing a multiscale segmentation. One builds a *hierarchical segmentation* so that any desired partition can be obtained efficiently. If all possible partitions are generated, then it is a *dense* hierarchy. But, if only a subset of the latter suffices, then it is *sparse* [3], as presented in Fig. 1. Conversely, *non-hierarchical* approaches build the necessary amount of partitions through one or several executions of the same algorithm. In the end, effective

hierarchical and non-hierarchical approaches aim to accurately delineate the objects of interest in all scales.

Although every hierarchical segmentation sequence is a multiscale one, the converse is not necessarily true since the former only made possible if every region on a finer scale is included in the subsequent coarser scale. By executing the same algorithm with different parameters, several approaches [4], [5] output multiscales that violate two principles in hierarchical segmentation [6]. If borders of coarser regions do not overlap those at finer scales, then it violates the *locality principle*. Also, if the quantity of regions varies in a non-strict monotonical behavior within a multiscale, then it violates the *causality* principle. As a result, numerous properties on hierarchies, which may assist in terms of efficiency and efficacy, are not applicable in such series of partitions, independently of the degree of violation. For instance, the task of generating scales presenting effective object delineation can be reduced as a meticulous node-selection procedure only in hierarchical approaches. Conversely, non-hierarchical strategies [5] allow object delineation corrections at each new execution of their strategy by not imposing a hierarchical nested relation between scales. As an example, by removing one region existent in a previous scale, the subsequent one may correct internal inaccurate borders by inducing a novel competition for the newly “orphan” elements. For hierarchical approaches, however, the existence of incorrect delineations is perpetuated or even aggravated as the hierarchy is being built [7], since their single region-operation is either merging or splitting regions. The latter occurs independently whether is a top-down or bottom-up approach. Furthermore, if no node presents accurate object delineation within a hierarchy, generating an output that maximizes delineation is challenging.

For a given non-hierarchical multiscale in which violations exist, it may be possible to extract information so that the objective is easily achievable. As an example, if there is a causality violation, one may order the series so that the quantity of regions is strictly monotonically decreasing. In contrast, for locality violations, one must evaluate their magnitude since they may be dismissed in non-crucial regions for solving a problem, especially when state-of-the-art delineation performance is achieved in such segmentation. If one aims to

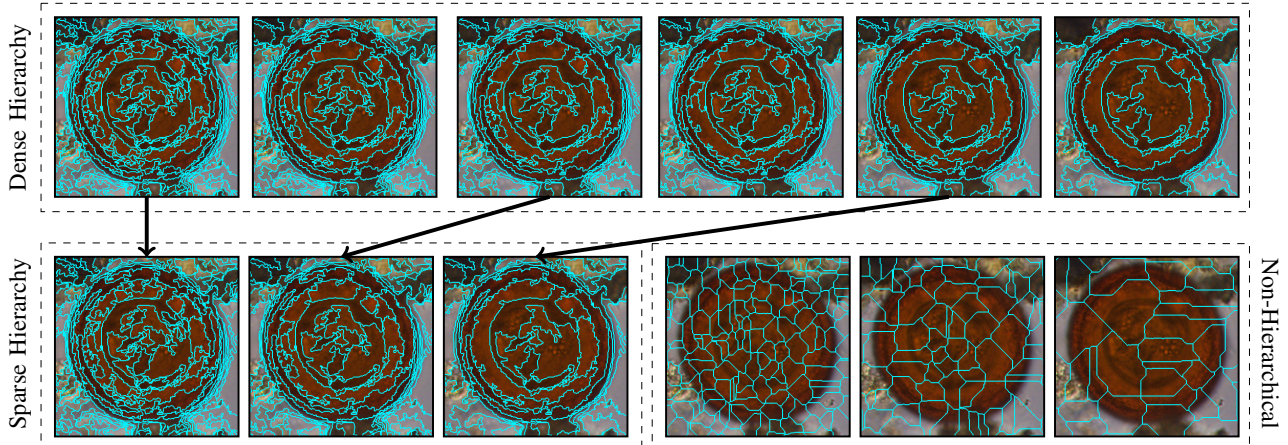


Fig. 1. Different multiscale segmentation outputs (borders depicted in cyan) of an image of a helminth egg. Images were cropped for visualization purposes.

hierarchically segment an object, it suffices to guarantee that object regions present a nested behavior in between partitions, implying that hierarchical background segmentation, although existent, may not be necessary. Consequently, one may exploit hierarchical properties for optimization and simplification of segmentation tasks in regions crucial for the user. If a measure determines the resemblance of a non-hierarchical multiscale segmentation resembles to a hierarchical one, one could relax several hierarchical definitions and exploit, up to some extent, hierarchical properties if such ratio is significantly low.

Although one may analyze the nested behavior between sequential partitions to verify whether a given multiscale segmentation is hierarchical or not, such binary measure does not provide insights into whether the given multiscale is “almost” a hierarchy (*i.e.* slight or severe violations). Even using hierarchical structures, violations may occur when defining objects [8]. Also, if one considers the possibility of adding, removing, or reordering scales, such problem becomes combinatorial. The latter stays unchanged even when considering only two subsequent partitions in the multiscale.

As consequence, and to the best of our knowledge, the existence of such a measure is unknown. Most of the works somewhat related require a ground-truth for segmentation quality assessment [9]–[11], but not for a hierarchiness measurement. Simply put, such proposals do not assist in determining whether a multiscale segmentation is close to a hierarchy, but whether a hierarchical segmentation achieves an effective object delineation. One could recur to our approach and of others for two, but different analysis. Although one may argue that, in our approach, the next scale is the contour ground-truth of its predecessor, we consider all borders equally relevant, differing from [12], which weights differently based on the object’s contours. Consequently, the study of strategies for transforming a multiscale segmentation into a hierarchical one are hampered by such challenge.

In this work, we characterize eight different modifications a region may present when considering two subsequent partitions: (i) full inflation; (ii) full merge; (iii) merge and inflation; (iv) instability; (v) stability; (vi) full deflation; (vii) full split;

and (viii) deflation and split. From such characterization, we propose three distinct measures for measuring the degree of locality violations in a series of partitions: (a) **nestedness**; (b) **inflation ratio**; and (c) **refinement error**. For evaluating whether a multiscale segmentation resembles a hierarchy, one may apply such measures for each pair of subsequential partitions in the sequence. Using **nestedness**, we estimate the ratio of nested regions between the segmentations, being maximum when it is a hierarchy. On the other hand, **inflation ratio** calculates the ratio of inflation, which leads to locality violations. Finally, **refinement error** is an error measure that estimates the cost of “correcting” the precedent scale to become nested in its subsequent one. Experimental results show that, using the proposed measures, it is possible to verify if a multiscale segmentation is a hierarchy and, if not, to measure the degree of locality errors from a local or global perspective, that is, to provide a deep analysis of the violations in between partitions in a multiscale segmentation, each measure can be computed with respect to each region or to the whole partition, respectively. Finally, from the information computed by each measure, one can assert the nature of the errors that prevent it from becoming hierarchical.

This work is organized as follows. Section II presents the necessary mathematical background for a full understanding of our proposal, which is presented and detailed in the subsequent section (*i.e.*, Section III). Using a state-of-the-art multiscale segmentation algorithm, experimental results are shown and discussed in Section IV and, finally, we draw some conclusions and possible future work in Section V.

II. MATHEMATICAL BACKGROUND

In this section, we recall mathematical definitions based on the work of [13], [14] necessary for comprehending our work. Throughout this paper, Υ denotes the universe of elements (*i.e.*, a *set*). The *size* (*i.e.*, number of elements) of Υ , for instance, is here denoted by $|\Upsilon|$. Also, let $\Pi : \Upsilon \mapsto \mathbb{P}(\Upsilon)$, being \mathbb{P} the *power set*, be a function which defines all *partitions* in Υ such that, $\forall P \in \Pi(\Upsilon)$, $\cup P = \Upsilon$ and $\cap P = \emptyset$. We term each element of a partition as *blocks* or *regions*. When

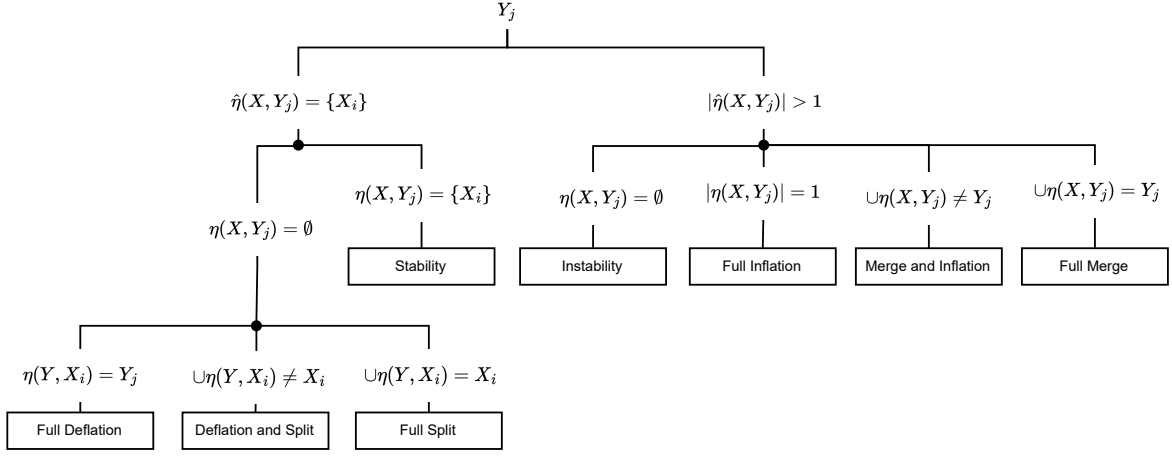


Fig. 2. Diagram depicting all eight possible cases for building a region $Y_j \in Y$ from the regions of the partition X .

each element $u \in \Upsilon$ is, also, a block in a partition $P \in \Pi(\Upsilon)$, then we say that P is the *identity partition*. Conversely, if $P = \{P_1\}$ is composed of a single block P_1 in which, $\forall u \in \Upsilon, u \in P_1$, then P is said to be the *universal partition*. Finally, given a *subset* $E \subseteq \Upsilon$, we may term $Q \in \Pi(E)$ and $Q \in \Pi^*(\Upsilon)$ as a *partial partition* of Υ , thus, $\Pi^*(\Upsilon)$ is the set of all partial partitions of Υ since it is a partition of E but not necessarily of Υ .

We may order the set $\Pi(\Upsilon)$ by *quantity*. Thus, given two partitions $P, Q \in \Pi(\Upsilon)$, we say that P is *bigger* than Q (denoted by $P \supseteq Q$) if, and only if, $|P| \geq |Q|$. Then, in such case, Q is said to be *smaller* than P (denoted by $Q \leq P$). Similarly, it is possible to partially order the set $\Pi(\Upsilon)$ by *refinement* such that, for instance, P is considered *finer* than Q (denoted by $P \preceq Q$) if and only if, for every region $p \in P$, exists a unique $q \in Q$ in which $p \subseteq q$. Thus, Q is said to be *coarser* than P and we write such information by $Q \succeq P$. In this work, we say that P is a *downscale* of Q and, thus, Q is an *upscale* of P . One can see that $P \preceq Q$ implies that $P \supseteq Q$, but the converse is not true. As one may note, the task of (*hard*) *segmentation* highly correlates with the aforementioned concepts of partition and order relations. For instance, a partition $X \in \Pi(\Upsilon)$ is then referred as a *segmentation* of Υ . Also, for a given series of segmentations $\mathcal{M} = \{S_1, \dots, S_k\}$ — which we usually term as a *multiscale* or *multilevel* segmentation — we may order them by their size (i.e., by $\langle \mathcal{M}, \supseteq \rangle$) such that $S_i \supseteq S_{i+1}$ for some $i \in [1, k-1]$ and $S_i \in \Pi(\Upsilon)$. On the other hand, if it is possible to order \mathcal{M} by refinement, thus, by $\langle \mathcal{M}, \preceq \rangle$, then we say that it is a *hierarchical segmentation*. In both cases, after ordering, we often refer to a segmentation $S_i \in \mathcal{M}$ as a *layer* or *level*, and S_{i-1} and S_{i+1} as the *previous* and *next* layer in \mathcal{M} .

III. HIERARCHINESS

In this section, we will introduce the concepts of *nucleus* and *cover*, which are crucial for identifying the specific case, amongst eight different ones (illustrated by Fig. 2), of a region in a partition $S_{i+1} = Y$ being built by the regions of $S_i = X$, given a multiscale segmentation $\mathcal{M} = \{S_1, \dots, S_k\}$ and $1 \leq$

$i < k \in \mathbb{N}$. Furthermore, with such definitions, we present three different measures for evaluating and analyzing whether \mathcal{M} is close to being a hierarchy.

A. Nucleus and Cover

One can perceive the *nucleus* of a coarser region as the group of those blocks at the finer scale totally included in the former (i.e., nested). In contrast, its *cover*, can be seen as the set of regions in the preceding scale necessary for building the coarser block. In this sense, building is the partial or total selection of elements in each region in the cover so that the result is equivalent. Mathematically speaking, we define the *nucleus* and *cover* of a region as follows. Given a multiscale segmentation \mathcal{M} and two subsequent layers $X = \{X_1, \dots, X_r\}$ and $Y = \{Y_1, \dots, Y_c\}$ in \mathcal{M} , we define the *nucleus* $\eta(X, Y_j)$ and the *cover* $\hat{\eta}(X, Y_j)$ of a region $Y_j \in Y$ with respect to X as $\eta(X, Y_j) = \{X_i \in X : X_i \subseteq Y_j\}$ and $\hat{\eta}(X, Y_j) = \{X_i \in X : X_i \not\subseteq Y_j\}$, respectively, for $1 \leq i \leq r = |X|$ and $1 \leq j \leq c = |Y|$, in which $X_i \not\subseteq Y_j$ indicates that X and Y *overlap* (i.e., $X_i \cap Y_j \neq \emptyset$). When this is not the case, we say that X_i and Y_j are *disjoint*. As one may note, $\cup \eta(X, Y_j) \subseteq Y_j \subseteq \cup \hat{\eta}(X, Y_j)$ and, consequently, $|\cup \eta(X, Y_j)| \leq |Y_j| \leq |\cup \hat{\eta}(X, Y_j)|$. Finally, both definitions are also applicable to any region $X_i \in X$ with respect to Y .

B. Region Cases

From Figure 2, we are able to identify eight possible cases, listed and defined below:

- *Full Deflation*: $\hat{\eta}(X, Y_j) = \{X_i\}$, $\eta(X, Y_j) = \emptyset$ and $\eta(Y, X_i) = \{Y_j\}$;
- *Deflation and Split*: $\hat{\eta}(X, Y_j) = \{X_i\}$, $\eta(X, Y_j) = \emptyset$ and $\cup \eta(Y, X_i) \neq X_i$;
- *Full Split*: $\hat{\eta}(X, Y_j) = \{X_i\}$, $\eta(X, Y_j) = \emptyset$ and $\cup \eta(Y, X_i) = X_i$;
- *Stability*: $\hat{\eta}(X, Y_j) = \eta(X, Y_j) = \{X_i\}$;
- *Instability*: $|\hat{\eta}(X, Y_j)| > 1$ and $\eta(X, Y_j) = \emptyset$;
- *Full Inflation*: $|\hat{\eta}(X, Y_j)| > 1$ and $|\eta(X, Y_j)| = 1$;
- *Merge and Inflation*: $|\hat{\eta}(X, Y_j)| > 1$, $|\eta(X, Y_j)| > 1$ and $\cup \eta(X, Y_j) \neq Y_j$; and

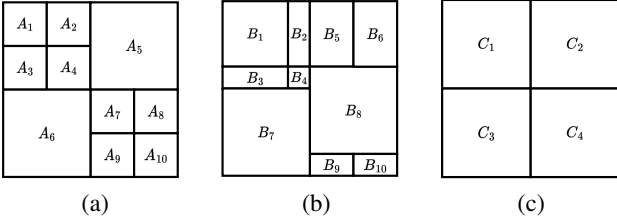


Fig. 3. Three distinct partitions of a 8×8 square universe Υ for illustrating the eight possible cases when evaluating two partitions.

- *Full Merge*: $|\eta(X, Y_j)| > 1$ and $\cup \eta(X, Y_j) = Y_j$;

We can exemplify each one of the aforementioned cases from the partitions depicted in Figure 3. For instance, while verifying whether B is an upscale of A , we can see that B_1 is built through full inflation, since $\eta(A, B_1) = \{A_1\}$. For B_2 and B_3 , one may notice that they are constructed by instability given that both present empty nuclei and more than one element in their cover. In contrast, B_7 is equivalent to A_6 (i.e., stable). Also, B_4 is constructed through full deflation because it has an empty nucleus, $\hat{\eta}(A, B_4) = \{A_4\}$ and that $B_4 \in \eta(B, A_4)$. Like B_4 , B_9 and B_{10} are built through full deflation. Both B_5 and B_6 are resultant from deflation and split. Finally, we can see that B_8 is built from merging and inflation. For illustrating the case of a full merge, one can refer to A_1, A_2, A_3, A_4 for building C_1 , and the converse exemplifies the case of a full split. Thus, when $X \preceq Y$, the sole case possible is the full merge of regions and, clearly, it is analogous for $X \succeq Y$ and full split (e.g., A and C).

C. Hierarchiness Measures

Based on the previous measures, we can define three distinct measures for evaluating two subsequent partitions X, Y in a multiscale segmentation \mathcal{M} . First, we recur to a $r \times c$ matrix $\mathbf{M}(X, Y) = [\mathbf{M}(X, Y)_{ij}]$ in which $r = |X|$, $c = |Y|$, and each element is defined by $\mathbf{M}(X, Y)_{ij} = |X_i \cap Y_j|$, given $X_i \in X$ and $Y_j \in Y$. Also, we may normalize each element by $|\Upsilon|$ thus resulting in the matrix $\mathbf{M}_\Upsilon(X, Y)$ whose elements are $\mathbf{M}_\Upsilon(X, Y)_{ij} = |X_i \cap Y_j|/|\Upsilon|$. Similarly, the elements of the matrix $\mathbf{M}_Y(X, Y)$ are defined as $\mathbf{M}_Y(X, Y)_{ij} = |X_i \cap Y_j|/|Y_j|$, and it is analogous for $\mathbf{M}_X(X, Y)$.

Given two regions $X_i \in X$ and $Y_j \in Y$, we may quantify the property of $X \in \eta(X, Y_j)$ through the *nestedness* measure $\mathbf{NE}(X_i, Y_j) \in \{0, 1\}$ defined by $\mathbf{NE}(X_i, Y_j) = \mathbf{1}(X_i \subseteq Y_j)$ or, equivalently, by $\mathbf{NE}(X_i, Y_j) = \mathbf{1}(\mathbf{M}_X(X, Y)_{ij} = 1)$, in which $\mathbf{1}$ is an *indicator function* that outputs “0” or “1” if the input boolean expression is false or true, respectively. The *inflation ratio* $\mathbf{IR}(X_i, Y_j) \in \{0, 1\}$ evaluates whether any inflation has through $\mathbf{IR}(X_i, Y_j) = \mathbf{1}(X_i \in \hat{\eta}(X, Y_j) \setminus \eta(X, Y_j))$, which can be computed by $\mathbf{IR}(X_i, Y_j) = \mathbf{1}(0 < \mathbf{M}_X(X, Y)_{ij} < 1 \wedge 0 < \mathbf{M}_Y(X, Y)_{ij} < 1)$, considering \wedge the logical operator AND. In contrast and inspired from [15], the *refinement error* $\mathbf{RE}(X_i, Y_j) \in [0, 1]$ can be defined by $\mathbf{RE}(X_i, Y_j) = \min\{|X_i \cap Y_j|, |X_i \setminus Y_j|\}/|X_i|$ and quantify the lowest “effort” to make $X_i \subseteq Y_j$. We can compute it, deriving from the latter equation, through $\mathbf{RE}(X_i, Y_j) = \min\{\mathbf{M}_X(X, Y)_{ij}, 1 - \mathbf{M}_X(X, Y)_{ij}\}$ but

imposing a harsher penalization by calculating it as $\mathbf{RE}(X_i, Y_j) = \min\{1, (1 - \mathbf{M}_X(X, Y)_{ij})/\mathbf{M}_X(X, Y)_{ij}\}$.

For two segmentations $X, Y \in \mathcal{M}$ of a given image containing $|\Upsilon|$ pixels, the effort to compute $\mathbf{M}(X, Y)$ and any presented measure is $O(|\Upsilon|^2)$ and $O(|X||Y|)$, respectively. Also, one can compute each measure for Y_j with respect to all partitions in X . For instance, $\mathbf{NE}(X, Y_j) = \sum_{i=1}^r \mathbf{NE}(X_i, Y_j) \mathbf{M}_Y(X, Y)_{ij}$, resulting in a normalized measure (with respect to Y_j). Similarly, it is possible to evaluate the $\mathbf{NE}(X, Y)$ between both partitions X and Y through $\mathbf{NE}(X, Y) = \sum_{i=1}^r \sum_{j=1}^c \mathbf{NE}(X_i, Y_j) \mathbf{M}_\Upsilon(X, Y)_{ij}$. As one may note, the aforementioned possibilities are also valid for \mathbf{IR} , and \mathbf{RE} .

When evaluating $X, Y \in \mathcal{M}$, if $X \preceq Y$, it is clear to see that $\mathbf{NE}(X, Y) = 1$, and $\mathbf{IR}(X, Y) = \mathbf{RE}(X, Y) = 0$. Conversely, if $X \succeq Y$, we have that $\mathbf{NE}(X, Y) = \mathbf{IR}(X, Y) = \mathbf{RE} = 0$. From Figure 3 and using B_8 as an example of merge and inflation, we have that $\mathbf{NE}(A, B_8) = 0.5$, $\mathbf{IR}(A, B_8) = 0.5$, and $\mathbf{RE}(A, B_8) = 0.5$, indicating that 50% of B_8 was built from inflation and 50% from merging. The refinement error indicates that 50% of B_8 present multiple region overlap. In contrast, for C_1 , which is built solely from full merge, $\mathbf{NE}(A, C_1) = 1$, $\mathbf{IR}(A, C_1) = 0$, and $\mathbf{RE}(A, C_1) = 0$.

IV. EXPERIMENTAL RESULTS

In this section, we provide the experimental setup and we analyze and discuss our findings. We chose two medical image datasets, named *Liver* [16] and *Parasites* [17], whose objects (i.e., human liver and intestinal parasite eggs) present smooth borders and impose a challenge in terms of delineation. Also, the object and background stability allows a more confident analysis, differently from natural image datasets in which both vary significantly. Since we propose non-optimized measures, we used all images in the dataset in the experiments. Also, in order to properly assess our proposed measures considering different multiscale segmentations (from different approaches), we selected four methods: (i) Voronoi; (ii) SICLE-COMP [5]; (iii) SICLE-IRREG [5]; and (iv) SH [18]. We use Voronoi as a representative of a method that minimizes nestedness between segmentations. For that, we generate random seeds and partition the image by minimizing the spatial distance to a seed. In contrast, Superpixel Hierarchy (SH) is a hierarchical approach and, thus, provides insight into hierarchical multiscale segmentation. Finally, SICLE-COMP and SICLE-IRREG are variants of a state-of-the-art method in superpixel segmentation named *Superpixels through Iterative CLEArcutting* (SICLE), which provides a non-hierarchical multiscale segmentation on-the-fly. The motivation relies on the resemblance of a hierarchical segmentation output, although such a condition is not assured. Finally, for all methods, we generated 750, 500, 250, and 100 regions in each segmentation and we analyzed, from finer to coarser, the following pairs of partitions: (a) (750, 500); (b) (500, 250); and (c) (250, 100). A demonstration code is

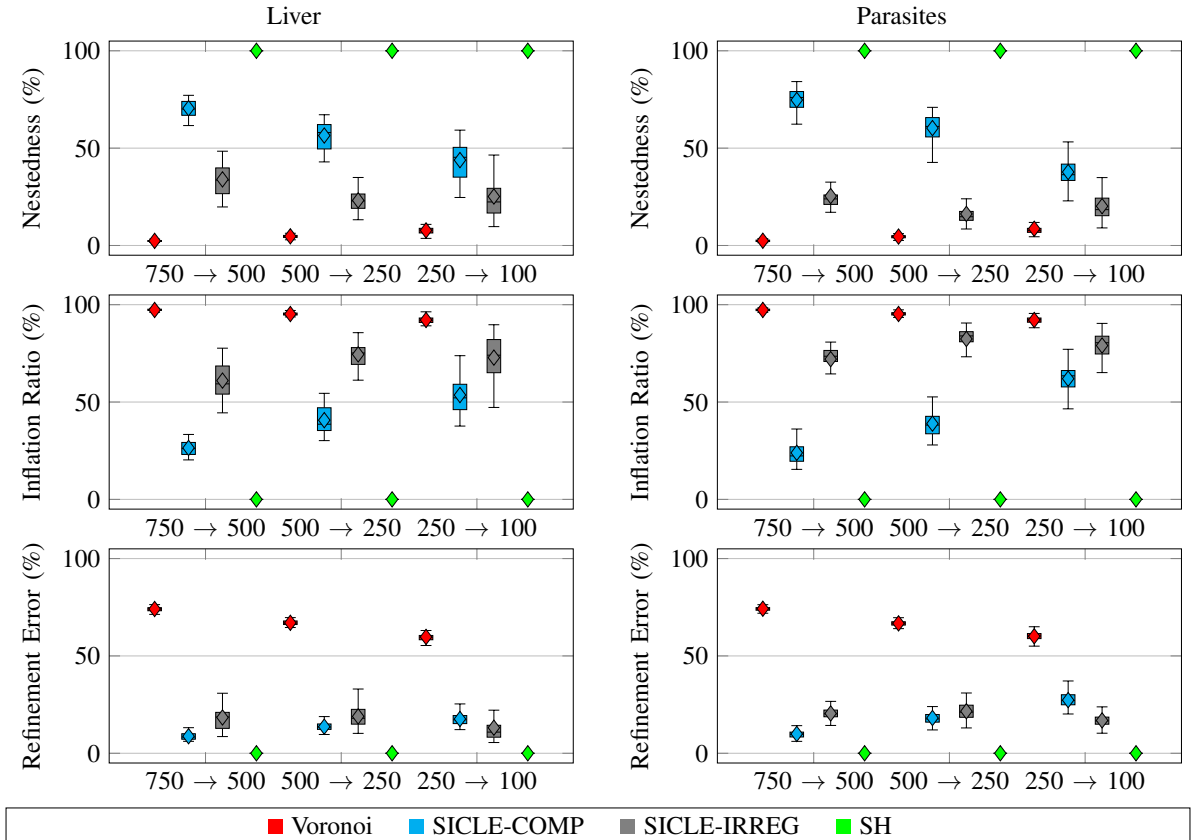


Fig. 4. Experimental results of the proposed measures on Liver and Parasites datasets (resp. left and right columns), whose average is depicted as a diamond.

publicly available*¹.

A. Quantitative Analysis

Figure 4 presents the average results of each method in Liver and Parasites datasets. It is clear to see that SH, a hierarchical method, achieves the expected behavior for all measures: maximum for nestedness and minimum for inflation ratio and refinement error. Likewise, Voronoi, due to its unstable partitioning strategy, undermines nesting regions which explains its almost zero nestedness and quasi-maximum inflation ratio. In terms of refinement error, we argue that the Voronoi results are high due to the significant impact that random sampling imposes on the closest points to a seed. When considering $750 \rightarrow 500$, the competition for conquering pixels is near the seed, implying that slight changes in the seed coordinates severely affect the region, being necessary to substantially be reshaped for being nested in some other region in the following segmentation. However, as the number of regions decreases, the competition is farther from the seed, limiting such impact mostly on the region’s borders, which demands fewer modifications.

When analyzing the results from SICLE variants, one may note that the behavior is similar between datasets. It is interesting to notice that SICLE-COMP, which aims for compact regions, achieves higher nestedness and lower inflation ratio than its irregular counterpart. We infer that, by imposing a

spatial regularity factor, regions tend to present similar shapes and be less impacted by the removal of regions far from them.

Conversely, slight modifications in the competition may alter drastically the regions’ shape when the goal is irregularity. Consequently, although both rarely promote merging behavior, the latter favors inflation whereas the former favors stability. The same argument goes for the refinement error. That is, by favoring stability, SICLE-COMP achieves lower errors with less variation than SICLE-IRREG, whose slightly higher results, but with higher variation, are a result of promoting inflation.

B. Qualitative Analysis

As one may note in Figs. 5(c-a), several regions in SICLE-COMP remained unaltered from one scale to the subsequent, achieving $\text{NE}(C_{500}, C_{250}) = 71\%$, $\text{IR}(C_{500}, C_{250}) = 30\%$, and $\text{RE}(C_{500}, C_{250}) = 14\%$, when considering C_{500} and C_{250} . However, as the number of regions decreases, regions tend to be closer to one another and, thus, removing one or more nearby regions may destabilize the ones left out. As an example, for C_{250} and C_{100} , we have that $\text{NE}(C_{250}, C_{100}) = 36\%$, $\text{IR}(C_{250}, C_{100}) = 64\%$, and $\text{RE}(C_{250}, C_{100}) = 27\%$.

On the other hand, seeing the finer segmentation from SICLE-IRREG in Figure 5(f-d), even when removed regions are far from those left out, it may affect local tie-zones which, for a function that maximizes irregularity, can significantly alter the regions’ shape in between partitions. Consequently,

¹* <https://github.com/IMScience-PPGINF-PucMinas/hierarchiness.git>

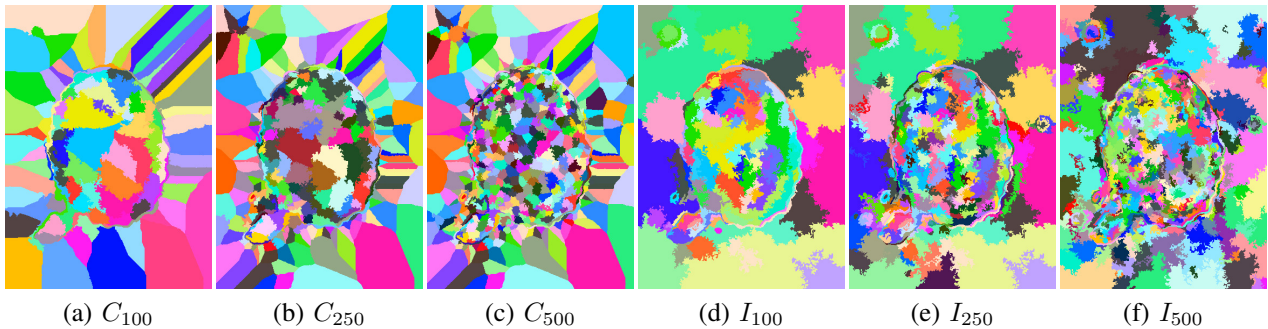


Fig. 5. Pseudocolored segmentations using SICLE-COMP and SICLE-IRREG, respectively, identified by C_n and I_n , for $n \in \{100, 250, 500\}$ regions.

for I_{500} and I_{250} , we have that $\mathbf{NE}(I_{500}, I_{250}) = 16\%$, $\mathbf{IR}(I_{500}, I_{250}) = 83\%$, and $\mathbf{RE}(I_{500}, I_{250}) = 34\%$. In contrast, when considering coarser scales, the amount of tie-zones is significantly diminished, leading to more nested regions (*i.e.*, unaltered) and less inflation and locality errors, specifically, for I_{250} and I_{100} , $\mathbf{NE}(I_{250}, I_{100}) = 33\%$, $\mathbf{IR}(I_{250}, I_{100}) = 67\%$, and $\mathbf{RE}(I_{250}, I_{100}) = 18\%$.

V. CONCLUSION AND FUTURE WORK

In this paper, we analyze the “hierarchiness” between two partitions in a multiscale segmentation and, by defining eight possible situations, we propose three different measures for assisting in the estimation of how far a multiscale segmentation resembles a hierarchy. Experimental results show that our proposals are consistent and offer a wide and complementary overview on each of the four algorithms considered. For future work, we aim not only to study and propose new measures but, also, to conceive new approaches for measuring the segmentation series integrally, alongside strategies for maximizing the hierarchiness of any multiscale segmentation. The latter may allow optimization solutions of partition-related problems, as in [19], [20].

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