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Identification of an Arbitrary-Surface Harmonic Magnetic Model from Close Measurements

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Decreasing spherical harmonic functions are widely used to identify and extrapolate the magnetic field produced by various devices. These functions allows to represent the sources as equivalent multipoles whose order is associated to a specific spatial decreasing rate. However this representation is not valid inside the Brillouin sphere, the smallest sphere enclosing the device. We introduce here the use of an alternative model to replace the spherical harmonic functions when the measurements are inside the Brillouin sphere. This representation corresponds to an harmonic basis of equivalent charges on a surface that reproduces the multipolar decomposition of the magnetic field outside the Brillouin sphere while being valid inside. We demonstrate here the ability of this model to identify and extrapolate the field from very close measurements.

Index Terms—Inverse problem, multipolar expansion, near magnetic field computation, single-layer potential, bayesian inference.

I. INTRODUCTION

FOR various applications (including electromagnetic compatibility [1], human exposure [2], ship silencing [3]), the magnetic field produced by a device must be determined all around it. Interpolation on a measured grid and direct numerical modelling of the sources are not always possible and other alternatives must be found. Identification of a compact model from close magnetic measurements is a common approach.

Classically, the external field is represented by a multipolar expansion using the decreasing spherical harmonic functions. Each multipole (e.g., dipole, quadrupole) is associated to a spatial harmonic which presents a specific spatial decreasing law (e.g., $1/r^3$, $1/r^4$, with r the distance to the multipole). Nevertheless, this representation is only valid outside the Brillouin Sphere [4], the smallest sphere that encloses all the sources, and cannot be used in the close vicinity of the device.

On the other hand, some other equivalent sources exist that produce the same field at any point all around it. We propose to study equivalent sources located on a given surface enclosing the device under investigation, that can be chosen as close as possible to it, and to compute the generated magnetic field in the external region without sources. It is thus well known that by exactly knowing the magnetic potential or the normal field on this surface, i.e. the limit conditions of the Laplace problem [5], the field can be determined. Other representations deal with single or double layer potentials on the surface [5]. But if the field can be computed anywhere, including inside the Brillouin sphere, the equivalent sources can not be separated according to their contribution to the device equivalent multipoles, i.e. to its equivalent dipole, quadripole, etc. In previous work [6], we have proposed an harmonic basis and its inner product, in order to project our single or double layer potentials on it and thus to determine the surfacic harmonic distribution whose order is associated to a specific decreasing spherical harmonic functions for the field. Then, for a given precision, the truncation of the source harmonic

source allows us to represent the sources both with a lower degree of freedom and an ordered contribution to the field. Moreover this original approach, by allowing to put sensors closer to source, helps to increase the signal to noise ratio.

In this paper, we propose to use our multipolar representation of equivalent charges to identify the sources from measurement located inside the Brillouin sphere and then to test the extrapolation of the field. In a first part, the construction of our multipolar basis for the single layer potential is presented for illustration. Then, the inverse problem is presented, with the introduction of a Bayesian prior in order to solve it. Finally, the approach is validated on an experimental mock-up.

II. MULTIPOLAR MODEL OF EQUIVALENT SOURCES

Any source can be represented on an enclosing surface S by equivalent charges (or single-layer potential) σ that produce the same field as the source outside this surface. The surface S can be chosen as close to the source as desired. The magnetic field \mathbf{H} and the magnetic scalar potential ϕ can then be computed at any point P outside the surface S [5]:

$$\mathbf{H}(P) = -\nabla \int_{M \in S} \frac{\sigma(M)}{4\pi \|\mathbf{PM}\|} dS \quad (1)$$

$$\phi(P) = \int_{M \in S} \frac{\sigma(M)}{4\pi \|\mathbf{PM}\|} dS \quad (2)$$

This single-layer potential can be decomposed into a multipolar basis $\{\sigma_{km}\}$ [6], [7]:

$$\sigma = \sum_{k=1}^{\infty} \sum_{m=-k}^k c_{km} \sigma_{km} \quad (3)$$

with c_{km} the harmonic coefficients that depend only on the source and σ_{km} the vectors of the basis $\{\sigma_{km}\}$ that depend only on the choice of the surface S and need to be precomputed like defined in [6] (see Fig. 1). In a practical case, the infinite sum is truncated to a given order K .

We recall here that the harmonic basis $\{\sigma_{km}\}$ is constructed by orthonormalizing an initial basis of equivalent charge distributions $\{\sigma_{km}^0\}$ with a specific inner-product [6]. The initial basis is derived from the increasing spherical harmonic functions, we express the distributions of the initial basis at any point (r, θ, φ) on the chosen surface:

$$\sigma_{km}^0(r, \theta, \varphi) = \nabla (r^k Y_k^m(\theta, \varphi)) \cdot \mathbf{n}(r, \theta, \varphi) \quad (4)$$

with \mathbf{n} the outward normal of the surface and $r^k Y_k^m(\theta, \varphi)$ the increasing spherical harmonic function of order k and degree m .

The inner-product to orthonormalize the basis or projected a given charge distribution onto the orthonormalized basis is expressed:

$$\langle \sigma_1 | \sigma_2 \rangle = \int_{M \in S} \sigma_1(M) \tau[\sigma_2](M) dS \quad (5)$$

with $\tau[\sigma_2]$ the distribution of normal dipoles that produced the same field as the distribution of charges σ_2 and that verifies $\int \tau[\sigma_2] dS = 0$.

The construction of the basis $\{\sigma_{km}\}$ (see Fig. 1) ensure the minimum decreasing rate of $1/r^{k+2}$ for the field produced by vector σ_{km} of order k while keeping the advantage of domain validity of an equivalent surface representation. Our goal is now to determine the coefficients c_{km} from a set of magnetic measurements.

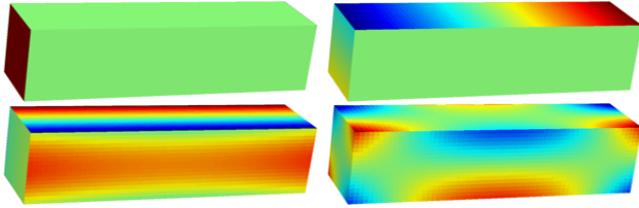


Fig. 1. Representation of vectors from the basis $\{\sigma_{km}\}$ on a cylinder. With from left to right and top to bottom: $k = 1$ and $m = 1$, $k = 2$ and $m = -2$, $k = 3$ and $m = 2$, $k = 4$ and $m = -1$.

III. INVERSE PROBLEM SOLVING

To identify the source harmonic coefficients c_{km} , we construct a linear problem $Ax = b$ where x is the vectors containing unknown source harmonic coefficients, b is the vector containing the field measurements and A is the matrix linking the coefficients to the measurements.

Each coefficient a_{ij} of this matrix (on row i and column j) is equal to the field produced by the j^{th} vector of the basis $\{\sigma_{km}\}$ at the point P_i of the i^{th} sensor, projected on the axis e_i of this sensor.

$$a_{ij} = - \left[\nabla \int_{M \in S} \frac{\sigma_{k(j)m(j)}(M)}{4\pi \|P_i M\|} dS \right] \cdot e_i \quad (6)$$

The harmonic model of equivalent charges can only represent the field produced by a contained source. In our application, the field seen by the sensors is the sum of the field produced by a localized source and the local field produced by the earth core. This local field can be considered known and removed from the measurements, we chose to add it as an

unknown to identify and have modeled it as a uniform field with 3 parameters that are added as unknown in the vector x . In this case, we must add three column to the matrix A , each row of this submatrix is filled with the components of the directions of the sensors e_i .

If there is not enough sensors or if they are not well distributed around the source, the problem is ill-posed and a least-square resolution might require regularization to achieve a satisfying identification.

A. Identification in a Bayesian framework

The Bayesian framework allows to add some prior information to help the identification. In this framework we consider the measurements and the unknown parameters as random variables whose distributions represent the known information on those variables. The Bayesian inference consist in starting from a prior information $P(x)$ on the unknown parameters we are identifying and update this distribution using the measurements information $P(b|x)$ to obtain the posterior distribution $P(x|b)$. Maximizing the posterior distribution is looking for a compromise between the prior and measurement information.

We assume distributions $P(x)$ and $P(b|x)$ as Gaussian, these distributions can then be characterized by a mean (x_0 for $P(x)$ and Ax for $P(b|x)$) and a covariance matrix (S_0 for $P(x)$ and S_m for $P(b|x)$). In this case, we can compute the maximum of the posterior distribution x_{MAP} , which is the state of the unknown parameters x that maximizes the posterior distribution [8]:

$$x_{\text{MAP}} = (A^t S_m^{-1} A + S_0^{-1})^{-1} (A^t S_m^{-1} b + S_0^{-1} x_0) \quad (7)$$

B. Construction of prior

The choice of prior plays a major role in the case of Bayesian inverse problem, and this choice is highly dependent on the studied system. Here we present a method allowing to transfer the prior on the studied system onto a prior on the harmonic coefficients, providing we have a numerical model of the studied system that is able to compute the magnetic field \mathbf{H} or the magnetic scalar potential ϕ .

In this case, for any given set of parameters of the numerical model, we are able to compute the magnetic field \mathbf{H} or scalar potential ϕ on the enclosing surface chosen to represent the harmonic basis of charges. Using (1) or (2), we can determine the equivalent charge distribution by solving a linear system. This charge distribution is then projected on the harmonic basis with respect to inner-product (5) to obtained the harmonic coefficients.

We can then from a prior on the parameters of the numerical model choose several scenarios and determine the coefficients associated to them and deduce a prior on the harmonic coefficients. As the computation of the coefficients from the parameters can be expensive, we want to limit the number of scenarios to run. The unscented transform [9] is a deterministic method to select the scenarios that needs only a few selected scenarios, if N is the number of uncertain parameters for the numerical model, only $2N + 1$ scenarios are necessary to construct the prior on coefficients.

IV. EXPERIMENTAL VALIDATION

A. The physical mock-up

We consider a magnetized open hollow steel cylinder with a diameter of 12 cm, a length of 50 cm and a width of 2 mm. It is equipped with two lines of 14 biaxial sensors which measure the longitudinal and orthoradial components of the field. Because a sensor has a broken axis, we total 27 axis measurements located at 8 mm far from the cylinder external surface, for the identification (Fig. 2). A triaxial sensor B1 (Fig. 2) placed at 8 cm under the cylinder is used to compare the extrapolation of the field on the Line 3, corresponding to a span of 3.4 m.

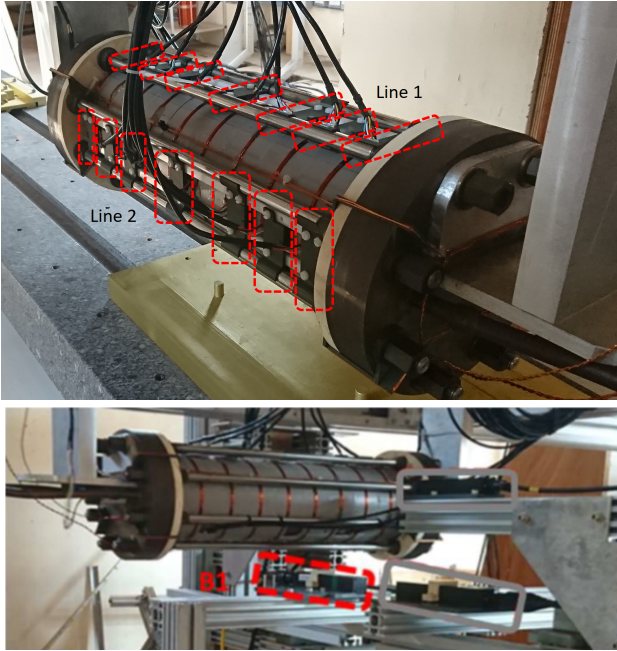


Fig. 2. Above: Hollow cylinder and identification sensors on Line 1 and 2, below: Line 3 is obtained by moving the cylinder along its longitudinal axis, above the sensor B1, for validation.

B. The numerical models

We present here the numerical models used to determine the prior (x_0 and S_0) for the harmonic coefficients. First of all, the basis is constructed on a parallelepiped of dimensions (50 cm \times 12 cm \times 12 cm) that encloses the cylinder and that is meshed with 2400 quadrangles (see Fig. 3). As the sensors are very close to the source we identify the harmonic coefficients c_{km} up to the 15th order (i.e. $K(K+2) = 255$ coefficients to identify).

Then, a numerical model of the cylinder is produced. As very thin, it is represented by a surface, meshed by 600 rectangular elements (see Fig. 4). Its magnetization state in the earth field is classically described by the sum of a permanent magnetization and a reversible magnetization. The reversible magnetization only depends on the reversible permeability $\mu_{r,rev}$ of the cylinder material and the ambient field H_0 . The permanent magnetization results from a magnetizing permeability $\mu_{r,pm}$ under a magnetizing field H_{pm} . The magnetization is then a combination of these two computations.

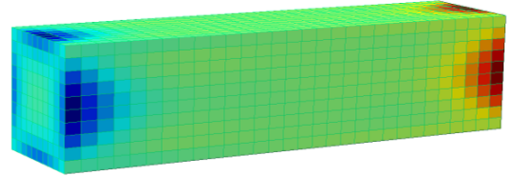


Fig. 3. Charge distribution constructed with the distributions of the basis $\{\sigma_{km}\}$ and the harmonic coefficients obtained from the inversion

Then, for a given magnetic state, we are able to compute the magnetic field \mathbf{H} or the magnetic scalar potential ϕ [10]. As the magnetic state is unknown, our prior will focus on the 8 parameters of $\mu_{r,rev}$ and $\mu_{r,pm}$, and the 6 components of H_0 and H_{pm} , which implies only 17 forward simulations with the use of the unscented transform [9], to compute the prior (x_0 and S_0).

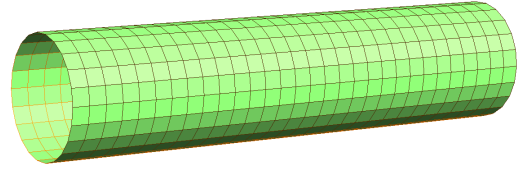


Fig. 4. Mesh of the cylinder for the numerical model.

C. Results

The measurements on the 2 close lines are used with the prior on the parameters to identify the c_{km} coefficients and the local field H_0 . The chosen prior correspond to Gaussian distributions for the 8 parameters (see Table I for means and standard deviations of the parameters). The obtained covariance matrix S_0 strongly links harmonic coefficients together to fit to plausible magnetic states of the cylinder.

TABLE I
MEANS AND STANDARD DEVIATIONS OF THE PARAMETERS OF THE NUMERICAL MOCK-UP TO CONSTRUCT THE PRIOR.

parameter	mean	standard deviation
$H_0 x$	0 μT	15 μT
$H_0 y$	1 μT	15 μT
$H_0 z$	0 μT	15 μT
$\mu_{r,rev}$	100	5
$H_{pm} x$	0 μT	15 μT
$H_{pm} y$	0 μT	15 μT
$H_{pm} z$	-1 μT	15 μT
$\mu_{r,pm}$	2000	500

The variance matrix S_m is chosen to take into account the measurement noise, here evaluated to a 20 nT standard deviation Gaussian noise, the measurements errors (offsets, position, alignment), here evaluated to a 500 nT standard deviation Gaussian noise, and the model errors (due to the truncation and precision of the basis). This model error is evaluated using the prior magnetic state of the cylinder, determining the associated charge distribution and harmonic coefficients, we then take as the evaluation of the model error the difference of the field on sensors given by the equivalent distribution and the projected distribution with a security multiplicative factor of 1.5. Here

we evaluated it to a $1 \mu\text{T}$ standard deviation Gaussian noise. In the model error we did not consider the error due to the discretisation as it is minor in this particular case.

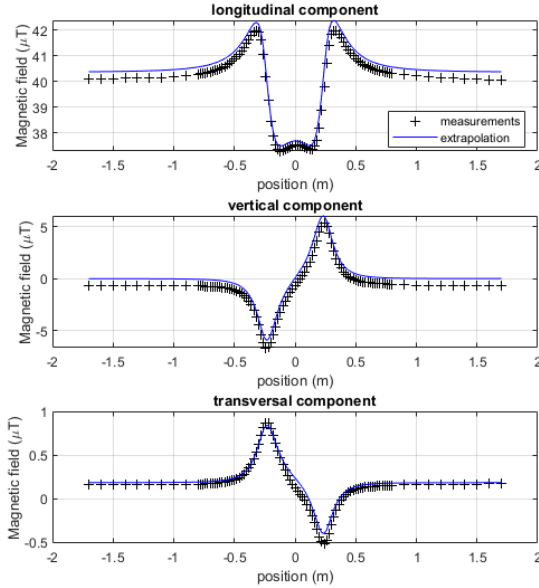


Fig. 5. Field produced by the cylinder on Line 3, comparison between the measurements and the model with the identified coefficients c_{km} (extrapolation)

Once the coefficients c_{km} and the local field H_0 are identified by (7), we extrapolate the field produced by the cylinder on Line 3 (see Fig. 5). We achieve an accurate extrapolation with less than 8.1% difference on every components of the magnetic anomaly, i.e. the field variation with respect to the ambient field (see Fig. 6 and Table II) demonstrating the good performance of this new representation in the case of very close measurements. The ambient field H_0 is also very well estimated with less than 1.6% relative error (Table II). The error on the anomaly components is computed with respect to the maximum anomaly modulus.

TABLE II

ERRORS ON THE LOCAL FIELD AND MAXIMUM ERRORS ON THE ANOMALY FOR EACH COMPONENT.

component	error (%)
$H_0 x$	0.77
$H_0 y$	1.6
$H_0 z$	0.026
max anomaly x	6.4
max anomaly y	8.1
max anomaly z	1.9

V. CONCLUSION

In this paper, the harmonic equivalent charge model we have proposed in [6] has been successfully tested to identify the magnetic state of a physical mock-up, by solving an inverse problem with very close magnetic field measurements and a Bayesian approach. The extrapolated magnetic anomaly has been determined with a maximum relative error inferior to 8.1% and the ambient field with less than 1.6% error, which is very satisfying.

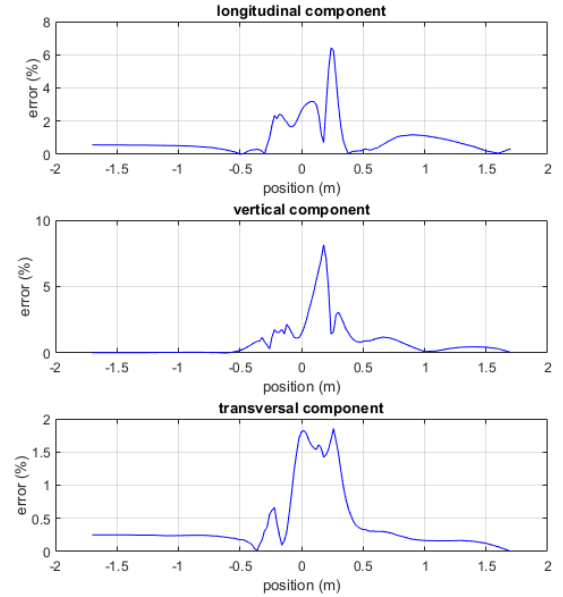


Fig. 6. Relative error of the anomaly of the extrapolated field.

We can note the high model error mainly due at this stage to the unstable orthonormalization process that causes a loss of orthogonality of the basis starting around the 5th to 10th order. Improving the basis should lead to a better identification.

Also the physical mock-up used has a simple geometry for which the construction of a good prior is easy, next steps should be about testing this method with a degraded prior on more complex objects.

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