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# Direct Model Reference Adaptive Control for Linear Fractional System of Commensurate Order with Time Delay Using Smith Predictor

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**Abstract**—In this research work, a model reference adaptive control design is introduced in order to deal with a class of fractional order systems of commensurate order with time delay. By using a Smith predictor (SP) design, slightly different from the conventional SP based on a fractional order inner model. The strategy for adjusting the controller parameters is the MIT rule. Two simulation examples are provided to show the efficiency of the proposed control configuration for this class of fractional-order systems.

**Index Terms**—delayed linear fractional system of commensurate order, direct model reference adaptive control, fractional Smith predictor, MIT rule.

## I. INTRODUCTION

Fractional order control is attracting great interest nowadays [1]–[4]. The main reason is that fractional order models are able to describe physical systems better than integer order ones. Examples of such models are numerous: dielectric polarization, semi-infinite transmission lines, viscoelasticity...etc [5], [6]. In the the field of control theory and applications, a great interest is directed to the synthesis and implementation of fractional order regulators with a growing number of control schemes and case studies [7], [8].

In the last decades, a the number of research works dealing with the stability analysis of fractional order time-delay systems took an exponential trend [9]. An investigation of Finite-time stability has been proposed in [10], while different stability criteria for this class of systems have been intensively discussed in literature [11]. In the field of control engineering, the introduction of fractional operators and systems has proven certain ability for performance enhancement. Podlubny was the first in 1999, who has proposed a generalized form for PID controllers to fractional order  $PI^\lambda D^\mu$ , which involves an integrator of order  $\lambda$  and a differentiator of order  $\mu$  [12]. He demonstrated the superiority of the fractional order PID control in termes of control system performance because of the extra adjustment parameters  $\lambda$  and  $\mu$ .

The problem of parameter and gains adjustment in control has found many solutions in literature, many of them do

not require any model of the process to control. Generally, the only needed information is the time response of the system. One can cite among these rules the method of Ziegler and Nichols, Cohen and Coon, and the Kappa-Tau rules. The most popular technique is the Ziegler-Nichols method that is widely used by engineers in industry. One modern technique that is taking a good place in the designers documents is based on the relay auto-tuning [13]. In this work, a fractional order adaptive control scheme based on MRAC configuration and associated with a fractional order Smith Predictor is designed in order to control a class of fractional order commensurate systems with time delays.

This paper is organized as follows: In Section II, Basic definitions and properties of fractional order systems are presented. The direct MRAC control configuration based on MIT rule is introduced in Section III, whereas the Smith Predictor for fractional order systems with delay is given in section IV. In section V, the proposed fractional order adaptive control design for fractional order commensurate systems is presented. Two simulation examples are given in Section VI. Finally, concluding remarks are given in Section VII.

## II. PRELIMINARIES ON FRACTIONAL ORDER SYSTEMS

### A. Definitions

1) *Caputo fractional derivative*: There are various ways to define fractional order systems, the most often used in the control field is the Caputo fractional derivative, and it's usually written as [12]:

$${}^C D_t^\mu f(t) = \frac{1}{\Gamma(n-\mu)} \int_{t_0}^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\mu-n+1}} d\tau, \quad (1)$$

${}^C D_t^\mu f(t)$  denotes the fractional derivative of order  $\mu$ , of a real function  $f$ , where  $t_0$  is the initial time and  $\mu$  is an arbitrary real number, satisfies  $n-1 < \mu < n$ ,  $n$  is a natural number, and  $\Gamma$  stands for the Euler gamma function and can be defined by the folowing mathematical formula:  $\Gamma(x) := \int_0^\infty t^{x-1} e^{-t} dt$ .

2) *Laplace transform of Caputo fractional derivative:* In order to find the Laplace transform formula for the Caputo fractional derivative, another definition of the operator 1 is given as:

$${}^C D_t^\mu f(t) = {}^C D_t^{-(n-\mu)} g(t) \quad (2)$$

where  $g(t) = f^{(n)}(t)$ , and  $\mu$  being any real number in the range  $(n-1, n]$ .

Computing the Laplace transform to both sides of 2, then using the formula (2.242) from ([12], p.104), we obtain:

$$L \{ {}^C D_t^\mu f(t) \} = s^{-(n-\mu)} . G(s) \quad (3)$$

where,  $G(s) = L \{ g(t) \} = L \{ f^{(n)}(t) \}$ .

According to the property given by equation (2.239) from ([12], p.104), we derive the following:

$$G(s) = s^n . F(s) - \sum_{k=0}^{n-1} s^{n-k-1} f^{(k)}(0) \quad (4)$$

Substituting the equation 4 in 3, gives:

$$L \{ {}^C D_t^\mu (f(t)) \} = s^{\mu-n} \left[ s^n F(s) - \sum_{k=0}^{n-1} s^{n-k-1} f^{(k)}(0) \right] \quad (5)$$

Rearranging equation 5 we get the standard formula:

$$L \{ {}^C D_t^\mu (f(t)) \} = s^\mu F(s) - \sum_{k=0}^{n-1} s^{\mu-k-1} f^{(k)}(0) \quad (6)$$

*B. Fundamental linear fractional system of commensurate order*

The fundamental linear time-invariant (LTI), single input single output (SISO) fractional system of commensurate order is represented by a linear fractional order differential equation of the following form (see [14]):

$$y(t) + aD^\mu y(t) = bu(t), \quad (7)$$

where  $y(t)$  is the output of the system,  $u(t)$  is its input, and  $\mu$  being any real number in the range  $(0,1)$  and is of commensurate order,  $a$  and  $b$  are constant parameters of real numbers. Suppose that,  $D^\mu y(t)$  is a fractional order derivative in the sence of Caupto. Then, applying the Laplace transforms (see section II-A2), and with zero initial conditions ( $t_0 = 0$ ), the transfer of the system takes the form:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{P(s^\mu)}{Q(s^\mu)} = \frac{b}{as^\mu + 1}, \quad (8)$$

We denote  $P(s^\mu)$  its numerator and  $Q(s^\mu)$  its denominator.

*C. Oustaloup recursive approximation*

The use of fractional, in more general terms non-integer, differentiation in real time operations, such as in physics and engineering sciences, requires an approximation of the fractional operator. Among nenerous methods, Oustaloup approximation ([15]) is commonly used in fractional calculus. It consists of synthesizing the fractional derivative by a recursive distribution of zeros and poles. In what follows, we give a brief discription of a generalized Oustaloup filter, which will be used later on in this paper.

The generalized Oustaloup filter can be configured in such a manner:

$$F(s) = K \prod_{k=1}^N \frac{s + z_k}{s + p_k}, \quad (9)$$

where  $z_k$  are the zeros of rank  $k$ ,  $p_k$  are the poles of rank  $k$  and  $K$  is the gain of the transmittance  $F(s)$ . These poles and zeros, in a given frequency range of interest ( $w_b, w_h$ ), are distributed geometrically around the gain frequency  $w_u$ , and can be determined based on the following calculus:

$$z_k = w_b w_u^{(2k-1-\mu)/N},$$

$$p_k = w_b w_u^{(2k-1+\mu)/N},$$

and

$$K = w_h^\mu,$$

where  $w_b$  and  $w_h$  are the low-and-high transitional frequencies,  $w_u = \sqrt{w_h/w_b}$  and  $\mu$  refers to fractional order derivative.

The designation of this filter, comes from the fact that the order of the filter say  $N$ , here, can be either odd or even integers.

### III. MODEL REFERENCE ADAPTIVE CONTROL BASED ON MIT RULE

#### A. Structure and principle of the conventional MRAC

The model reference adaptive control (MRAC) system, is one of the fundamental approaches to adaptive control. The MRAC technique consists on expressing the desired performances in terms of a reference model, while the structure of the plant is assumed to be known, with unknwon parameters. Then, on the basis of the output error (the error between the output of the system and that of the model) the controller parameters can be adjusted using an appropriate adaptation law. The closed loop system under the model reference adaptive control can be represented by Fig. 1 [16].

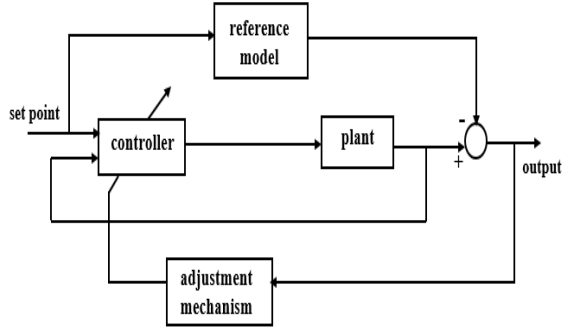


Fig. 1. Block diagram of a model reference adaptive control system.

### B. MIT rule

There are multiple ways to determine the adjustment mechanism, the method used in this paper is the MIT rule. The algorithm is as follows. The first step is to determine the tracking error, that is the output error, say  $e$ , as expressed in equation (10)

$$e(t) = y_p(t) - y_m(t), \quad (10)$$

where  $y_p$  and  $y_m$  are the output of the plant and the model respectively.  $e$ ,  $y_p$  and  $y_m$  are variables of the real time. The next step is to define the cost function, denoted by  $J$  which depends on the parameter to be updated identified by  $\phi$ . The standard cost function of the variable  $\phi$  can be expressed as:

$$J(\phi) = \frac{1}{2}e^2. \quad (11)$$

Based on this function, the adaptation law is that:

$$\frac{J(d\phi)}{dt} = -\gamma \frac{\partial J}{\partial \phi} = -\gamma e \frac{\partial e}{\partial \phi}, \quad (12)$$

where the components  $\frac{\partial e}{\partial \phi}$  are the sensitivity derivatives of the error with respect to  $\phi$ , and  $\gamma$  is the adaptation gain. As it can be seen from equation (12), this mechanism allows changing the parameters in the direction of the negative gradient of  $J$ . This is what we call MIT rule. In order to outline the algorithm, let us take an example of a first order system.

$$\frac{dy_p}{dt} + a_p y_p = b_p u, \quad (13)$$

where  $y_p$  is the output of the system and  $u$  is its input,  $a_p$  and  $b_p$  are unknown constants parameters of the system. For this plant, we propose the following model reference:

$$\frac{dy_m}{dt} + a_m y_m = b_m r, \quad (14)$$

where  $r$  is the reference input signal of the reference model,  $y_m$  is its output, and  $a_m$  and  $b_m$  are known constants. Perfect model-following can be achieved using the following control law:

$$u(t) = \phi_1 r(t) - \phi_2 y_p(t), \quad (15)$$

and the MIT rule can be applied using the following equations [17]:

$$\frac{d\phi_1}{dt} = -\gamma \left( \frac{1}{p + a_m} \right) r e, \quad (16)$$

and

$$\frac{d\phi_2}{dt} = \gamma \left( \frac{1}{p + a_m} \right) y_p e, \quad (17)$$

where  $p = d/dt$ .

Finally, the parameters  $\phi_1$  and  $\phi_2$  in the frequency domain are obtained by an integration as follows:

$$\phi_1(t) = -\frac{\gamma}{s} \left( \frac{1}{s + a_m} \right) r e, \quad (18)$$

and

$$\phi_2(t) = \frac{\gamma}{s} \left( \frac{1}{s + a_m} \right) y_p e, \quad (19)$$

## IV. SMITH-PREDICTOR FOR FRACTIONAL ORDER SYSTEMS WITH TIME DELAY

### A. Conventional Smith predictor

When analysing most of industrial and non-industrial plants, one finds dead times in their dynamic behavior. This makes them difficult to stabilize and control using classical feedback control techniques. In this case, a predictor structure allows the improvement of the closed-loop system performance. These controllers based on prediction are the dead-time compensators (DTC). One of the most popular compensating techniques is the Smith Predictor (SP). This algorithm is widely used in industry.

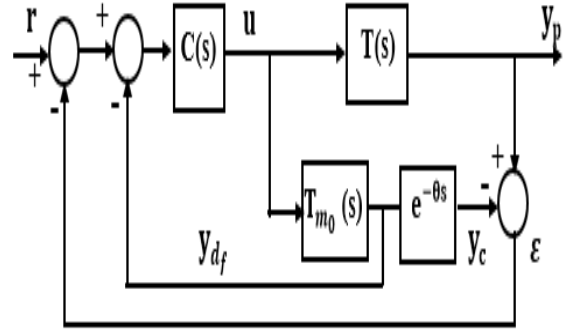


Fig. 2. Closed loop system based on Smith predictor scheme.

1) *Smith predictor structure*: As it can be seen in Fig. 2, the structure of the SP is divided into two parts: the controller  $C(s)$  and the predictor structure [18]. The predictor is composed of the model of the plant without dead time  $T_{m_0}(s)$  and a model of a dead time  $e^{-\theta s}$ . Therefore, the complete process model is:

$$T_m(s) = T_{m_0}(s)e^{-\theta s}. \quad (20)$$

$r$  and  $u$  correspond to the set point reference and the system input respectively.  $y_p$ ,  $y_c$  and  $y_{df}$  are the system output, model output and delay-free output of the model

respectively.  $\varepsilon$  is the modeling error, that is,  $\varepsilon = y_p - y_c$ . If there are no modeling error,  $\varepsilon = 0$ , then the predictor output signal  $y_c$  will be the dead-time free output of the system.

2) *Smith predictor principle*: The Smith predictor uses an internal model  $T_{m_0}$  to predict the delay-free response  $y_{d_f}$  of the process. It then compares this prediction  $y_{d_f}$  with the desired reference  $r$ . In order to decide what adjustments are needed for the control law. The Smith predictor, also, compares the actual process output with a prediction  $y_c$ , that takes the dead time into account, to prevent instabilities and reject external disturbances if the latter are presented. The gap  $\varepsilon$  is fed back to contributes to the overall error signal.

### B. Smith predictor for fractional order systems

For a fractional system of commensurate order (8) with time delay, the structure of the Smith predictor is similar to that of the conventional one (2) [19]. Nevertheless, the dead-time-free model of the plant has a fractional order transfer, which can be represented by the following equation:

$$T_{m_0}(s) = \frac{b_c}{a_c s^\mu + 1}, \quad (21)$$

where  $a_c$  and  $b_c$  are constants parameters of the model, and  $\mu$  being any real number in the range  $(0, 1)$ .

## V. ADAPTIVE CONTROL DESIGN FOR A FRACTIONAL ORDER SYSTEM WITH TIME DELAY

### A. Statement of the problem

Consider the open-loop plant represented by the fractional delay system with commensurate order transfer below:

$$T_d(s) = \frac{b}{a s^\mu + 1} e^{-\theta}, \quad (22)$$

where  $a$  and  $b$  are real constants parameters of the system,  $\mu$  the fractional commensurate order which is a real number in the range  $(0, 1)$ , an  $\theta$  is the pure time delay. Suppose we have a reference model of the form:

$$M(s) = \frac{b_m}{a_m s^\mu + 1} e^{-\theta}, \quad (23)$$

where  $a_m$ ,  $b_m$  are real constants parameters of the model reference,  $\mu$  is a real number such that  $0 < \mu < 1$ , and  $\theta$  is the pure time delay.

Therefore, the objective is to make the output of the plant follows the output of the reference model.

### B. Control using conventional MRAC

In order to achieve our objective, that is, the system output tracks the desired reference model output we use a conventional MRAC design as is shown in Fig. 1. Then, a control law of the form (15) is applied to the dead-time-free process (22). The adjustment mechanism to be used is the MIT rule previously discussed in section (III-B).

### C. SP-MRAC design for a fractional system of commensurate order with time delay

In section (V-B) the control law is adjusted in a absence of the time delay. Then, in the presence of dead-time, a dead-time compensator, that is, a Smith predictor, is added to the loop. Because we are dealing with a fractional system of commensurate order, a fractional inner model of the form (21) is used. A compensation of the time delay of the whole process can be accomplished by the present design.

## VI. SIMULATION EXAMPLE

This section presents two numerical simulation examples to prove the efficiency of the proposed control solution configured as direct MRAC and applied to the special class of fractional order commensurate systems with delay.

### A. Example 1

We consider a fractional system of commensurate order with time delay of the form:

$$T_1(s) = \frac{26}{16.2s^{0.3}} e^{-0.5s}. \quad (24)$$

We propose the model reference to be as follows:

$$M_1(s) = \frac{1}{10s + 1} e^{-0.5s}. \quad (25)$$

The Oustaloup approximation values are: the approximation order  $N = 10$ ,  $w_b = 10^{-5}$  and  $w_h = 10^5$ .

With the proposed SP-MRAC scheme to the class of system (24), simulation results are given in Fig. (3).

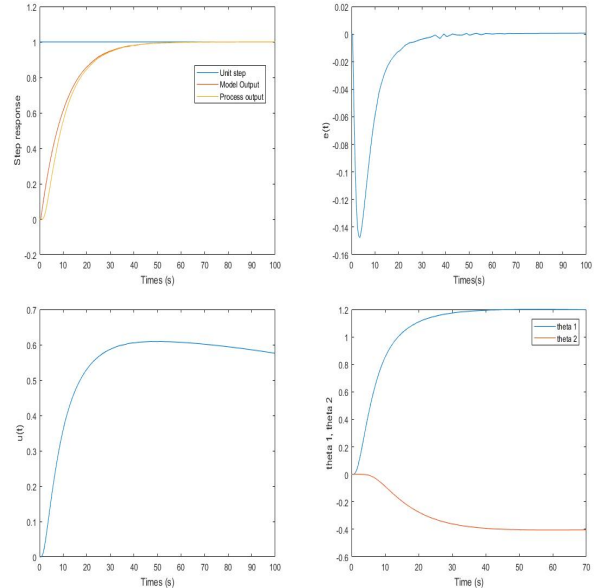


Fig. 3. Simulation results for the proposed control design

Fig. 3-(a) shows the step response, where the system output tracks the model output. Fig. 3-(b), the tracking error, while Fig.3-(c) and (3,d) present the evolution of the

control signal and the adaptation parameters respectively. From the above results, it can be seen that the tracking error tends to zero.

### B. Example 2

In this second case study, we consider the following fractional system of commensurate order with time delay:

$$T_2(s) = \frac{25}{s^{2.61} + 6s^{1.74} + 30s^{0.87} + 25} e^{-0.5s}. \quad (26)$$

The model reference proposed in this section is as follows:

$$M_2(s) = \frac{1}{10s^2 + 10s + 1} e^{-0.5s}. \quad (27)$$

The Oustaloup approximation values are: the approximation order  $N = 4$ ,  $w_b = 10^{-4}$  and  $w_h = 10^4$ .

Simulation results with a similar control design for a plant given by (26) are given by Fig. 4.

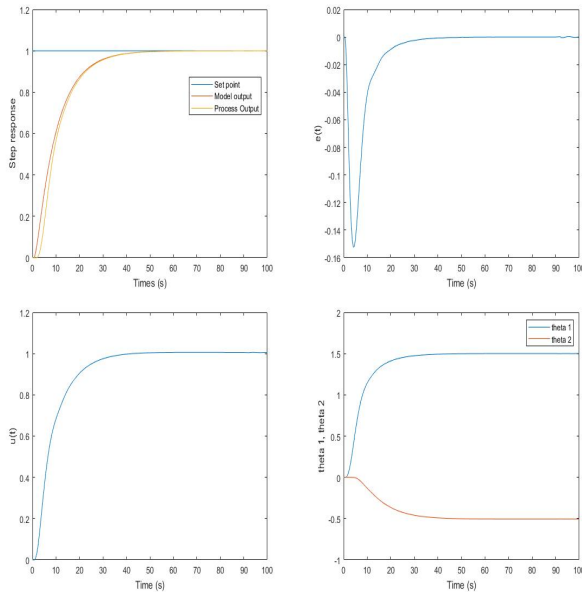


Fig. 4. Simulation results for the proposed control design

Fig. 4-(a) shows the step response, where the system output tracks the model output. Fig. 4-(b), the tracking error, while Fig. 4-(c) and (4,d) present the evolution of the control signal and the adaptation parameters respectively. From the above results, it can be seen that the tracking error tends to zero.

## VII. CONCLUSION

In this study, a fractional order SP-based MRAC control design is proposed for dead-time compensation of a class of fractional systems of commensurate orders and time delay. The structure of the FSP, is similar to the conventional one, the difference lies in the inner model of the dead-time compensator. Using a conventional MRAC design to control the system, the adaptation law is obtained based on the MIT rule. Two numerical simulations examples have been presented to outline the efficiency of the proposed

FSP-MRAC design. Hence, a perfect model following of the plant is achieved.

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