A Simple Revolute Joint with Coactivation Principle

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Abstract. Designing joints with adjustable stiffness is of great interest in human-robot collaboration tasks. In addition, remote cable actuation allows inertia reduction. Actuating a revolute joint with a pulley and two antagonistic cables is a simple solution, but stiffness modulation is possible only with nonlinear springs or other additional devices. This paper proposes a simple solution to overcome this limitation. It consists in using an eccentric pulley. We show that this solution allows coactivation as in biological joints, i.e. increasing antagonistic forces increases stiffness.

Keywords: antagonist actuation, stiffness modulation, cable actuated joint, eccentric pulley.

1 Introduction

Designing joints with adjustable stiffness is of high interest in human-robot collaboration tasks or rehabilitation [1]. In addition, remote actuation with cables allows inertia reduction. In [2], different tendon-actuated four-bar mechanisms were compared. It was shown that their stiffness can be adjusted or not depending on the cable attachment scheme. Variable stiffness actuators (VSA) allow one to modulate stiffness in situations with human-robot interactions [3]. Hardware implementations of VSA have been proposed with appropriate spring arrangement [4–6] or using cams to change the transmission angle [7]. Other solutions use antagonistic cable actuation together with springs [8]. In such solutions, several pulleys are needed and stiffness modulation uses rather complex solutions. Finally, [9] use soft-material based joints that combine pneumatic and cable actuation mechanisms. In biological joints, antagonistic muscles generally work in opposite ways: when one contracts, the other relaxes, so that energy consumption is minimized [10]. If necessary, antagonistic muscles can be activated simultaneously for a short period of time. Known as *coactivation*, this principle aims to increase the stiffness of the joint [10]. The use of a pulley is an easy choice to drive a revolute joint with cables and constant lever arm. However, a simple circular pulley does not allow coactivation [11]. To alleviate this limitation, a pulley with an elliptical shape was proposed in [12]. It was shown that under certain design hypotheses, an elliptical pulley can allow coactivation as in biological joints, i.e., increasing stiffness by increasing antagonistic cable forces. In this paper, we propose an even simpler solution to actuate a revolute joint with antagonistic cables while allowing for coactivation. The proposed solution uses an eccentric circular pulley. Eccentric pulleys have been used for torque modulation but, to the best of our knowledge, not for coactivation.

The rest of this paper is organized as follows. Section 2 reviews the coactivation principle and the use of antagonistic forces in circular pulleys. The eccentric pulley is introduced in section 3 and studied in section 4. Section 5 concludes this paper.

2 Coactivation factor and the classical pulley

Biological joints are often modeled with revolute joints actuated by antagonistic cables [13] but, unfortunately, the increase in cable forces decreases stiffness [14], which is in contradiction with the coactivation principle. In cable-driven revolute joints or four-bar mechanisms, the cable attachment points must be defined with care to avoid singularities [11]. An alternative solution is to attach the cables to a pulley. This solution avoids singularities, produces a constant lever arm and mimics bones swelling at their extremities [12]. Its main drawback is that stiffness cannot be modulated without using additional devices such as nonlinear springs [3]. However, we will show that a single eccentric circular pulley can produce coactivation. Consider the circular pulley of radius r actuated by two antagonistic cables shown in Fig. 1, which rotates a rod attached to it. The rotation angle α is measured counterclockwise from the vertical orientation.

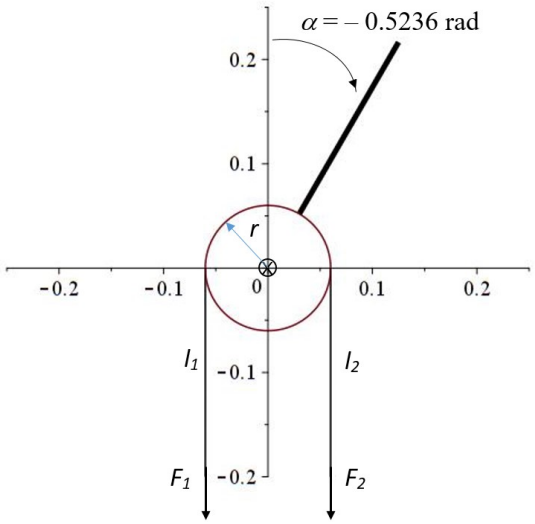

Fig. 1: A pulley operated by two antagonistic cables and rotating a rod

Let us now define a coactivation factor. The potential energy U of the system can be written as $U = U_g - F_1 l_1 - F_2 l_2$, where U_g is associated with gravity, l_1, l_2 are the cable lengths and F_1, F_2 are magnitudes of the cable forces.

The equilibrium equation $\frac{\mathrm{d}U}{\mathrm{d}\alpha} = 0$ is:

$$G = \frac{\mathrm{d}l_1}{\mathrm{d}\alpha} F_1 + \frac{\mathrm{d}l_2}{\mathrm{d}\alpha} F_2 \tag{1}$$

where $G = \frac{\mathrm{d}U_g}{\mathrm{d}\alpha}$. The equilibrium solution with minimal forces s.t. $0 \leq F_i \leq F_{max}$ is obtained when one of the forces is zero. To reach a desired configuration α^d such that, for example, $G(\alpha^d) < 0$ (i.e. $\alpha < 0$ if the center of mass of the system is above the center of rotation), the minimum force solution is:

$$F_1^* = \frac{G(\alpha^d)}{\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}}, \qquad F_2^* = 0 \tag{2}$$

If coactivation is desired, the solutions for F_2 are:

$$F_1 = \frac{G(\alpha^d) - \frac{\mathrm{d}l_2}{\mathrm{d}\alpha}F_2}{\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}} = F_1^* - \frac{\frac{\mathrm{d}l_2}{\mathrm{d}\alpha}}{\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}}F_2 \tag{3}$$

The stiffness K of the joint can be obtained as:

$$K = \frac{\mathrm{d}^2U}{\mathrm{d}\alpha^2} = \frac{\mathrm{d}G}{\mathrm{d}\alpha} + \frac{\mathrm{d}^2l_1}{\mathrm{d}\alpha^2}F_1 + \frac{\mathrm{d}^2l_2}{\mathrm{d}\alpha^2}F_2 \tag{4}$$

Let us define the coactivation factor κ as:

$$\kappa = \left(\frac{\frac{\mathrm{d}^2l_2}{\mathrm{d}\alpha^2}\frac{\mathrm{d}l_1}{\mathrm{d}\alpha} - \frac{\mathrm{d}^2l_1}{\mathrm{d}\alpha^2}\frac{\mathrm{d}l_2}{\mathrm{d}\alpha}}{\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}} \right) \tag{5}$$

Then, upon reporting F_1 from Eq. 3, Eq. 4 can be rewritten as

$$K = K^* + \kappa F_2 \tag{6}$$

where K^* is the stiffness with minimal forces:

$$K^* = \frac{\mathrm{d}G(\alpha^d)}{\mathrm{d}\alpha} - \frac{\frac{\mathrm{d}^2l_1}{\mathrm{d}\alpha^2}G(\alpha^d)}{\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}} \tag{7}$$

Whenever $\kappa > 0$, coactivation is possible and the higher κ, the higher the stiffness for a given increase in antagonistic forces.

If $G(\alpha^d) > 0$, it is easy to show that κ takes the following symmetric expression:

$$\kappa = \left(\frac{\frac{\mathrm{d}^2l_1}{\mathrm{d}\alpha^2}\frac{\mathrm{d}l_2}{\mathrm{d}\alpha} - \frac{\mathrm{d}^2l_2}{\mathrm{d}\alpha^2}\frac{\mathrm{d}l_1}{\mathrm{d}\alpha}}{\frac{\mathrm{d}l_2}{\mathrm{d}\alpha}} \right) \tag{8}$$

For a classical pulley of radius r shown in Fig. 1, when the output rotates of an angle $\delta\alpha$, the length variation of the left (resp. right) cable is $-r\delta\alpha$ (resp. $r\delta\alpha$). Then, the second derivative vanishes and, therefore, $\kappa = 0$. This confirms that a classical pulley antagonistically actuated with 2 cables does not support coactivation.

3 The eccentric pulley

From the expression of the coactivation factor derived above, the cable length expressions l_1 and l_2 must be non-linear in α for this factor to be non zero. A first solution is to modify the shape of the pulley [12]. We now show that another solution, even simpler, is to shift the rotation center. This solution is used in nature, for example in the bird's leg, to ensure stability [15]. Figure 2 shows an eccentric pulley for two different sets of design parameters. Each cable is attached to the pulley so that it remains wound on the pulley as it rotates. In this study, we assume that the center of mass of the pulley/rod system is always above the center of rotation. This will be the case if the mass of the rod is sufficiently large compared to the mass of the pulley, or if the rod is sufficiently long.

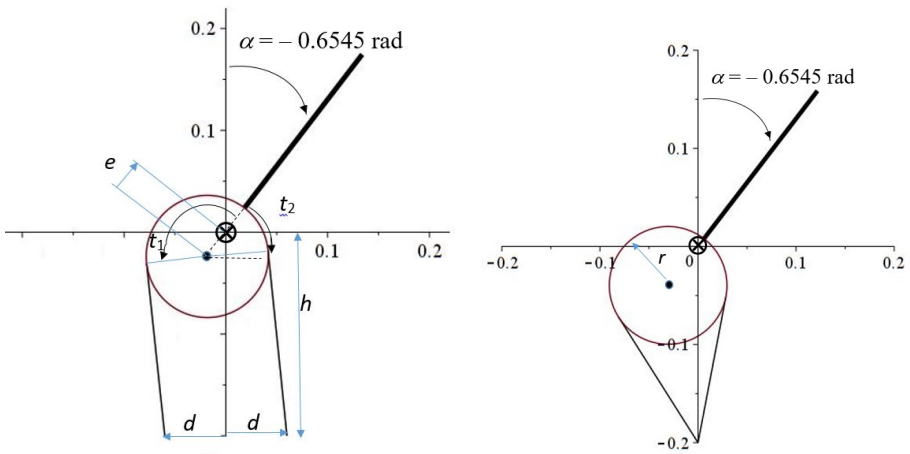

Fig. 2: The eccentric pulley ($r = 0.06$m, $h = 0.2$m). The rotation center, marked by $\otimes$, is at the frame origin and offset by $e > 0$ from the geometric center of the pulley. Left: $d = 0.06$m, $e = 0.03$m. Right: $d = 0$, $e = 0.05$m.

Assume that the left (resp. right) cable is attached at $(-d, -h)$ (resp. $(d, -h)$). The coordinates x_1 and y_1 (resp. x_2 and y_2) of the tangent point of the eccentric pulley and the left (resp. right) cable as a function of α are:

$$\begin{cases} x_i = -\cos(\alpha)\, r \sin(t_i) + \sin(\alpha)\,(e - r\cos(t_i)) \\ y_i = -\sin(\alpha)\, r \sin(t_i) - \cos(\alpha)\,(e - r\cos(t_i)) \end{cases} \tag{9}$$

where t_i, $i = 1, 2$ are the angles associated with the tangent points measured from the base point of the rod:

$$\begin{cases} t_1 = 2\arctan\left(\dfrac{-\cos(\alpha)d - h\sin(\alpha) + \sqrt{-r^2+e^2-2\cos(\alpha)he+2\sin(\alpha)de+h^2+d^2}}{\cos(\alpha)h - \sin(\alpha)d - r - e}\right) \\ t_2 = 2\arctan\left(\dfrac{-\cos(\alpha)d + h\sin(\alpha) + \sqrt{-r^2+e^2-2\cos(\alpha)he-2\sin(\alpha)de+h^2+d^2}}{-\cos(\alpha)h - \sin(\alpha)d + r + e}\right) \end{cases} \tag{10}$$

The length $l_i = \sqrt{(x_i + d)^2 + (y_i + h)^2} \pm rt_i$ of each cable i is obviously non-linear in α. This is confirmed by Fig. 3, left, which shows the plot of l_1 and l_2 for the eccentric pulley of Fig. 2, left ($r = 0.06$m, $e = 0.05$m, $d = 0.06$m and $h = 0.2$m).

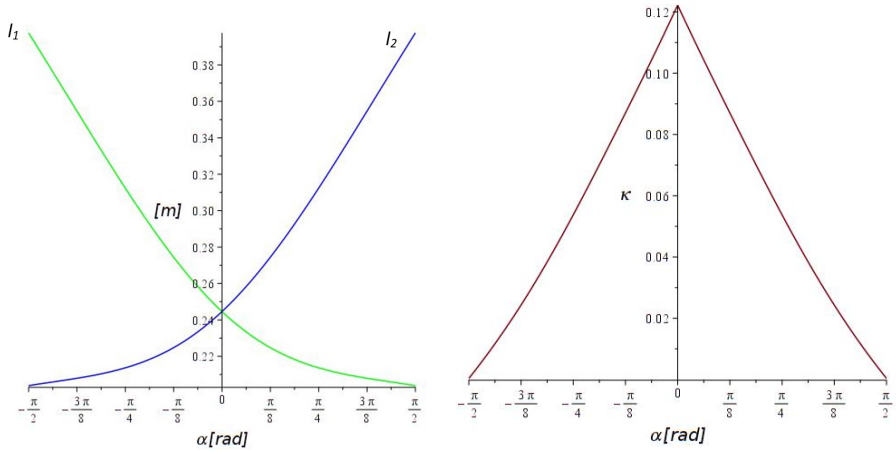

Fig. 3: Left: plot of the coiled cable length. Right: plot of coactivation factor κ

Figure 3, right, shows the plot of the coactivation factor κ ($r = 0.06$m, $e = 0.05$m, $d = 0.06$m and $h = 0.2$m). It turns out that κ is positive in almost the full range $-\pi/2 < \alpha < \pi/2$ and coactivation is thus possible in this range.

4 Parameter analysis

We now investigate the influence of the geometric parameters of the eccentric pulley on the coactivation factor. The expression of κ is too complex to perform an analytical study. We will therefore perform graphical analyses. There are 4 geometric parameters r, h, d, e but h can be fixed (to a sufficiently large value) without loss of generality since only the cable orientation matters. Moreover, we can fix one of the parameters, say the pulley radius $r = 0.06$m, and study the influence of the others with respect to this value. We first study the effect of the pulley offset e. Figure 4, left, shows the 3D plot of κ versus α and e for $h = 0.2$m and $d = 0.06$m. It shows that the higher the offset, the higher the maximum of the coactivation factor but the smaller the coactivation range. This is even clearer in Fig. 4, right, which shows the plots of κ for $e = 0.06$m (in red), $e = 0.05$m (in green), $e = 0.03$m (in blue) and $e = -0.03$m (in dark). A negative value of e, which means that the rotation center of the pulley is below its center of symmetry, provides negative values of κ, namely, stiffness decreases as the antagonistic forces increase. Note also that e affects the torque transmission. As $|e|$ increases, so does the variation of the lever arm and thus the variation of the transmitted torque. The limit is $|e| = r$ since in this case, the cables pass through the center of rotation when $\alpha = \pm\pi/2$

and cannot transmit any torque at these angles. Figure 5 shows the plot of the bounds of the transmitted torque for $e = 0.06$m (in red), $e = 0.05$m (in green) and $e = 0.03$m (in blue) when $F_{max} = 100$N.

We now study the influence of d, i.e. the horizontal position of the cable base points, which affects the orientation of the pulling cables. We take $e = 0.05$m and keep $h = 0.2$m. Figure 6, left, shows the 3D plot of κ versus α and d. The highest κ is obtained for $d \approx 0.05$m. Figure 6, right, shows the plot of κ for 4 values of d (a negative value of d means that cable directions cross). A compromise choice might be $d = 0$, which provides both a full range of coactivation and $\max_{\alpha \in [-\pi/2, \pi/2]} \kappa$ close to the highest. This pulley is shown in Fig. 2, right.

An optimal design of the eccentric pulley can be obtained on the basis of a relevant criterion, under the constraint of a minimal torque transmission factor. One possibility is to maximize $\max_{\alpha \in [-\pi/2, \pi/2]} \kappa$ but one may also want to optimize the coactivation range, i.e. the range of α where κ is positive. This choice depends on the application.

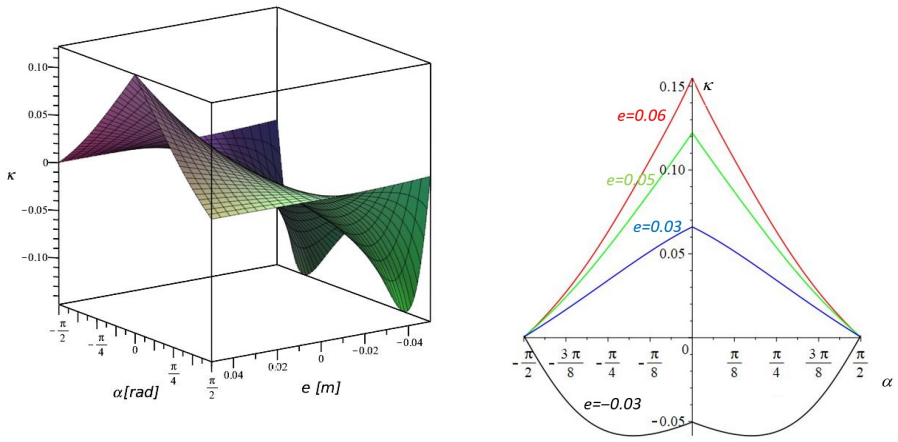

Fig. 4: Influence of the eccentricity on the coactivation factor: 3D plot of κ (left) and plot of κ for 4 values of e.

5 Conclusion

Stiffness modulation is useful in collaborative robotics. It can be obtained with antagonistic actuation but only under certain conditions. In this work, we have proposed a simple way to design a revolute joint that supports coactivation, e.g. stiffness increases as antagonistic forces increase. It consists in using an eccentric pulley. We have studied the influence of the geometric parameters, namely, the pulley offset and the position of

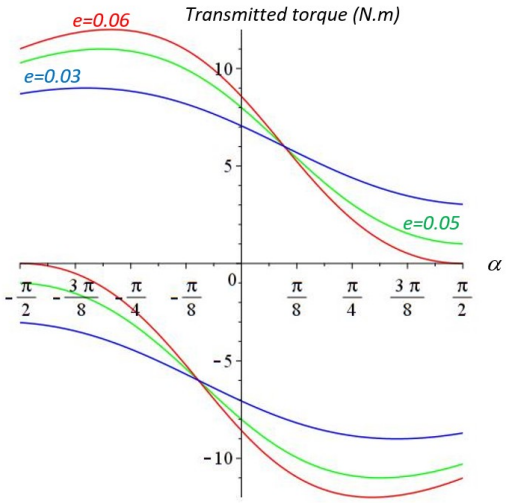

Fig. 5: Bounds of the transmitted torque by the eccentric pulley ($r = 0.06m, d = 0, h = 0.2m$) for 3 values of e with $F_{max} = 100$N.

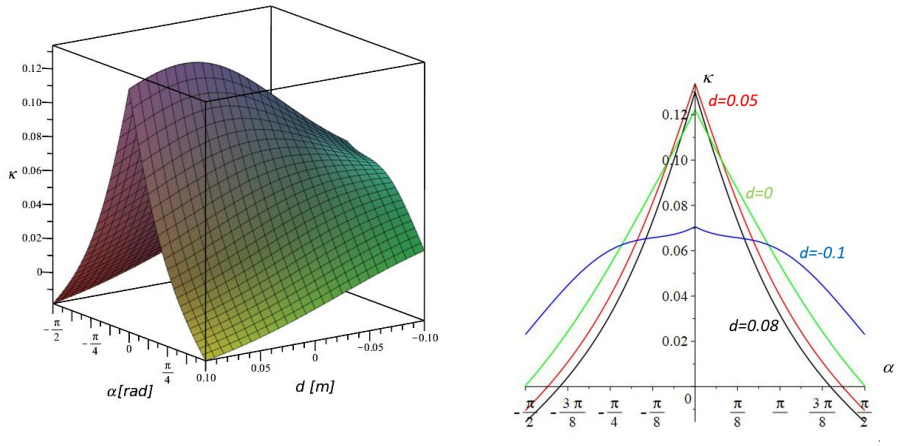

Fig. 6: Influence of d on the coactivation factor: 3D plot of κ (left) and plot of κ for 4 values of d (right).

the cable base points. As compared to the elliptic pulley, the eccentric pulley exhibits wider coactivation ranges, close to $[-\pi/2, \pi/2]$, while there are close to $[-\pi/4, \pi/4]$ for the former [12]. The proposed joint is simple and compact and it does not need any additional devices like non linear springs. It provides a low-cost actuation solution with variable stiffness and can be used in collaborative robots, humanoid robots, or rehabilitation devices.

Design optimization was not performed due to space limitations and is left for future work.

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