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► **To cite this version:**

Hugo Fauvel, Jean-Christophe Cexus, Abdelmalek Toumi, Ali Khenchaf. Change Detection in Polarimetric and Multilook SAR imagery using Stochastic Distance. IEEE INTERNATIONAL CONFERENCE RADAR 2024, Oct 2024, Rennes, France. hal-04813386

HAL Id: hal-04813386

<https://hal.science/hal-04813386v1>

Submitted on 1 Dec 2024

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Change Detection in Polarimetric and Multilook SAR imagery using Stochastic Distance

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Abstract— Change detection is useful for monitoring environmental evolution or tracking anthropogenic changes over time. One of the promotion of satellite monitoring is advocated for its ability to revisit the same sites periodically and gives us a growth data from different sensors like Radars. Synthetic Aperture Radar (SAR) data is preferred because weather conditions (e.g. cloud cover) do not hamper the ability to perform soil measurements. However, traditional change detection methods are sensitive to speckle noise. We propose to compare several statistical change detection methods in order to reduce false alarm.

In this paper, the data used are polarimetric and multilook which are simulated by their covariance matrices following a complex Wishart law. Change detection methods rely on the pixel patches whose size we fix. After estimating the parameters of the Wishart law on patches of pixels from the same area but acquired on two different dates, we compare these two patches of pixels with three stochastic divergences: Kullback-Leibler, Bhattacharyya, and Hellinger. The decision threshold is then analyzed by comparing two methods, one by the Otsu segmentation method, the other by the Chi-square distribution. We evaluate these different methods by the number of true positives, false positives, and the number of targets identified.

On simulation data, Bhattacharyya distance with the Chi-square threshold gives a higher true positive rate and a lower false alarm rate than other methods. Moreover, we note that the smaller the size of the pixel patches, the more the number of detected targets increases, but with a greater number of false alarms. Due to the strong influence of speckle, which causes false alarms, a post-processing method based on the concentration index of Getis and Franklin is proposed.

Keywords— SAR, change detection, multilook polarimetric synthetic aperture radar (PolSAR) data, stochastic distance.

I. INTRODUCTION

Change detection methods have a wide range of applications in remote sensing [1]. These methods can be used to track the evolution a target for a short period or the evolution of an area over a longer period. Classically, Synthetic Aperture Radar (SAR) imagery is particularly valuable for change detection compared to optical imagery [2]. SAR images are not affected by weather conditions like cloud cover, which can obstruct the study area. Examples of applications using change detection methods include monitoring forest cover changes, tracking crop development in agricultural fields, and detecting algae blooms or pollutions in the ocean [3]-[6].

By contrast, the presence of multiplicative noise in the SAR data makes it more difficult to identify changes between two images than in optics. This speckle noise leads to a significant number of false alarms. Adapted methods are

employed with SAR imagery to reduce this effect [1], [7], [8]. In particular, two methods are commonly used. The first is to rely on multi-look data, that is considering several images taken at the same date in order to reduce the influence of speckle. The second principle is to use the statistical information theory, that is to say by modeling the statistical distribution of several geographically neighboring data. From this modeling, we can apply a stochastic distance between pixel patches covering the same area at two different times to reduce the effect of noise [8]-[11].

SAR data contains rich information consisting of four polarizations, S_{hh} , S_{hv} , S_{vh} and S_{vv} summarized in the scattering matrix, named Polarimetric SAR (PolSAR), with the assumption that cross-coefficient are equal, so $\dim(S) = 3$. It describes the relationship between the incident electric field E_i and the received electric field E_s :

$$E_s = S E_i, \quad (1)$$

with:

$$S = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix}. \quad (2)$$

Change detection is restricted here to the comparison between two dates t_1 and t_2 . The objective is to determine if a pixel, associate with its scattering matrix S , changes between these two dates. In addition, in order to reduce the influence of speckle, we use multilook data. Several matrices $(S_i)_{i \in [1,L]}$, where $L > \dim(S)$, are acquired for each pixel at the same dates t_1 and t_2 . Then, we estimate the correlation matrix C at these two times in order to compare them:

$$C = \frac{1}{L} \sum_{i=1}^L S_i S_i^H, \quad (3)$$

where H is the Hermitian operator.

The covariance matrix LC is modeled by a complex Wishart distribution, $LC \sim W_C(n, \Sigma)$, with n is an Equivalent Number of Looks (ENL) [10]. In the following, we will denote LC by C , neglecting the proportionality term. It is a widely used model for multilook full polarimetric SAR data. This means that the data are polarimetric since the Wishart distribution applies in the theory of random and multilook matrices, with at least four acquisitions for one instant, since a covariance matrix is estimated. The Wishart complex law can be seen as a generalization of the Gamma law, knowing that the coefficients of the matrix follow this last stochastic distribution.

In Section II, we provide a comprehensive explanation of the methodology. Initially, we detail the data simulation process using Wishart's complex law. Subsequently, we delve into parameters estimation, focusing mainly on Equivalent Number of Looks (ENL) estimators and stochastic distances. Section III is dedicated to the discussion of the results obtained and the evaluation of the effectiveness of the proposed approach. Finally, we conclude with a summary and outline potential future directions.

II. DESCRIPTION OF THE METHODOLOGY

A. Data Simulation from a Complex Wishart Law

To evaluate the proposed methods, the simulated data from the complex Wishart law are obtained. Several methods have been proposed to generate complex Wishart samples. One of them proposed by Sawyer [12] is based on the generation of two samples following Gaussian and Gamma laws. For a matrix following a complex Wishart distribution, a matrix Z_i , with i in $[1, n]$, following a Gaussian distribution is drawn. Then, a matrix A supposed to follow a Gamma distribution is calculated from the Σ matrix. This matrix A is the Cholesky decomposition of Σ :

$$\Sigma = A'A, \quad (4)$$

with lower diagonal matrix.

To avoid redundancy in quadratic operations, it's recommended to perform this calculation once to minimize computation time. Then, the Eq. (5) is applied to obtain the desired matrix for a matrix $C \sim W_c(n, \Sigma)$.

$$C = \sum_{i=1}^n (AZ_i)(AZ_i)' \quad (5)$$

Hagedorn *et al.* [13] describe that the coefficients of a random matrix following a complex Wishart distribution follow a Gamma distribution, that is: $c. \sim \Gamma(\sigma, ENL)$. To verify that the generated matrix follows a Wishart complex law, a Kolmogorov-Smirnov test is applied on these coefficients. The test does not allow us to reject that the coefficients of the matrix follow a gamma law, with an average p-value of 0.8. By extension the matrix does indeed follow a complex Wishart distribution.

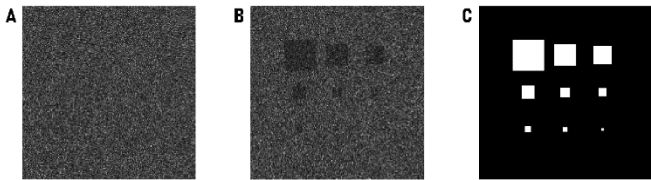


Fig. 1. Simulation of two images governed by a complex Wishart law. Image at date 1 (A), image at date 2 (B) and the ground truth change between the two dates (C).

B. Estimation of Parameters

For the first parameter of the Wishart complex law, we estimate the expectation of the covariance matrix C . We use the maximum likelihood estimator given by Eq. (6).

$$\hat{\Sigma} = \frac{1}{L} \sum_1^N C_i \quad (6)$$

The ENL is the second parameter of the random law. Several ENL estimators are discussed in the literature [10]. In particular, the Conventional Estimator (CV) and the Fractional Moment estimator (FM) [11]. They give close result with a relatively same and low complexity in computation time. However, they have a non-zero bias which often overestimates the ENL [10]. This bias remains acceptable and has little influence as it will be shown later on the sensitivity of stochastic distances.

We note $c.$ the coefficients of the correlation matrix C . For CV estimator, we replace the estimator of the first moment and the variance in Eq. (8), giving the estimator (Eq. (9)):

$$ENL = \frac{E(c.)^2}{Var(c.)}, \quad (7)$$

$$ENL^{(CV)} = \frac{\langle c. \rangle^2}{\langle c.^2 \rangle - \langle c. \rangle^2}, \quad (8)$$

where $\langle . \rangle$ denoted the mean of $(.)$.

The analytical expression of FM comes from the following expression:

$$E(c.^2) = \frac{\Gamma\left(ENL + \frac{1}{2}\right)}{\Gamma(ENL)} \sqrt{\frac{\sigma}{ENL}} \quad (9)$$

The ENL is then estimated by solving the following equation:

$$\frac{\Gamma\left(ENL^{(FM)} + \frac{1}{2}\right)}{\Gamma(ENL^{(FM)})} \sqrt{\langle c. \rangle} - \langle \sqrt{c.} \rangle = 0 \quad (10)$$

C. Hypothesis test theory

The methods of change detection can be divided into two families, on the one hand the hypothesis test theory, on the other hand by the statistical information theory according to N. Bouhlef *et al.* [9]. Among the first family, the best known are the determinant test, the trace test and the Barlett test. Each one is based on a ratio between the correlation matrices at time t_1 and t_2 in order to quantify the separation between these two matrices. Then, a threshold is applied in order to separate the pixels considered as having changed and those not having changed between two dates. These methods are interesting when there is a need to have a method requiring few computational resources. On the other hand, they are not very suitable for SAR images because of their strong dependence on speckle. The number of false alarms is so important that a post-processing step is often necessary.

D. Statistical information theory

Stochastic distances are a way to reduce the influence of speckle when comparing two data from different dates. Indeed, the methods based on these methods use the information contained on a set of pixels and not on a single pixel.

The principle is to take a patch with neighboring pixels around the analyzed pixel. The histograms of the values taken by the coefficients of the covariance matrices is then modeled by a scaled complex Wishart law. The larger the patch size, the less sensitive the comparison between two instants will be to noise. On the other hand, it is more difficult to locate small areas that have changed. Many stochastic distances are used: Kullback-Leibler, Bhattacharyya and Hellinger. The proposed distances do not respect all the conditions to be distances, in particular they are not always symmetrical, that is why they are rather divergences; by abuse of language, we will call them distance.

Since we model the covariance matrices for each pixel by a known stochastic distribution, it is possible to access an analytical formulation of these distances [11]. These formulas then depend only on the parameters of the Wishart complex law, its covariance matrix and the ENL. Three cases can be analyzed, they are listed below, with Σ . and ENL are the two parameters of the complex Wishart distribution at date (.):

- Case (i): $\hat{\Sigma}_1 \neq \hat{\Sigma}_2, ENL_1 \neq ENL_2$
- Case (ii): $\hat{\Sigma}_1 \neq \hat{\Sigma}_2, ENL_1 = ENL_2$
- Case (iii): $\hat{\Sigma}_1 = \hat{\Sigma}_2, ENL_1 \neq ENL_2$

E. Change detection threshold

Calculating a stochastic distance between two patches in the same region on two different dates quantifies the observed statistical difference. In the absence of change between the two dates, this distance is close to zero. On the contrary, in the presence of a change, the distance is high. A threshold is applied to consider from what value a change is supposed to be observed. We use two approaches to determine this threshold automatically. The first uses the Otsu algorithm [14]. The method estimates a constant threshold for all pixels in an image. It is based on the hypothesis that at least one pixel of this image is changed between the two dates.

Based on the theory of hypothesis testing, Salicru *et al.* [15] proposed a method to obtain a local threshold, varying from one pixel to another. This threshold comes from the following lemma:

Lemma 1:

$$\text{If } \frac{N_1}{N_1 + N_2} \xrightarrow{N_1, N_2 \rightarrow +\infty} \lambda \in [0, 1], \text{ and } C_1 = C_2$$

Then,

$$S_h^\phi(\hat{\Sigma}_1, \hat{\Sigma}_2) = \frac{2N_1N_2}{N_1 + N_2} \frac{d_h^\phi(\hat{\Sigma}_1, \hat{\Sigma}_2)}{h'(0)\phi''(1)} \xrightarrow{N_1, N_2 \rightarrow +\infty} \chi_M^2 \quad (11)$$

$S_h^\phi(\hat{\Sigma}_1, \hat{\Sigma}_2)$ converge in distribution toward χ_M^2 , with M degrees of freedom, $d_h^\phi(\cdot, \cdot)$ is a stochastic distance between two matrices, h and ϕ are two functions depending on the stochastic divergence.

A.-C. Frery *et al.* [11] proposes pairs of functions for each stochastic distance. This function and their values are given in the following tables 1 and 2.

TABLE 1 – FUNCTION h AND ϕ ACCORDING TO THE STOCHASTIC DISTANCE FROM [11].

(h, ϕ) -distance	$h(y)$	$\phi(x)$
Kullback-Leibler	$y/2$	$(x-1) \log x$
Rényi (order β)	$\frac{1}{\beta-1} \log((\beta-1)y+1), 0 \leq y < \frac{1}{1-\beta}$	$\frac{x^{1-\beta} + x^\beta - \beta(x-1)^{-2}}{2(\beta-1)}, 0 < \beta < 1$
Bhattacharyya	$-\log(-y+1), 0 \leq y < 1$	$-\sqrt{x} + \frac{x+1}{2}$
Hellinger	$y/2, 0 \leq y < 2$	$(\sqrt{x}-1)^2$

TABLE 2 – VALUES OF h AND ϕ .

	$h'(0)$	$\phi''(0)$
Kullback-Leibler	0.5	2
Bhattacharyya	1	0.25
Hellinger	0.5	0.5

Lemma 1 allows us to set a threshold by a Chi-square test. The calculation of the statistic S for each pixel is compared to a threshold α , given by a reference table of the test, for example, in the case of the study, $M = 2$, for a confidence of 95%, $\alpha = 5.991$. The null hypothesis $C_1 = C_2$, that is the pixel to change between the dates t_1 and t_2 , can be rejected at level α if:

$$P(\chi_M^2 > S_h^\phi(\hat{\Sigma}_1, \hat{\Sigma}_2)) \leq \alpha \quad (12)$$

From the threshold locally determined by Lemma 1, it is possible to visualize the sensitivity of the change detection algorithm.

F. Evaluation of change detection methods

Two tests are performed. One where the estimate of the mean of the covariance matrix is tainted by an error on one of its coefficients, the other where the estimate of the ENL is incorrect for one of the two dates.

In the first test, the aim is to predict the behavior of the algorithm when the estimation of the mean of the distribution is tainted by error,. For this, a reference matrix on the first parameter will be fixed for the first time. For the second instant, the coefficient in position (1, 1), corresponding to the variance of the HH polarization. For the sensitive test, the Σ parameters are as follows, with $x = 360932$ for the first date, and the ENL is equal to 8.

$$\Sigma(x) = \begin{pmatrix} x & 11050 + 3759i & 63896 + 1581i \\ & 98960 & 6593 + 6868i \\ & & 208843 \end{pmatrix} \quad (13)$$

In the second test, the number of ENL are considered constant for the first date and varies for the second date in order to simulate an error in estimation of this parameter. The Σ parameters are fixed with $x = 36093$. The mean of the covariance matrix will be equal to the two times.

To compare the stochastic divergences between them, two metrics are exploited. One is based on the number of true detections and false detections pixel by pixel. The other determines the number of targets actually located by these methods (figures 4 and 5). These two types of measurements are performed in two different tests, the first by varying the ENL, the second by varying the size of the analysis window.

III. RESULTS

A. Sensitivity of stochastic distance

In the first test, when x moves away from the value of the same coefficient at date t_1 , the statistic obtained by the Kullback-Leibler distance is larger than that obtained by Bhattacharyya and close with Hellinger. This suggests that the distances Kullback-Leibler and Hellinger have a similar behaviour and are more sensitive to a change. On the one hand, we therefore expect to better locate small targets by these distances, but the risk of false alarm is increased.

This is also the case for the second test as shown in figure 2, when the number of ENLs estimated for the second image moves away from the first equal to 8, the statistic is larger for Kullback-Leibler. That is, the Bhattacharyya distance will be less likely to consider a pixel as having changed for the same estimates of the probability law parameters. The Hellinger distance has a behavior very close to the Kullback-Leibler distance. Moreover, the sensitivity of all distances is greater when x is smaller than the value at date t_1 , as well as when the estimated ENL is smaller. Thus, underestimation of the parameters results in a greater influence on the decision that there has been a change.

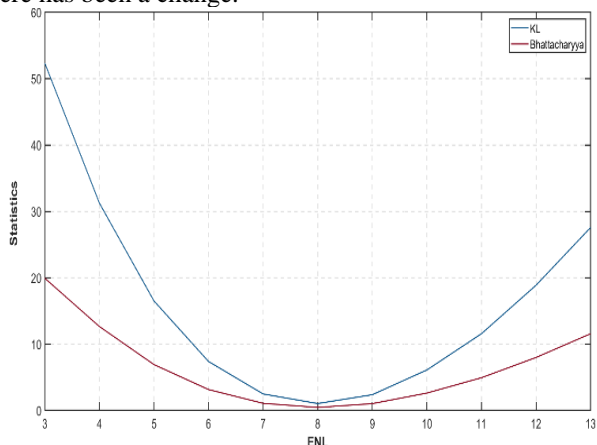


Fig. 2. Sensitivity of Kullback-Leibler and Bhattacharyya distances with ENL variation. The reference ENL being equal to 8.

B. Performance of the stochastic distance

A visual comparison of the results obtained by hypothesis testing theory, with the determinant, trace and Bartlett tests, and statistical information theory, with Kullback-Leibler, Bhattacharyya and Hellinger distances, illustrates the advantage brought by the latter (figure 3). Indeed, the number of false detections is very high in the first techniques because of their sensitivity to speckle. We do not find as many of these false detections in the case of stochastic distances.

In figure 4, we plot the metrics of true and false decisions as a function of ENL variation and patch size. In the case of ENL variation, we notice that increasing ENL causes little change in the good decision rate. On the other hand, the ENL must be high enough. The best results are obtained for an ENL of at least 6.

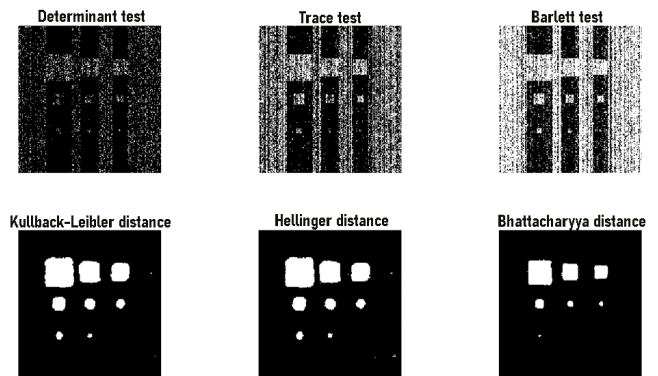


Fig. 3. Comparison between methods from hypothesis testing theory, top, and statistical information theory by stochastic distances, bottom, (Kullback-Leibler dist., Hellinger dist., and Bhattacharyya dist., respectively).

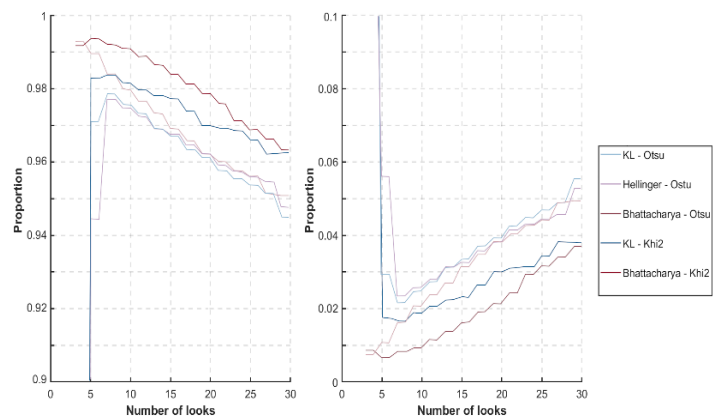


Fig. 4. Proportion of correctly classified pixels, left, and not correctly classified, right, with methods using stochastic distances by varying ENL.

The metrics are not invariant to the size of the patch as illustrated in figure 5. As the patch size increases, the results decrease. It could be counterintuitive to have these results since the parameter estimates are more accurate as the number of pixels increases for a homogeneous area. But since the areas are not homogeneous, the probability that a patch analyzes a heterogeneous area, consisting of at least two targets with different characteristics, the proportion of false decision increases. Consequently, the choice of patch size is an important parameter to consider. A smaller patch size may result in less precise estimates of the parameters of the random law, while a larger patch size may lead to , interference between the nearby targets and an increase in computation time.

As mentioned in the sensitivity test, the distances of Kullback-Leibler and Hellinger have close behaviors relative to the distance of Bhattacharyya. This behavior can be seen in figure 2, where the first two distances give close results whatever the test performed. This results in a significant difference between these distances when the patch size is small. Indeed, the number of pixels on which the parameters are estimated have a high variance and therefore can give rise to false alarms. These false alarms are present mainly for the more sensitive distances, but not present for the distance of

Bhattacharyya which is less tolerant of a greater variation of the parameters of the complex Wishart law.

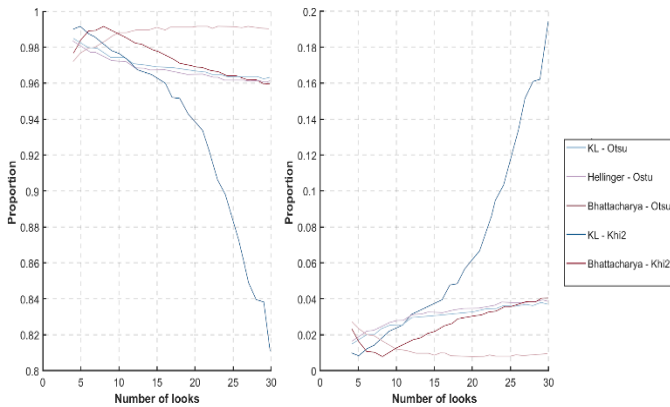


Fig. 5. Proportion of correctly classified pixels, left, and not correctly classified, right, with methods using stochastic distances by varying patch size.

However, when using the second metric (figures 6 and 7), we see that the advantage of using the more sensitive distances is that they locate better the small targets. Bhattacharyya distance does not detect changes in a small group of pixels. The general trend of this test shows that the larger the patch size, the smaller targets are not detected as having changed. In the case of close targets, the sensitive distances are even more likely to fail to distinguish between two targets. Their separating power is weaker than the distance of Kullback-Leibler.

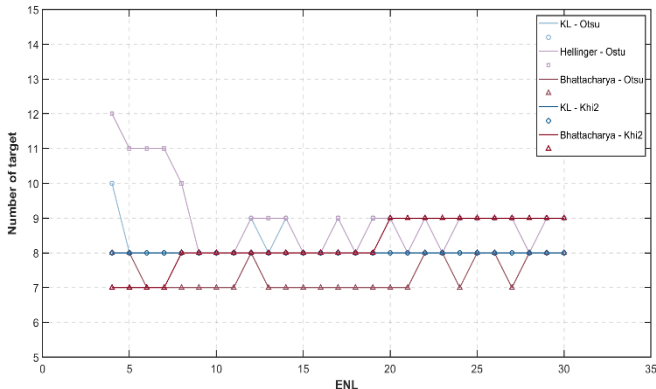


Fig. 6. Number of detected on the nine presents with the methods using the stochastic distances by varying ENL.

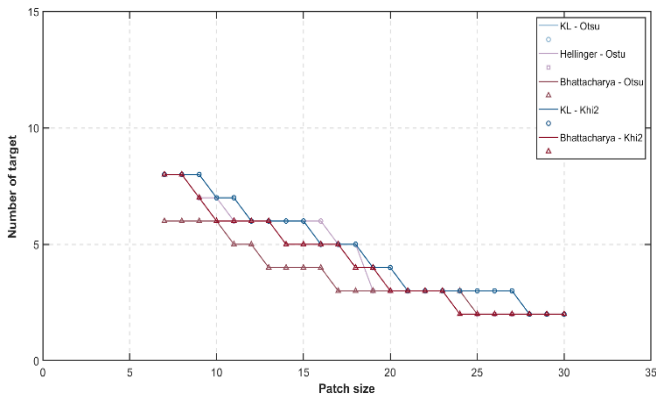


Fig. 7. Number of detected on the nine presents with the methods using the stochastic distances by varying the patch size.

C. Performance of the threshold

The use of a thresholding by the χ_2^2 law allows to obtain better results. The reason is that the segmentation of the targets is better done. But, the Kullback-Leibler distance diverges for a large number of ENL.

Thresholding by the χ_2^2 law also gives slightly better results on the number of targets detected. When the patch size varies, the results are generally similar than those with the Otsu method. When the number of ENL varies, the number of false alarms is however more important with the calculated statistic. This metric gives the equivalent result to the first one that the number of ENL varies little on the results whatever the distance.

IV. DISCUSSION

A. Toward a more complex stochastic distribution ?

In the study conducted, only simulated data were used in the tests. In the presence of real data, we can expect a degradation of the results, without questioning the analysis made so far on the behavior of the methods used. On the other hand, it may be interesting to review the assumption that the covariance matrix follows a complex Wishart distribution. In order to take into account the field reality, factorizing of the scattering matrix S into one term containing the speckle τ and the other the texture information of the target X can be done [9].

$$S = \sqrt{\tau} X \quad (14)$$

In particular, by using (14), N. Bouhlef *et al.* [9] use a \mathcal{G}_d^0 distribution. A comparison with the complex Wishart distribution is illustrated on the image of figure 8. On this example, two remarks can be made. On the one hand, the modeling by the law of \mathcal{G}_d^0 detects the target of the middle contrary to the complex Wishart distribution. On the other hand, the number of false alarms is slightly lower for the first distribution.

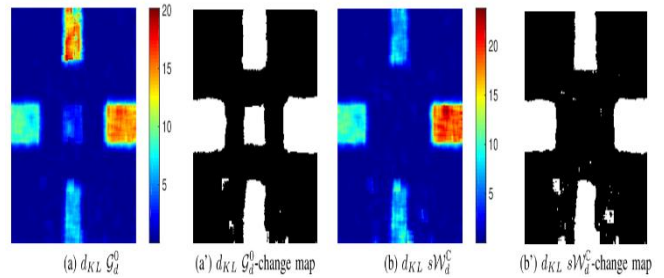


Fig 8 - Comparison of the complex Wishart and \mathcal{G}_d^0 distributions as a model of the covariance matrix using a Kullback-Leibler distance.

B. Post-treatment of false alarm

In some cases, despite the lesser influence of speckle, the number of false alarms can be significant. Especially if there is no presence of these false alarms, there is a risk of not detecting small targets. This is why post-processing to remove these errors is to be favored.

Very often, these false alarms are local, or even isolated, iso that. they do not form groups with many pixels, unlike the targets. A low pass filter is therefore used to remove these

errors, like a median filter. We propose a more suitable approach using the theory of geostatistics. It consists of considering that a pixel has changed between two dates if some of its neighboring pixels have changed. A concentration of change calculation is used. The estimator of concentration has been proposed by Getis *et al.* [16]. Such a method has been applied to images from a complex Wishart law and a law of G_d^0 with a large number of false alarms. The visual results are presented on figure 9.

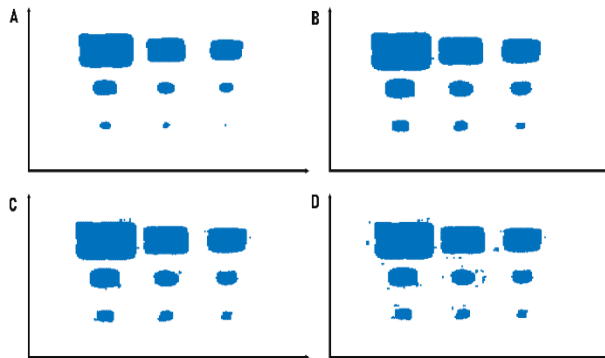


Fig. 9. Application of a filter to remove false alarms by the Getis and Franklin estimator. From top to bottom and from left to right, thresholding for a distance of 4 pixels (A), for a distance of 6 pixels (B), for a distance of 8 pixels (C), image without filtering (D).

V. CONCLUSIONS

The interest of using stochastic distances to detect changes in SAR images between two dates shows its interest in its ability to be little affected by speckle. The discussion should however focus on the parameters to be used for these statistical methods, in particular the choice of the random distribution to model the data, the size of the patch as the analysis window or the thresholding to be applied on the distances. In addition, in this study, only simulated data were used to evaluate and compare the methods when using multilook and polarimetric data. A similar study on real data is to be planned in order to confirm the results obtained.

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