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# Frequency–Fresnel Domain Equalization for Underwater Acoustic OCDM Systems

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**Abstract**—Orthogonal Chirp Division Multiplexing (OCDM) is a promising multi-carrier modulation for Underwater Acoustic (UWA) systems, as several low complexity OCDM receivers capable of exploiting the channel diversity have already been proposed. However, OCDM receivers are often designed and studied over time-invariant channels.

This paper proposes a hybrid Frequency-Fresnel domain equalizer to enhance the system performance over the doubly selective UWA channel. Our method first estimates the Channel Impulse Response (CIR) using pilot blocks and the Least Square (LS) criterion. The CIR is then sparsified by removing coefficients below a threshold, and the receiver performs equalization in the frequency domain. To address residual InterSymbol Interference (ISI) in doubly selective UWA channels, we propose to equalize again the obtained symbols in the Fresnel domain and perform jointly phase synchronization. Simulations both on synthetic channels and a real underwater channel show that the proposed algorithm improves significantly the BER performance compared to existing methods.

## I. INTRODUCTION

Orthogonal Chirp Division Multiplexing (OCDM) has been recently proposed as a new multi-carrier modulation scheme; based on a series of orthogonal chirp signals, it spreads the information symbols both in the time and the frequency domain [1]. The OCDM modulation was tested over some UnderWater Acoustic (UWA) channels, and was shown to be robust to different kinds of imperfections such as the use of insufficient guard intervals [1]. It also leads to better system performance when compared to Orthogonal Frequency Division Multiplexing (OFDM) in several scenarios in terms of Bit Error Rate (BER) [1]–[3].

Over frequency-selective channels, OCDM signals can be equalized using a Time Domain Equalizer (TDE), a Frequency Domain Equalizer (FDE) [4], [5], [5], or other advanced detectors [1]. The FDE has been privileged in the literature [1]–[3], [7] as it leads to good BER performance and is well suited to physical implementation. The performance of the FDE considerably depends on the performance of the channel estimation. However, channel estimation is challenging for OCDM systems as the symbols are not separable in both time and frequency domains. [7] proposed several frequency domain pilot multiplexing techniques to enable independent channel estimation and detection. In [8], two pilots block are attached at the head and the tail of data blocks, and are

designed so as to optimally minimise the Mean Square Error (MSE) of the Channel Impulse Response (CIR) estimation; however the MSE tends to degrade significantly as the channel varies [8], [9]. Also, the synchronization must be carefully devised in a channel as difficult as the UWA channel [10]–[14].

In this paper, we thus propose a hybrid Frequency-Fresnel domain equalizer. First the receiver estimates the Doppler shift and resamples the received symbols accordingly. Then, it estimates the Channel Impulse Response (CIR) based on the pilots blocks and the Least Square (LS) criterion. Given the sparsity of the UWA channel, the estimated CIR is made sparse by zeroing out the coefficients of the estimated CIR that are below a predefined threshold. Since the UWA channel is doubly selective, the CIR is not necessarily constant over one block leading to residual InterSymbol Interference (ISI); we thus propose to develop a joint Frequency and Fresnel domain equalizer with a limited number of coefficients and a separate Phase-Locked Loop.

The reminder of this paper is organized as follows. Section II describes the system model. Section III presents the proposed hybrid equalization algorithm. Numerical results obtained over some time-varying channels and a real UWA channel are displayed in Section IV. Finally, some conclusions are drawn in Section V.

## II. SYSTEM MODEL

Information bits  $b$  are first channel encoded and interleaved, and then mapped into possibly complex symbols. Thereafter, the symbols are separated into blocks of  $N_s/N_u$  symbols, where  $N_u$  is an integer divisor of  $N_s$  for convenience; it represents the number of chirp to be modulated by the coded symbols. For an underloaded OCDM transmission [2], the symbols to be transmitted in the  $i$ -th block  $d^{(i)}$  can be expressed as:

$$\mathbf{d}^{(i)} = [d_0^{(i)} \quad \mathbf{0}_{1 \times N_s/N_u - 1} d_1(i) \cdots d_{N_u - 1}^{(i)} \quad \mathbf{0}_{1 \times N_s/N_u - 1}]^T. \quad (1)$$

Pilots symbols are then added such that the  $i$  th transmitted symbol block can be expressed as:  $\mathbf{x}^{(i)} = [\mathbf{p}_0^T \mathbf{d}^{(i)} \mathbf{p}_1^T]^T$ ; in our simulations, we consider the following pilot design [8]:  $\mathbf{p}_0 = [b, \mathbf{0}_{K-1}^T]^T$ , and  $\mathbf{p}_1 = \mathbf{0}_{K-1}$ , with  $\|b\|^2 = \mathcal{P}_b$  where  $\mathcal{P}_b$  is the power allocated to the pilots. The time-domain OCDM

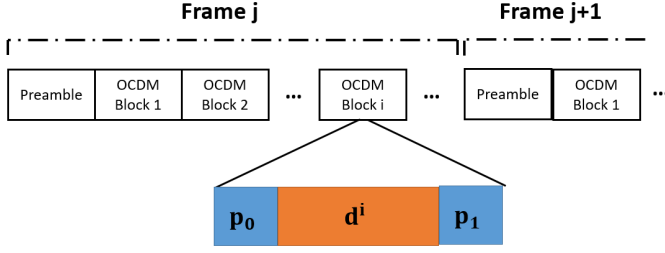


Fig. 1. The frame structure.

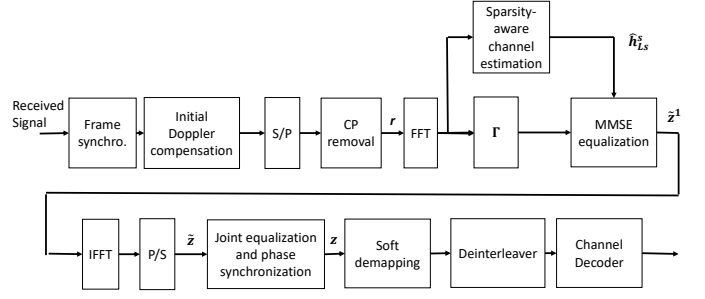


Fig. 2. The proposed receiver design.

symbol block, denoted by  $\mathbf{s}^{(i)}$  of length  $N = N_s + 2K - 1$ , can be then written as:

$$\mathbf{s}^{(i)} = \Phi^H \mathbf{x}^{(i)}, \quad (2)$$

where  $\Phi$  is an  $N \times N$  discrete Fresnel transform (DFnT) matrix whose  $(m, n)$  th entry is defined as:

$$[\Phi]_{m,n} = \frac{e^{-j\frac{\pi}{4}}}{\sqrt{N}} \begin{cases} e^{j\frac{\pi}{N}(m-n)^2} & N \equiv 0(\text{mod}2) \\ e^{j\frac{\pi}{N}(m+\frac{1}{2}-n)^2} & N \equiv 1(\text{mod}2). \end{cases} \quad (3)$$

Finally, a cyclic prefix larger than the maximum delay spread of the channel  $L_h$  is added at the beginning of the frame. Figure 2 illustrates the frame structure in the Fresnel domain. In the sequel of this paper, the block index  $i$  is dropped for simplicity reasons.

At the receiver side, the receiver first estimates the Doppler shift and resamples the received symbols accordingly. Thereafter, it uses our proposed hybrid equalization algorithm that we detail in the next section. Finally, the equalized symbols are fed to the channel decoder to obtain LLR values [15]–[17] and to estimate the information symbols.

### III. THE PROPOSED FREQUENCY-FRESNEL DOMAIN EQUALIZATION ALGORITHM

#### A. Sparsity aware channel estimation and Frequency domain equalization

At the receiver side, after frame synchronisation and initial Doppler correction [18] and assuming the channel to be constant within one block, the received observations for a single block can be expressed as:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} = \mathbf{H}\Phi^H \mathbf{x} + \mathbf{w}, \quad (4)$$

where  $\mathbf{H}$  is the CIR matrix and  $\mathbf{w}$  is an additive Gaussian noise. The CIR matrix  $\mathbf{H}$  is circulant: its first column is  $\mathbf{h} = [h^0, h^1, \dots, h^{L_h-1}, 0, \dots, 0]^T$ , where  $h^l$  are the CIR taps. The receiver then applies a Fast Fourier Transform (FFT), leading to:

$$\mathbf{r}_1 = \mathbf{F}\mathbf{H}\Phi^H \mathbf{x} + \mathbf{F}\mathbf{w} = \mathbf{\Gamma}^H \mathbf{\Lambda} \mathbf{F}\mathbf{x} + \mathbf{w}_1; \quad (5)$$

where  $\mathbf{F}$  is the Fourier matrix of size  $N$ ,  $\mathbf{\Lambda} = \mathbf{F}\mathbf{H}\mathbf{F}^H$ , and  $\mathbf{\Gamma}$  is a diagonal matrix with  $\{\mathbf{\Gamma}\}_{n,n} = e^{-j\frac{\pi}{N}n^2}$ .

As, the statistics of the UWA channels are quite hard to obtain, we thus estimate the channel  $\hat{\mathbf{h}}_{LS}$  using the Least

Square (LS) criterion.

The receiver then obtains the sparse CIR estimate  $\hat{\mathbf{h}}_{LS}^S$  from  $\hat{\mathbf{h}}_{LS}$ ; indeed, we propose to take under consideration the sparsity of the UWA CIR by setting coefficients of  $\hat{\mathbf{h}}_{LS}$  to zero if they fall below a predefined threshold.

In our simulations, only coefficients with power exceeding 30% of the average coefficient power are considered for the equalization of the received signal. Then the receiver performs channel equalization in the Frequency domain as:

$$\tilde{\mathbf{z}}^{Freq} = \mathbf{G}\mathbf{\Gamma}\mathbf{r}_1, \quad (6)$$

where  $\mathbf{G}$  is a diagonal equalization matrix. For instance, if the Zero forcing criterion is used, it can be expressed as:

$$\mathbf{G}_{ZF}(k) = \tilde{\Lambda}^{-1}(k). \quad (7)$$

Under the Minimum MSE (MMSE) criterion,  $\mathbf{G}(k)$  is given by:

$$\mathbf{G}_{MMSE}(k) = \frac{\tilde{\Lambda}^*(k)}{|\tilde{\Lambda}(k)|^2 + \rho^{-1}}, \quad (8)$$

where  $\mathbf{\Lambda}$  is the estimated Channel Frequency Response (CFR) and  $\rho$  is the Signal-to-Noise Ratio (SNR): it can be estimated using conventional methods such as described in [3], [19].

#### B. Fresnel domain equalization

The equalized observations can be obtained in the Fresnel domain:

$$\tilde{\mathbf{z}} = \mathbf{F}^H \tilde{\mathbf{z}}^{Freq}. \quad (9)$$

In practice, channel estimation cannot be perfectly accurate, and the channel may fluctuate within one symbol block. As a result, residual ISI may persist after frequency equalization, necessitating an additional equalization.

The symbols at the output of the FDE can be modeled in the Fresnel domain as:

$$\tilde{z}_n = \tilde{h}_0 x_n + \sum_{l \neq 1} \tilde{h}_l x_{n-l} + w_2 \quad (10)$$

Therefore, we propose to perform jointly equalization and phase synchronization of the symbols  $\tilde{z}_n$ . We then obtain:

$$z_n = P_n^T \tilde{Z}_n e^{-i\varphi_n}, \quad (11)$$

where  $\tilde{Z}_n = [\tilde{z}_{n+L}, \dots, \tilde{z}_n, \dots, \tilde{z}_{n-L}]^T$ ,  $\varphi_n$  is the estimated phase error, and  $f_n = [f_{-L}(n), \dots, f_0(n), \dots, f_L(n)]^T$  is a transversal filter of length  $2L+1$ ,

The coefficients of the filters are updated over the known symbols in order to minimize the mean-square-error (MSE) criterion defined as [20]–[24]:

$$MSE = \mathbb{E}(|z_n - x_n|^2), \quad (12)$$

The filter coefficients  $P_n$  and the estimated phase error  $\phi_n^I$  are updated as:

$$\begin{cases} P_{n+1} = P_n - \mu \nabla_{P_n} (|z_n - x_n|^2), \\ \phi_{n+1} = \phi_n - \gamma \nabla_{\phi_n} (|z_n - x_n|^2), \end{cases} \quad (13)$$

The step sizes  $\mu$  and  $\gamma$  are updated over all known symbols as:

$$\begin{aligned} \mu_{k+1} &= \mu_k - \alpha \nabla_{\mu} (|z_n - x_n|^2), \\ \gamma_{n+1} &= \gamma_n - \beta \nabla_{\gamma} (|z_n - x_n|^2), \end{aligned} \quad (14)$$

where  $\alpha$  and  $\beta$  are the second-order step-sizes. Over information symbols, the joint equalization and phase synchronization algorithm operates in a Decision-Directed (DD) mode.

Finally, the receiver performs BICM decoding to recover the transmitted bits.

## IV. SIMULATIONS RESULTS

### A. The BCH1 channel [25]

In our simulations, the information data were coded by a 1/2 rate convolutional code with generator polynomials equal to (37,33). The length of the pilot symbols  $p_0$  (resp. feedforward equalizer  $L$ ) is set to 65 (resp. 16), and we consider  $N = 1024$ ,  $N_u/N_s = 1/2$ .

Figure 2 compares the performance of our proposed method to the solution used in [2], [3] consisting of an MMSE equalization followed by a PLL. It was obtained over a real UWA channel (BCH1) measured off the commercial harbour of Brest [25]. The BCH1 is a rather a stable channel; Fig. 3 show some impulse response at the different time instants of the BCH1 channel.

We can observe that our proposed method achieves a gain of about 1.5 dB compared with the conventional method.

### B. Synthetic time varying channel

In addition to the WATERMARK real recorded channel BCH1, we obtained similar results on a 3 tap synthetic channel where each tap is given by [26]:

$$h_l(k) = \sqrt{\frac{P_l}{I}} \sum_{i=1}^I e^{j(k\epsilon\pi \cos \psi_{l,i} + \zeta_{l,i})}, \quad (15)$$

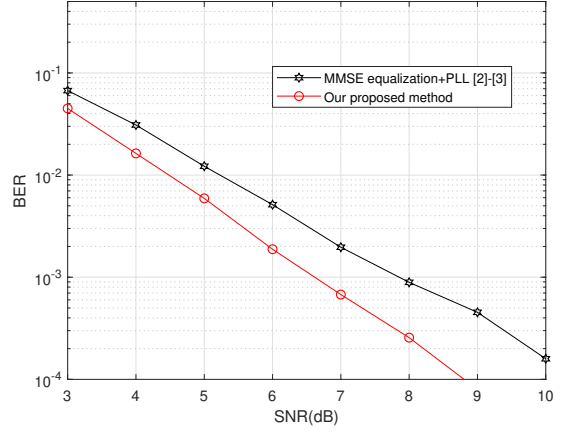


Fig. 3. BER comparison over the BCH1 channel [25].

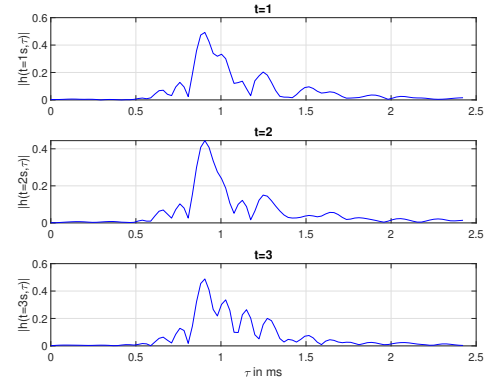


Fig. 4. Impulse responses at different time instants of the BCH1 channel.

where the Doppler shift is characterised by  $\epsilon$ ,  $P_l$  denotes the mean power of the  $l$ -th channel path,  $\psi_{l,i}$  and  $\zeta_{l,i}$  are uniform random variables over  $[0, 2\pi[$ ;  $I$  is fixed to 10 and (15) has three paths having all the same mean power [26] in our simulation. The length of the pilot symbols  $p_0$  (resp. feedforward equalizer  $L$ ) is set to 23 (resp. 6), and we consider  $N = 1024$ ,  $N_u/N_s = 1/2$ .

Figure 4 compares our proposal with the conventional method [2], [3] over this channel with two different Doppler shifts. It can be observed that our method achieves a better BER performance in particular over fast time varying channels as it surpasses the conventional method by 0.8 dB for  $\epsilon = 1e^{-4}$  and by more than 1.5 dB for  $\epsilon = 3e^{-4}$ .

## V. CONCLUSION

In this paper, we proposed a hybrid Frequency-Fresnel domain equalizer for Orthogonal Chirp Division Multiplexing (OCDM) to improve the BER performance over time-varying Underwater Acoustic (UWA) channels. First, the receiver performs frequency domain equalization based on the estimated

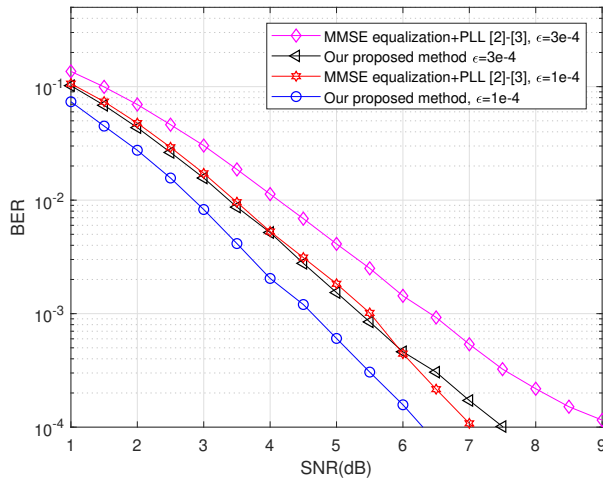


Fig. 5. BER comparison over a 3 tap synthetic channel (15).

sparse channel impulse response.

Moreover, we propose to deal with residual ISI by equalizing the obtained observation again in the Fresnel domain by using a transversal filter that operates jointly with a phase locked loop to better track phase variations over time. Simulations including a real underwater channel, showed significant gains in BER performance compared to conventional equalization techniques.

In the future, we would like to study the OCDM system performance using rotated constellations [27]–[32].

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