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### **CLASSROOM SIMULATORS: A NEW TRAINING APPROACH TO INVESTAGATE TEACACHERS PROFESSIONNAL KNOWLEDGE AND SUPPORT ITS DEVELOPMENT**

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*This paper offers an empirical study for supporting teachers' educators. It presents a new form of professional learning based on the use of a classroom simulator in training courses. Among the various research questions that this training approach makes it possible to tackle, we choose to focus in this paper on the way in which teachers' beliefs and geometry knowledge can be revealed by means of simulation.*

#### **INTRODUCTION**

This paper offers an empirical study for supporting teachers' educators. It presents a new form of professional learning based on the use of a classroom simulator in training courses. The design of the simulator is guided both by the outcomes of research into the characteristics and needs of geometric learning at lower secondary school level, and by the effective difficulties experienced by these students in solving geometric tasks. The simulated class situation consists of solving a problem within a Dynamic Geometry Environment (DGE) starting by drawing and manipulation, to pass afterwards to conjecturing and analytic visualization and then to proving.

Among the various research questions that this training approach makes it possible to tackle, we choose to focus in this paper on the way in which teachers' beliefs and geometry knowledge can be revealed by means of simulation. Indeed, Thomas & Palmer (2014) highlights that teachers not only need a special kind of knowledge for technology implementation, but that beliefs play a crucial role since they frame, guide and filter situations, actions and intentions. Training courses that directly address teachers' competencies are therefore regarded as important in supporting the development of knowledge, beliefs and practices (Hegedus et al. 2017). The specific research question we address is: what do trainee teachers' simulated practices reveal about their professional knowledge and beliefs about geometry and its teaching and particularly about the role of DGEs in solving geometric problems, focusing on drawing, manipulating, and conjecturing processes?

The rationale behind our questioning is informed by a range of factors and contextual elements. First, in French curriculum, students are expected to know how to draw figures before engaging in geometrical reasoning. Constructing with instruments is compulsory and is expected to help students passing from tangible to abstract and understanding geometric concepts. DGEs could be the appropriate place to make this passage. Constructing tasks play thus a key role in curricula, as they help students to move from spatial to geometrical thinking from "seeing on a drawing" to proving (Mithalal & Balacheff, 2019). Second, it is nowadays widely shared that DGEs could contribute in developing geometrical learning, particularly in supporting solving geometrical problems. It is acknowledged that they have the potential to "encourage both exploration and proof, because it makes it so easy to pose and test conjectures" (Hanna, 2000, p.13). Nevertheless,

researchers, such as Mariotti (2006) underline that even if dragging within DGEs provides the students with strong perceptual evidence that a certain property is true, this may reinforce a critical point in the teaching of geometry concerning the relationship between empirical evidence and theoretical reasons. Finally, one of the challenges that research has attempted to meet is to design and/or analyze situations where the connection between spatial and geometric reasoning is effective. Fischbein (1993) has emphasized that one of the main tasks of mathematics education (in the domain of geometry) is "to create types of didactical situations which would systematically ask for a strict cooperation between the two aspects [image and concepts], up to their fusion in unitary mental objects" (p 161). However, many researchers highlighted that although the situations designed and analyzed were very promising, they have been little used in teaching. The problem of the usability of these situations and "their communication to teachers has not been yet solved" (Bloch & Pressiat, 2009, p 66 [personal translation]).

To address these issues in the context of teacher training, we designed and used a computerized classroom simulator (SIC). The advantage of using such simulator is that it enables teachers to experiment classroom practices in an environment that is both safe - all types of experiments are allowed (no impact on real students) - and allows experiments to be repeated. Indeed, simulation is often used in professional training because it is less risky and speeds up the process of acquiring experience (Pastré, 2005). Moreover, the designed training course includes an analysis of the geometric problem, as well as a collective discussion of the concepts involved and the role of DGEs. The moments we will analyze in this paper are precisely the moments of collective discussion during and after the use of SIC.

#### **LEARNING GEOMETRY IN THE TRANSITION FROM PRIMARY TO SECONDARY SCHOOL**

This section is far from being exhaustive, its aim is rather to highlight some of the research findings on which our work is based, firstly with regard to the learning of geometry in general, and secondly in the specific context of DGEs. A significant amount of research has been devoted to studying students' learning in the transition from the tangible geometry of elementary school to the reasoning geometry of early secondary school; characteristics and difficulties have been identified.

Within French research, two main conceptual perspectives inform the field of geometry didactics. Berthelot & Salin (1998) differentiate between spatial and geometric knowledge. They stress that the main source of many of learning difficulties met by secondary school students is the lack of sufficient account of the spatial knowledge that are embedded in learning at elementary school level. Houdement & Kuzniak (1998) distinguish three paradigms of geometry: natural geometry (GI), natural axiomatic geometry (GII) and formalist axiomatic geometry (GIII). The passage from GI to GII is identified as difficult to implement in the classroom, even though it is decisive for students' learning. Furthermore, even if students entering secondary school have certain knowledge within GI, they don't necessarily link geometric constructions to the properties and theorems that justify them; their geometry tends to amalgamate GI and GII. In a similar vein, Parzysz (1991) shows that spatial objects and physical representations play an important role in the conceptualization of geometrical concepts. According to Fischbein (1993), the relationship between figural and conceptual components of geometry is not at all spontaneous and must be grounded in a long construction. Moreover, Sinclair et al. (2019) underline the strong link between drawing,

spatial reasoning and the learning of geometry. In analyzing the drawing processes, the authors show that the use of language or gesture can change the way that students see, draw and describe.

This leads us to reflect on the place of construction in geometric work. It is not the precise technical mastery of drawing processes with instruments, but rather the mental objects constructed by the student to accomplish this process, and what ensues for a better apprehension of the figure (Duval, 2006) that is at stake in the school-level transition that interests us here.

The use of DGEs in geometry learning and teaching is nowadays recognized as offering novel ways of carrying out geometrical activities in mathematics education (Laborde et al. 2006) and participating to the progress in students' conceptualization, influencing thereby different aspects of geometry learning. Within an exploratory approach, research studies stress the key role of dragging in forming a mathematical conjecture (Healy, 2000). By making it possible to drag and drop points and to multiply experiments, DGEs encourage access to conjecture and possibly reasoning to validate this conjecture. Bokosmaty, Mavilidi, & Paas (2017) show that manipulating a geometric figure in a DGE or seeing the teacher manipulating it limits cognitive load and improves memorization and appropriation of properties. Clements et al. (2008) indicate that students' understanding of geometric concepts can be enhanced by the dynamic environment, which improves visualization skills and the ability to focus on the interrelationships between different parts of geometric figures. Furthermore, Hoyles and Jones (1998) claim that dynamic geometry, supported by "what if" and "what if not" questions, has the potential to promote links between empirical and deductive reasoning.

Yet, teaching approaches involving a cycle of exploration-conjecturing-proving require thoughtful design; "engaging students in situations which make them aware of the constructive character of mathematical activities, especially those involving conjecture and proof, possess complex challenges" (Durand-Guerrier et al, 2012, p. 364). Even if dragging is a powerful tool for conjecturing, the resistance of the objects drawn brings a conviction that could slow down students' understanding of the need to prove by arguments. Similarly, while the above comments show the importance of manipulating geometric figures in a DGE, constructing these figures using the software's tools is a separate task from the conjecturing/proving process.Thus, the importance of designing appropriate tasks and the role the teacher should play in their implementation seem even more important in order to meet some learning aims. This is the subject of the following section.

#### **SIMULATOR AND TEACHER TRAINING COURSE DESIGN**

#### **From tasks for working on transition between spatial and geometric reasoning with Dynamic Geometry to tasks implemented in SIC**

Designing a classroom simulator means first and foremost finding a teaching situation that is conducive to raising the professional and mathematical questions that will become the focus of training. To encourage teachers to question the role of geometric construction within DGEs in the proof process, we used the problem described in Figure 1. This task considers the epistemological aspects identified by Lesnes-Cuisiniez (2021) in his synthesis of research on the double break between physical and theoretical geometry. These include the need to distinguish figure drawing, instrumental mobilization, heuristic arguments, and theoretical validation. We also added a promising idea: using a problem in which perception is challenged to highlight the need to rely on

reasoning rather than measurement or perception. For SIC, we have chosen a problem where perception is distorted (the red segment seems larger than the green segment, Figure 1), which is reinforced by the question posed which is not: "are the lengths the same?" but "Say which of the two segments, red or green, is larger".



Figure 1: Problem implemented in SIC.

This problem is available in several resources for lower secondary school teachers in paper-andpencil; we have adapted it to be solved in a DG environment. It is considered both in these resources and by the teachers who have tested it as an open problem that is simple to understand and enables the learner to engage in an experimental approach.

To begin, the student is asked to draw the figure in a DGE. This requires specific knowledge that is not the same as that required to draw the figure in paper-and-pencil, notably the order of construction steps, the commands available in the software (the software commands that the teacher can choose to authorize or not such as "perpendicular bisector") and the robustness of drawing under dragging (Healy 2000). Afterwards, the student is asked to conjecture the answer to the question: "Say which of the two segments, red or green, is larger". Finally, the student is asked to explain why, which refers to the process of argumentation and/or proof.

From a didactic point of view, there are several analyzable difficulties. The one we are aiming at here is that there is little connection between the construction work in the DGE and the conjecture work. In fact, the conjecture can be made using the "measurement" command, by moving the points connected to the circle to superimpose them, and not by using construction commands ("point on", for example, adds nothing to the conjecture). The proof phase requires to have noticed that the two segments are the diagonals of two rectangles and that the second one is in fact the radius of the circle. The construction does not allow identifying the rectangles (since it rather directs towards the idea of an orthogonal projection). In conclusion, being able to manipulate the figure in the DGE is important and sufficient to conjecture and prove; constructing it step by step doesn't provide necessarily additional means to complete these two tasks.

#### **The design of SIC**

Our goals in building SIC (available at http://www.fabien-emprin.ovh) are to get trainees to reflect on their practices, using their professional knowledge to solve a professional problem and to test several hypotheses by having the possibility to try again and again. To achieve these goals, we used part-scale simulators, which enable the user to make choices (what the teacher says, to whom he/she says it: one student or the whole classroom, what he/she does: act directly on a student's screen or projecting a same screen on the white board, etc.) and to see the effect of his/her choices on what students do (actions on the DGE, verbal responses, etc.). According to Pastré (2005), it is a way of "impoverishing the situation to make it more accessible to learning" (p. 27). We use a non-random model in which an action always has the same effect each time it is repeated, with no introduction of random phenomena, enabling analysis between trials by the same user and between users. In this sense, it's an experience-building accelerator.

To design a simulation, combined with a tool for computing these interactions, we needed to define the possible choices of the teacher and the effects of these choices on students in real classrooms. Our method was based on recording enough classroom settings. To do this, we set a learning situation and a support-resource accessible to the teacher to implement in his/her classroom (which could have been found in an educational resource (textbook, website)). We carried out an a priori analysis of the situation, based on didactic knowledge about geometry, highlighting possible choices in terms of tasks and class management. We then provided the situation to teachers, who were free to interpret it according to how they plan to carry it out in their classrooms. By observing these classrooms' settings, we gathered information that enabled us to compare the a priori analysis with actual implementations, and to identify the training knowledge that can be updated in the simulation. We observed around ten classroom settings, then built a simulation reproducing the teachers' choices and the students' reactions, while respecting the average proportion observed.

The feedback provided by the simulator is twofold: during the simulated session, the user can access the student's work, either by observing it (on the DGE screen, for example) or by questioning it; at the end of the session, the user can access what each student found (his/her construction, his/her answer to the question) and what the student remembers one week after the session. The latter could help the user measure the effect on students 'learning.

#### **A TRAINING COURSE USING SIC**

First, we would like to make it clear that we do not consider that SIC has any intrinsic value. It's not the act of playing out the simulated situation that enables the professional skills to be worked on. It's the debriefing that follows the simulation that allows questions and analyses to emerge and brings out the trainees' professional knowledge and thus possibly enabling effective training to take place (Pastré, 2005). Secondly, we consider that the trainee teacher's professional knowledge develops through an interaction between the existing knowledge/know-how and the information derived from the interactions with SIC, and then by the interaction with other trainees (with and without the trainer's interventions)

The training course centered on the simulator, as described below, aims both to immerse teachers in a (simulated) professional context and to provide reflective feedback on their choices and actions with the simulator. An alternation of moments of experimentation and discussions is therefore supposed to develop the teachers' professional knowledge.

The course is organized in five phases following the more general training approach proposed by Abboud et al. (2022). This process is presented to trainees at the beginning of the course. The aim of the first phase is to make trainees analyze the mathematical task possibly by carrying it out themselves and anticipating the possible difficulties students may encounter. The second phase takes the form of a group discussion on what the trainees were able to anticipate throughout the task analysis: what problems and difficulties the students are likely to encounter, what objective should be assigned to the teaching/learning situation.... During the third phase, the trainees use SIC. Thanks to the fact that the time is simulated (independent of real time), they can make several trials,

but without going back and completing the simulation each time. At the end of each simulation, they obtain summary information on the students' learning, which enables them to adapt their choices and test new hypotheses during the next trial. The fourth phase is again a collective discussion on the performance of the virtual students and the comparison with the real students (their own) and ways of helping them to learn geometry when accomplishing the task. Finally, during the fifth phase, the trainees are asked to design a class session based on the geometric task they had tried out and its implications in terms of promoting students' geometric reasoning.

It is during phase 2 and 4 that the trainer can pick up on elements of the teachers' practices and can intervene by supplying/sharing knowledge for professional development. It is these two phases that interest us in this article. In what follows we propose to analyze some exchanges from phase 2 and others from phase 4 (the page limitation of this paper doesn't allows us to provide the corresponding transcripts)

#### **Analysis of interactions from phase 2**

After clarifying, at the request of the trainees, that the virtual students had already experienced the DGE, the discussion turns to the difference between construction within a DGE and within paperand-pencil environment; the trainer provide then several clarifications on this point. Given the risk of multiple difficulties, one of the trainees suggests that the drawing should be carried out on paperand-pencil instead. This first exchange informs that the teachers are primarily concerned with the students' ability to construct the figure correctly so that they can then engage in conjecture. They have the feeling that using the DGE generated several difficulties. This led them to favor to abandon its use for the construction phase. Another exchange takes place with the trainer on the construction process, which is different in paper-and-pencil, particularly about constraints on the order of construction. We can notice here that, although the task was chosen for its potential in the DG environment in terms of two aspects - construction and manipulation - the teachers seemed doubtful about the usefulness of the first and preferred to start with paper-and-pencil to ensure less difficulty in the construction phase and favorable conditions to engage in the manipulation one. In the course of the ensuing discussion, we observe an exchange relating to the conjecture and the usefulness of using a DGE to "see it" in the sense highlighted by Mariotti (2006) where dragging provides the students with strong perceptual evidence that a certain property is true. The trainees emphasis the dynamic nature of the software and the fact that it allow to visualize an infinite number of figures (thus permitting the generalizing the observed property).

The whole discussion in phase 2 thus seems to show teachers' beliefs that distinguish between the role of the DGE in the drawing process and its role in the conjecturing own. In the first case, it seems more efficient to go back to the usual paper-and-pencil tools to make sure that the students are constructing the figure correctly (by using skills they have already acquired) before engaging in a more complex (for this class level) conjecturing task, where the dynamic aspect of DGEs seems to be unequalled in paper-and-pencil and is therefore very useful.

#### **Analysis of interactions from phase 4**

Following the same thread, we observe an exchange in phase 4 initiated by a trainee who suggests to give the students the "right" figure from the start, so that they can move on to conjecture. The trainer then informs him that there were three variations of this proposal within SIC to help the

students make the right figure, without giving them everything straight away. This gives rise to a discussion about how to help the students make the conjecture. In the ensuing discussion we observe the trainer attempting to provoke a reflection on the fact that giving the students the correct figure without them having constructed it themselves doesn't guarantee that they can engage in the conjecturing process. This attempt fails to achieve its objective, and the trainees prefer to direct the discussion towards ways of finding and/or validating the conjecture. They suggest providing "instrumental" help to students: make the software display the measurements so that equality can be seen. Another possibility of instrumental help was raised in the discussion: asking students to superimpose the points, but recognizing it as a special case that doesn't lead generalizing the conjecture.

Using the simulator seems thus to makes it possible to question the trainees' professional knowledge and make them aware of the link that can exist between the construction phase and the conjecture phase and the role that the DGE's functionalities can play in the latter. Furthermore, as SIC allows to keep track, in the form of a chronological table, of the actions taken by the trainee teacher during the various trails made during phase 3, we also analyzed these tracks. This analysis shows that the trainees who appropriate the simulator as a tool for testing and experimenting, make trials with a very short construction phase and realize that it is the manipulation of the figure and not the drawing that is useful for the proof.

#### **CONCLUSION**

This exploratory study highlights the potential of simulation-based training approach to reveal teachers' professional knowledge of geometry and particularly about supporting students to link geometrical construction and geometrical reasoning.

The aim of using SIC was also to engage teachers in collective reflection on their classroom practices in geometry, with SIC acting as a revealer of these practices. We show how it has revealed to the trainer some of the trainees' geometric beliefs and practices. We also look at the knowledge that the trainer considered relevant to contribute in-situ to promote professional learning on key elements to geometry teaching enhanced by dynamic geometry.

While geometric construction work, under certain conditions, is conducive to the transition from GI to GII, the technical skill of drawing, whether on paper-and-pencil environment or in a DGE, is a specific skill. However, simulated practices show that teachers remain attached to this activity, with certain confusion between constructing and conjecturing. Feedback from the simulator may lead them to reconsider this role.

This study shows the potential of simulated practices for both the trainer and the researcher. For the moment, the data extracted from the software can only be used by the researcher because of its complex format, but it is possible to imagine transforming it into learning analytics so that the trainer can also use it directly in his/her training sessions.

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