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## RESEARCH ARTICLE

# Corporate probability of default under an energy transition scenario with business model adaptation to the transition

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### ARTICLE HISTORY

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### ABSTRACT

The energy transition generates for the financial system the so-called '*transition risks*', leading to the development of Climate Stress-Tests. We propose a firm-level corporate credit risk model that accounts for business model evolution in a transition scenario for Climate Stress-Tests. It is a structural and path-dependent model with stochastic total assets and debt and it integrates all the transition risks drivers as well as physical risks. Simulations show that reducing emissions intensity may improve leverage ratios despite the mitigation costs. However, often-used strategies such as aligning with sectoral averages, increase credit risk, with default probabilities up to 4 times higher in orderly transitions. Constant market share assumptions underestimate default risk for high polluters and overestimate it for low polluters. Tailored business model strategies are essential for managing credit risk effectively during the energy transition.

### KEYWORDS

Climate transition and physical risks, business model adaption, Credit Risk Stress-Test, default probability, numerical computation, Monte-Carlo simulation.

### JEL CLASSIFICATION

Q54, G32, C61, G38.

### AMS CLASSIFICATION

91B38, 91G40.

## 1. Introduction

### 1.1. *The context of Climate Stress-Tests.*

The increasing focus on climate-related financial risks has led to the emergence of Climate Stress-Tests, a forward-looking risk assessment that aims to evaluate how financial entities would fare under different climate transition scenarios [2, 3]. Credit Risk Stress-Tests typically assess the impact of adverse but plausible short-term macroeconomic shocks on financial actors' credit portfolios and compute inherent financial risks' metrics, mainly the probabilities of default term structures. However, as [16] points out, the methods used for traditional Stress-Tests are unfit for capturing the long-term and potentially abrupt nature of transition risks. The complexity of Climate Stress-Testing is compounded by the time horizons involved and the need

for finer granularity to account for the various drivers of transition risks (policy, technology and sentiment, [5]) that break past relations in the bank’s book. The lack of historical data makes it difficult to apply traditional statistical models to estimate the relationships between these factors, and as the energy transition progresses, these relationships are expected to evolve.

### *1.2. State of the art on Corporate Credit Risk Stress-Testing.*

Incorporating climate Stress-Tests into Credit Risk Stress-Testing frameworks requires new models that integrate climate risk drivers and satisfy Climate Stress-Tests requirements. A first attempt by [15] proposed a bottom-up approach by augmenting the long-used Asymptotic Single Factor Risk model [24] with a climate risk factor but assumed static correlations that may not hold under transition risks. Moreover, it is not fit for the counterparty-level evaluation of the credit risk required by regulators in this context (see as an example [3]). On a finer granularity, [11] refines the model from [13] with a stochastic component to evaluate short-term transition risks for SMEs but is unsuitable for long-term Climate Stress-Tests due to the static nature of their modeling. In particular, they do not account for a change of business model (i.e. the way the firm makes profits) for the SMEs, which is a determinant of the energy transition.

Additionally, [16] thoroughly details the financial cash flows that would be impacted by an energy transition. On the one hand, profits would be reduced due to increased carbon taxation and augmentation of input prices. However, transition-compliant firms could see their net sales revenues rise thanks to inflated demand. This will directly impact the total assets of the company. On the other hand, debt stock would need to expand for the deployment of new technologies to meet the new market requirements. The choice of new technologies, either in the production process or as the object of production itself, constitutes a shift in the company’s business model induced by the transition.

Other works, such as [10] and [7], extend the well-known Merton model by [18] by introducing carbon price shocks, but they often excluded business model adaptation and its impact on debt. Note that these models only focus on one driver of transition risk. Models like [1, 4, 14] encompass the business model adaptation using deterministic projections calibrated on historical data and based on a constant market shares assumption. While this allows a dynamic debt, this hypothesis limits the effects of consumer sentiment on demand for “brown” goods and services the associated credit risk augmentation. For instance, car manufacturers, while having low direct emissions, remain highly vulnerable to changes in consumer sentiment. Furthermore, they use scoring models calibrated on historical data to compute the default probability, similarly to [16]. The key question remains: how to compute probabilities of default for corporates under a transition scenario, encompassing all transition risk drivers and business model adaptation strategies?

### *1.3. Our contribution.*

We propose a structural and path-dependent model for corporate credit risk assessment in Climate Stress-Tests. This scenario and sector invariant stochastic model encompasses the different transition risks’ drivers as described by [5] and allows for market share reallocation throughout the scenario with respect to the company’s emissions relative to the rest of its sector. It also enables the design and assessment of different types of business model adaptation strategies to the energy transition and scenarios [9]. We use a financial cash flow logic to compute both sides of the firm’s balance sheet as stochastic processes that depend on the scenario and the company’s business model. The model also takes into account physical shocks with increasing frequency. Using Nested Monte Carlo simulations, we evaluate different business model tran-

sition strategies in terms of corporate default probabilities under two transition scenarios: an orderly and a disorderly transition. We find that in a market sensitive to the consumer sentiment channel, companies that lower their intensity are rewarded with a better leverage ratio, despite the convex costs of such reduction. However, if the business model adaptation plan is not chosen wisely, it may worsen the company’s credit risk. Costs-based strategies perform better in terms of the probability of default. Specifically, following the same growth rate as the sector’s average intensity provided by the scenario systematically increases the credit risk, due to the required investment. It can yield PDs up to 4 times greater than other strategies. Moreover, contrary to similar works that rely on macroeconomic simulations or the assumption of optimal and homogenous firm-level behaviors, we focus on the impact of a firm’s individual decisions that may not be optimal. This allows interesting perspectives for banks’ risk management thanks to informed dialogue with their clients.

#### *1.4. Organisation of the paper.*

In Section 2, the default event and its probability are defined in a general setting. Next, the various cash flows are outlined in Section 3, leading to the computation of the firm’s total assets and debt as stochastic variables. Results based on different fictitious firms and transition scenarios are presented in Section 5.

## **2. Definition of the Default and the Probability of Default**

As scenarios are the central component of Stress-Tests, we begin with a given energy transition scenario, namely a deterministic set of likely energy transition pathways for main macroeconomic variables ranging on the period  $[0, T]$ , where  $T$  is the end of said scenario. Consider an energy transition risk-vulnerable firm that evolves in that same scenario. Precisely, its financial cash flows and business decisions will depend on the scenario on  $[0, T]$ . Our goal is to compute the company’s probability of default term structure, as the probabilities of defaulting within the next period at each date, given the state of the macroeconomy described by the scenario, as well as its business decisions at  $N$  fixed dates  $i = 0 : N - 1$ , with constant time step  $\delta$ . Thus, an important first step is to define the default event. We denote by  $A_i$  and  $D_i$  the company’s total assets and debt at date  $i$  respectively, which are stochastic variables depending on the scenario and the firm’s business decisions.

Following [8], we define the company’s default as the first time the total assets are no longer sufficient to cover the debt commitments, given it has never been the case beforehand. In other terms, the default of the firm is the random time  $\tau$  defined as follows:

$$\{\tau = i\} = \{i \geq 1, A_i < D_i, A_j \geq D_j, \forall j = 0, \dots, i - 1\}. \quad (2.1)$$

At each decision date  $i$ , we wish to compute the probability of the company defaulting within the next period, given that it has not occurred before. To do so, consider a filtered probability space  $(\Omega, (\mathcal{F}_i)_{i \geq 0}, \mathbb{P})$  where,  $(\mathcal{F}_i)_{i \geq 0}$  represents the information flow.  $(A_i)_{i \geq 0}$  and  $(D_i)_{i \geq 0}$  are  $\mathcal{F}$ -adapted processes. To get the probability of default term structure, it is necessary to compute for each  $i = 0 : N - 1$  the probability of defaulting in the following period. Hence, we leverage the definition given in (2.1). The 1-period probability of default at date  $i = 0 : N - 1$  denoted

by  $\text{PD}_i$  is:

$$\begin{aligned}\text{PD}_i &:= \mathbb{P}(\tau = i + 1 \mid \mathcal{F}_i) \\ &= \mathbb{P}(A_{i+1} < D_{i+1} \mid A_j \geq D_j, \forall j = 0 : i) \mathbf{1}\{A_j \geq D_j, \forall j = 0 : i\}.\end{aligned}$$

This is the conditional probability of the company defaulting in  $i + 1$ , evaluated in  $i$  and given that it has not defaulted before. We discuss the firm's dynamics in the next section.

### 3. Financial cash flows and balance sheet's dynamics

#### 3.1. Business model adaptation

We assume the company will adapt to the transition by reducing its relative emissions uniquely. Doing so will not only reduce its carbon tax costs, but also increase its sales revenues. The firm's sole lever of action is the intensity reduction rate process  $(\gamma_i)_{i=0:N-1}$ .

Moreover, the climate policy is usually disclosed in advance. For firms to enclose future carbon price shocks or technology bans in the design of their business model. Therefore, we assume the company tailors its own carbon emissions mitigation plan in date 0. This plan may be constructed with respect to the scenario and the company's characteristics or may be completely exogenous. Furthermore, it may either be a deterministic or constant function of some parameters or even stochastic.

At each date  $i = 0 : N - 1$ , the company may choose whether to invest in its own carbon emissions mitigation plan for the next period  $[i, i + 1]$  by selecting its intensity reduction effort  $\gamma_i \in \mathbb{R}^+$ . If  $\gamma_i = 0$ , the company's effort is null for the next period, and there is no "green" investment. However, if  $\gamma_i > 0$ , this implies that the company has chosen to invest in its emissions mitigation plan. We assume that large investment strategies are not feasible either due to the lack of technological progress or a constrained number of available units, thus  $\gamma_i \leq \gamma_{max}$ , with  $\gamma_{max} > 0$ .

The company's carbon emissions plan is  $\boldsymbol{\pi} := (\gamma_i)_{i=0:N-1}$  a vector containing all the intensity reduction rates over the scenario. For the following, we will refer to it as a "*carbon reduction strategy*" (or "*strategy*"). We give a more thorough definition:

**Definition 3.1.** A strategy is a sequence  $\boldsymbol{\pi} = (\gamma_0, \gamma_1, \dots, \gamma_{N-1})$  where, each  $\gamma_i$  is a universally measurable stochastic kernel of the history  $(\mathbf{X}_0, \gamma_0, \dots, \gamma_{i-1}, \mathbf{X}_i) \in (\mathbb{R}^+)^2 \times \mathbb{R}^+ \times \dots \times \mathbb{R}^+ \times (\mathbb{R}^+)^2$  with values in  $\mathbb{R}^+$ , such that:

$$\gamma_i([0, \gamma_{max}] \mid \mathbf{X}_0, \gamma_0, \dots, \gamma_{i-1}, \mathbf{X}_i) = 1,$$

where  $\gamma_k, k = 0, \dots, i - 1$  are the past realizations of the control process.

In other words, each  $\gamma_i, \forall i = 0 : N - 1$  conditional probability of  $\gamma_i$  has support in  $[0, \gamma_{max}]$ . For all  $i = 0 : N - 1$ , they each are a stochastic function of the history, i.e., a stochastic function of all  $\mathbf{X}_j, \forall j \leq i$  and  $\gamma_j, \forall j < i$ . We may consider weaker definitions of such a strategy. This is the object of the next definitions.

**Definition 3.2.** A Markovian strategy is a sequence  $\boldsymbol{\pi} = (\gamma_0, \gamma_1, \dots, \gamma_{N-1})$  such that, for each  $i = 0, \dots, N - 1$ ,  $\gamma_i$  is a universally measurable stochastic kernel on  $(\mathbb{R}^+)^2$  satisfying:

$$\gamma_i([0, \gamma_{max}] \mid \mathbf{X}_i) = 1.$$

### 3.2. Financial Cash Flows

We leverage the model from [20] to define the firm's financial cash flows under a given business model strategy. Let  $\Pi$  be the set of admissible strategies  $\boldsymbol{\pi} = (\gamma_i)_{i=0:N-1}$ , i.e. Markov,  $\mathcal{F}$ -adapted and such that each  $\gamma_i \in [0, \gamma_{max}]$ ,  $\forall i = 0 : N - 1$ . To compute the probability of default under admissible strategy  $\boldsymbol{\pi}$ , we need to introduce the next notation: Let  $\mathbf{X}_i^\boldsymbol{\pi} = (I_i^\boldsymbol{\pi}, S_i^\boldsymbol{\pi})$  be the value of the state variable at date  $i$  computed with strategy  $\boldsymbol{\pi}$  for  $i = 0 : N$ . Firstly, we denote the firm's intensity under strategy  $\boldsymbol{\pi} = (\gamma_i)_{i=0:N-1} \in \Pi$  as the next stochastic process:

$$I_{i+1}^\boldsymbol{\pi} = \begin{cases} I_i^\boldsymbol{\pi} e^{-\gamma_i \delta} \times e^{\sigma_I \Delta \varepsilon_i^I - \psi_i^I(\sigma_I)}, & i = 0 : N - 1 \\ I_N^\boldsymbol{\pi}, & i \geq N, \end{cases} \quad (3.1)$$

with  $I_0 > 0$ ,  $\gamma_i \in \mathbb{R}^+$  the intensity reduction effort rate and  $\sigma_I$  a positive constant. Here,  $\Delta \varepsilon_i^I$  should be viewed as the time-increment of a random process  $\Delta \varepsilon_i^I$ , which models the uncertainty in the evolution of the emission intensity. We assume that  $(\Delta \varepsilon_i^I)_i$  are independent and bounded for technical reasons. The additional factor  $\psi_i^I(\sigma_I)$  is the log moment generating function of  $\Delta \varepsilon_i^I$  at point  $\sigma_I$ :

$$e^{\psi_i^I(\sigma_I)} = \mathbb{E} \left[ e^{\sigma_I \Delta \varepsilon_i^I} \right],$$

which ensures that the uncertainty factor in (3.1) has unit mean. Secondly, the company's sales revenues  $(S_i)_{i \geq 0}$  are given by the following dynamics:

$$S_{i+1}^\boldsymbol{\pi} = \begin{cases} S_i^\boldsymbol{\pi} \frac{\bar{S}_{i+1}}{\bar{S}_i} e^{-\kappa(I_i^\boldsymbol{\pi} - I_i^{\text{ref}})\delta} \times e^{\sigma_S \Delta \varepsilon_i^S - \psi_i^S(\sigma_S)}, & i = 0 : N - 1 \\ S_N^\boldsymbol{\pi}, & i \geq N. \end{cases}$$

where  $\bar{S}$  are the sales revenues of the reference market given by the scenario in **current prices**, and  $I^{\text{ref}}$  is the market reference for the intensity. We assume that the market shares are reallocated with respect to the company's carbon efficiency relative to its market: this is reflected through the first exponential, where  $\kappa \geq 0$  is the sensitivity to the company's relative intensity with respect to the market. This captures both the consumer sentiment and demand-price effects due to the carbon cost pass-through (see [21]). As for (3.1),  $\sigma_S$  is a constant real number and the firm's idiosyncratic disturbances  $(\Delta \varepsilon_i^S)_i$  are independent random variables with log moment generating function  $\psi_i^S(\cdot)$ :

$$e^{\psi_i^S(\sigma_S)} = \mathbb{E} \left[ e^{\sigma_S \Delta \varepsilon_i^S} \right].$$

We also assume that  $\Delta \varepsilon_i^S$  is bounded for technical reasons. Because the intensity is by definition the ratio of emissions over sales revenues, we add the following assumption:

$$\text{Cov}(\Delta \varepsilon_i^I, \Delta \varepsilon_i^S) < 0.$$

For the following, we will denote the variables needed to retrieve the absolute carbon emissions as one single vector in  $\mathbb{R}^+ \times \mathbb{R}^+$ :  $\mathbf{X}_i := \{(I_i, S_i)\}_{i \geq 0}$ . Moreover, we will write as  $\mathbf{X}^{(1)} = I$  and  $\mathbf{X}^{(2)} = S$  for the two components of  $\mathbf{X}$ . The production of goods or services needed to earn  $S_i$  yields costs that directly depend on the quantity effectively produced and the price of used inputs. Here, we use the sales revenues as a proxy for the volume of production since the sales revenues are the product of the volume of sold goods and their selling price. Indeed, it enables

to account for both the eventual changes in the volume of production as well as the inputs' price's inflation that is assumed to be the same as the selling price's.

$$T^C(i, S, I) = \text{cp}_i IS + kS^\nu. \quad (3.2)$$

with  $k$  a positive constant standing for the coefficient factor of variable costs and  $\nu \in \mathbb{R}^+$  the exponent of the sales revenues, and  $(\text{cp}_i)_{i=0:N}$  the deterministic carbon price trajectory given by the scenario. Aside from costs, production also requires long-term capital. Let  $(K_i^\pi)_{i \geq 0}$  the required productive capital under  $\pi$  to produce  $S_i^\pi$  and  $(P_i)_{i \geq 0}$  the selling price of the firm's production. Assuming the relation between the production volume  $(Y_i^\pi)_{i \geq 0}$  and the productive capital is a Cobb-Douglas function [12] with constant labor stock and

Let  $a \in \mathbb{R}^+$  be the productivity rate of capital and  $\theta \in \mathbb{R}$  the exponent of the capital. Given that the selling price evolves with respect to a deterministic inflation rate trajectory  $(I_i^R)_{i=0:N}$  extracted from the scenario, we get the next dynamics for the selling price:  $P_i := P_{i-1}(1 + I_i^R)$  with  $P_0$  set to 1. The firm's capital under strategy  $\pi$  is:

$$K_i^\pi = \left( \frac{Y_i^\pi}{a} \right)^{1/\theta} = \left( \frac{S_i^\pi}{aP_i} \right)^{1/\theta}.$$

Considering the depreciation rate  $d \in [0, 1]$ , the capital investments are obtained as follows:

$$I^{NG}(i, \pi) = (K_i^\pi - (1 - d\delta)K_{i-1}^\pi)^+, \quad (3.3)$$

where  $x \mapsto (x)^+ = \max(x, 0)$ .

### 3.3. The firm's balance sheet

As in [18], we assume the company's total assets equal its value. Moreover, we insert our model in a Modigliani-Miller framework [19], i.e., the value of the firm is independent of its capital structure and the physical assets are modeled through the flow of profits they yield. Given a deterministic risk-free interest rate  $r > 0$  we model  $(A_i^\pi)_{i=0:N}$  as the conditional expectation of the sum of future discounted profits [17]:

$$A_i^\pi = \mathbb{E} \left[ \sum_{j=i}^{\infty} \frac{\text{Profits}_j^\pi}{(1 + r\delta)^{j-i}} \mid \mathcal{F}_i \right].$$

The firm's profit between  $i - 1$  and  $i$  is the difference of the sales revenues  $S_i^\pi$ , the company's operating costs (3.2) and physical risks damages that we model as an inhomogeneous compound Poisson process  $\sum_{q=n_{i-1}+1}^{n_i} Z_q$ . Note that, each  $n_i$  is an inhomogeneous Poisson process with intensity  $\lambda_i := \lambda_0 g^i, g \geq 1, i \geq 0$ <sup>1</sup> and  $(Z_t)_{t \in \mathbb{N}^*}$  is an i.i.d. stochastic process with positive values independent from  $(n_i)_{i > 0}$ ,  $(\Delta \varepsilon^I, \Delta \varepsilon^S)_{0 \leq i \leq N}$  and  $(\mathcal{F}_i)_{i \geq 0}$ . The profit is thus given by:

$$\text{Profits}_i^\pi = S_i^\pi - T^C(i, S_i^\pi, I_i^\pi) - \sum_{q=n_{i-1}+1}^{n_i} Z_q. \quad (3.4)$$

Using the definition of profits given in (3.4), the independence of  $(n_i)_{i > 0}$  and  $(Z_t)_{t > 0}$  from  $(\Delta \varepsilon^I, \Delta \varepsilon^S)_{0 \leq i \leq N}$ , equation (??), and the properties of the sum of an infinite number of consec-

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<sup>1</sup> $n_0 = 0$  by convention

utive terms of a geometric sequence with reason lower than 1, we get:

$$A_i^\pi = \mathbb{E} \left[ \sum_{j=i}^{N-1} \frac{S_j^\pi - T^C(j, S_j^\pi, I_j^\pi)}{(1+r\delta)^{j-i}} + \frac{S_N^\pi - T^C(N, S_N^\pi, I_N^\pi)}{r\delta(1+r\delta)^{N-i-1}} \mid \mathcal{F}_i \right] - \bar{Z}_i,$$

where:

$$\bar{Z}_i = \mathbb{E} \left[ \sum_{j \geq i} \frac{\sum_{q=n_{j-1}+1}^{n_j} Z_q}{(1+r\delta)^{j-i}} \right] = \begin{cases} \lambda_0 \frac{(1+r\delta)}{\delta \ln(g)} \mathbb{E}[Z_1], & \text{if } g = 1, \\ \lambda_0 \frac{[g^{i\delta} - g^{(i-1)\delta}]}{\delta \ln(g)} \times \frac{1+r\delta}{1+r\delta - g^\delta} \mathbb{E}[Z_1], & \text{if } g > 1. \end{cases}$$

We assume that the company contracts debt for the sole purpose of fully financing long-term investments such as in [1]. Thus, there is no self-financing or shareholder's equity augmentation possible for the payment of investments costs. The investments are the sum of green investments and non-green investments (or capital investments, given by (3.3)). The green investment costs  $I^G$  are the ones generated by the intensity reduction strategy  $\pi$ . They are used to green the producing capabilities that are already set up in place, meaning they aim to produce  $S_{i-1}^\pi$  with a lesser intensity whose reduction rate is determined by  $\gamma_{i-1}$ . Following Nordhaus' Dynamic Integrated Climate-Economy (DICE) model [22], we denote the green investment costs by:

$$I^G(i, S, \gamma) = S \times c \times \alpha^{i\delta} \times \frac{(1 - e^{-\gamma\delta})^\beta}{\beta},$$

where  $c > 0$  is the unit abatement cost of emissions in *USD*,  $\alpha \in [0.95, 1]$  the factor of autonomous cost decrease over time, and  $\beta \geq 2$  the exponent of the emissions reduction rate as a percent of total emissions  $1 - e^{-\gamma\delta}$ . With  $\zeta \in (0, 1)$  being the constant amortization rate for the debt, we define the debt dynamics as:

$$D_i^\pi = (1 - \zeta\delta)D_{i-1}^\pi + I^G(i-1, S_{i-1}^\pi, \gamma_{i-1}) + \left( \left( \frac{S_i^\pi}{aP_i} \right)^{1/\theta} - (1 - d\delta) \left( \frac{S_{i-1}^\pi}{aP_{i-1}} \right)^{1/\theta} \right)^+.$$

#### 4. Computation of the Probability of Default Term Structure

For the sake of clarity, we will write as  $\mathbf{X}_j^\pi(i, \mathbf{x}) = \left( I_j^\pi(i, \mathbf{x}), S_j^\pi(i, \mathbf{x}) \right)$ ,  $\forall i = 0 : N, j = 0 : N, i \leq j$  the value of the state variable in date  $j$  starting with value  $\mathbf{x} \in (\mathbb{R}^+)^2$  at date  $i$  and computed with strategy the admissible strategy  $\pi \in \Pi$ .

- (1) Simulate  $M_1 \gg 1$  trajectories  $\{(\mathbf{X}_i^{\pi,m}(0, \mathbf{X}_0))_{i=0:N}\}_{m=1:M_1}$  with starting value  $\mathbf{X}_0 = (I_0, S_0)$  strategy  $\pi(0, X_0) = (\gamma_i)_{i=0:N-1}$ .
- (2) For  $i = 0 : N - 1$ :
  - (a) Compute the expected sum of future discounted physical damages  $\bar{Z}_i$ .
  - (b) For each simulated value  $\mathbf{X}_i^{\pi,m}(0, \mathbf{X}_0)$ ,  $m = 1 : M_1$ , do:
    - (i) Consider an admissible strategy  $\pi_i(\mathbf{X}_i^{\pi,m}(0, \mathbf{X}_0)) = (\gamma_j(\cdot))_{j=i:N-1}$  and compute  $\left\{ \left( \mathbf{X}_j^{\pi_i,m,l}(i, \mathbf{X}_i^{\pi,m}(0, \mathbf{X}_0)) \right)_{j=i:N} \right\}_{l=1:M_2}$ , with  $M_2 \gg 1$ .
    - (ii) Compute the assets  $A_{i+1}^{\pi_i,m,l}$  and the debt  $D_{i+1}^{\pi_i,m,l}$  for trajectory  $m$  with the newly simulated trajectories.



(c) For each trajectory  $m$ , compute the default frequency as:

$$\widehat{\text{PD}}_i^{\pi,m} = \frac{1}{M_2} \sum_{l=1}^{M_2} \mathbf{1}\{A_{i+1}^{\pi,m,l} < D_{i+1}^{\pi,m,l}\}.$$

We thus get:

$$\widehat{\text{PD}}_i^{\pi} = \frac{1}{M_1} \sum_{m=1}^{M_1} \widehat{\text{PD}}_i^{\pi,m}.$$

(d) Because the trajectories such that  $\widehat{\text{PD}}_i^{\pi,m} = 1$  are defaulted a.s., they can no longer be considered in the computation of the default frequency due to the default condition (2.1). Hence, we need to delete them and reduce the number of trajectories  $M_1$  accordingly:  $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1} \mathbf{1}\{\widehat{\text{PD}}_i^{\pi,m} = 1\}$ .

We detail this methodology in Algorithm 1:

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**Algorithm 1: Computation of the Probability of Default Term Structure with Nested Monte Carlo Simulations**


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**Input:** Number of Trajectories:  $M_1 \gg 1$ ,  
Number of Nested Simulations:  $M_2 \gg 1$ ,  
Strategy:  $\boldsymbol{\pi}(\mathbf{X}_0) = (\gamma_i)_{i=0:N-1}$ .

**Output:** PD term structure:  $(\widehat{\text{PD}}_i^\boldsymbol{\pi})_{i=0:N-1}$ .

1 Compute  $M_1$  Monte Carlo paths for

$\{(\mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0))_{i=1:N}\}_{m=1:M_1} = \{(I_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0), S_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0))_{i=0:N}\}_{m=1:M_1}$  with strategy  $\boldsymbol{\pi}$ .

2 for  $i = 0 : N - 1$  do

3     Compute  $\bar{Z}_i$ .

4     for  $m = 1 : M_1$  do

5         for  $l = 1 : M_2$  do

6             Set strategy  $\boldsymbol{\pi}_i(\mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)) := (\gamma_i(\cdot), \gamma_{i+1}(\cdot), \dots, \gamma_{N-1}(\cdot))$ .

7             Simulate a trajectory  $(\mathbf{X}_j^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)))_{j=i:N}$  starting with value  $X_i^{\boldsymbol{\pi},m}$  in  $i$ .

8             Compute the assets and debt :

$$A_{i+1}^{\boldsymbol{\pi},m,l} := \sum_{j=i+1}^{N-1} \frac{S_j^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)) - T^C(j, S_j^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)), I_j^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)))}{(1 + r\delta)^{j-i-1}} \\ + \frac{S_N^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)) - T^C(N, S_N^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)), I_N^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)))}{r\delta(1 + r\delta)^{N-i-2}} \\ - \bar{Z}_i,$$

$$D_{i+1}^{\boldsymbol{\pi},m,l} := (1 - \delta\zeta)D_i^{\boldsymbol{\pi},m,l} + I^G \left( i, S_i^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)), \gamma_i(\mathbf{X}_i^{\boldsymbol{\pi},m}(0, \mathbf{X}_0)) \right) \\ + \left( \left( \frac{S_{i+1}^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, X_0))}{aP_{i+1}} \right)^{1/\theta} - (1 - \delta d) \left( \frac{S_i^{\boldsymbol{\pi}_i, m, l}(i, \mathbf{X}_i^{\boldsymbol{\pi},m}(0, X_0))}{aP_i} \right)^{1/\theta} \right)^+.$$

end

end

9     Compute the conditional default frequency:  $\widehat{\text{PD}}_i^{\boldsymbol{\pi},m} = \frac{1}{M_2} \sum_{l=1}^{M_2} \mathbf{1}\{A_{i+1}^{\boldsymbol{\pi},m,l} < D_{i+1}^{\boldsymbol{\pi},m,l}\}$ .

10     Compute  $\widehat{\text{PD}}_i^\boldsymbol{\pi} = \frac{1}{M_1} \sum_{m=1}^{M_1} \widehat{\text{PD}}_i^{\boldsymbol{\pi},m}$ .

11     Delete all trajectories such that  $\widehat{\text{PD}}_i^{\boldsymbol{\pi},m} = 1$ .

12     Update  $M_1 \leftarrow M_1 - \sum_{m=1}^{M_1} \mathbf{1}\{\widehat{\text{PD}}_i^{\boldsymbol{\pi},m} = 1\}$ .

end

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**Table 1.** Summary of the different NGFS Scenarios studied for the analysis. CDR means carbon Dioxide Removal technologies (Source: NGFS ).

Scenario	Temperature increase in 2100	Policy type	Technology	CDR	Regional Policy Variation
B2C	1.6°C	Immediate and smooth	Moderate change	Medium-high	Low variation
DT	1.6°C	Delayed	Slow then Fast change	Low-medium	High variation

## 5. Application to fictitious companies

### 5.1. Description of the scenarios

We have based our application on two NGFS phase II energy transition scenarios [23], namely, *Below 2 Degrees* (B2C), and *Delayed Transition* (DT) (see Table 1). Both show different pathways running from 2020 to 2050. B2C is a smooth (or orderly) transition scenario, with a gradual increase of the carbon price until 2050. DT is a disorderly transition scenario, with constant carbon price until 2030, where a carbon price shock occurs with a dramatic increase of the carbon price until 2050. It goes from USD10 to USD1000 at the end of the scenario. We focused on a transition vulnerable sector “D35: Electricity, gas, steam, and air conditioning supply” for France.

### 5.2. Choice of parameters

We have considered a 6 dates problem ( $N = 6, \delta = 5$ ), and have ran tests on three fictitious companies that only differ in starting point intensity:  $I_0^1 = 1.16E-3$  (brown),  $I_0^2 = 7.71E-4$  (yellow),  $I_0^3 = 3.86E-4$  (green). Table 2 summarizes all other parameters used. We ran  $M_1 = 10E4$  simulations with  $M_2 = 200$  nested simulations.

### 5.3. Description of the strategies

For each scenario, we have computed the results for four strategies. The first one is entitled the *uncontrolled strategy*, i.e. with  $\pi := \mathbf{0}_N$ . Then, we consider the *exogenous* (exo) strategy consisting in selecting the same reduction rate as the reference intensity with the yearly reduction rate capped to 40% (see  $\gamma_{max}$  in Table 2). The third strategy is the *myopic* (myo) strategy and consists of selecting the reduction rate such that the investment at date  $i$  is the same as the cost of paying the carbon tax at date  $i$ :

$$\begin{aligned} \pi^{\text{exo}} &:= (\gamma_i^{\text{exo}})_{i=0:N-1}, & e^{-\gamma_i^{\text{exo}}\delta} &= \max\left(\frac{I_{i+1}^{\text{ref}}}{I_i^{\text{ref}}}, e^{-\gamma_{max}\delta}\right), \\ \pi^{\text{myo}} &:= (\gamma_i^{\text{myo}}(I))_{i=0:N-1}, & \gamma_i^{\text{myo}}(I) &= \min\left(-\frac{1}{\delta} \ln\left(1 - \left(\beta \frac{\text{cp}_i I}{c\alpha^i}\right)^{\frac{1}{\beta}}\right), \gamma_{max}\right), \end{aligned}$$

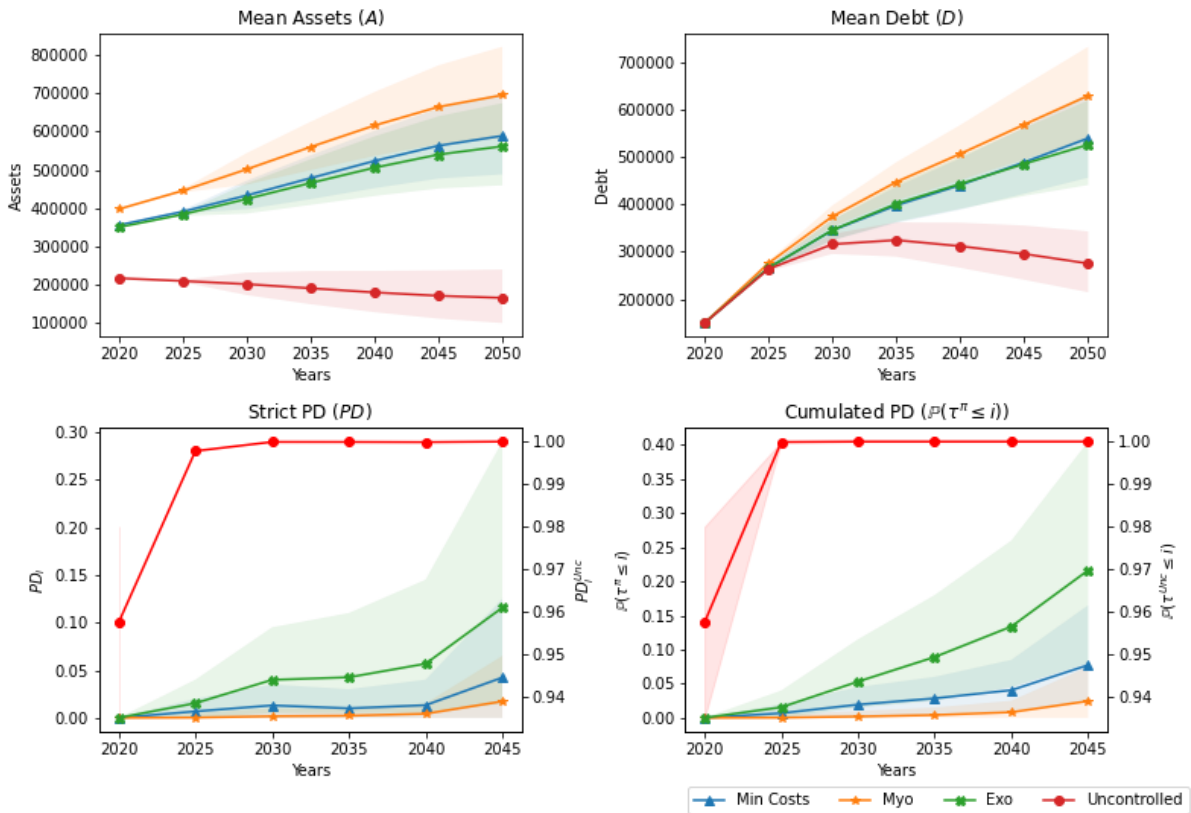
for all  $i = 0 : N - 1$ . Lastly, let us introduce the minimum transition costs strategy (“*min costs*”). It is defined so as to minimize the firm’s future carbon cost in the cheapest way over the time horizon  $[0, T]$  in [20].

**Table 2.** Values of the model parameters for the toy example.

$\sigma_I$	$S_0$	$\sigma_S$	$\kappa$	$\alpha$	$\beta$	$c$	$r$	$\gamma_{max}$	$k$	$\nu$	$a$	$\theta$	$\mathbb{E}[Z_1]$	$\lambda_0$	$g$	$D_0$	$\zeta$	$d$
2%	8.4E5	2%	30	0.95	2.8	1.26	6%	$-\log(0.6)$	1	1	1	1	100	1/5	1	1.5E5	12%	5.5%

#### 5.4. Results in the ‘Below 2 Degrees’ scenario

As expected, the uncontrolled strategy largely degrades the company’s financials. The company’s total assets’ growth is smothered, while the debt costs augment until 2035. This results in a starting level PD of 95.74%. Because the firm’s sales are rapidly decaying, there is no need for more productive capital investments, thus, the company gradually pays back its debt from 2035 onwards, should it have survived until then. As shown in [20], the myopic strategy provides the lowest intensity levels throughout the B2C scenario with the brown company, thus leading to higher sales and lower carbon costs, but also higher green investments  $I^G$ . On the one hand, the total assets growth is driven up by higher gross revenues and lower carbon costs simultaneously (Fig. 1). On the other hand, the debt stock augments rapidly due to the inflated green investments, but also non-green investments required to follow the production’s needs. The solidity of the total assets supersedes the debt stock augmentation, leading to a stable probability of default.

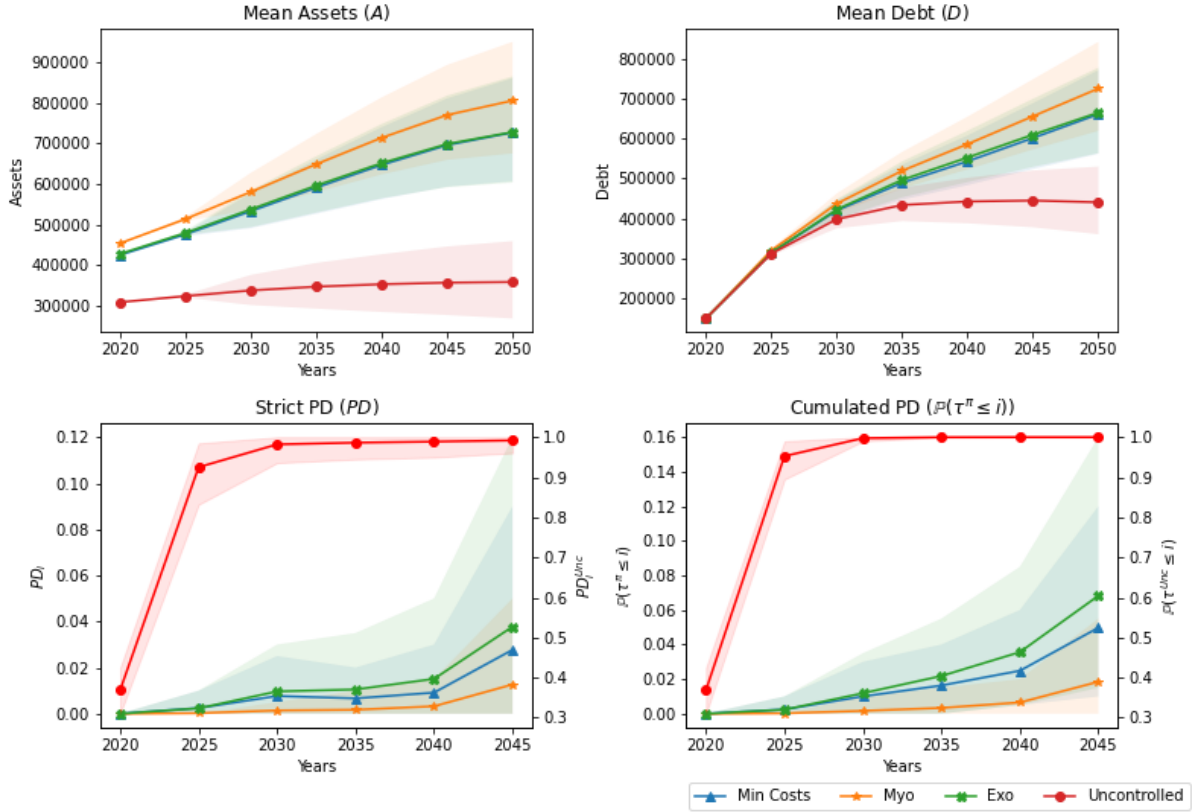


**Figure 1.** Simulated results for the **brown company** ( $I_0^1 = 1.16E-3$ ) in scenario **B2C**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.

The minimum costs strategy takes advantage of the autonomous factor of cost decrease  $\alpha$  to schedule higher intensity mitigation when it is cheaper. This leads to similar debt costs than the exogenous strategy while greater total assets. In turn, the PD under the exogenous strategy surpasses the minimum transition costs’ for the brown firm.

In Figure 2, results are shown for the yellow company. First, the uncontrolled PD starts largely degraded in 2020 (36.9%). Then, it immediately shoots up to 92.5% in 2025, leading to default chances near 100% by the end of the scenario. The myopic strategy still supersedes the other strategies thanks to boosted sales due to  $\kappa$ . In 2025, it is more than 9 times lower

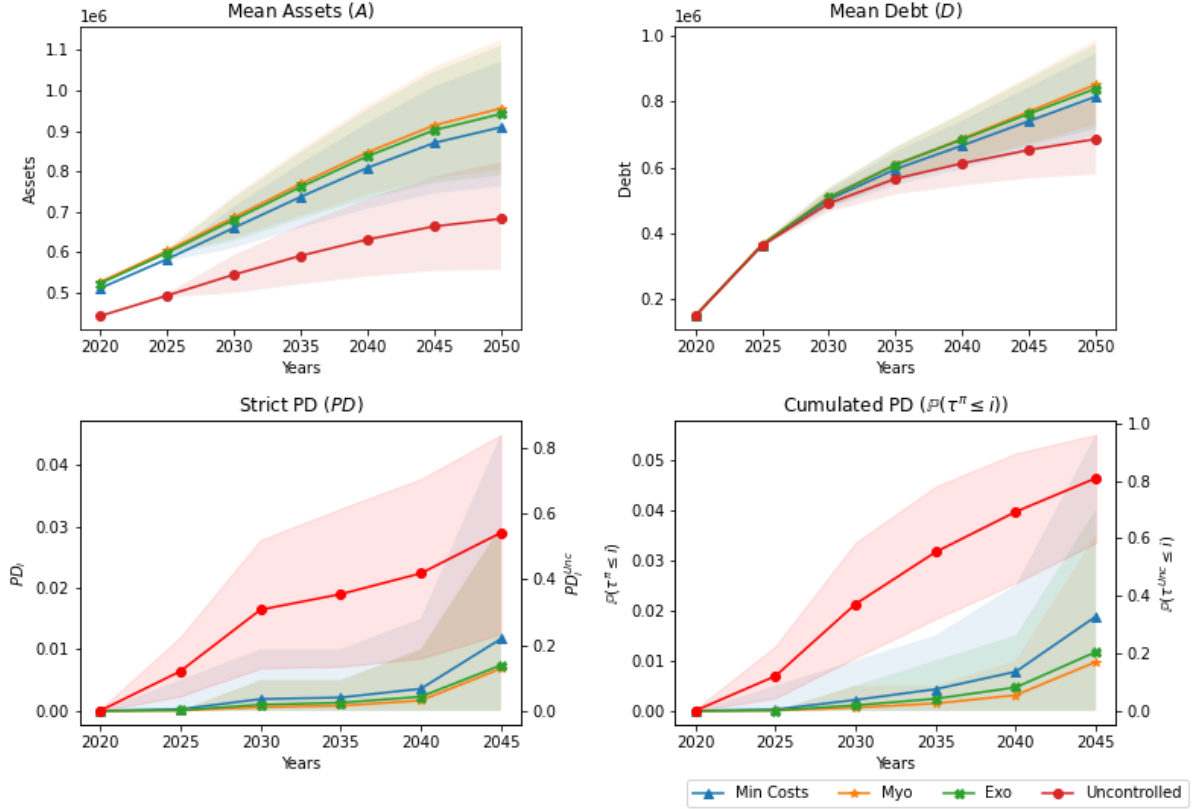
than both the minimum transition costs and the exogenous PDs. The gap narrows over the scenario, until it becomes around twice as much as the other PDs in 2050. The exogenous and minimum transition costs PD are close in the first periods considered. Then, because of large green investment costs, the exogenous debt grows faster than the minimum transition costs strategy's, leading to a 1.6 times higher PD in 2035.



**Figure 2.** Simulated results for the **yellow company** ( $I_0^2 = 7.71E-4$ ) in scenario **B2C**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.

The green company starts off with an advantage in terms on intensity. However, it would catch up with the scenario-provided sector average intensity pathway around 2035 under the uncontrolled strategy. This would lead to significant carbon tax costs, that thus decreases the company's profit. In turn, the total assets under the uncontrolled strategy are much lower than the others (Fig. 3). The firm's uncontrolled PD is almost 450 times higher than the second largest PD in 2035 (minimum transition costs). However, the green company's uncontrolled PD is still much lower than the other two companies studied.

Nevertheless, the green company's financial health would benefit from participating in the transition. In addition, the more it invests in reducing its carbon emissions, the smaller its PD gets. Precisely, contrarily to the ulterior cases, despite being much more onerous than the minimum transition costs strategy in terms of investment, it displays lower PDs (Fig. 3). The gap widens with time due to higher carbon tax and lower gross revenues generated by the gradually increasing difference in intensity. In a similar fashion, the myopic PD yields once again the lower PDs despite requiring the most green investments. This suggests that overzealous but carefully crafted green investment leads to better PD thanks to lower intensity driving sales up and and cutting costs for green firms.



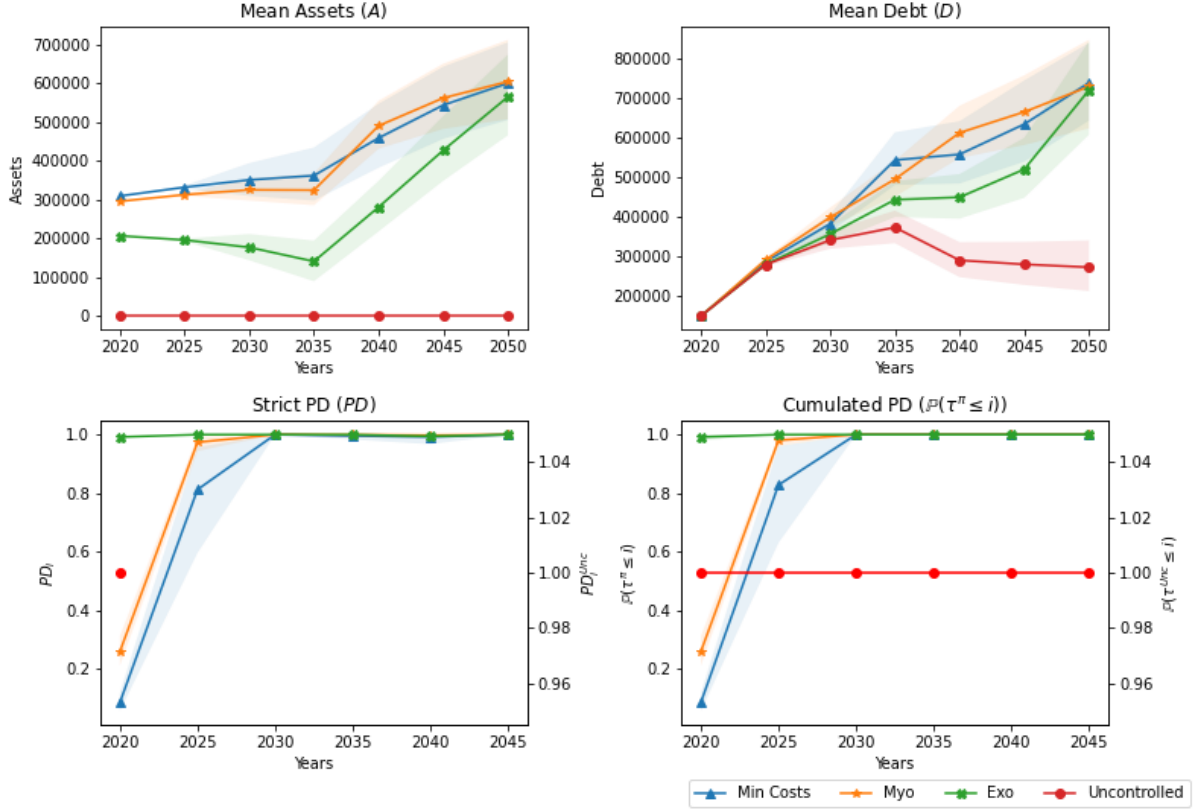
**Figure 3.** Simulated results for the **green company** ( $I_0^3 = 3.86E-4$ ) in scenario **B2C**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.

### 5.5. Results in the ‘Delayed Transition’ scenario

The brown firm is expected to fail almost surely under the uncontrolled strategy (Fig. 4). Indeed, its floored assets fall to 0 as soon as 2020. Thanks to the investment in carbon mitigation, the other strategies cope better under this stringent scenario. For instance, the exogenous strategy’s carbon costs reach  $1.75E5$  in 2035, compared to  $4E5$  for the uncontrolled strategy. However, this causes a bump in green investments that is then passed on the debt stock in 2035 (Fig. 4). At the same time, the perspective of high carbon costs in this scenario degrows the total assets from 2020 to 2035. This causes the exogenous PD to flirt with 100%.

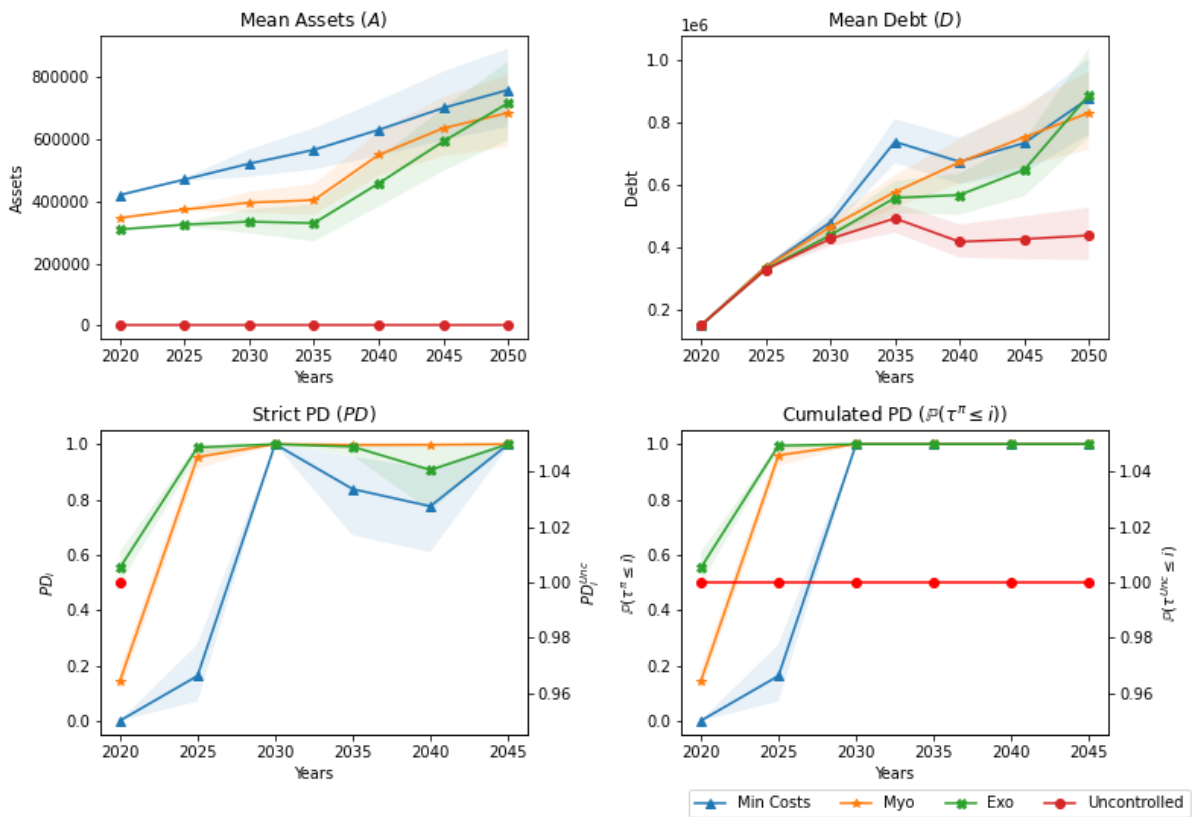
Regarding the other two strategies, they have lower odds of leading to bankruptcy at the beginning of the scenario, as they take the costs of reducing the intensity into account. Thanks to its forward-looking feature, the minimum transition costs strategy acts much earlier and harder so as to provide a buffer to lessen the 2035 carbon shock impact (see [20]). In turn, the impact of the 2030 carbon price on total assets is smoother. Nonetheless, this colossal investment effort degrades the leverage ratio. However, this strategy still performs better than the myopic in this setting. This is explained by the fact that the myopic strategy happens in reaction to the scenario’s shocks. Thus, not only would the firm suffer the 2030 carbon price shock on its profits, and thus total assets, but also it will need to invest drastically to match the carbon tax.

Figure 5 shows that all the uncontrolled trajectories have almost surely defaulted by 2035 for the yellow company due to the anticipation of the 2035 carbon price shock and the slow down in sales due to higher intensity. Once again, the exogenous strategy puts the company in



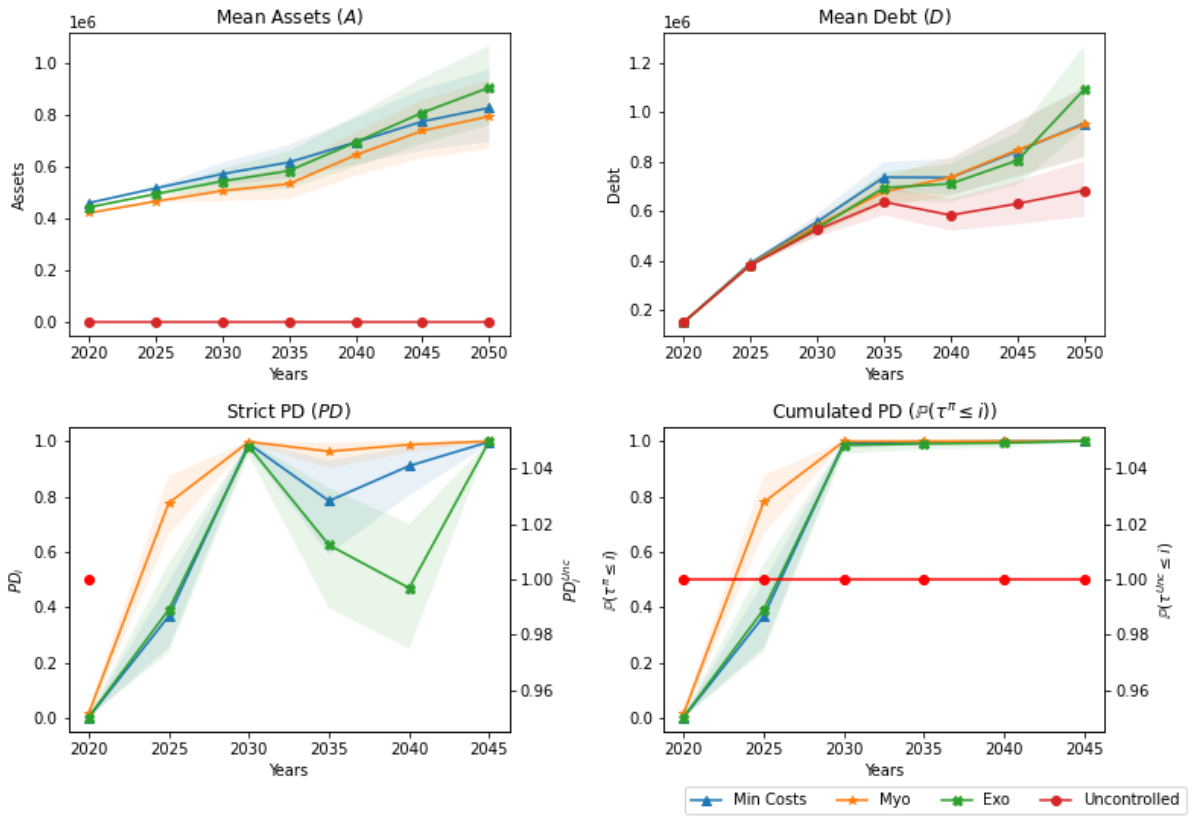
**Figure 4.** Simulated results for the **brown company** ( $I_0^1 = 1.16E-3$ ) in scenario **DT**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.

default in 2030 almost surely in this scenario. Similarly, the 2030 carbon shock also inflates the minimum transition costs PD despite a strong start in 2020 (0.01%). Lastly, the myopic PD suffers a first shock in 2025 due to the future carbon price sharp increase. It goes from 14.34% in 2020 to 95.31%, whereas the minimum transition costs PD is equal to 16.2% in 2025. Thanks to its starting level intensity, the green company's PDs under the different strategies are lower than that of the other two companies at starting point (Fig. 6). What stands out is that the exogenous strategy now yields smaller PDs than the myopic strategy. They are in fact close to that of the minimum costs strategy. What's more is that the minimum costs PDs are thrice higher with the green company than the yellow company (0.01% vs 0.03% in 2020, 16.2% vs 36.7%). This is because the intensity levels yielded by the minimum transition costs strategy for the green and yellow company are similar as of 2035. In turn, the green company's assets growth joins that of the yellow, whereas it needed to contract more debt in the first half of the scenario to follow the sales' boost.



**Figure 5.** Simulated results for the **yellow company** ( $I_0^2 = 7.71E-4$ ) in scenario **DT**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.





**Figure 6.** Simulated results for the **green company** ( $I_0^3 = 3.86E-4$ ) in scenario **DT**. Shaded area: 90% confidence interval for each strategy. Probabilities of default (strict and cumulated) for the uncontrolled strategy are to be read on the RHS  $y$ -axis.

## 6. Discussion

### 6.1. Computation of the PD.

We define the default probability based on a stronger default condition than that of Merton [18]. The probability of default we get is thus path-dependent. One major issue is that the number of trajectories accounted for the computation of the expected default frequency will decrease through time. Thus, the precision of the algorithm also decreases. In the banking industry, PDs are around  $50E-4$ , which means the number of defaulted trajectories is negligible with respect to the total number of simulated points. However, in some of our simulations, the resulting PDs largely surpass this value. This is the reason why we have systematically presented the cumulated PDs trajectories for each. Moreover, we give the computation for a 1-period probability of default. If  $\delta \neq 1$ , this does not amount to the 1-year probability of default usually required in stress tests. One workaround would be to use a linear interpolation to compute the values of the assets and debt 1 year after the considered date.

### 6.2. The debt dynamics.

We consider that the entirety of investments, whether green or for production, are financed by debt. This is in line with real-life practice, where corporations finance considerable investments through debt. The same assumption is found in the literature [1, 6]. However, equity augmentation, as well as self-financing, can also be used. Moreover, we designed the debt dynamics using a constant amortization rate, independent from the firm's profits. One can consider that the relationship between the debt and the firm's profits can be retrieved in the leverage ratio during the PD computation. Indeed, if the firm's profits perspective is unfavorable, the total assets will be devaluated, leading to a degraded leverage ratio. Thus, by considering debt stocks with respect to the firm's profit, the impact on its credit risk is clear. The debt does not depend on its costs either. Similarly to profits, the relationship between both appears in the credit risk evaluation. Indeed, the risk-free interest rate directly impacts the firm's valuation, thus the firm's PD.

### 6.3. The total assets.

We define the assets as the expectation of future discounted profits. However, the estimation of profits after the end of the scenario is an arduous task. We have assumed constant sales revenues for all  $i > N$ . However, this may underestimate the assets at the end of the scenario. A deterministic growth rate could be added. Note that this choice would be another source of model risk. Furthermore, the link between the total assets and the capital  $(K_i)_{i \geq 0}$  is implicit. Since both are estimated based on the sales revenues trajectory, a clear relationship does appear. Moreover, for firms with decreasing sales, the need for non-green investment gradually decrease, while the previously owned capital is depreciated. The same effect is seen through total assets that gradually decrease due to the perspective of decaying future profits through  $(S_i)_{i \geq 0}$ .

### 6.4. Considered strategies.

We have considered four strategies: uncontrolled, myopic, exogenous and minimum transition costs. The first three were chosen because they have been used in the literature. The latter was designed to resemble the way scenarios are computed. For instance, GCAM, one of the IAMs used for the computation of the NGFS scenarios, use a minimum cost assumption to derive

the likely transition pathways. Other strategies could be considered, such as “*max profits*” or “*min PD*”. However, such strategies require more work on their resolution, specifically on the existence of their solution.

## 7. Conclusion

We have proposed a model for computing probabilities of default alongside a transition scenario for credit risk stress testing. The foundation of this model is based on the work of [20], which presents a framework for modeling a firm’s sales revenues and relative emissions, conditional on a transition scenario. This approach particularly fits credit risk stress testing, as it accounts for the various transmission drivers of transition risk to credit risk, as outlined in [5]. Additionally, the model allows for the analysis of the firm’s business model adaptation to the energy transition.

The business model variables from [20] have been extended to incorporate transition-contingent financials while adapting market shares according to the company’s relative emissions. Using a discounted cash flows approach, the firm’s total assets have been computed as a stochastic process. Given that transition risks and physical risks are complementary, physical risk shocks have been incorporated into the firm’s profits equation, thereby directly linking total assets to physical risks. On the liabilities side, the company’s dynamics have been modeled by directly estimating the firm’s total investments. A notable feature of this model is that both total assets and debt are stochastic processes that evolve in response to both the scenario and the company’s choice of business model. Due to the model’s complexity, a numerical methodology for computing probability of default term structures has been proposed, using Nested Monte Carlo simulations. Then, probability of default term structures for companies with varying starting levels of relative emissions under two transition scenarios from the NGFS Phase II have been computed with the purpose of comparing the credit risks associated with different business models.

We found that companies that keep a static business model have higher probabilities of default, regardless of their starting point emissions level. This amplifies with time. Thus, PD models that disregard company transition are not suitable for long-term credit risk estimation, as is the case in Climate Stress-Tests. Moreover, the strategy consisting of following the same decrease rate as the scenario’s sectoral average emissions augments the credit risk due to the investment costs it engages. Forward-looking strategies perform best in disorderly transition scenarios.

## Disclosure statement

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## References

- [1] S. Alogoskoufis, N. Dunz, T. Emambakhsh, T. Hennig, M. Kaijser, C. Kouratzoglou, M.A. Muñoz, L. Parisi, and C. Salleo, *ECB economy-wide climate stress test: Methodology and results*, <https://www.ecb.europa.eu/pub/pdf/scpops/ecb.op281-05a7735b1c.en.pdf> (2021).
- [2] Autorité de Contrôle Prudentiel et de Résolution, *Scénarios et hypothèses principales de l'exercice pilote climatique*, [https://acpr.banque-france.fr/sites/default/files/medias/documents/20200525\\_principales\\_hypotheses\\_pour\\_lexercice\\_pilote\\_climatique\\_vf.pdf](https://acpr.banque-france.fr/sites/default/files/medias/documents/20200525_principales_hypotheses_pour_lexercice_pilote_climatique_vf.pdf) (2020).
- [3] Bank of England, *Discussion paper: the 2021 biennial explanatory scenario on the financial risks from climate change*, <https://www.bankofengland.co.uk/-/media/boe/files/paper/2019/the-2021-biennial-exploratory-scenario-on-the-financial-risks-from-climate-change.pdf> (2019).
- [4] Barclays PLC, *Corporate transition risk forecast model*, <https://home.barclays/content/dam/home-barclays/documents/citizenship/ESG/2021/Corporate-Transition-Forecast-Model-2021.pdf> (2021).
- [5] Basel Committee on Banking Supervision, *Climate-related risk drivers and their transmission channels*, <https://www.bis.org/bcbs/publ/d517.pdf> (2021).
- [6] S. Battiston, A. Mandel, I. Monasterolo, and A. Roncoroni, *Climate credit risk and corporate valuation*, Available at SSRN 4124002 (2023).
- [7] M. Belloni, F. Kuik, and L. Mingarelli, *Euro area banks' sensitivity to changes in carbon price*, <https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2654-9a537f810a.en.pdf> (2022).
- [8] F. Black and J.C. Cox, *Valuing corporate securities: Some effects of bond indenture provisions*, *The Journal of Finance* 31 (1976), pp. 351–367.
- [9] A. Boirard, D. Gayle, T. Löber, L. Parisi, C. Payerols, E. Schets, M. Spaggiari, A. Bavandi, C. Bertram, M.D. Paries, *et al.*
- [10] V. Bouchet and T. Le Guenedal, *Credit risk sensitivity to carbon price*, *SSRN Electronic Journal*, <https://ssrn.com/abstract=3574486> (2020).
- [11] G. Bouveret, J.F. Chassagneux, S. Ibbou, A. Jacquier, and L. Sopgoui, *Propagation of a carbon price in a credit portfolio through macroeconomic factors* (2024). Available at <https://arxiv.org/abs/2307.12695>.
- [12] C.W. Cobb and P.H. Douglas, *A theory of production*, *The American Economic Review* 18 (1928), pp. 139–165. Available at <http://www.jstor.org/stable/1811556>.
- [13] A. Devulder and N. Lisack, *Carbon tax in a production network: Propagation and sectoral incidence*, <https://publications.banque-france.fr/sites/default/files/medias/documents/wp760.pdf> (2020).
- [14] T. Emambakhsh, M. Fuchs, S. Kordel, C. Kouratzoglou, C. Lelli, R. Pizzeghello, C. Salleo, and M. Spaggiari, *The road to Paris: stress testing the transition towards a net-zero economy*, <https://ssrn.com/abstract=4564374> (2023).
- [15] J. Garnier, J.B. Gaudemet, and A. Gruz, *The Climate Extended Risk Model (CERM)*, <https://arxiv.org/abs/2103.03275> (2022).
- [16] M. Hayne, S. Ralite, J. Thomä, and D. Koopman, *Factoring transition risks into regulatory stress-tests: The case for a standardized framework for climate stress testing and measuring impact tolerance to abrupt late and sudden economic decarbonization*, *ACRN Journal of Finance and Risk Perspectives* 8 (2019), pp. 206–222.
- [17] L. Kruschwitz and A. Löffler, *Stochastic discounted cash flow: a theory of the valuation of firms*, Springer Nature, 2020.
- [18] R.C. Merton, *On the pricing of corporate debt: The risk structure of interest rates*, *The Journal of finance* 29 (1974), pp. 449–470.
- [19] F. Modigliani and M.H. Miller, *The cost of capital, corporation finance and the theory of investment*, *The American Economic Review* 48 (1958), pp. 261–297. Available at <http://www.jstor.org/stable/1809766>.
- [20] E. Ndiaye, A. Bezat, E. Gobet, C. Guivarch, and Y. Jiao, *Optimal business model adaptation plan for a company under a transition scenario*, 2024. Available at <https://hal.science/hal-04682824>,

- working paper or preprint.
- [21] K. Neuhoff and R.A. Ritz, *Carbon cost pass-through in industrial sectors*, Tech. Rep., Energy Policy Research Group, University of Cambridge, 2019. Available at <http://www.jstor.org/stable/resrep30282>.
  - [22] W.D. Nordhaus, *Revisiting the social cost of carbon*, Proceedings of the National Academy of Sciences 114 (2017), pp. 1518–1523.
  - [23] O. Richters, C. Bertram, E. Kriegler, J. Anz, T. Beck, D.N. Bresch, M. Charles, L. Clarke, and R. Cui, *NGFS climate scenario database: Technical documentation v2. 2*, <https://data.ece.iiasa.ac.at/ngfs-phase-2/#/downloads> (2022).
  - [24] O. Vasicek, *Limiting loan loss probability distribution*, KMV corporation (1991).