

# HOW TO MODIFY THE TREE OF SHAPES OF AN IMAGE: CONNECTED OPERATORS WITHOUT GRADIENT INVERSION

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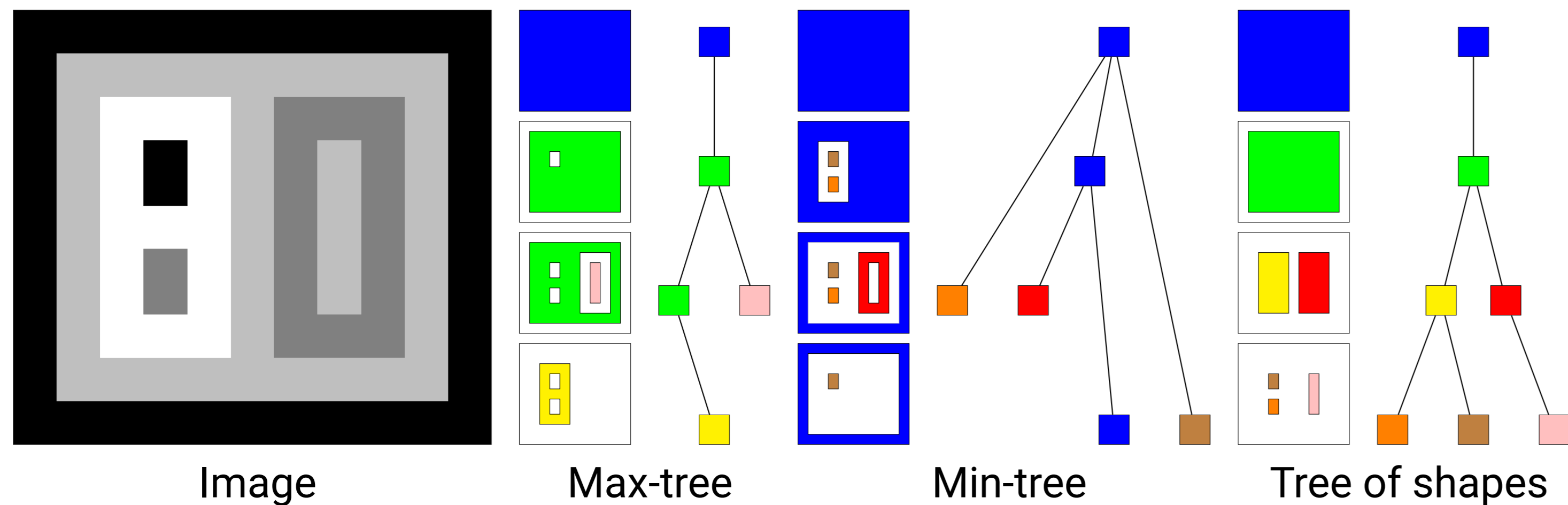
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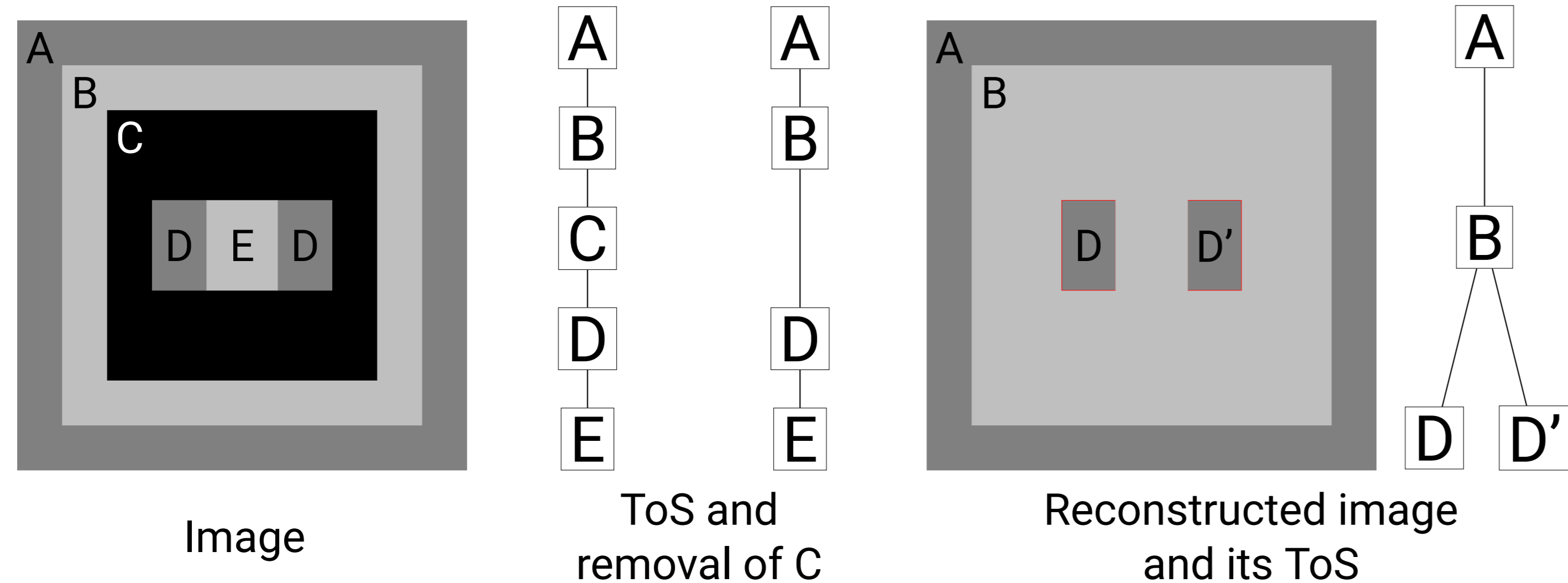


## Motivations

The tree of shapes (ToS) [1] is a **hierarchical data structure** that models a grey-level image via the inclusion of its **level lines**. It is often described as the self-dual version of the min- and max-trees.



- The **modification** (shifting of the altitude of one or several nodes) of the tree of shapes allows to design **connected operators**.
- If carried out uncarefully, the resulting structure **may not be the ToS of the modified image** and **this image may present differential inconsistencies**.

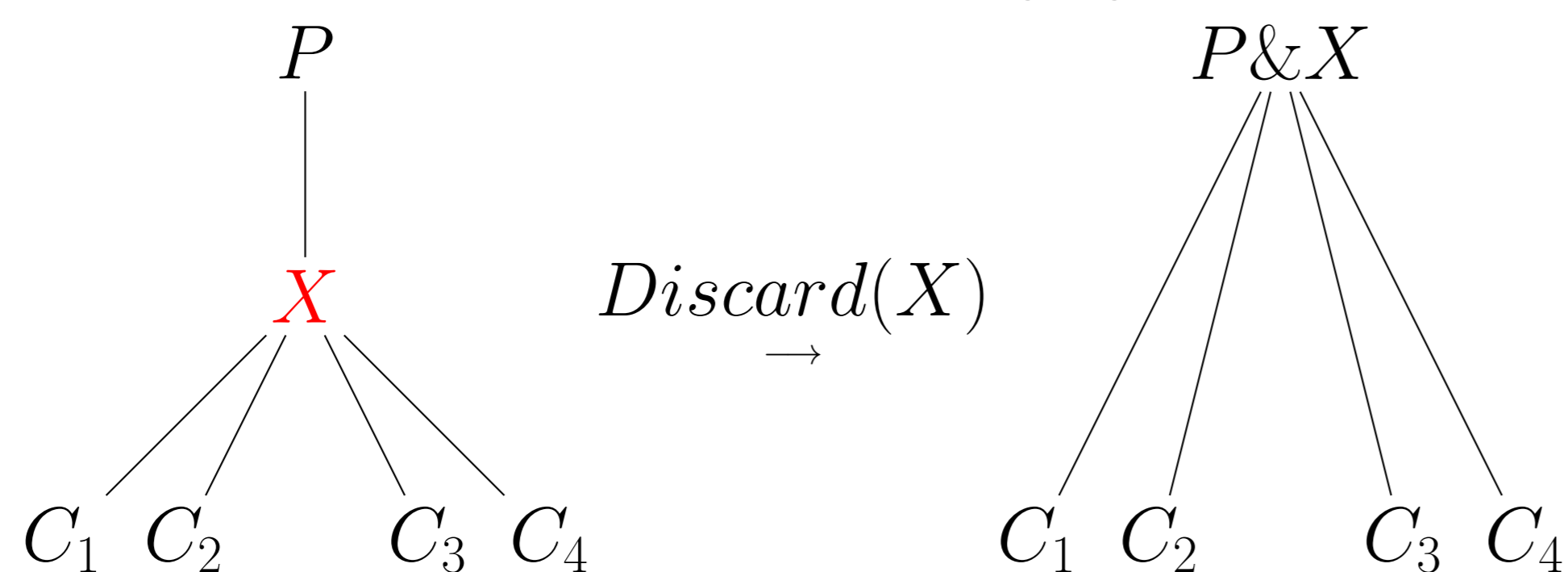


The proposed modifications rules guarantee that:

- the modified tree of shapes **is still the tree of shapes** of the modified image;
- the **signs of the gradients** of the resulting image are not inverted.

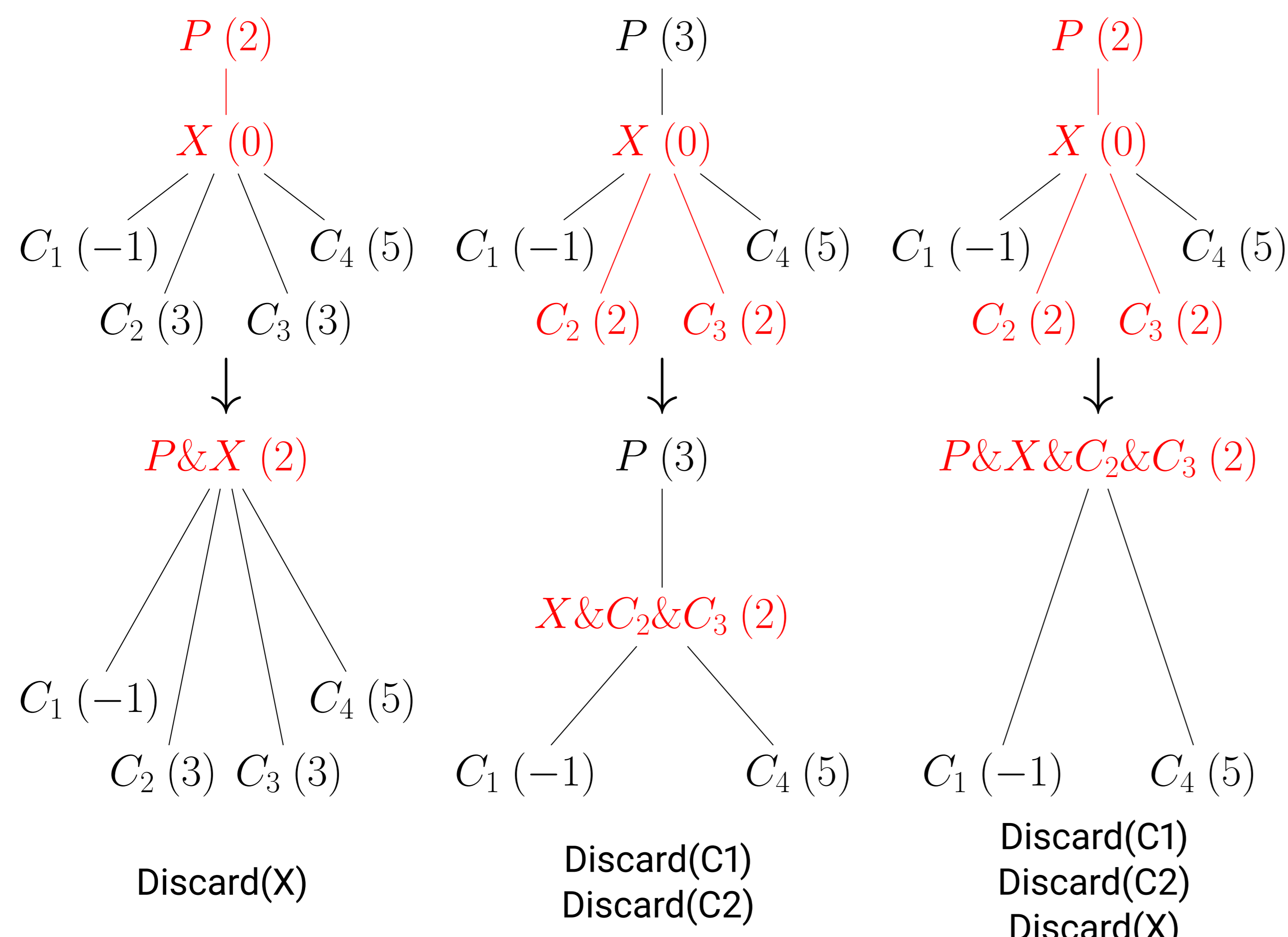
## Short range altitude shifting

The *Discard* function is the **atomic operation** merging a node to its parent.



**Short range shift:** modify the altitude of a node to a target altitude within that of its **nearest neighbouring nodes** (parent and children).

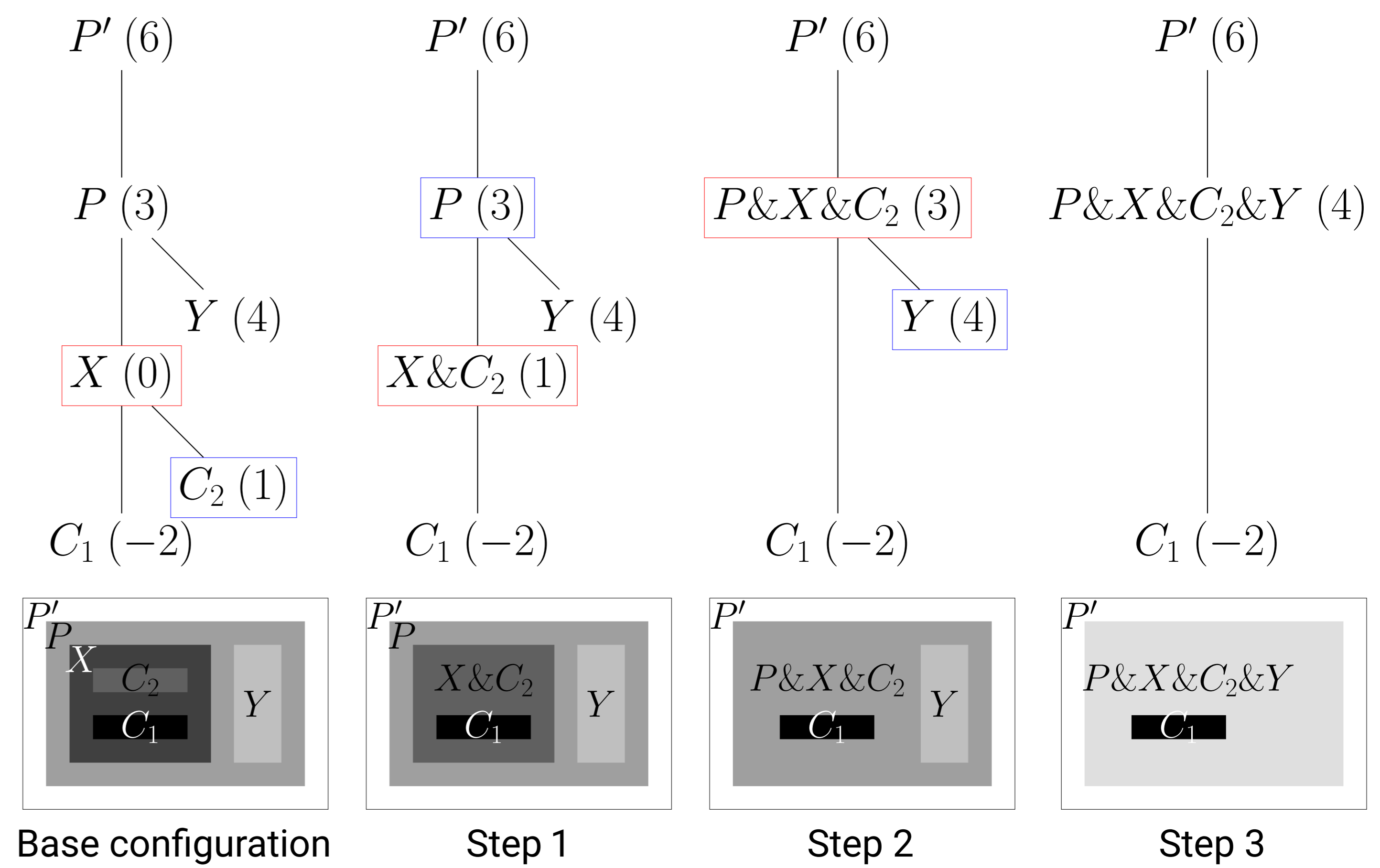
The short range shifting of node  $X$  with a target altitude of 2 can lead to the following configurations:



## Long range altitude shifting

**Long range shift:** modify the altitude of a node to **any target altitude** within the grey level range of the image.

- **Gradually** progress toward the target altitude by applying **successive short range shifts** until the target altitude is reached.
- Results in the progressive flattening of the surrounding regions.



## Applications

Defining a **filtering criterion** allows the modification of specific nodes of the tree.

- Depending on the **filtering strategy**, the defined operators can satisfy specific properties (extensivity, antiextensivity, quantification, etc.).
- Example of an area opening, removing nodes of the image with **small proper part**:

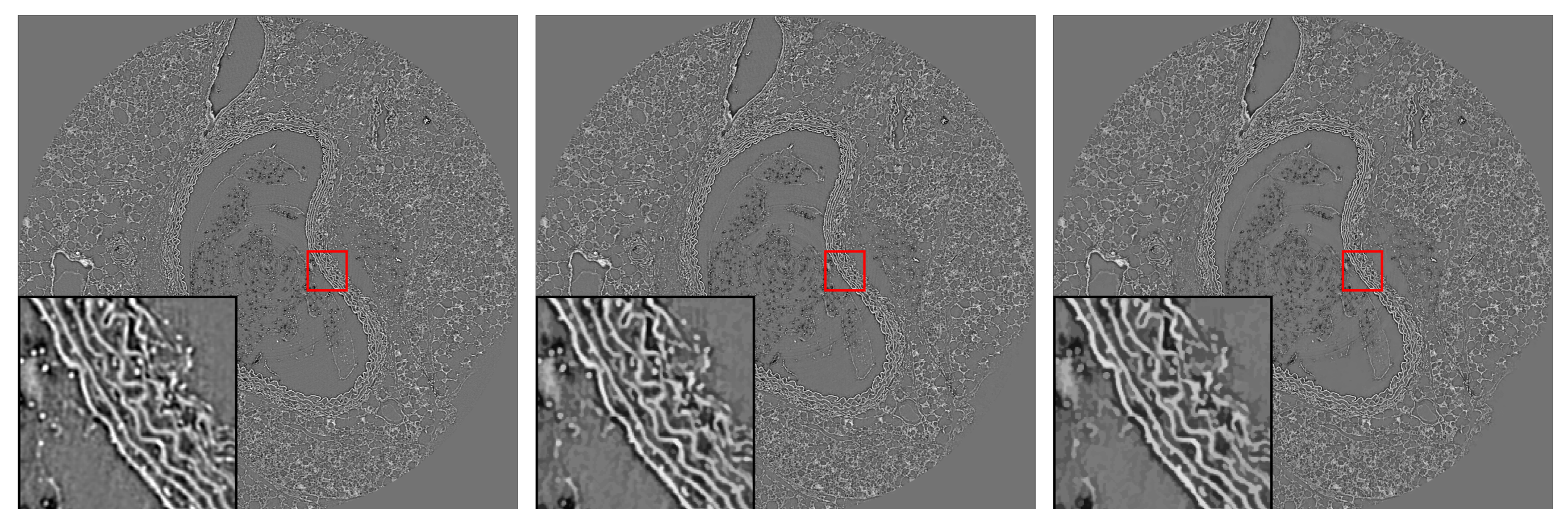


Satellite image

Proper part < 3

Proper part < 10

- One can define strategies to **simplify** a tree of shapes.
- Each node is assigned the mean altitude value of its neighbouring nodes.
- Apply this mean filtering  $n$  times:



Biological image [2]  
4.2 · 10<sup>6</sup> pixels,  
ToS : 1.2 · 10<sup>6</sup> nodes

$n = 5$   
SSIM = 0.981  
ToS : 4.7 · 10<sup>5</sup> nodes

$n = 10$   
SSIM = 0.910  
ToS : 2.3 · 10<sup>5</sup> nodes

[1] Caselles, V., Coll, B., Morel, J. : Topographic maps and local contrast changes in natural images. International Journal of Computer Vision, 33:5-27 (1999)

[2] Synchrotron microtomography, SOLEIL ANATOMIX, project #20211303, courtesy S. Almagro Work funded by Région Normandie (thesis grant RIN), Partenariats Hubert Curien (Sakura, 49674RK) and Agence Nationale de la Recherche (ANR-20-CE45-0011, ANR-22-CE45-0034, ANR-23-CE45-0015). ANATOMIX is an EQUIPEX funded by the Investments for the Future program of the Agence Nationale de la Recherche (ANR-11-EQPX-0031).