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► **To cite this version:**

Camille Cornand, Rodolphe Dos Santos Ferreira. Central bank's communication and stabilization policies under firms'motivated beliefs. *Economic Theory*, In press, 10.1007/s00199-024-01598-1 . hal-04808798

**HAL Id: hal-04808798**

**<https://hal.science/hal-04808798v1>**

Submitted on 28 Nov 2024

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# Central bank's communication and stabilization policies under firms' motivated beliefs\*

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July 6, 2024

## Abstract

Using a simple microfounded macroeconomic model with price making firms and a central bank maximizing the welfare of a representative household, we show that the presence of firms' motivated beliefs has stark consequences for central banks' optimal communication and stabilization policies. Under pure communication, motivated beliefs overweighting the accuracy of firms' private information may reverse the bang-bang solution of transparency found in the literature under objective beliefs and lead to intermediate levels of communication. Similarly, when communication and stabilization policies are combined, motivated beliefs overweighting firms' ability to process idiosyncratic information in general may reverse the bang-bang solution of opacity applying under objective beliefs, leading again to intermediate levels of communication and stabilization.

**JEL codes:** D83, D84, E52, E58.

**Keywords:** motivated beliefs, public and private information (accuracy), overconfidence, communication policy, stabilization policy.

Declarations of interest: none

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\*We thank participants to the 2022 Joint Banque de France - Deutsche Bundesbank Spring Conference on Monetary Policy and Expectations of Households and Firms, the 2022 ASFEE conference, the internal seminar at GATE, the CLE online seminar at the Catholic University in Milan, the online micro seminar at Indiana University, the online Bordeaux School of Economics seminar, and in particular Klaus Adam, Ala Avoyan, Peter Andre, Domenico Delli Gatti, Zeno Enders, Gianluca Femminis, Daniela Puzzello, and Louis Raffestin for useful feedback. We also thank Alistair Cameron for his careful proofreading.

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# 1 Introduction

Central banks communication plays a decisive role in the way to achieve effective monetary policy. Ben Bernanke, the former Chair of the Federal Reserve, put this point sharply, at the same time bringing to the fore the dilemma of transparency versus opacity arising in the way to communicate:

"When I was at the Federal Reserve, I occasionally observed that monetary policy is 98 percent talk and only two percent action. The ability to shape market expectations of future policy through public statements is one of the most powerful tools the Fed has. The downside for policymakers, of course, is that the cost of sending the wrong message can be high. Presumably, that's why my predecessor Alan Greenspan once told a Senate committee that, as a central banker, he had 'learned to mumble with great incoherence'" (Bernanke, 2015).

New information technologies and a growing claim for transparency and accountability have led to a rapid evolution in central banks communication.<sup>1</sup> Over the last three decades, central banks have gone from absolute secrecy to vague statements and then to transparency. The growing popularity of inflation targeting – in which communication plays a central role – partly explains this increase in transparency. Yet, if transparency is generally suitable (especially in terms of accountability) and optimal in a world free of imperfections where the first best is attainable, full transparency is questionable in an imperfect world where only a second best is within reach. We purpose to revisit this issue under a slight deviation from rational expectations.

One characteristic of many expectations is that they are held in an overconfident manner,<sup>2</sup> possibly responding to motivated cognition (Bénabou and Tirole, 2016).<sup>3</sup> Firms may be tempted to interpret or process their information so as to increase their anticipatory utility, meaning that firms' managers experience pleasant emotions from thinking that they can reach high profits by choosing to perceive their information as more accurate than it really is or by considering their abilities to process information as greater than they really are. Such motivated beliefs maximizing utility benefits of good outcomes characterize wishful thinking (or willful blindness) in firms' information accuracy or processing. They naturally affect the way firms' managers set their prices, possibly leading them to put too much weight

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<sup>1</sup>On the recent changes of central banks communication see for instance Macklem and Vardy (2023).

<sup>2</sup>A vast experimental and empirical literature shows that economic agents generally overestimate the accuracy of their expectations (Dunning et al., 1990). See Barrero (2022) for an application to firms' growth outcomes. The literature on the overconfidence bias also calls it 'overprecision' bias or 'miscalibration' (Ben-David et al., 2013).

<sup>3</sup>Bénabou and Tirole (2016, p. 145) point out that "individuals will overestimate or underestimate their own abilities depending upon which distortion is advantageous."

in privately owned or privately processed information, thus reaching but a suboptimal degree of mutual coordination. Under these circumstances, as central banks disclose information by their communication (and stabilization) policies, they certainly increase firms' access to public information, but they also modify – possibly increase – the weight put in private information. The aim of this paper is to study how central banks can and should adjust to such induced distortion in the price setting of incompletely informed and overconfident firms – a distortion that alters the terms of the dilemma of transparency versus opacity.

Most economic models, considering the role of heterogeneous and dispersed information, derive policy recommendations on the assumption that private agents form objective beliefs. In the realm of monetary policy, for example, motivated beliefs about private agents' information knowledge or processing have not yet received attention. This paper determines optimal communication and stabilization policies when firms exhibit motivated subjective beliefs about (i) the quality of their *own* private information on the fundamental (preference) shocks affecting the economy or (ii) their ability to process information in general, whether private or disclosed by the central bank. Using a simple model with price setting firms, shocks that do not affect market power and a policy maker maximizing the welfare of a representative household, it is shown that the presence of endogenously motivated subjective beliefs has stark consequences for the conduct of optimal communication and stabilization policies.

In this model, each firm derives anticipatory utility from its profit prospects, and accordingly faces a trade-off: it can accept the grim implications of either inaccurate own information about the fundamental shock or poor capacity to process that information as well as central bank public disclosures, and act in conformity, or else maintain hopeful beliefs by discounting and denying the fact that its information is inaccurate or its abilities in information processing are low, at the risk of making overoptimistic decisions. Sticking to hopeful beliefs is costly, so much so that at the limit of an infinite cost firms would prefer to form objective beliefs. However, when such cost is limited, firms exhibit in equilibrium overconfidence in the accuracy of their private signals (i.e. they overestimate the precision of their private information)<sup>4</sup> or in their ability to process the information they receive (i.e. they believe they can extract the appropriate signal from their own private information or from the noisy public information sent by the central bank). In both cases, they may accordingly rely too much on idiosyncratic information to set their price, which can raise price dispersion and decrease welfare.

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<sup>4</sup>We focus on the overestimation of precision, putting aside the possibility of biased beliefs. See Barrero (2022) who establishes, by observing managerial expectations for year-ahead sales growth (i) that "managers do not appear to be overoptimistic: sales growth expectations on average do not exceed realizations" and (ii) that they "underestimate the volatility of future own-firm sales growth, which they reveal by overestimating their forecasts' accuracy."

We formalize the cost of being irrational by distinguishing the *size* of the belief distortion which, according to Caplin and Leahy (2019), is commonly captured by the Kullback-Leibler divergence, and the *concern* for that distortion, which we assume to depend upon the firm’s environment. This feature is a novelty in comparison to the extant literature, which assumes it to be constant. The idea is that the extent to which the concern for the belief distortion modulates its cost depends on whether the distortion is impactful or inconsequential. In other words, the incentive to reduce the belief distortion is adjusted to its relevance for firms’ decisions. The trade-off between the pleasure generated by overconfidence and the cost of parting with reality is therefore assumed to depend upon the relative quality of private or privately-processed information with respect to total available information. Everything else equal, signals of (presumed) higher quality will be more accounted for in the pricing rule. If their quality is affected by overconfidence, the cost associated with belief distortion will be high. Conversely, the distortion by overconfidence of signals of low quality induces a lower cost, because these signals are less accounted for.

This formalization is key to our results because it is by influencing the relative quality of public and private information that central bank communication and stabilization policies affect the cost of belief distortion. We shall distinguish three regimes of central bank intervention, with the central bank:

- (i) taking an action (e.g. setting a stabilization rule) that signals its economic assessment to market participants, implicitly revealing its information set (*signalling stabilization*),
- (ii) abstaining from taking any action, only disclosing information to market participants about its own information set (*pure communication*), or
- (iii) disclosing information about its information set when taking an action (*communication and stabilization*).

The regime of *signalling stabilization* has been analyzed by Baeriswyl and Cornand (2010) who argue that taking an action inevitably provides public information because it signals to firms the belief of the central bank, thus denying the possibility of taking an action under opacity. Under objective beliefs, Baeriswyl et al. (2020) show that the optimal stabilization policy is indeterminate. We show that this result is robust to both types of motivated beliefs.

The regime of *pure communication* has been explored under objective beliefs by Morris and Shin (2002), who show in an abstract beauty contest game that disclosing public information when agents additionally receive private signals on the fundamentals can decrease welfare. However, Angeletos and Pavan (2007) have explained that transparency is always beneficial in a microfounded set-up, as the

equilibrium degree of coordination is lower than the efficient degree of coordination. Under motivated beliefs about the accuracy of firms' information precision, we qualify the latter result. Under plausible assumptions about the cost of parting with objective beliefs and about the precisions of private and public signals, we obtain an optimal interior degree of transparency. Intuitively, by increasing the precision of its disclosure, the central bank makes firms more informed about the fundamental shocks and therefore reduces the cost of firms mistakenly believing that their private information is precise. Firms thus tend to be even more overconfident in their private information and tend to overly rely on this information when setting their prices. This raises price dispersion and decreases welfare. Under realistic conditions, the optimal communication policy is therefore an intermediate level of transparency: the central bank faces a trade-off, instead of the dilemma opposing transparency and opacity.

Finally, the regime of *communication and stabilization* has been analyzed under objective beliefs by James and Lawler (2011), who show that full opacity is optimal. Indeed, by taking an action that is hidden from the public, the central bank succeeds in stabilizing the economy without creating overreaction to any disclosure. This result is robust to a microfounded set-up (Baeriswyl et al., 2020), but must be qualified when firms hold motivated beliefs about their capacity to process their own private information as well as central bank public disclosures. The rationale for this qualification is that by being opaque, the central bank reduces the cost of firms mistakenly believing that they are able to efficiently process semi-public information. Firms thus tend to be more confident in this ability, therefore relying more both on the semi-public signal disclosed by the central bank and on their own private information. More reliance on idiosyncratic information decreases welfare. In this case, it is optimal for the central bank to set an intermediate level of transparency, while also implementing an intermediate level of stabilization policy. Again, the central bank faces a trade-off rather than a dilemma.

To summarize, the paper considers three regimes of central bank policies: "pure communication" (when the central bank only communicates about its own information set), "signalling stabilization" (when the central bank takes an action - e.g. sets its stabilization rule - that can also be read as information about its information set) and "communication and stabilization" (when the central bank does both of the above). The main result of the paper is that while under signalling stabilization, the optimal central bank policy continues to be indeterminate (like under objective beliefs), the bang-bang solutions conventionally obtained under objective beliefs for pure communication as well as communication and stabilization must be qualified under motivated beliefs, which typically lead to interior solutions for optimal central bank policies.

The questioning of bang-bang solutions for optimal communication policy is a central issue in the literature. The literature has shown how different trade-offs may lead to interior solutions.<sup>5</sup> Motivated beliefs introduce a novel trade-off because the provision of information no longer serves exclusively to increase firms' information sets, but also recalibrates the cost-benefit structure of distorting firms' beliefs. Individual decisions result from the trade-off between the supposed benefit of being overconfident and the cost of parting with objectivity. Because of motivated beliefs, outcome prospects are enhanced by overconfidence (either in own information or in ability to process information), which may end up in more price dispersion and decrease welfare. In our model, this new trade-off induces interior solutions regarding *both* communication and stabilization policies: the central bank modulates this trade-off by manipulating its stabilization instrument and the signal to noise ratio. More generally, an important implication of our and others' related contributions for the design of central bank policies is that some distance should be taken with the all-or-nothing approach to the dilemma of transparency versus opacity suggested by the conventional theory.

The paper is organized as follows. Section 2 relates the paper to the literature. Section 3 describes the economy. Section 4 presents the information and belief structures, the timing of the game and solves for the equilibrium behavior of firms. Section 5 discusses central bank communication and stabilization policies under objective and motivated beliefs. Section 6 concludes the paper.

## 2 Related literature

Our paper is composed of three main ingredients: (i) coordination games (beauty contest framework) with heterogeneous and dispersed information (in the vein of Morris and Shin, 2002, and Angeletos and Pavan, 2007) applied to a macro-setting, (ii) central bank's monetary (stabilization) policy, and (iii) motivated beliefs or wishful thinking (Bénabou and Tirole, 2016), which translates in our framework into firms' overestimation of signals' precision or processing.

### 2.1 Beauty contest with heterogeneous and dispersed information

A growing literature has addressed the issue of central banks' communication in coordination games with heterogeneous and dispersed information. Morris and Shin (2002) presented a Keynesian beauty contest game where the equilibrium behavior of economic agents is driven by both a fundamental and a

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<sup>5</sup>For recent instances, see e.g. Gáti (2023) where the trade-off stems from a dynamic set-up or Herbert (2022) where it comes from dispersed information with heterogeneous priors.

coordination motive. The focal role that public information exerts on higher-order beliefs of agents gives rise to an overreaction, which may be detrimental to welfare. If public information is not accurate, it distorts the market outcome away from economic fundamentals, challenging the presumed benefit of central bank transparency. Svensson (2003, 2006) questioned this anti-transparency result. He showed that in the beauty contest model à la Morris and Shin, for transparency to be detrimental to welfare, public information had to be much less accurate than private information, which is very unlikely. Hellwig (2005) also questioned the anti-transparency result of Morris and Shin and showed that public announcements by the central bank always improve welfare because they lead to lower price dispersion. Angeletos and Pavan (2007) gave a general explanation to this result by pointing out the disutility from cross-sectional price dispersion which heightens social aversion to dispersion, thereby contributing to a higher optimal degree of coordination than the one reached in equilibrium. Angeletos, Iovino, and La'O (2016) studied the welfare consequences of public disclosures in microfounded business cycle models.

Hellwig and Veldkamp (2009) extended the quadratic Gaussian model by adding an information acquisition stage, in which agents can improve the quality of their information at some cost. They showed that strategic complementarities in actions induce strategic complementarity in private information acquisition. Maćkowiak and Wiederholt (2009) obtained a similar result in a macroeconomic model with rational inattention. Myatt and Wallace (2012) considered information acquisition in a beauty contest framework where agents have access to an arbitrarily large number of information sources and the precision of each source depends on both the accuracy of the source and the attention that an agent devotes to the source. The choice of attention is continuous implying that the symmetric equilibrium is unique (while there are multiple symmetric equilibria in Hellwig and Veldkamp, 2009). Colombo et al. (2014) illustrated the implications of a continuous choice of attention in a monetary economy with price rigidities and dispersed information about productivity shocks.

There is some similarity between the information acquisition approach and the motivated beliefs approach explored in the present paper. While the former has agents facing a trade-off between the benefit of having access to more precise information and the cost of acquiring it, the latter has agents facing a trade-off between the illusory benefit created by overconfidence in information precision and the presumed cost of parting with objectivity.



## 2.2 Central bank’s monetary policy under heterogeneous and dispersed information

While Morris and Shin (2002) referred to the case where the provider of public information only consid-  
180 ers the possibility of disclosing information, James and Lawler (2011) analyzed the optimal disclosure  
strategy when the central bank takes an action and find that full opacity is optimal. Some microfounded  
macroeconomic models also gave a role to central bank stabilization policy (possibly in addition to com-  
munication policy) under dispersed and heterogeneous information (e.g. Woodford, 2003; Adam, 2007;  
Baeriswyl and Cornand, 2010; Lorenzoni, 2010; Paciello and Wiederholt, 2014; Angeletos and La’O,  
185 2020; Baeriswyl et al., 2020; Benhima and Blengini, 2020; Chahrour and Ulbricht, 2023; Femminis and  
Piccirilli, 2023).

## 2.3 Motivated and subjective beliefs

Our paper contributes to this literature on communication and stabilization policies under dispersed  
and heterogeneous information but departs from it by relaxing the key assumption of rational expecta-  
190 tions (i.e., the knowledge of the true joint distribution of signals and fundamentals), instead allowing  
for subjective beliefs, so that firms may incorrectly interpret the information available to them. Fol-  
lowing Bénabou and Tirole (2016), these subjective beliefs are not given in an exogenous manner but  
are instead endogenized through motivated reasoning. Banerjee et al. (2022) extended the generalized  
quadratic-Gaussian model of Angeletos and Pavan (2007) to allow for motivated belief choice about the  
195 precision of public information on fundamentals.<sup>6</sup> By considering a flexible setting that encompasses  
strategic substitutability and complementarity, they highlight how agents’ subjective beliefs about both  
types of information give rise, at the same time, to overconfidence in private signals and disagreement  
about the interpretation of public information across a wide range of (especially) industrial organization  
applications. We differ from them in three main respects. First, while their set-up is abstract (so that  
200 central bank policy is outside their scope), we derive the consequence of motivated beliefs in a fully mi-  
crofounded model and obtain analytical results concerning the role of central bank communication and  
stabilization policies. Second, they apply motivated beliefs principally to public information and only  
quite marginally to private information. Instead, we apply motivated beliefs to firms’ own private infor-  
mation and to their ability to process central bank disclosure, following the literature on overconfidence

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<sup>6</sup>Other works that generate endogenously optimistic biases in beliefs, by taking into account the utility benefits of good  
outcomes include Brunnermeier and Parker (2005), Brunnermeier et al. (2007), and Caplin and Leahy (2019).

in traders' private information and their ability to interpret available information (e.g. Odean, 1998; 205  
Angrisani et al., 2021). A third major difference is the way the cost of being irrational is formalized.  
Banerjee et al. assume players maximize a weighted average of their "anticipatory utility" (stemming  
from subjective beliefs) and their expected "experienced utility" (stemming from objective beliefs). The  
relative weight they put on the latter, which may be seen as expressing the disutility level of the belief  
distortion, is assumed to be a fixed datum. Instead, we assume a variable concern for the belief distor- 210  
tion, depending upon how much this distortion is impactful, or on the contrary inconsequential, in firms'  
pricing decisions.

Finally, our work relates to the literature that introduces overconfidence bias in models with het-  
erogeneous and dispersed information. While there is a vast psychological literature on overconfidence,  
especially in finance, the overconfidence bias is rather novel in macro models. Benigno and Karantounias 215  
(2019) and Broer and Kohlhas (2022) represent two recent contributions aimed at refining models to  
account for empirical evidence. Our paper differs from these as, instead of imposing an exogenous bias,  
over-precision of private information results from an endogenous choice of firms that maximize their  
anticipatory profits and incur a cost from deviating from rational expectations. In equilibrium, we find  
overconfidence in information precision and processing. Moreover, we study the optimal communication 220  
and stabilization policies of the central bank.

### 3 The economy

We use a simple New Keynesian model, which is a variant of Adam (2007). We assume in particular  
that shocks on fundamentals concern the marginal rate of substitution between consumption and labor  
without affecting the price elasticity of product demand, hence firms' market power and the ensuing price 225  
distortion. Further, following Maćkowiak and Wiederholt (2009), we have, first, systematically resorted  
to second-order Taylor approximations of objective functions rather than linear approximations, so as  
not to play down the role of the curvature of those functions and, second, applied Gaussian noises  
to deviations in the *logarithms* of the variables, thus avoiding violations of non-negativity constraints.  
Finally, contrary to standard practices in the literature, we have refrained from approximating by the 230  
arithmetic mean the generalized means issued from CES aggregators, an approximation which loses  
information on dispersion and its damaging effects on welfare.

### 3.1 The household

The representative household derives utility from consuming a volume  $C$  of a composite good and disutility from supplying an amount  $L$  of homogeneous labor:

$$U(\Theta C, \Theta L) = \frac{(\Theta C)^{1-\xi}}{1-\xi} - \Theta L, \text{ with } \xi \in (0, 1). \quad (1)$$

The parameter  $\xi$ , which is the coefficient of relative risk aversion, has been restricted to be smaller than 1, resulting in prices being strategic complements, the case on which we want to focus. The variable  $\Theta$  is a random variable allowing for the introduction of preference shocks around 1 ( $\mathbb{E}(\Theta) = 1$ ). The first order condition for the maximization in  $(C, L)$  of utility  $U$  under the budget constraint  $PC \leq WL + \Pi$  (where  $W$  is the competitive money wage,  $P$  the price index of the components of the composite good and  $\Pi$  the nominal aggregate profit received by the household) is

$$(\Theta C)^\xi = \frac{W}{P}, \quad (2)$$

determining consumption  $C$ . Labor supply  $L$  is then computed by insertion of  $C$  in the budget equation.

The volume  $C$  of the composite good is a CES aggregate of a continuum of differentiated products represented by the unit interval:

$$C = \left( \int_0^1 C_i^{\frac{s-1}{s}} di \right)^{\frac{s}{s-1}}, \quad (3)$$

where  $C_i$  is the output of firm  $i$  and  $s$  is the constant elasticity of substitution between the differentiated goods, which we will assume to be larger than 1 (the case of substitutable goods). The consumer minimizes the expenditure  $\int_0^1 P_i C_i di$  required to ensure a volume  $C$  of consumption, that is, under the constraint (3). The first order condition for this minimization gives the demand for each good  $i$ :

$$C_i = \left( \frac{P_i}{P} \right)^{-s} C, \text{ with } P = \left( \int_0^1 P_i^{1-s} di \right)^{\frac{1}{1-s}} \quad (4)$$

as the price index of all the differentiated goods, so that  $\int_0^1 P_i C_i di = PC = Z$ , the nominal expenditure that we initially assume to be given, and then endogenize to be under the control of the central bank.

Each firm  $i \in [0, 1]$  produces the quantity  $C_i$  of a single differentiated good with  $C_i$  units of labor. Hence, equilibrium in the labor market requires the condition  $L = \int_0^1 C_i di$ . In a symmetric equilibrium,  $L = C$ , but without symmetry across firms, the arithmetic mean  $L = \int_0^1 C_i di$  of the employment in all

firms ( $L_i = C_i$  in firm  $i$ ) is higher than the mean  $C = \left( \int_0^1 C_i^{\frac{s-1}{s}} di \right)^{\frac{s}{s-1}}$  obtained by applying the CES aggregator, which is intermediate between the two limits of the arithmetic mean (when  $s \rightarrow \infty$ ) and of the geometric mean (when  $s \rightarrow 1$ ).<sup>7</sup> As a consequence, output asymmetry is welfare reducing: as dispersion of the output levels across firms increases, the same level of consumption requires more and more labor to be feasible. Let us emphasize that this effect is ignored when the volume of the composite output is approximated by the arithmetic mean of the quantities produced by all the firms. 255

### 3.2 The firms 260

The information structure assumed in this paper will be made explicit in the next section. Now it suffices to say that neither the firms nor the central bank observe the realizations of the fundamental  $\Theta$ ; instead, they receive some signals on  $\Theta$ .

Each firm  $i \in [0, 1]$  is assumed to set its price  $P_i$  so as to maximize its expected *real* profit (deflated by the price index  $P$ ), conditional on its information set  $\Gamma_i$  (which allows the formation of expectations of  $C$  and  $P$ ): 265

$$\mathbb{E} \left[ \left( \left( \frac{P_i}{P} \right)^{1-s} - (\Theta C)^\xi \left( \frac{P_i}{P} \right)^{-s} \right) C \middle| \Gamma_i \right]. \quad (5)$$

Following Maćkowiak and Wiederholt (2009), we work with a log-quadratic approximation of the profit function around the nonstochastic solution of the model, obtained under certainty ( $\Theta = 1$ ), perfect information and symmetry, namely

$$C^* = (1 - 1/s)^{1/\xi} \quad \text{and} \quad P^* = W^* / (1 - 1/s) = \bar{Z} / C^*, \quad (6)$$

where  $\bar{Z}$  is some nominal expenditure, given in the absence of any central bank manipulation. Notice that  $1/s$  is the firms' degree of monopoly, which introduces a price distortion, contracting consumption. If competition were perfect, we would indeed obtain the efficient equilibrium values  $C^{**} = 1$  and  $P^{**} = W^{**} = \bar{Z}$ . 270

Using (2) and taking a lower case letter to denote the log-deviation of the variable from its value at the nonstochastic solution, the real profit can be rewritten as 275

$$\pi(p_i - p, c, \theta) = (1 - 1/s)^{1/\xi} \left[ e^{c - (s-1)(p_i - p)} - (1 - 1/s) e^{\xi\theta + (1+\xi)c - s(p_i - p)} \right], \quad (7)$$

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<sup>7</sup>See Appendix A.

where  $\theta = \ln \Theta - \ln 1 = \ln \Theta$ ,  $p_i = \ln P_i - \ln P^*$  and  $p = \ln P - \ln P^*$ , so that  $p_i - p = \ln P_i - \ln P$ . We show in Appendix B that the second order Taylor approximation of this function at the origin is given (up to a constant with respect to  $p_i - p$ ) by

$$\tilde{\pi}(p_i - p, c + \theta) = (s - 1)(p_i - p) \left( \xi(c + \theta) - \frac{1}{2}(p_i - p) \right). \quad (8)$$

The first order condition for the maximization in  $p_i - p$  of this function can be formulated as the log-linear  
 280 equation:

$$\arg \max_{p_i} \tilde{\pi}(p_i - p, c + \theta) = p + \xi(c + \theta) \equiv \hat{p}. \quad (9)$$

Given some realization of the fundamental  $\theta$ , setting a price  $p_i$  that differs from  $\hat{p}$  leads to a profit loss which, by (8), can be written:

$$\tilde{\pi}(\hat{p} - p, c + \theta) - \tilde{\pi}(p_i - p, c + \theta) = \frac{s - 1}{2} (p_i - \hat{p})^2. \quad (10)$$

Now, since  $p$ ,  $c$  and  $\theta$  are not known with certainty, firm  $i$  sets a price maximizing its *expected* profit, hence equal to its expectation of  $\hat{p}$ , conditional on its information:

$$p_i = \mathbb{E}_i[\hat{p}] = \mathbb{E}_i[p + \xi(c + \theta)], \text{ with } \mathbb{E}_i \equiv \mathbb{E}[\cdot | \Gamma_i]. \quad (11)$$

285 As  $p + c = z$ , where  $z$  is the log-deviation of the nominal aggregate expenditure  $Z$  (0 if  $Z = \bar{Z}$ , a constant), we can alternatively take as firm  $i$ 's pricing rule

$$p_i = (1 - \xi) \mathbb{E}_i[p] + \xi \mathbb{E}_i[z + \theta]. \quad (12)$$

In other words, the expected profit maximizing price (deviation) is a convex combination of the expected mean price (deviation), reflecting a *coordination motive*, and of the expected sum of the fundamental and policy deviations, reflecting a *fundamental motive*. By this pricing rule, the price set by firm  $i$  responds  
 290 positively to a demand pull (triggered by  $z$ ), to a cost push (triggered by  $\theta$ ) and, through  $p$ , to others' price strategies (the assumption of a positive weight  $1 - \xi$  on the coordination motive translates into strategic complementarity, as stated above).

### 3.3 The central bank

The central bank seeks to maximize the expected welfare of households conditional on the information it receives about the fundamental  $\theta$ . Indeed, the central bank receives a noisy signal  $y = \theta + \eta$ , with  $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$ . The welfare function is described in subsection 3.3.1. To maximize the expected welfare, the central bank can disclose to the firms information about the realization of the fundamental and/or take an action affecting  $z$ , depending on the considered policy regime (Baeriswyl et al. 2020). These policy regimes are presented in subsection 3.3.2.

#### 3.3.1 The welfare function

Central bank policy decisions are assumed to be taken so as to maximize a social welfare function. Since profits are entirely distributed to the representative household, welfare can be simply identified with household's utility  $U(\Theta C, \Theta L)$  at equilibrium. As shown in Appendix C, once expressed in terms of log-deviations from the nonstochastic solution of the model, this utility is a function  $v(p_{-s} - p_{1-s}, c + \theta)$  of two variables. The first one, a difference of two different price CES means,<sup>8</sup> opens the way to the negative effect on welfare of price dispersion, hence output dispersion, already pointed out in subsection 3.1: the same level of (composite) consumption requires more and more (homogeneous) labor as output dispersion increases. The second term expresses the shocks on the fundamental. It is further shown in Appendix C that, after approximating at the second order the function  $v$ , both arguments are themselves expressed as linear functions of the price variance  $\sigma_{\mathbf{P}}^2$ , which thus appears as the sole intermediate target of the central bank policy. More precisely, after neglecting a constant term as well as the remainder term of the Taylor approximation, we may refer to the transformed welfare function

$$V(\sigma_{\mathbf{P}}^2) = -\frac{s\xi - 1}{\xi} \left(\frac{\sigma_{\mathbf{P}}^2}{2}\right) - \frac{1}{2} \left(\frac{(s-1)(3s\xi - 1)}{\xi^2} + s^2\right) \left(\frac{\sigma_{\mathbf{P}}^2}{2}\right)^2. \quad (13)$$

By the conventional condition  $s \geq 1/\xi$ ,<sup>9</sup>  $V$  is a decreasing function, so that the objective of the central bank is to minimize the price variance  $\sigma_{\mathbf{P}}^2$ .

<sup>8</sup>Let us emphasize, once again, that any welfare effect of price dispersion would vanish under the systematic use of arithmetic means and linear approximations. The influence of price variance works through the curvature of the relevant functions.

<sup>9</sup>The strict inequality  $s > 1/\xi$  corresponds to the conventional condition that the elasticity of substitution between the differentiated goods be larger than the elasticity of substitution between the composite consumption good and leisure (Dixit and Stiglitz, 1977, p.299).

### 315 3.3.2 Three policy regimes

We will distinguish three policy regimes, two with a single policy instrument and one with two policy instruments.

The first is *signalling stabilization*: the central bank cannot blur the public signal it receives, which is common knowledge among the firms, thanks to the full observation of the stabilization action  $z$  taken by the central bank under a policy rule which is also common knowledge. This situation corresponds to the framework of Baeriswyl and Cornand (2010), examining how the central bank should optimally take its action when it is perfectly observable by the firms. The central bank sets  $z$ , aiming at the stabilization of the economy, by neutralizing the shocks on the fundamental. Ideally, under perfect information, the central bank would choose  $z = -\theta$ . However, as the central bank only has the information  $y$  on the realization  $\theta$  of the fundamental, it chooses instead  $z = -(\rho y + (1 - \rho)0) = -\rho y$ , where 0 is the non-stochastic fundamental value and  $\rho \in [0, 1]$  is the value of the policy instrument, set by the central bank and known by the firms.<sup>10</sup>

The second regime is *pure communication*, an example of which is forward guidance, whereby the central bank influences the behavior of economic agents by simply disclosing information. This situation corresponds to the framework of Morris and Shin (2002), examining how the central bank should optimally communicate when it takes no stabilizing action (implying  $z = 0$  in our context). Following Baeriswyl and Cornand (2014), to allow for an intermediate level of disclosure, we assume that the central bank chooses the variance  $\sigma_\phi^2$  of the idiosyncratic noise affecting the signal  $y_i = y + \phi_i$ , with  $\phi_i \sim \mathcal{N}(0, \sigma_\phi^2)$ , that it communicates. This noise captures the idea that each firm may interpret differently the same equivocal statement made by the central bank, rather than the idea that the central bank discloses a specific signal to each firm. It thus formalizes the notion that the central bank communicates its information  $y$  with more or less ambiguity. The signal  $y_i$  can be considered as a semi-public signal.<sup>11</sup> Under *full transparency*, all firms interpret without ambiguity the same unequivocal signal ( $\sigma_\phi^2 = 0$ ). The central bank disclosure  $y$  is then a public signal that is common knowledge among firms. Under *full opacity*, each firm interprets differently its individual signal that contains an infinite idiosyncratic noise ( $\sigma_\phi^2 \rightarrow \infty$ ), and the central bank disclosure does not contain any valuable information.

In the case of a two-instruments policy, covering *communication and stabilization*, the central bank

<sup>10</sup>We follow the literature (e.g. James and Lawler, 2011; Baeriswyl and Cornand, 2010) in assuming a linear rule for the central bank.

<sup>11</sup>Other ways have been exploited to model intermediate degrees of transparency in beauty contest games. They add to precision the extent of information disclosure (Cornand and Heinemann, 2008), the receiver noise (Myatt and Wallace, 2012) or cross-sectional correlations (Myatt and Wallace, 2014). Grout et al. (2015) apply to the context of dispersed preferences the tools thus developed to analyze coordination in a context of dispersed information.

takes an action and discloses information. This regime corresponds to the framework of James and Lawler (2011), examining how the central bank should optimally combine its action and disclosure.<sup>12</sup> In the present context, this means that the central bank chooses  $\sigma_\phi^2$  and  $\rho$ .

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## 4 Information and beliefs, timing and equilibrium

We start by making explicit the information structure and introducing the notion of firms' motivated beliefs concerning the quality of their private information or their ability to process it. Then we describe the timing of the game. Finally, we will characterize symmetric equilibria when firms' price strategies are linear affine with respect to the (private and semi-public) signals they receive.

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### 4.1 Information structure and motivated beliefs

We first consider the information structure. The random variable  $\theta$  taken as the fundamental follows a normal distribution with zero mean and a variance which is unknown to all the players, including the central bank. Taken to be infinite by the players, this variance does not play any role in our analysis. The central bank does not observe the realized value  $\theta$  of the fundamental – a preference shock – but receives a signal  $y = \theta + \eta$ , where  $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$  is white Gaussian noise. Each firm  $i$  does not observe the realized preference shock either, but receives a *private* signal  $x_i = \theta + \varepsilon_i$ , where  $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$  is white Gaussian idiosyncratic noise, and a *semi-public* signal  $y_i = y + \phi_i = \theta + \eta + \phi_i$ , where  $\phi_i \sim \mathcal{N}(0, \sigma_\phi^2)$  is also white Gaussian idiosyncratic noise, the variance of which may be under the control of the central bank. All the noises are independently distributed.

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Firms may form two types of subjective beliefs. First, firm  $i$  may form subjective beliefs about the objective quality of its *private* information. In this case, firm  $i$  perceives the variance of its *own* private signal as  $\sigma_\varepsilon^2/\delta$ . The benchmark of rational expectations is captured by  $\delta = 1$ . When  $\delta$  is larger than one, firm  $i$  overweighs the private information when forming expectations, meaning that it believes the signal to contain less noise than it objectively contains, and conversely when  $\delta$  is smaller than one.

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Second, firm  $i$  may form subjective beliefs about its ability to tackle any kind of idiosyncratic noise, meaning that the coefficient  $\delta$  will apply to both variances  $\sigma_\varepsilon^2$  and  $\sigma_\phi^2$  (while it only applies to the variance  $\sigma_\varepsilon^2$  in the case of subjective beliefs on the quality of private information). To cover both cases, it will be convenient to start by distinguishing two coefficients  $\delta_\varepsilon$  and  $\delta_\phi$ , according to the specific noise to which each one applies,  $\varepsilon_i$  or  $\phi_i$ . The coefficient  $\delta$  is adopted by each particular firm  $i$ , but we will omit the

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<sup>12</sup>Note that it differs though from James and Lawler (2011) in allowing for an intermediate degree of transparency.



index  $i$  for simplicity of notation. As we will keep our analysis restricted to symmetric equilibria, this omission should not be harmful, under some caveats when considering deviations from equilibrium.

Subjective beliefs may be motivated, or endogenized, by making firms choose their preferred value of  $\delta$ . By (10), firm  $i$  will be assumed to minimize its expected profit loss, resulting from a price set on the basis of its subjective belief  $\delta$  and of its information  $(x_i, y_i)$  yet to come.<sup>13</sup> Ideally, it would like to make equal to zero the expectation of its profit loss.

The choice of the preferred value of  $\delta$ , characterizing *wishful thinking*, responds to what we may be tempted to view as an instance of the “pleasure principle”. This principle must however be confronted with the “reality principle” commanding  $\delta$  to be equal to 1, the rational expectations benchmark. So, firms will have to find a trade-off between the two principles, by minimizing a sum of the expected profit loss function (given by (10)) and of some function representing the cost of being irrational. This cost is assumed to have two components: the size of the distortion introduced by the subjective beliefs and the concern for that distortion. The former can be measured (as in Caplin and Leahy, 2019) by the *Kullback-Leibler divergence* between the two probability distributions, from the one under objective to that under subjective beliefs.<sup>14</sup>

As to the concern for the belief distortion, we will allow it to depend upon the firm environment. Take the case where firm  $i$  forms subjective beliefs about the objective quality of its private information. This firm should not incur the same cost for a given size of the belief distortion if that quality is high (if the variance  $\sigma_\varepsilon^2$  is small), inducing a strong reactivity to the private signal  $x_i$  relative to the semi-public signal  $y_i$  in the pricing rule (as described below by (20)), or if on the contrary that quality is poor, inducing a low relative reactivity to signal  $x_i$ . Taking the concern for the belief distortion as constant would unduly efface the difference between the two situations (high precision, hence strong reactivity, versus low precision, hence weak reactivity), the same penalty applying when the belief distortion is impactful or on the contrary inconsequential. The same argument applies of course to the case where the firm forms subjective beliefs about its objective ability to process idiosyncratic information of any kind.

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<sup>13</sup>As highlighted by Banerjee et al. (2022), a complementary approach to modeling motivated beliefs is ambiguity aversion, where the belief choice of an ambiguity-averse firm is such that it *maximizes* over a set of reasonable subjective beliefs the minimized expected profit loss from its price choice. Instead, under motivated beliefs, the firm is supposed to love ambiguity, that is, to exploit ambiguity so as to *minimize* over reasonable subjective beliefs the minimal expected profit loss from its price decision.

<sup>14</sup>Notice that, in the present context where the distortion is induced by the sole parameter  $\delta$ , we might alternatively take any strictly quasi-convex function of  $\delta$  with a zero minimum at  $\delta = 1$ , hence increasing (resp. decreasing) for  $\delta > 1$  (resp.  $\delta < 1$ ).

## 4.2 Timing

Before we proceed to the analysis of the game, we want to define thoroughly its timing by characterizing its successive stages:

1. On the basis of its knowledge of the laws of distribution of signals  $y$  and  $x_i$ , and anticipating the later stages of the game, the central bank chooses the value of one or both of the two policy instruments: (i) the variance  $\sigma_\phi^2$  of the idiosyncratic white Gaussian noise blurring the public signal  $y$  and/or (ii) the stabilization rule  $\rho$  governing the response  $-\rho y$  to that signal. 400
2. Knowing the laws of distribution of public and private information, as well as the values of the policy instruments chosen by the central bank, each firm  $i$  adopts its subjective belief concerning either (i) the quality of the information to which it has a private access ( $\delta_\varepsilon = \delta$ , with  $\delta_\phi = 1$ ) or (ii) its ability to treat idiosyncratic information ( $\delta_\varepsilon = \delta_\phi = \delta$ ). The belief motivation involves linear affine pricing responses to private and semi-public information available in the future:  $p_i = \kappa_0 + \kappa_1 y_i + \kappa_2 x_i$ . 405
3. Nature chooses a realization  $\theta$  of the fundamental and sends specific noisy signals of this realization  $y = \theta + \eta$  to the central bank and  $x_i = \theta + \varepsilon_i$  to each particular firm  $i$ . The central bank discloses its information, although possibly blurring it by a white Gaussian noise, with each firm  $i$  eventually receiving a signal  $y + \phi_i = \theta + \eta + \phi_i$ . 410
4. Firms set the coefficients  $\kappa_0$ ,  $\kappa_1$  and  $\kappa_2$  of their linear affine pricing responses to the signals  $(x_i, y_i)$  they receive, conditional on their adopted subjective beliefs  $\delta$  and on the central bank policy  $(\sigma_\phi^2, \rho)$ . 415
5. The representative household supplies labor and consumes products at the prices set by the firms.

As usual, we will consider these different stages backwards. Stage 5 has already been treated in subsection 3.1. Stages 4 and 2 will successively be examined in subsections 4.3 and 4.4, respectively, stage 3 not requiring additional developments. Stage 1 will be analyzed in section 5.

## 4.3 Equilibrium in linear price strategies

We have assumed that, at stage 4, each firm  $i$  sets its price as a linear affine function of the two signals it receives:

$$p_i = \kappa_0 + \kappa_1 y_i + \kappa_2 x_i. \tag{14}$$

Of course, the values of the coefficients  $\kappa_0$ ,  $\kappa_1$  and  $\kappa_2$  must be compatible with the pricing rule (12) established in the former section. To determine these values, we must formulate the expression for the expected price of the composite good, taking into account the linear price strategies of the other firms. Sticking to symmetry<sup>15</sup> throughout the paper, we suppose that every other firm uses the same triple of coefficients  $\bar{\kappa}_0$ ,  $\bar{\kappa}_1$  and  $\bar{\kappa}_2$ . By equation (39) in Appendix A, we may write for the second order Taylor approximation of  $p$  (denoting by  $\mathcal{P}_1(\mathbf{p})$  the arithmetic mean of the  $p_j$ 's)

$$p \equiv p_{1-s} \simeq \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2} \sigma_{\mathbf{p}}^2 = \bar{\kappa}_0 + \bar{\kappa}_1 y + \bar{\kappa}_2 \theta - \frac{s-1}{2} (\bar{\kappa}_1^2 \sigma_{\phi}^2 + \bar{\kappa}_2^2 \sigma_{\varepsilon}^2). \quad (15)$$

Notice that, in presence of dispersed information, resorting to the *arithmetic* mean  $\mathcal{P}_1(\mathbf{p})$  of the log-deviations of the individual prices  $p_j$ 's would introduce a positive bias relative to the log-deviation  $p$  of the appropriate CES price index, a bias which would vanish only in the limit case  $s \rightarrow 1$  (excluded by the condition  $s \geq 1/\xi > 1$ ). At this limit case the price index  $P$  would be equal to the *geometric* mean of the individual prices  $P_j$ 's.

The expectation  $\mathbb{E}_i[p]$  of the price  $p$  of the composite good conditional on the information of firm  $i$  involves of course  $\mathbb{E}_i[y]$  and  $\mathbb{E}_i[\theta]$ :

$$\mathbb{E}_i[p] = \mathbb{E}[p|x_i, y_i] \simeq \bar{\kappa}_0 + \bar{\kappa}_1 \mathbb{E}_i[y] + \bar{\kappa}_2 \mathbb{E}_i[\theta] - \frac{s-1}{2} (\bar{\kappa}_1^2 \sigma_{\phi}^2 + \bar{\kappa}_2^2 \sigma_{\varepsilon}^2). \quad (16)$$

Firm  $i$ 's expectation of the fundamental shock  $\theta$  when its information is reduced to the two signals  $x_i$  and  $y_i$  is

$$\mathbb{E}_i[\theta] = \mathbb{E}[\theta|x_i, y_i] = \underbrace{\frac{\sigma_{\varepsilon}^2/\delta_{\varepsilon}}{\sigma_{\varepsilon}^2/\delta_{\varepsilon} + \sigma_{\eta}^2 + \sigma_{\phi}^2/\delta_{\phi}}}_{\lambda} y_i + \underbrace{\frac{\sigma_{\eta}^2 + \sigma_{\phi}^2/\delta_{\phi}}{\sigma_{\varepsilon}^2/\delta_{\varepsilon} + \sigma_{\eta}^2 + \sigma_{\phi}^2/\delta_{\phi}}}_{1-\lambda} x_i. \quad (17)$$

Like Morris and Shin (2002), we suppose that firms have no *ex ante* information about the distribution of the fundamental. Their information is only *ex post*, on the realized value  $\theta$ . Similarly, firm  $i$ 's expectation

<sup>15</sup>Symmetry applies of course to the pricing rules, not to the prices themselves, whose asymmetry at equilibrium will reflect the asymmetry of firms' information.

of the central bank information  $y$  is

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$$\begin{aligned}
\mathbb{E}_i[y] &= \mathbb{E}[y|x_i, y_i] = \underbrace{\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_\mu y_i + \underbrace{\frac{\sigma_\phi^2/\delta_\phi}{\sigma_\eta^2 + \sigma_\phi^2/\delta_\phi}}_{1-\mu} \mathbb{E}_i[\theta] \\
&= (\mu + (1-\mu)\lambda) y_i + (1-\mu)(1-\lambda) x_i \\
&= \underbrace{\frac{\sigma_\eta^2 + \sigma_\varepsilon^2/\delta_\varepsilon}{\sigma_\eta^2 + \sigma_\varepsilon^2/\delta_\varepsilon + \sigma_\phi^2/\delta_\phi}}_\nu y_i + \underbrace{\frac{\sigma_\phi^2/\delta_\phi}{\sigma_\eta^2 + \sigma_\varepsilon^2/\delta_\varepsilon + \sigma_\phi^2/\delta_\phi}}_{1-\nu} x_i.
\end{aligned} \tag{18}$$

Referring to the pricing rule (12) which ensures that firm  $i$  sets a profit maximizing price  $p_i$ , and further referring to the expectation expressions (16), (17) and (18), we then obtain, for  $z = -\rho y$ ,

$$\begin{aligned}
p_i &= (1-\xi) \mathbb{E}_i[p] + \xi \mathbb{E}_i[z + \theta] \\
&= (1-\xi) \left( \bar{\kappa}_0 - \frac{s-1}{2} (\bar{\kappa}_1^2 \sigma_\phi^2 + \bar{\kappa}_2^2 \sigma_\varepsilon^2) \right) + ((1-\xi) \bar{\kappa}_1 - \xi \rho) \mathbb{E}_i[y] \\
&\quad + ((1-\xi) \bar{\kappa}_2 + \xi) \mathbb{E}_i[\theta],
\end{aligned} \tag{19}$$

so that, at a *symmetric equilibrium* where  $(\bar{\kappa}_0, \bar{\kappa}_1, \bar{\kappa}_2) = (\kappa_0, \kappa_1, \kappa_2)$ , we obtain by identification:

$$\begin{aligned}
\kappa_0 &= -\frac{s-1}{2} \frac{1-\xi}{\xi} (\kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2) \equiv \kappa_0(\boldsymbol{\delta}), \text{ with } \boldsymbol{\delta} = (\delta_\varepsilon, \delta_\phi), \\
\kappa_1 &= \lambda - \rho \nu = \frac{(1-\rho) \sigma_\varepsilon^2/\delta_\varepsilon - \rho \xi \sigma_\eta^2}{\xi \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi + \sigma_\varepsilon^2/\delta_\varepsilon} \equiv \kappa_1(\boldsymbol{\delta}), \\
\kappa_2 &= (1-\lambda) - \rho(1-\nu) = \frac{\xi \sigma_\eta^2 + (1-\rho) \sigma_\phi^2/\delta_\phi}{\xi \sigma_\eta^2 + \sigma_\phi^2/\delta_\phi + \sigma_\varepsilon^2/\delta_\varepsilon} \equiv \kappa_2(\boldsymbol{\delta}),
\end{aligned} \tag{20}$$

verifying  $\kappa_1(\boldsymbol{\delta}) + \kappa_2(\boldsymbol{\delta}) = 1 - \rho$ . Notice that, for  $\rho > (\sigma_\varepsilon^2/\delta_\varepsilon) / (\xi \sigma_\eta^2 + \sigma_\varepsilon^2/\delta_\varepsilon) \equiv \underline{\rho}$ ,  $\kappa_1(\boldsymbol{\delta}) < 0$ .

#### 4.4 Motivation of beliefs

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We have assumed that firms' beliefs are adopted at stage 2 by minimizing the sum of the expected profit loss  $((s-1)/2) \mathcal{L}(\boldsymbol{\delta})$  and of an expected cost depending upon both the size of the distortion  $\mathcal{D}(\boldsymbol{\delta})$  introduced by the subjective beliefs and the concern  $\psi$  for that distortion:

$$\min_{\boldsymbol{\delta} \in \mathbb{R}_+} \left\{ \frac{s-1}{2} \mathcal{L}(\boldsymbol{\delta}) + \psi \mathcal{D}(\boldsymbol{\delta}) \right\}. \tag{21}$$

Here  $\boldsymbol{\delta} = (\delta_\varepsilon, \delta_\phi)$  may be alternatively equal to  $(\delta, 1)$  or to  $(\delta, \delta)$ , so that the minimization takes place  
 450 over one variable only. The functions  $\mathcal{L}$ ,  $\mathcal{D}$ , but also  $\psi$ , depend directly or indirectly upon the variances  $\sigma_\varepsilon^2$ ,  $\sigma_\phi^2$  and  $\sigma_\eta^2$ , and  $\mathcal{L}$  depends in addition upon the parameter  $\xi$  and the instrument variable  $\rho$ .

#### 4.4.1 The expected profit loss under subjective beliefs

The first component of the objective function (the expected profit loss under subjective beliefs) is, by  
 (10), proportional to the expected squared error loss (in pricing decisions)

$$\mathcal{L}(\boldsymbol{\delta}) = \mathbb{E}_i \left( (p_i(\boldsymbol{\delta}) - \hat{p})^2 \right), \quad (22)$$

455 where, by (14) and then by (12) and (16),

$$\begin{aligned} p_i(\boldsymbol{\delta}) &= \kappa_0(\boldsymbol{\delta}) + \kappa_1(\boldsymbol{\delta})(\eta + \phi_i) + \kappa_2(\boldsymbol{\delta})\varepsilon_i + (1 - \rho)\theta \text{ and} \\ \hat{p} &= (1 - \xi) \left( \bar{\kappa}_0 + \bar{\kappa}_1\eta - \frac{s-1}{2} (\bar{\kappa}_1^2\sigma_\phi^2 + \bar{\kappa}_2^2\sigma_\varepsilon^2) + (1 - \rho)\theta \right) \\ &\quad + \xi(-\rho\eta + (1 - \rho)\theta). \end{aligned} \quad (23)$$

Applying to the expected squared error loss these expressions for  $p_i(\boldsymbol{\delta})$  and  $\hat{p}$ , we obtain:

$$\begin{aligned} \mathcal{L}(\boldsymbol{\delta}) &= \left( \kappa_0(\boldsymbol{\delta}) - (1 - \xi)\bar{\kappa}_0 + (1 - \xi)\frac{s-1}{2} (\bar{\kappa}_1^2\sigma_\phi^2 + \bar{\kappa}_2^2\sigma_\varepsilon^2) \right)^2 \\ &\quad + (\kappa_1(\boldsymbol{\delta}) - (1 - \xi)\bar{\kappa}_1 + \xi\rho)^2 \sigma_\eta^2 + (\kappa_1(\boldsymbol{\delta}))^2 \sigma_\phi^2 / \delta_\phi \\ &\quad + (\kappa_2(\boldsymbol{\delta}))^2 \sigma_\varepsilon^2 / \delta_\varepsilon. \end{aligned} \quad (24)$$

The loss is directly decreasing in the parameters of subjective beliefs  $\delta_\varepsilon$  and  $\delta_\phi$  and also indirectly dependent, through the coefficients  $\kappa_0, \kappa_1$  and  $\kappa_2$ , on these parameters.

In order to investigate the conditions for firm  $i$  to optimize its belief  $\boldsymbol{\delta}$ , we consider the gradient of  
 460 the loss function:

$$\begin{aligned} \nabla \mathcal{L}(\boldsymbol{\delta}) &= 2 \left( \kappa_0(\boldsymbol{\delta}) - (1 - \xi)\bar{\kappa}_0 + (1 - \xi)\frac{s-1}{2} (\bar{\kappa}_1^2\sigma_\phi^2 + \bar{\kappa}_2^2\sigma_\varepsilon^2) \right) \nabla \kappa_0(\boldsymbol{\delta}) \\ &\quad + 2 \left( \begin{array}{l} (\kappa_1(\boldsymbol{\delta}) - (1 - \xi)\bar{\kappa}_1 + \xi\rho) \sigma_\eta^2 + \\ \kappa_1(\boldsymbol{\delta}) \sigma_\phi^2 / \delta_\phi - (1 - \rho - \kappa_1(\boldsymbol{\delta})) \sigma_\varepsilon^2 / \delta_\varepsilon \end{array} \right) \nabla \kappa_1(\boldsymbol{\delta}) \\ &\quad - \left( (\kappa_2(\boldsymbol{\delta}))^2 \sigma_\varepsilon^2 / \delta_\varepsilon^2, (\kappa_1(\boldsymbol{\delta}))^2 \sigma_\phi^2 / \delta_\phi^2 \right), \end{aligned} \quad (25)$$

which, by the envelope theorem, reduces to

$$\nabla \mathcal{L}(\boldsymbol{\delta}) = - \left( (\kappa_2(\boldsymbol{\delta}))^2 \sigma_\varepsilon^2 / \delta_\varepsilon^2, (\kappa_1(\boldsymbol{\delta}))^2 \sigma_\phi^2 / \delta_\phi^2 \right), \quad (26)$$

as can be easily checked using (20) at a symmetric profile of belief choices by all the firms, where  $(\bar{\kappa}_0, \bar{\kappa}_1, \bar{\kappa}_2) = (\kappa_0(\boldsymbol{\delta}), \kappa_1(\boldsymbol{\delta}), \kappa_2(\boldsymbol{\delta}))$ .

Now, as announced in subsection 4.1, we are interested in two cases of subjective beliefs: (i) the firm's belief in the quality of the information to which it has access, which may be characterized by  $\delta_\varepsilon = \delta$  and  $\delta_\phi = 1$ , and (ii) the firm's belief in its ability to treat any kind of idiosyncratic information, which we may identify with  $\delta_\varepsilon = \delta_\phi = \delta$ . In both cases the minimization involves a single variable, so that we have to refer in fact to the total derivative of the loss function

$$\frac{d\mathcal{L}(\boldsymbol{\delta})}{d\delta} = \begin{cases} -\frac{(\kappa_2(\delta, 1))^2 \sigma_\varepsilon^2}{\delta^2} & \text{if } \boldsymbol{\delta} = (\delta, 1) \\ -\frac{(\kappa_2(\delta, \delta))^2 \sigma_\varepsilon^2 + (\kappa_1(\delta, \delta))^2 \sigma_\phi^2}{\delta^2} & \text{if } \boldsymbol{\delta} = (\delta, \delta) \end{cases}. \quad (27)$$

We see that, as long as firms react to dispersed private information ( $(\kappa_2(\boldsymbol{\delta}))^2 \sigma_\varepsilon^2 > 0$ ), a symmetric interior equilibrium in motivated beliefs cannot be obtained on the basis of the minimization of the sole loss function  $\mathcal{L}$  (i.e. when  $\psi = 0$ ), since the incentive to increase  $\delta$  remains indefinitely present ( $d\mathcal{L}(\boldsymbol{\delta})/d\delta < 0$ ), pushing  $\delta$  to infinity. This tendency is countervailed by the cost of being irrational, firms minimizing instead the sum of the expected profit loss and that cost.

#### 4.4.2 The size of the belief distortion

We take this cost to be equal to the product of the concern  $\psi$  for the belief distortion and its size  $\mathcal{D}$ , as measured by the Kullback-Leibler divergence from the objective probability distribution of  $\theta$  to the subjective one (conditional on the random vector  $(x_i, y_i)$ ). In the present context, these probability distributions are bivariate normal distributions, with equal means  $\boldsymbol{\theta} = (\theta, \theta)$  and diagonal covariance matrices

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\eta^2 + \sigma_\phi^2 \end{bmatrix} \text{ and } \boldsymbol{\Sigma}_\delta = \begin{bmatrix} \sigma_\varepsilon^2 / \delta_\varepsilon & 0 \\ 0 & \sigma_\eta^2 + \sigma_\phi^2 / \delta_\phi \end{bmatrix}, \quad (28)$$

480 respectively. The KL divergence is accordingly given by

$$\begin{aligned}
D_{\text{KL}}(\mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_\delta) \parallel \mathcal{N}(\boldsymbol{\theta}, \boldsymbol{\Sigma}_1)) &= \frac{1}{2} \left( \text{tr}(\boldsymbol{\Sigma}_1^{-1} \boldsymbol{\Sigma}_\delta) - 2 + \ln \left( \frac{\det \boldsymbol{\Sigma}_1}{\det \boldsymbol{\Sigma}_\delta} \right) \right) \\
&= \frac{1}{2} \left( \ln \delta_\varepsilon + \frac{1}{\delta_\varepsilon} + \ln \left( \frac{\sigma_\eta^2 + \sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2 / \delta_\phi} \right) + \frac{\sigma_\eta^2 + \sigma_\phi^2 / \delta_\phi}{\sigma_\eta^2 + \sigma_\phi^2} \right) - 1 \equiv \mathcal{D}(\boldsymbol{\delta})
\end{aligned} \tag{29}$$

(see Zhang et al., 2022). Hence,

$$\frac{d\mathcal{D}(\boldsymbol{\delta})}{d\delta} = \begin{cases} \frac{\delta-1}{2\delta^2} & \text{if } \boldsymbol{\delta} = (\delta, 1) \\ \frac{\delta-1}{2\delta^2} \left( 1 + \frac{\sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2} \frac{\sigma_\phi^2 / \delta}{\sigma_\eta^2 + \sigma_\phi^2 / \delta} \right) & \text{if } \boldsymbol{\delta} = (\delta, \delta) \end{cases}. \tag{30}$$

The sign of the derivative of  $((s-1)/2)\mathcal{L}(\boldsymbol{\delta}) + \psi\mathcal{D}(\boldsymbol{\delta})$  with respect to  $\delta$  is then  $-1$  if  $\delta \leq 1$ , otherwise

$$\text{sgn} \left( \psi(\delta-1) - (s-1)(\kappa_2(\delta, 1))^2 \sigma_\varepsilon^2 \right) \text{ if } \boldsymbol{\delta} = (\delta, 1), \tag{31}$$

and

$$\text{sgn} \left( \psi(\delta-1) \left( 1 + \frac{\sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2} \frac{\sigma_\phi^2 / \delta}{\sigma_\eta^2 + \sigma_\phi^2 / \delta} \right) - (s-1) \left( (\kappa_1(\delta, \delta))^2 \sigma_\phi^2 + (\kappa_2(\delta, \delta))^2 \sigma_\varepsilon^2 \right) \right) \text{ if } \boldsymbol{\delta} = (\delta, \delta). \tag{32}$$

This sign is, in both cases, equal to  $+1$  for  $\delta$  large enough, thus entailing an interior minimum  $\delta^* \in (1, \infty)$ , tending to infinity as  $\psi \rightarrow 0$ .

#### 4.4.3 The concern for the belief distortion

As to the concern  $\psi$  for the belief distortion, we have suggested in subsection 3.4.1 that it should be taken as decreasing when belief distortion becomes more and more inconsequential, because firms become less and less reactive to private information (when  $\boldsymbol{\delta} = (\delta, 1)$ ) or to idiosyncratic information (when  $\boldsymbol{\delta} = (\delta, \delta)$ ). Formally, we will assume in the former case that  $\psi$  is a decreasing function of  $\lambda = \sigma_\varepsilon^2 / (\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2)$ . In the extreme case where  $\lambda \rightarrow 0$ , that is, where the relative precision of the private signal tends to infinity, making private information all-important, we assume that  $\psi \rightarrow \infty$ , imposing rationality of beliefs. In the other extreme case where  $\lambda \rightarrow 1$ , that is, where the relative precision of the private signal tends to zero, we assume a vanishing concern for belief distortion, meaning  $\psi \rightarrow 0$ . For simplicity of the discussion to follow, we will take  $\psi$  as iso-elastic in  $\lambda/(1-\lambda)$ :  $\psi(\lambda) = \beta(\lambda/(1-\lambda))^{-\alpha}$ , with positive parameters  $\alpha$ , an index of the *sensitivity* of the concern for belief distortion to the relative

precision of the subjectively assessed information, and  $\beta$ , an index of the *intensity* of that concern. When  $\delta = (\delta, \delta)$ , we will take the same specification of  $\psi$ , but now with  $\lambda = (\sigma_\varepsilon^2 + \sigma_\phi^2) / (\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2)$ :

$$\psi = \begin{cases} \beta \left( \frac{\sigma_\varepsilon^2}{\sigma_\eta^2 + \sigma_\phi^2} \right)^{-\alpha} & \text{if } \delta = (\delta, 1) \\ \beta \left( \frac{\sigma_\varepsilon^2 + \sigma_\phi^2}{\sigma_\eta^2} \right)^{-\alpha} & \text{if } \delta = (\delta, \delta) \end{cases} .$$

## 5 Central bank policy

In this section, we analyze the first stage of the game, namely central bank's communication and stabilization policies. Recall from subsection 3.3 that the central bank aims at maximizing welfare, which translates into minimizing price dispersion

$$\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2, \quad (33)$$

depending directly upon the central bank policy instrument  $\sigma_\phi^2$  and indirectly, through the coefficients  $\kappa_1$  and  $\kappa_2$ , again on  $\sigma_\phi^2$  but also on  $\rho$ .

Observe that, under *homogeneous information* ( $\sigma_\varepsilon^2 = 0$ ), by choosing *not to implement* an active stabilization policy ( $\rho = 0$ ), the central bank ensures that, whatever the value of  $\sigma_\phi^2$ , firms set  $\kappa_0 = \kappa_1 = 0$  and  $\kappa_2 = 1$ , which leads to the absence of price dispersion ( $\sigma_{\mathbf{p}}^2 = 0$ ) and no welfare loss. In what follows, we will consequently assume heterogeneous information, with a positive variance  $\sigma_\varepsilon^2$  of the private signal, a case where an active central bank policy may be welcome.

### 5.1 Signalling stabilization: confirmed indeterminacy

First consider what we have called the *signalling stabilization* regime, where the central bank's stabilization policy  $-\rho y$  is directly and perfectly observed by firms, which allows them, on the basis of their knowledge of the adopted value for the stabilization instrument  $\rho$ , to infer the central bank's information  $y$  about the fundamental shock.<sup>16</sup> The communication instrument is then neutralized at  $\sigma_\phi^2 \equiv 0$ , implying price dispersion  $\sigma_{\mathbf{p}}^2 = \kappa_2^2 \sigma_\varepsilon^2$ , with  $\kappa_2 = \xi \sigma_\eta^2 / (\xi \sigma_\eta^2 + \sigma_\varepsilon^2 / \delta)$ , so that welfare does not depend upon the stabilization policy anymore. Any value for the stabilization instrument  $\rho$  (including  $\rho = 0$ ) yields the

<sup>16</sup> Empirical evidence about the signalling role of monetary policy actions is provided by Romer and Romer (2000).



505 same welfare, the optimal stabilization strategy being indeterminate. Price dispersion is then given by

$$\sigma_{\mathbf{p}}^2 |_{\sigma_{\phi}^2=0} = \kappa_2^2 \sigma_{\varepsilon}^2 = \left( \frac{\xi \sigma_{\eta}^2}{\xi \sigma_{\eta}^2 + \sigma_{\varepsilon}^2 / \delta} \right)^2 \sigma_{\varepsilon}^2, \quad (34)$$

which may be heightened by firms' overconfidence (if  $\delta > 1$ ), but which cannot be manipulated by the instrument  $\rho$ .

So, let us next consider the more interesting case where the central bank can manipulate its communication instrument  $\sigma_{\phi}^2$ , alone or in conjunction with its stabilization instrument  $\rho$ , in order to minimize the price variance  $\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2 \sigma_{\phi}^2 + \kappa_2^2 \sigma_{\varepsilon}^2$ . We shall first examine the regime of pure communication (with  $\rho \equiv 0$ ), and then the regime combining communication and stabilization.

## 5.2 Pure communication: contested transparency

In the *pure communication* regime, the central bank discloses information while abstaining to take any stabilization action ( $\rho \equiv 0$ ). Full transparency allows minimization of the direct impact of noisy central bank communication on price dispersion, given by (33). Besides, by (20), full transparency also minimizes the value of  $\kappa_2$  under objective beliefs, thus further contributing to reduced price dispersion.<sup>17</sup> The optimality of a fully transparent pure communication policy extends to a situation where motivated beliefs are not very sensitive to the central bank's instrument  $\sigma_{\phi}^2$  but is destroyed when the concern for belief distortion is strong enough and sufficiently elastic to the relative precision of private information. The following proposition expresses the condition for this change in results.

**Proposition 1** *In the pure communication regime, full transparency ( $\sigma_{\phi}^2 = 0$ ) is the optimal central bank policy both under exogenous beliefs (not necessarily objective) and under motivated beliefs, when firms are only moderately overconfident, because of a high intensity  $\beta$  of concern for the belief distortion (resulting in  $\delta^*$  close enough to 1). However, if this concern has a low intensity  $\beta$  (resulting in a high enough  $\delta^*$ ) and is sufficiently sensitive to firms's overconfidence on the quality of private vs. quasi-public information ( $\alpha > 1/\xi$ ), an intermediate level of transparency ( $0 < \sigma_{\phi}^2 < \infty$ ) is the optimal central bank policy.*

**Proof.** See Appendix D. ■

<sup>17</sup>As underlined in the literature (e.g. Hellwig 2005 and Baeriswyl et al. 2020), in a microfounded macroeconomic model, transparency always improves welfare by reducing price dispersion across firms. According to Angeletos and Pavan (2007), the reason is that the equilibrium degree of coordination is lower than the efficient degree of coordination.

Opacity cannot be optimal under motivated beliefs about the accuracy of private information, just as it cannot be optimal under objective beliefs. By making firms more dependent upon private information and accordingly making more costly any misjudgment about it, the central bank can only reinforce the importance given to the more informative private signals. More reliance on private information enhances price dispersion and decreases welfare. However, full transparency is not always optimal either, under firms' motivated beliefs. By reducing the variance  $\sigma_\phi^2$ , the central bank makes firms overall more informed about the fundamental shocks and therefore reduces the importance they give to the consequences of mistakenly believing that their *private* information is very precise. Allowing themselves to become even more confident in this information, firms may overcompensate the direct impact of the reduction of  $\sigma_\phi^2$  on the weight  $\kappa_2(\delta, 1)$  put on private information. In this case, increasing the precision of semi-public information ends up in increasing price dispersion.

By contrast, an intermediate level of transparency balances the benefit of increasing firms' overall information on the fundamental shocks (making them rely objectively less on their private information) and the detrimental effect of firms' subjective overconfidence (making them rely more on that information). By not being fully opaque, the central bank makes firms more informed and less dependent on private information, and by not being fully transparent, the central bank imposes a larger cost on overconfidence in the precision of private information.

### 5.3 Communication and stabilization: contested opacity

In practice, central banks do not only communicate, they also implement a stabilization policy. In the *communication and stabilization* regime, the central bank chooses jointly its optimal stabilization policy  $\rho$  and its communication policy  $\sigma_\phi^2$  in order to minimize  $\sigma_p^2$ . A simple inspection of (20) shows that, under exogenous beliefs (be they objective or not), the central bank, by choosing full opacity ( $\sigma_\phi^2 \rightarrow \infty$ ), leads firms to set  $\kappa_1 = 0$ , which results in  $\kappa_1^2 \sigma_\phi^2 = 0$ . The coefficient  $\kappa_2 = 1 - \rho$  can then also be made equal to zero by setting  $\rho = 1$ . Welfare loss is thus minimized at zero, with no price dispersion. As soon as the central bank can use its two instruments, full opacity and complete stabilization are its two optimal policies, leading to the full achievement of the central bank objective.

Taking an action is thus more efficient for maximizing welfare than just disclosing information. If the central bank were not fully opaque, firms would overreact to public disclosure due to strategic complementarities in price setting: they would make inefficient use of information. As emphasized by Baeriswyl et al. (2020), reducing price dispersion does not require public information *per se* but a weaker

response to private information, which the central bank can achieve through its stabilization policy, by  
 560 setting  $\rho = 1$ . In conformity with the Lucas critique, the manipulation of the nominal expenditure  
 ( $z = -\rho y$ ) does not stabilize the economy by compensating the consumers' preference shocks, it does so  
 by moderating the firms' response to the information on those shocks ( $\kappa_1 + \kappa_2 = 1 - \rho$ ).

Does the optimality of full opacity stand in the case of motivated beliefs? Clearly, when firms are  
 overconfident about the quality of their private information, namely when  $\delta_\varepsilon = \delta$  and  $\delta_\phi = 1$ , then  
 565  $\sigma_\phi^2/\delta_\phi = \sigma_\phi^2$  and the same argument applies. By contrast, in the case of firms overconfident about  
 their ability to process information, namely in the case  $\delta_\varepsilon = \delta_\phi = \delta$ , then  $\sigma_\phi^2/\delta_\phi = \sigma_\phi^2/\delta^*$  and, by the  
 specification of  $\psi$  in this case,  $\psi \rightarrow 0$  as  $\sigma_\phi^2 \rightarrow \infty$ , pushing  $\delta^*$  to infinity. As a consequence,  $\sigma_\phi^2/\delta^*$  ( $\sigma_\phi^2$ )  
 may actually be decreasing (if the elasticity of  $\delta^*$  with respect to  $\sigma_\phi^2$  is larger than 1), full opacity ceasing  
 then to induce the absence of firms' response to the semi-public information disclosed by the central  
 570 bank. The following proposition formulates the condition for this outcome.

**Proposition 2** *In the communication and stabilization regime, full opacity ( $\sigma_\phi^2 = \infty$ ) coupled with full  
 stabilization ( $\rho = 1$ ) is the optimal central bank policy both under exogenous beliefs (not necessarily  
 objective) and under motivated beliefs, when firms overweight the quality of private vs. quasi-public  
 information. However, when firms overweight their ability to process idiosyncratic information, full  
 575 opacity ceases to be the optimal policy for an elastic concern for belief distortion ( $\alpha > 1$ ). An intermediate  
 stabilization level ( $0 < \rho < 1$ ) and, under a high concern  $\psi$  for belief distortion (a large  $\beta$ ), also an  
 intermediate communication level ( $0 < \sigma_\phi^2 < \infty$ ) are then optimal. Under a low enough concern for  
 belief distortion (a small  $\beta$ ), full transparency ( $\sigma_\phi^2 = 0$ ) coupled with vanishing stabilization ( $\rho \rightarrow 0$  as  
 $\beta \rightarrow 0$ ) is the optimal choice for the central bank.*

580 **Proof.** See Appendix D. ■

Full opacity under a two-instruments policy is not always optimal when firms' subjective beliefs  
 concern their ability to process information, contrasting with the case of objective beliefs. Indeed, by  
 increasing the variance  $\sigma_\phi^2$ , hence diminishing the relative precision of the subjectively assessed (idiosyn-  
 cratic) information, the central bank actually reduces the cost for firms of mistakenly believing that they  
 585 are able to process this idiosyncratic information. Firms thus tend to be even more confident in their  
 ability to process idiosyncratic information and may consequently overcompensate the direct impact of  
 the increase in  $\sigma_\phi^2$  on the weight  $\kappa_2(\delta, \delta)$  put on private information. Hence, decreasing the precision of  
 semi-public information ends up in increasing price dispersion.

By optimally setting an intermediate degree of transparency and an intermediate level of stabilization

policy, the central bank balances the benefit of increasing firms' information on the fundamental  $\theta$  (making them rely less on private information) and the detrimental effect of firms' overconfidence in their ability to process idiosyncratic information. Large overconfidence resulting from a low concern for belief distortion even opens the way to a complete reversal of the recommended central bank's communication policy, full transparency rather than full opacity being then the optimal choice.

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## 6 Conclusion

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Firms' motivated beliefs about the precision of their private information or about their ability to process the information that underlies their pricing decisions can impact the optimal policy of the central bank. First, motivated beliefs may make monetary policy less efficient without however commanding any change in the optimal behavior of the central bank. This happens in the case of signalling stabilization. The absence of idiosyncratic noise affecting the public information displayed by the central bank deprives the latter, in this case, of the capacity to modulate motivated beliefs.

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Second, motivated beliefs may, by contrast, require the optimizing behavior of the central bank to be adjusted. Firms' outcome prospects are enhanced by overconfidence in the quality of their own information, which ends up in more price dispersion and sub-optimal coordination of individual decisions. These decisions result however from a trade-off between the supposed benefit created by overconfidence and the cost of parting with objectivity. If this trade-off is modulated by the signal to noise ratio of central bank communication, the manipulation of this ratio can induce effects sufficiently distorted to temper or even to reverse the direction of optimal monetary policy.

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The way the trade-off is modulated depends upon the object of motivated beliefs. If they address the quality of firm's own information relative to that of the semi-public information provided by the central bank, firms' overconfidence is dampened when the latter is noisier, hence less decisive as a basis for pricing decisions. As a consequence, the central bank may weaken price dispersion by abandoning full transparency (optimal under objective beliefs) and introducing some idiosyncratic noise. By contrast, if motivated beliefs address the ability for each firm to extract a signal from noisy information, whatever its source, by providing noisier information the central bank can only reinforce firms' overconfidence, making them wishfully perceive a signal even under full opacity. Then opacity, combined with full stabilization (in order to weaken firms' reactivity), may cease to be optimal, some transparency being welcome to reduce price dispersion.

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Overall, motivated beliefs tend to give rise to intermediate solutions for the central bank (instead of

620 corner, bang-bang, solutions under objective beliefs), due to effects that go in two opposite directions and that compensate each other. As such, an intermediate level of transparency balances the benefit of increasing firms' information on the fundamental shocks and the detrimental effect of firms' subjective overconfidence.

## Appendix

### 625 A Generalized means

A *generalized* (or *power*) *mean* of a continuum of non-negative values  $\mathbf{P} = (P_i)_{i \in [0,1]} \in \mathbb{R}_+^{[0,1]}$  is defined, for some non-zero real number  $a$ , by the equality

$$\mathcal{P}_a(\mathbf{P}) = \left( \int_0^1 P_i^a di \right)^{1/a}. \quad (35)$$

Limit cases are  $\mathcal{P}_0(\mathbf{P}) = \prod_{i=0}^1 P_i$  (the geometric mean),  $\mathcal{P}_{-\infty}(\mathbf{P}) = \min(P_i)$  and  $\mathcal{P}_{+\infty}(\mathbf{P}) = \max(P_i)$ . Two currently used special cases are  $\mathcal{P}_1(\mathbf{P}) = \int_0^1 P_i di$  (the arithmetic mean) and  $\mathcal{P}_{-1}(\mathbf{P}) = \left( \int_0^1 P_i^{-1} di \right)^{-1}$  (the harmonic mean). The CES price aggregators are generalized means with  $a$  restricted to  $(-\infty, 1)$  in the case where goods are substitutes ( $s > 1$ ). An important property of the family of generalized means is that

$$a < b \implies \mathcal{P}_a(\mathbf{P}) \leq \mathcal{P}_b(\mathbf{P}) \quad (36)$$

with equality if and only if  $P_i = P_j$  for any  $i$  and  $j$  in  $[0, 1]$ .

A transformation of the variables into their log-deviations from a reference fixed value  $P^*$ , with  $p_i \equiv \ln P_i - \ln P^*$ , gives:

$$p_a \equiv \ln \mathcal{P}_a(\mathbf{P}) - \ln P^* = \frac{1}{a} \ln \int_0^1 e^{a p_i} di \quad (37)$$

and, taking a second order Taylor approximation of  $p_a$  around  $p^* = 0$ ,

$$p_a \simeq \int_0^1 p_i di + \frac{a}{2} \left( \int_0^1 p_i^2 di - \int_0^1 \int_0^1 p_i p_j di dj \right) = \mathcal{P}_1(\mathbf{p}) + \frac{a}{2} \sigma_{\mathbf{p}}^2, \quad (38)$$

where  $\mathcal{P}_1(\mathbf{p})$  and  $\sigma_{\mathbf{p}}^2$  are the arithmetic mean and the variance of  $\mathbf{p}$ , respectively. As  $\mathcal{P}_a(\mathbf{P}) = P^* e^{\mathcal{P}_1(\mathbf{p}) + (a/2)\sigma_{\mathbf{p}}^2}$ , we thus retrieve the property that  $\mathcal{P}_a(\mathbf{P})$  is increasing in  $a$ , except when  $\sigma_{\mathbf{p}}^2 = 0$ .

An application of formula (38) concerns the CES price aggregator

$$p \equiv p_{1-s} \simeq \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2} \sigma_{\mathbf{p}}^2 \quad (39)$$

which, in the case of the linear pricing rule  $p_j = \bar{\kappa}_0 + \bar{\kappa}_1 y_j + \bar{\kappa}_2 x_j$  for any firm  $j$ , becomes

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$$\begin{aligned} p = & \bar{\kappa}_0 + \bar{\kappa}_1 \left( y + \int_0^1 \phi_j dj \right) + \bar{\kappa}_2 \left( \theta + \int_0^1 \varepsilon_j dj \right) \\ & - \frac{s-1}{2} \int_0^1 \left( \bar{\kappa}_1 \left( \phi_j - \int_0^1 \phi_k dk \right) + \bar{\kappa}_2 \left( \varepsilon_j - \int_0^1 \varepsilon_k dk \right) \right)^2 dj, \end{aligned}$$

## B The profit function

Take the expression of the real profit in terms of log deviations from the non-stochastic solution:

$$\pi(p_i - p, c, \theta) = (1 - 1/s)^{1/\xi} \left[ e^{c-(s-1)(p_i-p)} - (1 - 1/s) e^{\xi\theta + (1+\xi)c - s(p_i-p)} \right]. \quad (40)$$

The second order Taylor approximation of this function at the origin is given by

$$\begin{aligned} \frac{\pi(p_i - p, c, \theta)}{(1 - 1/s)^{1/\xi}} & \simeq 1/s + \frac{\partial \pi}{\partial (p_i - p)} (p_i - p) + \frac{\partial \pi}{\partial c} c + \frac{\partial \pi}{\partial \theta} \theta \\ & + (p_i - p) \left( \frac{1}{2} \frac{\partial^2 \pi}{\partial (p_i - p)^2} (p_i - p) + \frac{\partial^2 \pi}{\partial (p_i - p) \partial c} c + \frac{\partial^2 \pi}{\partial (p_i - p) \partial \theta} \theta \right) \\ & + c \left( \frac{1}{2} \frac{\partial^2 \pi}{\partial c^2} c + \frac{\partial^2 \pi}{\partial c \partial \theta} \theta \right) + \frac{1}{2} \frac{\partial^2 \pi}{\partial \theta^2} \theta^2 + R_2(p_i - p, c, \theta), \end{aligned} \quad (41)$$

where  $R_2$  is the remainder term at the second order. By computing the second order derivatives at the origin, we obtain the log-quadratic approximation of the profit function

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$$\begin{aligned} \frac{\pi(p_i - p, c, \theta)}{(1 - 1/s)^{1/\xi}} & \simeq \underbrace{(s-1)(p_i - p) \left( \xi(c + \theta) - \frac{1}{2}(p_i - p) \right)}_{\tilde{\pi}(p_i - p, c + \theta)} \\ & \quad + \frac{1}{s} + \frac{1 - (s-1)\xi}{s} c - \frac{(s-1)\xi}{s} \theta + \frac{s - (s-1)(1+\xi)^2}{2s} c^2 \\ & \quad - \frac{(s-1)(1+\xi)\xi}{s} c\theta - \frac{(s-1)\xi^2}{2s} \theta^2 + R_2(p_i - p, c, \theta), \end{aligned} \quad (42)$$

where all but the first term do not depend upon  $p_i - p$  or involve a higher order of approximation. Hence, we may take

$$\tilde{\pi}(p_i - p, c + \theta) = (s - 1)(p_i - p) \left( \xi(c + \theta) - \frac{1}{2}(p_i - p) \right) \quad (43)$$

as the approximated profit function to be maximized in  $p_i - p$ .

## C The welfare function

650 Welfare is identified with the household's utility

$$U(\Theta C, \Theta L) = \frac{(\Theta C)^{1-\xi}}{1-\xi} - \Theta \int_0^1 \left( \frac{P_i}{P} \right)^{-s} C di = \frac{(\Theta C)^{1-\xi}}{1-\xi} - \Theta C \mathcal{P}_{1-s}(\mathbf{P})^s \mathcal{P}_{-s}(\mathbf{P})^{-s}, \quad (44)$$

using the generalized mean  $\mathcal{P}_a(\mathbf{P}) \equiv \left( \int_0^1 P_i^a di \right)^{1/a}$  (see Appendix A). Equivalently, in log-deviations from the nonstochastic solution of the model,

$$v(p_{-s} - p_{1-s}, c + \theta) = (1 - 1/s)^{1/\xi} \left( \frac{e^{(1-\xi)(c+\theta)}}{(1-\xi)(1-1/s)} - e^{c+\theta-s(p_{-s}-p_{1-s})} \right). \quad (45)$$

The second order Taylor approximation of this function, at the origin, is

$$\begin{aligned} \frac{\tilde{v}(p_{-s} - p_{1-s}, c + \theta)}{(1 - 1/s)^{1/\xi}} &= \underbrace{\frac{1 + \xi(s-1)}{(1-\xi)(s-1)}}_{\tilde{v}(\mathbf{0})/(1-1/s)^{1/\xi}} + \frac{c + \theta}{s-1} + s(p_{-s} - p_{1-s}) \\ &+ \frac{1}{2} \left( \frac{1-s\xi}{s-1} (c + \theta)^2 - s^2 (p_{-s} - p_{1-s})^2 + 2s(p_{-s} - p_{1-s})(c + \theta) \right) \\ &+ R_2(p_{-s} - p_{1-s}, c + \theta). \end{aligned} \quad (46)$$

where  $R_2$  is the remainder term at the second order.

655 This welfare function has two arguments, which may be seen as expressing coordination and stabilization intermediate objectives of the central bank policy. By equation (38), its first argument,  $p_{-s} - p_{1-s}$ , is indeed always non-positive, attaining its maximum at zero, when all the prices are identical:

$$p_{-s} - p_{1-s} = -\frac{1}{2} \sigma_{\mathbf{P}}^2. \quad (47)$$

As to the second argument of the approximated welfare function, we can use the equality  $c + \theta = z + \theta - p$ ,

with  $z$  possibly under the control of the central bank. By (39) and then (12), we have at equilibrium:

$$p = \mathcal{P}_1(\mathbf{p}) - \frac{s-1}{2}\sigma_{\mathbf{p}}^2 = (1-\xi)p + \xi(z+\theta) - \frac{s-1}{2}\sigma_{\mathbf{p}}^2, \quad (48)$$

so that

$$p = z + \theta - \frac{s-1}{2\xi}\sigma_{\mathbf{p}}^2, \quad (49)$$

and finally

$$c + \theta = \frac{s-1}{2\xi}\sigma_{\mathbf{p}}^2. \quad (50)$$

The second potential intermediary objective of the central bank, stabilization, is out of reach, leaving us with coordination. This is of course in line with the Lucas critique.

By (46) and neglecting the constant term as well as the remainder term at the second order, we may refer to the transformed welfare function

$$V(\sigma_{\mathbf{p}}^2) = -\frac{s\xi-1}{\xi}\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right) - \frac{1}{2}\left(\frac{(s-1)(3s\xi-1)}{\xi^2} + s^2\right)\left(\frac{\sigma_{\mathbf{p}}^2}{2}\right)^2, \quad (51)$$

which is clearly a decreasing function under the assumption  $s\xi \geq 1$ .

## D Communication policy under motivated beliefs

We recall that the loss function to be minimized by the central bank is the price variance  $\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2\sigma_{\phi}^2 + \kappa_2^2\sigma_{\varepsilon}^2$ . Motivated beliefs impose a constraint on this minimization, taking into account the dependency of  $\delta^*$  upon the instrument  $\sigma_{\phi}^2$ . By totally differentiating the objective  $\sigma_{\mathbf{p}}^2$  of the central bank with respect to its instrument  $\sigma_{\phi}^2$ , we obtain:

$$\begin{aligned} \frac{d\sigma_{\mathbf{p}}^2(\sigma_{\phi}^2, \delta^*(\sigma_{\phi}^2))}{d\sigma_{\phi}^2} &= \kappa_1^2 + 2(\kappa_1\sigma_{\phi}^2 - \kappa_2\sigma_{\varepsilon}^2)\left(\frac{\partial\kappa_1}{\partial\sigma_{\phi}^2} + \frac{\partial\kappa_1}{\partial\delta}\frac{d\delta^*}{d\sigma_{\phi}^2}\right) \\ &= \kappa_1^2 \underbrace{\left[1 + 2\left(1 - \frac{\kappa_2\sigma_{\varepsilon}^2}{\kappa_1\sigma_{\phi}^2}\right)\left(\frac{\partial\kappa_1}{\partial\sigma_{\phi}^2}\frac{\sigma_{\phi}^2}{\kappa_1} + \left(\frac{\partial\kappa_1}{\partial\delta}\frac{\delta^*}{\kappa_1}\right)\left(\frac{d\delta^*}{d\sigma_{\phi}^2}\frac{\sigma_{\phi}^2}{\delta^*}\right)\right)\right]}_{f(\sigma_{\phi}^2, \delta^*)}, \end{aligned} \quad (52)$$

with sign equal to the sign of  $f(\sigma_{\phi}^2, \delta^*)$ . Two cases must be distinguished, corresponding to  $\boldsymbol{\delta} = (\delta, 1)$  and  $\boldsymbol{\delta} = (\delta, \delta)$ . In both cases, we take  $\delta \geq 1$ . If  $\delta < 1$ , both  $\mathcal{L}$  and  $\mathcal{D}$  are decreasing in  $\delta$ . Consequently,



a value  $\delta^*$  minimizing the firms' loss function cannot be smaller than one.

## 675 D.1 When firms overweigh the quality of private vs. quasi-public information

In this case, where  $\boldsymbol{\delta} = (\delta, 1)$  and  $\psi = \beta \left( \sigma_\varepsilon^2 / (\sigma_\eta^2 + \sigma_\phi^2) \right)^{-\alpha}$ , we have

$$\frac{\kappa_2}{\kappa_1} = \frac{\xi \sigma_\eta^2 + (1 - \rho) \sigma_\phi^2}{(1 - \rho) \sigma_\varepsilon^2 / \delta^* - \rho \xi \sigma_\eta^2}, \quad (53)$$

$$\frac{\partial \kappa_1}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\kappa_1} = - \frac{\sigma_\phi^2}{\xi \sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 / \delta^*}, \quad (54)$$

$$\frac{\partial \kappa_1}{\partial \delta} \frac{\delta^*}{\kappa_1} = \frac{\sigma_\varepsilon^2 / \delta^*}{\xi \sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 / \delta^*} - \frac{(1 - \rho) \sigma_\varepsilon^2 / \delta^*}{(1 - \rho) \sigma_\varepsilon^2 / \delta^* - \rho \xi \sigma_\eta^2}, \quad (55)$$

and hence

$$\begin{aligned} & f(\sigma_\phi^2, \delta^*) \quad (56) \\ = & 1 + 2 \frac{\xi \sigma_\eta^2 (\rho \sigma_\phi^2 + \sigma_\varepsilon^2) + (1 - \rho) (1 - 1/\delta^*) \sigma_\varepsilon^2 \sigma_\phi^2}{(\xi \sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 / \delta^*) ((1 - \rho) \sigma_\varepsilon^2 / \delta^* - \rho \xi \sigma_\eta^2)} \left( 1 + \frac{\xi \sigma_\eta^2 + (1 - \rho) \sigma_\phi^2}{(1 - \rho) \sigma_\varepsilon^2 / \delta^* - \rho \xi \sigma_\eta^2} \frac{\sigma_\varepsilon^2 / \sigma_\phi^2}{\delta^*} \frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} \right). \end{aligned}$$

Under pure communication ( $\rho = 0$ ), the policy framework in which we will be essentially interested  
680 in this case, the function

$$f(\sigma_\phi^2, \delta^*) = 1 + 2 \frac{\delta^* \xi \sigma_\eta^2 + (\delta^* - 1) \sigma_\phi^2}{\xi \sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2 / \delta^*} \left( 1 + (1 + \xi \sigma_\eta^2 / \sigma_\phi^2) \frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} \right) \quad (57)$$

is positive ( $\sigma_{\mathbf{p}}^2$  is increasing in  $\sigma_\phi^2$ ) if  $(d\delta^* / d\sigma_\phi^2) (\sigma_\phi^2 / \delta^*)$  is non-negative or negative but small enough in absolute value. Full transparency ( $\sigma_\phi^2 = 0$ ) is then the optimal central bank communication policy, a result already established in the literature for objective (more largely, exogenous) beliefs, such that  $(d\delta^* / d\sigma_\phi^2) (\sigma_\phi^2 / \delta^*) = 0$ .

685 Endogenous beliefs can however reverse this result, if the elasticity of  $\delta^*$  with respect to  $\sigma_\phi^2$  is negative and large in absolute value. To determine the sign of this elasticity, we differentiate the firm's first order condition for minimizing  $((s - 1) / 2) \mathcal{L}(\boldsymbol{\delta}) + \psi \mathcal{D}(\boldsymbol{\delta})$  in  $\delta$  (that the sign in (31) be equal to zero) and

obtain (for  $\rho = 0$ )

$$\frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} = - \frac{\left(\partial\psi/\partial\sigma_\phi^2\right)\left(\sigma_\phi^2/\psi\right) - 2\left(\partial\kappa_2/\partial\sigma_\phi^2\right)\left(\sigma_\phi^2/\kappa_2\right)}{\delta^*/(\delta^* - 1) - 2(\partial\kappa_2/\partial\delta)\left(\delta^*/\kappa_2\right)} \quad (58)$$

$$= - \frac{\sigma_\phi^2}{\xi\sigma_\eta^2 + \sigma_\phi^2} \frac{\alpha \frac{\xi\sigma_\eta^2 + \sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2} - 2\kappa_1}{\frac{\delta^*}{\delta^* - 1} - 2\kappa_1}, \quad (59)$$

with the denominator of the last term  $(\delta^*/(\delta^* - 1) - 2\kappa_1)$  positive by the corresponding second order condition, hence with  $\left(d\delta^*/d\sigma_\phi^2\right)\left(\sigma_\phi^2/\delta^*\right) < 0$  for  $\alpha$  large enough. By inserting this expression in (57), we obtain

$$f(\sigma_\phi^2, \delta^*) = 1 + 2 \frac{\delta^* \xi \sigma_\eta^2 + (\delta^* - 1) \sigma_\phi^2 \frac{\delta^*}{\delta^* - 1} - \alpha \frac{\xi \sigma_\eta^2 + \sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2}}{\xi \sigma_\eta^2 + \sigma_\phi^2 + \sigma_\varepsilon^2/\delta^*} \frac{\delta^*}{\delta^* - 1} - 2\kappa_1, \quad (60)$$

such that  $\lim_{\delta^* \rightarrow 1} f(0, \delta^*) > 0$  but  $\lim_{\delta^* \rightarrow \infty} f(0, \delta^*) = \text{sgn}(1 - \alpha\xi) \infty$ , destroying the result of full transparency optimality for  $\alpha > 1/\xi$  and  $\beta$  small enough, hence  $\delta^*$  large enough.

## D.2 When firms overweigh their ability to process idiosyncratic information

Now, with  $\boldsymbol{\delta} = (\delta, \delta)$  and  $\psi = \beta \left( (\sigma_\varepsilon^2 + \sigma_\phi^2) / \sigma_\eta^2 \right)^{-\alpha}$ , we have

$$\frac{\kappa_2}{\kappa_1} = \frac{\xi\sigma_\eta^2 + (1 - \rho)\sigma_\phi^2/\delta^*}{(1 - \rho)\sigma_\varepsilon^2/\delta^* - \rho\xi\sigma_\eta^2}, \quad (61)$$

$$\frac{\partial\kappa_1}{\partial\sigma_\phi^2} \frac{\sigma_\phi^2}{\kappa_1} = - \frac{\sigma_\phi^2/\delta^*}{\xi\sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*}, \quad (62)$$

$$\frac{\partial\kappa_1}{\partial\delta} \frac{\delta^*}{\kappa_1} = \frac{(\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*}{\xi\sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*} - \frac{(1 - \rho)\sigma_\varepsilon^2/\delta^*}{(1 - \rho)\sigma_\varepsilon^2/\delta^* - \rho\xi\sigma_\eta^2}, \quad (63)$$

and hence

$$f(\sigma_\phi^2, \delta^*) = 1 + 2 \left[ \frac{\rho\sigma_\phi^2/\delta^* + \sigma_\varepsilon^2/\delta^*}{(1 - \rho)\sigma_\varepsilon^2/\delta^* - \rho\xi\sigma_\eta^2} \frac{\xi\sigma_\eta^2}{\xi\sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*} \cdot \left( 1 + \frac{\xi\sigma_\eta^2(\rho + \sigma_\varepsilon^2/\sigma_\phi^2)}{(1 - \rho)\sigma_\varepsilon^2/\delta^* - \rho\xi\sigma_\eta^2} \frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} \right) \right]. \quad (64)$$

Under exogenous beliefs, if  $\delta^* = \delta$ , constant,  $\lim_{\sigma_\phi^2 \rightarrow \infty} f(\sigma_\phi^2, \delta^*) < 0$  for  $\rho > \underline{\rho} \equiv (\sigma_\varepsilon^2/\delta^*) / (\xi\sigma_\eta^2 + \sigma_\varepsilon^2/\delta^*)$  (entailing the negativity of  $\kappa_1$ ). This outcome is compatible with the optimality of full opacity but can be destroyed under endogenous beliefs, when  $\delta^*$  has an elasticity with respect to  $\sigma_\phi^2$  larger than one,

700 reversing the sense of variation of  $\sigma_\phi^2/\delta^*$ .

By differentiation of the firm's first order condition for minimizing  $((s-1)/2)\mathcal{L}(\delta) + \psi\mathcal{D}(\delta)$  in  $\delta$ , (that the sign in (32) be equal to zero), we obtain

$$\frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} = \frac{\alpha \frac{\sigma_\phi^2}{\sigma_\varepsilon^2 + \sigma_\phi^2} - \frac{(1-\mu(1))(1-\mu(\delta^*))}{1+(1-\mu(1))(1-\mu(\delta^*))} (\mu(1) + \mu(\delta^*)) + \left( \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\sigma_{\mathbf{P}}^2} \right)}{\frac{\delta^*}{\delta^* - 1} - \frac{(1-\mu(1))(1-\mu(\delta^*))}{1+(1-\mu(1))(1-\mu(\delta^*))} \mu(\delta^*) - \left( \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \delta} \frac{\delta^*}{\sigma_{\mathbf{P}}^2} \right)}, \quad (65)$$

705 with  $\mu(\delta) \equiv \sigma_\eta^2 / (\sigma_\eta^2 + \sigma_\phi^2/\delta)$ . The denominator of the RHS is positive by the corresponding second order condition. Hence,  $(d\delta^*/d\sigma_\phi^2) (\sigma_\phi^2/\delta^*) > 1$  (implying that  $\sigma_\phi^2/\delta^*$  ( $\sigma_\phi^2$ ) is decreasing in  $\sigma_\phi^2$ ) if the numerator of the RHS is larger than the corresponding denominator:

$$\begin{aligned} & \alpha \frac{\sigma_\phi^2}{\sigma_\varepsilon^2 + \sigma_\phi^2} \\ & > \frac{(1-\mu(1))(1-\mu(\delta^*))}{1+(1-\mu(1))(1-\mu(\delta^*))} \mu(1) + \frac{\delta^*}{\delta^* - 1} - \left( \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \delta} \frac{\delta^*}{\sigma_{\mathbf{P}}^2} + \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\sigma_{\mathbf{P}}^2} \right). \end{aligned} \quad (66)$$

We can formulate a sufficient condition for this inequality to be satisfied by neglecting the last term, if positive. Indeed,

$$\frac{\partial \sigma_{\mathbf{P}}^2}{\partial \delta} \frac{\delta^*}{\sigma_{\mathbf{P}}^2} = 2 \frac{(\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*}{\xi \sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*} - \frac{2}{\kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2} \left( \kappa_1^2 \sigma_\phi^2 \frac{(1-\rho) \sigma_\varepsilon^2/\delta^*}{(1-\rho) \sigma_\varepsilon^2/\delta^* - \rho \xi \sigma_\eta^2} + \kappa_2^2 \sigma_\varepsilon^2 \frac{(1-\rho) \sigma_\phi^2/\delta^*}{\xi \sigma_\eta^2 + (1-\rho) \sigma_\phi^2/\delta^*} \right) \quad (67)$$

$$\frac{\partial \sigma_{\mathbf{P}}^2}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\sigma_{\mathbf{P}}^2} = -2 \frac{\sigma_\phi^2/\delta^*}{\xi \sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*} + \frac{1}{\kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2} \left( \kappa_1^2 \sigma_\phi^2 + 2 \kappa_2^2 \sigma_\varepsilon^2 \frac{(1-\rho) \sigma_\phi^2/\delta^*}{\xi \sigma_\eta^2 + (1-\rho) \sigma_\phi^2/\delta^*} \right) \quad (68)$$

$$\frac{\partial \sigma_{\mathbf{P}}^2}{\partial \delta} \frac{\delta^*}{\sigma_{\mathbf{P}}^2} + \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\sigma_{\mathbf{P}}^2} = 2 \frac{\sigma_\varepsilon^2/\delta^*}{\xi \sigma_\eta^2 + (\sigma_\phi^2 + \sigma_\varepsilon^2)/\delta^*} + \frac{\kappa_1^2 \sigma_\phi^2}{\kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2} \left( 1 - 2 \frac{(1-\rho) \sigma_\varepsilon^2/\delta^*}{(1-\rho) \sigma_\varepsilon^2/\delta^* - \rho \xi \sigma_\eta^2} \right) > 0 \quad (69)$$

provided  $\rho > \underline{\rho} \equiv (\sigma_\varepsilon^2/\delta^*) / (\xi \sigma_\eta^2 + \sigma_\varepsilon^2/\delta^*)$ . Now, if  $\sigma_\phi^2 \rightarrow \infty$  and consequently  $\mu(1) \rightarrow 0$  and  $\psi \rightarrow 0$ , so that  $\delta^* \rightarrow \infty$  and  $\underline{\rho} \rightarrow 0$ , the condition for  $(d\delta^*/d\sigma_\phi^2) (\sigma_\phi^2/\delta^*)$  to be larger than 1 becomes (for  $\rho > 0$ ):

710  $\alpha > 1$ .

### D.3 Pure communication

**Proof of Proposition 1.** The case for full transparency, associated with positivity of  $d\sigma_{\mathbf{P}}^2/d\sigma_\phi^2$  for any  $\sigma_\phi^2$  when  $d\delta^*/d\sigma_\phi^2 = 0$  (exogenous beliefs) results immediately from the positivity of  $f(\sigma_\phi^2, \delta^*)$  when  $\rho = 0$  and  $d\delta^*/d\sigma_\phi^2 = 0$ . Under motivated beliefs this is still true for a high cost  $\beta$  of being

irrational, keeping  $\left(d\delta^*/d\sigma_\phi^2\right)\left(\sigma_\phi^2/\delta^*\right)$  weak enough. Indeed, by (58) and (65),  $\left(d\delta^*/d\sigma_\phi^2\right)\left(\sigma_\phi^2/\delta^*\right) \rightarrow 0$  as  $\delta^* \rightarrow 1$  (because  $\beta \rightarrow \infty$ ). When firms overweigh the quality of their private information ( $\delta = (\delta, 1)$ ), we obtain from (60) that  $f(0, \delta^*)$  becomes negative for  $\alpha > 1/\xi$  and  $\beta \rightarrow 0$  hence  $\delta^* \rightarrow \infty$ , excluding optimality of full transparency. At the other extreme, as  $\sigma_\phi^2 \rightarrow \infty$ ,  $\psi \rightarrow \infty$  hence  $\delta^* \rightarrow 1$ ,<sup>18</sup> so that  $\lim_{\sigma_\phi^2 \rightarrow \infty} \left(\sigma_\phi^2, \delta^* \left(\sigma_\phi^2\right)\right) \rightarrow 1$ , excluding optimality of full opacity. As a consequence, the optimal value of  $\sigma_\phi^2$  must then belong to  $(0, \infty)$ .

#### D.4 Stabilization and communication

**Proof of Proposition 2.** Full opacity ( $\sigma_\phi^2 = \infty$ ) coupled with full stabilization ( $\rho = 1$ ) imply, by (20),  $\sigma_{\mathbf{p}}^2 \simeq \kappa_1^2 \sigma_\phi^2 + \kappa_2^2 \sigma_\varepsilon^2 = 0$ , as long as  $\delta_\phi$  remains constant when varying  $\sigma_\phi^2$ , hence in particular in both cases of exogenous beliefs and overconfidence in the quality of private information. The case left to be examined is the one with  $\delta = (\delta, \delta)$  and  $\psi = \beta \left( \left( \sigma_\varepsilon^2 + \sigma_\phi^2 \right) / \sigma_\eta^2 \right)^{-\alpha}$ . In this case, making  $\sigma_\phi^2$  tend to infinity does not ensure, as shown in subsection D.2, that  $\sigma_\phi^2/\delta^* \left(\sigma_\phi^2\right)$  tends itself to infinity (which would entail the preceding result), not even that it increases, if the elasticity of  $\delta^*$  with respect to  $\sigma_\phi^2$  is larger than 1 (at least for large  $\sigma_\phi^2$ ). More precisely, it was shown that the condition  $\alpha > 1$  implies  $\lim_{\sigma_\phi^2 \rightarrow \infty} \left( \left( d\delta^*/d\sigma_\phi^2 \right) \left( \sigma_\phi^2/\delta^* \right) \right) > 1$ , hence  $\lim_{\sigma_\phi^2 \rightarrow \infty} \left( \sigma_\phi^2/\delta^* \left( \sigma_\phi^2 \right) \right) = 0$ , which itself implies  $\lim_{\sigma_\phi^2 \rightarrow \infty} f \left( \sigma_\phi^2, \delta^* \left( \sigma_\phi^2 \right) \right) = 1$ , hence  $\lim_{\sigma_\phi^2 \rightarrow \infty} \left( d\sigma_{\mathbf{p}}^2 \left( \sigma_\phi^2, \delta^* \left( \sigma_\phi^2 \right) \right) / d\sigma_\phi^2 \right) > 0$ . Thus,  $\sigma_{\mathbf{p}}^2$  is eventually increasing in  $\sigma_\phi^2$  and full opacity cannot be optimal.

The optimal choice of the two instruments  $\rho$  and  $\sigma_\phi^2$  results from the minimization of  $\sigma_{\mathbf{p}}^2$  under the constraint imposed by beliefs motivation, that the sign of the derivative of the firms' loss function with respect to  $\delta$  as expressed in (32) be zero. Thus, using the corresponding Lagrangian, we can refer to the minimization in  $(\rho, \sigma_\phi^2, \delta)$  of the function

$$\sigma_{\mathbf{p}}^2 + \zeta \left( \psi (\delta - 1) \left( 1 + \frac{\sigma_\phi^2}{\sigma_\eta^2 + \sigma_\phi^2} \frac{\sigma_\phi^2/\delta}{\sigma_\eta^2 + \sigma_\phi^2/\delta} \right) - (s - 1) \sigma_{\mathbf{p}}^2 \right), \quad (70)$$

where  $\zeta$  is the Lagrange multiplier. The first order condition relative to  $\rho$  is  $(1 - \zeta(s - 1)) \partial \sigma_{\mathbf{p}}^2 / \partial \rho = 0$ , hence

$$\rho = \frac{2\xi \sigma_\eta^2 + \sigma_\varepsilon^2/\delta + \sigma_\phi^2/\delta}{(\xi \sigma_\eta^2 + \sigma_\varepsilon^2/\delta)^2 / (\sigma_\varepsilon^2/\delta) + \sigma_\phi^2/\delta}, \quad (71)$$

which is such that  $\rho \rightarrow 1$  as  $\sigma_\phi^2/\delta \rightarrow \infty$  and  $\rho \rightarrow (2 - \underline{\rho}) \underline{\rho} > \underline{\rho} \equiv (\sigma_\varepsilon^2/\delta^*) / (\xi \sigma_\eta^2 + \sigma_\varepsilon^2/\delta^*)$  as  $\sigma_\phi^2/\delta \rightarrow 0$ .

<sup>18</sup> Notice that  $\lim_{\sigma_\phi^2 \rightarrow 0} \psi = 0$  in the case where firms overweigh their ability to process idiosyncratic information, so that the same argument does not prevail in this case.

Thus, the optimal value of  $\rho$  belongs to the interval  $[(2 - \underline{\rho}) \underline{\rho}, 1] \subset [\underline{\rho}, 1] \subset (0, 1]$ . Also, recall that  
740  $\sigma_\phi^2 = \infty$  cannot be optimal if  $\alpha > 1$ , further restricting the optimal  $\rho$  to belong to the interval  $(0, 1)$ .  
At the other extreme, using the corresponding optimal value of  $\rho$ , namely  $(2 - \underline{\rho}) \underline{\rho}$ , we obtain from (67)  
and (68)

$$\lim_{\sigma_\phi^2 \rightarrow 0} \left( \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \delta} \frac{\delta^*}{\sigma_{\mathbf{P}}^2} \right) = 2\underline{\rho} \text{ and } \lim_{\sigma_\phi^2 \rightarrow 0} \left( \frac{\sigma_\varepsilon^2}{\sigma_\phi^2} \frac{\partial \sigma_{\mathbf{P}}^2}{\partial \sigma_\phi^2} \frac{\sigma_\phi^2}{\sigma_{\mathbf{P}}^2} \right) = -\underline{\rho}^2, \quad (72)$$

hence, by (65), with  $\mu(1) = \mu(\delta^*) = 1$ ,

$$\lim_{\sigma_\phi^2 \rightarrow 0} \left( \frac{\sigma_\varepsilon^2}{\sigma_\phi^2} \frac{d\delta^*}{d\sigma_\phi^2} \frac{\sigma_\phi^2}{\delta^*} \right) = \frac{\alpha - \underline{\rho}^2}{\frac{\delta^*}{\delta^* - 1} - 2\underline{\rho}}, \quad (73)$$

so that, by (64),

$$f(0, \delta^*) = -\frac{\frac{\delta^*}{\delta^* - 1} - 2\alpha/\underline{\rho}}{\frac{\delta^*}{\delta^* - 1} - 2\underline{\rho}}, \quad (74)$$

745 the denominator on the RHS being positive by the second order condition for the minimization in  $\delta$  of the  
firm's loss function. Hence, for large  $\beta$  and correspondingly small  $\delta^*$  ( $\delta^* \rightarrow 1$  as  $\beta \rightarrow \infty$ ),  $f(0, \delta^*) < 0$ ,  
price variance being then decreasing in  $\sigma_\phi^2$  at the origin, so that the optimal value of  $\sigma_\phi^2$  is interior.  
By contrast, as  $\beta \rightarrow 0$ , implying  $\delta^* \rightarrow \infty$  and  $\underline{\rho} \rightarrow 0$ , we have  $\lim_{\delta^* \rightarrow \infty} f(\sigma_\phi^2, \delta^*) = \infty$ , so that price  
variance is always increasing in  $\sigma_\phi^2$ . Hence, for small  $\beta$ , full transparency ( $\sigma_\phi^2 = 0$ ) coupled with vanishing  
750 stabilization ( $\rho \rightarrow 0$  as  $\beta \rightarrow 0$ ) is the optimal choice for the central bank, resulting in  $\sigma_{\mathbf{P}}^2 \rightarrow \sigma_\varepsilon^2$ .

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