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ARTICLE TYPE

DEM Filtration Modeling for Granular Materials: Comparative Analysis of Dry and Wet Approaches

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Summary

A detailed study comparing two - dry and wet - numerical approaches to model filtration processes at stake in actual granular filters is presented using the discrete element method. In the first approach, the migration of fines is provided by gravitational forces, while in the second, hydrodynamic forces induce their movement. Numerical filtration tests were performed on granular filters involving materials with different gradings and porosities. The study demonstrated that the wet filtration approach generates higher tortuosity due to the possibility for fines to deviate from direct paths towards more open sideways. It leads to a lower coefficient of retention for the filter than if it were characterised using a dry filtration approach. However, the intensity of this feature greatly depends on the grading and the porosity of the granular filter. Finally, an enhanced dry filtration model designated as the "equivalent cyclic wet filtration model" is presented, which better mimics the results obtained through the preferable wet filtration model compared to the original dry filtration model. This new model constitutes a valuable alternative tool for studies of filtration properties in granular materials.

KEYWORDS:

Granular filters, solid-fluid coupling, pore, constriction, tortuosity

1 | INTRODUCTION

The use of granular filters to remove suspended fine particles from fluid streams is a critical process across multiple industries, including geotechnical and environmental engineering, water treatment, mining, and oil industry¹. Designing an efficient and cost-effective filtration system is essential not only for ensuring the quality of the process but also for safety reasons. For instance, filter fouling can have a significant impact on filtration operations, particularly when the applied pressure is driving the flow. When filters become clogged, they can directly reduce the rate of filtrate flux and increase the applied pressure, thereby hindering the overall effectiveness of the filtration process². Similarly, in hydraulic structures and behind retaining walls, clogged filters can unexpectedly create high hydraulic loads on the structure, potentially leading to structural failure^{3,4}. In this case, it may imply critical consequences both in terms of human life loss and economic costs.

In the past, a multitude of experimental tests were conducted to investigate various features. Some focused on studying and characterising the fundamental mechanisms involved in filtration processes, while others were specifically carried out for engineering applications, such as developing filtration criteria for designing granular filters. For example, Ghidaglia⁵ carried out experimental tests to study the penetration depth of fine particles in deep bed filtration for two different particle injection

methods under hydrodynamic and gravitational forces (wet and dry conditions, respectively). The results show that in dry conditions (only gravitational forces drive the movement of fines), the fine particles penetrated slightly less deep than in the case where fine particles were dragged by hydrodynamic forces (due to the existence of a water seepage). Furthermore, in the latter case, they remarked that fine particles are trapped not only because of geometrical restrictions existing in the filter, but also because some particles are blocked in “hydrodynamic” sites (associated with stagnation points in the flow). These sites capture the fine particles in a very unstable regime where sharp changes in the flow characteristics due to changes in the inlet property can relaunch these particles. Within the framework of engineering applications, Terzaghi⁶ conducted several filtration tests on granular filters to develop a filter criterion that can be used for granular filters installed in hydraulic structures such as dams and dikes. Since it was difficult to directly quantify the filter void space and especially the constriction sizes (narrowest paths linking neighbouring pores) within the filter, Terzaghi indirectly related the capacity of a granular filter to retain some class of particles to an equivalent filter particle size. Recently, advanced techniques have been developed to provide more insight on local phenomena involved in the filtration processes, including particle image velocimetry^{7,8} and high-speed photography^{9,10}. However, these experimental methods typically offer qualitative information but often fall short in providing quantitative information about fluid-particle interaction^{11,12}.

Alongside lab experiments involving samples with actual granular materials, researchers have tried to use numerical modelling approaches to complement experimental data and gain a deeper understanding of complex filtration mechanisms. One of the most useful numerical tools is the discrete element method (DEM) coupled with a flow model, as it can provide detailed information on the filtration mechanism at the pore scale. However, the multiphysics coupling scheme remains challenging both in terms of the physical interaction modelling and in terms of computation time. Indeed, while the movement of fine particles is efficiently governed by the DEM computations, the calculation of fluid motion requires the solution of the Navier-Stokes equations. This process can be computationally demanding, particularly when high levels of accuracy are required to represent the fluid flow behaviour.

Wautier *et al.*¹³ listed the different fluid-solid coupling approaches that can be used to model the flow, such as resolved and unresolved computational fluid dynamics (CFD)^{14,15,16}, lattice Boltzmann methods (LBM)^{17,18,19} and pore network models^{20,21}. They emphasised that the first three approaches have a prohibitive computational cost when modelling filtration processes in granular materials, while pore network models can offer a cost-effective way to model fluid and grain interactions at a microscale level with fully coupled simulations. Within this framework, Chareyre²² proposed an effective up-scaled fluid-flow pore network model called the pore-scale finite volume method (PFV). However, when a large number of filter particles are involved and the moving fine particles are small in size, this technique remains computationally expensive. Moreover, this approach based on the Delaunay-segmentation of the space can only be used for filters composed of perfect spheres.

To overcome this limitation, different authors bypassed the solution of the fluid dynamics equations and substituted the use of hydrodynamic forces as driving forces on suspended particles with that of only gravitational forces^{23,24,25}. The underlying statement was that the main features observed in filtration processes involving water seepage may be found in dry filtration under gravity. There was no proof of the validity of this statement and specially when quantitative results are expected. Indeed, gravitational forces act only in one direction, while hydrodynamic forces influenced by local maximum fluxes in the porous medium can redirect fine particles in multiple directions, allowing them to travel a longer distance in the filter²⁶. In fact, there is currently no comprehensive numerical analysis that quantitatively assesses the differences between the two approaches and the equivalence between a filtration process mainly driven by hydrodynamic forces and that driven by only gravitational forces.

Building on the insights gained from these previous works, the present study aims to run a thorough analysis of the differences between wet and dry filtration using DEM simulations. This study allows us to outweigh the drawbacks of dry filtration, which is much less time-consuming than wet filtration, particularly when the primary focus only lies on the retention of particles in the filter. Moreover, the notion of cyclic wet filtration tests is introduced. This kind of simulation tries to mimic the natural cyclic variability of hydraulic head in actual hydraulic structures that may disrupt existing unstable retention sites caused by sedimentation or “hydrodynamic” capture^{5,27}.

Lastly, this study presents an enhanced dry filtration model, denoted herein as an “equivalent cyclic wet filtration model” able to compensate for the intrinsic limits of dry filtration tests as a model for wet filtration tests. This new approach will be

validated based on DEM-PVF simulations of cyclic wet filtration tests. The paper is structured as follows: First, the procedure for generating the numerical sample (granular filter) using DEM is described. Then, dry and wet filtration models, including cyclic wet filtration, are introduced. The difference between these approaches and their impact on filtration results in terms of tortuosity and coefficient of retention are assessed. Next, the equivalent cyclic wet filtration model is presented and validated. Finally, the key findings and conclusions of this study are provided.

2 | GENERATION OF NUMERICAL SAMPLES

To investigate the transportation mechanism of fine particles in porous media, granular filters were numerically modelled as an assembly of spheres using the DEM open-source framework YADE²⁸. In DEM, the numerical sample is defined as packed assemblies of individual interacting particles. Then, local features including pore space properties are well conserved with respect to laboratory experiments if the modelled particles hold the same shape as actual ones. As a result, modelling the movement of the fine particles throughout the filter and recording all necessary information, such as the position, size, and velocity of each fine particle, can be easily processed²⁹.

An elastic-frictional model limited by the Coulomb criterion was chosen for the interaction at contact between particles. The contact is characterised by a normal stiffness k_n , a tangential stiffness k_t , and a Coulomb inter-particle friction φ_c . The normal stiffness k_n and tangential stiffness k_t were set to 5.65×10^5 N/m, and the specific weight of particles was assigned as $2,650 \text{ kg/m}^3$. To reduce non-frictional energy and particle bouncing during the preparation phase, a local damping coefficient of 0.7 was introduced²⁹.

The numerical filter samples were created by gravity deposition of a group of spheres inside a well-defined box, where this deposition was designed to mimic the filter installation procedure in actual engineering projects. To minimise wall boundary effects and reduce the number of particles involved in the computations, lateral periodic boundary conditions were implemented. The preparation process was considered complete when the assembly achieved a stable equilibrium state, characterised by an average unbalanced force ratio of less than 0.05²⁶. This ratio is defined as the ratio between the mean resultant forces at contact divided by the mean contact force over the sample. After equilibrium was obtained, the coarse matrix was fixed since filters are supposed to be internally stable. It allows for the optimisation of computation times since in this case, contact forces and kinematic torques for coarse particles are not updated throughout a filtration test.

To provide a comprehensive overview of the investigation, three distinct types of granular materials with different gradings were used. It involves a narrowly-graded (NG) material, with a coefficient of uniformity of 1.1 encompassing a limited range of particle sizes; an intermediate-graded (IG) material, with a coefficient of uniformity of 2.2, representing a middle ground with a moderate variety of particle sizes; and a well-graded (WG) material, exhibiting a coefficient of uniformity of 3.9 and containing a broader range of particle sizes (Figure 1). The range of particle diameters [D0;D100] for each material, where D0 is the minimum particle size and D100 is the maximum particle size, was chosen to be equal to [5.5 mm; 14.3 mm], [1.25 mm; 20 mm], and [0.7 mm; 10 mm] for NG, IG, and WG, respectively. In order to model the loosest and densest states of the filter for each material type, two samples were prepared with inter-particle friction φ_c set to 0.3 and 0, respectively²⁶. The values of the model parameters used for generating the numerical samples are given in Table 1.

The dimensions of the boxes were chosen to be greater than the representative elementary volume (REV). A preliminary study was conducted to confirm the appropriate sample size, using a methodology similar to that employed by Sjah *et al.*²⁴. First, each sample was divided into layers with thicknesses equal to the largest particle diameter D_{100} (Figure 2-b). Subsequently, the porosity of each layer was computed along the lateral direction with an increasing volume control width. The REV was determined by pinpointing the control volume width, where the porosity stabilised around a mean value with a coefficient of variation smaller than 0.02. In Figure 2b, the mean porosity of the horizontal layers is depicted along with the associated standard deviation for the three materials in their loose state. Notably, the average porosity stabilised when the volume control width was approximately $2.5(D_{100})$. This observation aligns with the findings reported by Sjah *et al.*²⁴, and any sample size greater than this may be suitable. The accompanying standard deviation error bars depict the heterogeneity observed between the layers. For NG materials, this heterogeneity diminishes significantly beyond $4D_{100}$. However, for both

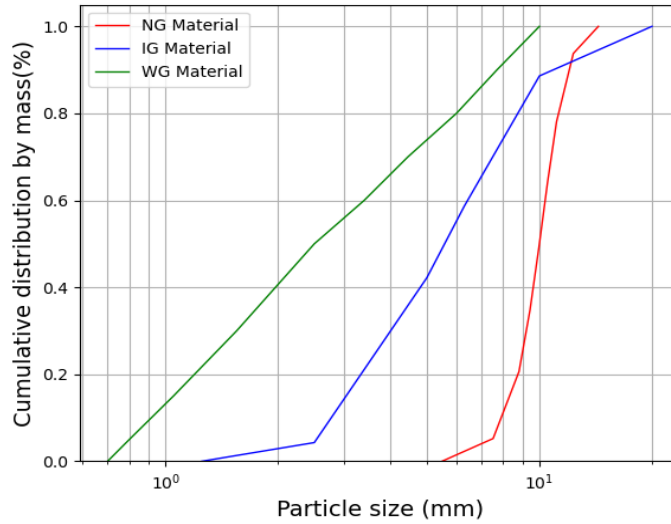


Figure 1 Particle size distributions of the numerical filters.

Table 1 Mechanical and numerical parameters for DEM simulations in the generation phase

Parameter	Magnitude
Normal stiffness (K_n)	5.5×10^5 N/m
Tangential stiffness (K_t)	5.5×10^5 N/m
Specific weight of spheres (ρ)	2650 kg/m ³
Global damping (α)	0.7
Inter-particle friction (μ)	0.3 (loosest state) 0 (densest state)

IG and WG materials, there was no noticeable decrease in heterogeneity after $2.5(D_{100})$ as it fluctuated around a value equal to 0.013, even though a significantly larger number of particles were involved in the horizontal layers. Based on these insights, the thicknesses of the samples determined were $7(D_{100})$ for NG, $3(D_{100})$ for IG, and $4(D_{100})$ for WG. The chosen thicknesses were optimised to obtain a manageable number of particles and prevent excessive computational delays. This balance ensured both the representativeness and efficiency of the simulations.

Selecting the appropriate filter thickness and the required number of coarse particles to create the samples can be challenging. Indeed, as the filter thickness increases, the number of constrictions along a pathway also increases. As a result, it increases the probability of fine particles being trapped in the filter. It means that different retention properties can be assigned to a filter depending on its thickness. However, above a certain thickness, a further increase does not significantly affect the results³⁰. Sherard³¹ stated that fine particles can pass through all possible combinations and permutations of pore network sizes and shapes after travelling a relatively short distance of approximately $5D_{15}$. According to Seblany²⁶, for the results to be reliable, a fine particle needs to pass through at least 40 constrictions. The transcription of the number of required constrictions on a pathway into a sample thickness can be performed assuming that the pore space is a multilayered cubic network model of pores interconnected by constrictions, as proposed by Schuler³². Then, the computation of a sample thickness H is defined by the relationship $H = N_c * s$ where N_c denotes the number of constrictions in a given straight direction and s the average pore diameter, which is herein also the average distance between two successive constrictions. It can be calculated by the equation (1) designed by Wu³³ and improved by Seblany³⁴. This formula takes into account both the grading of the material and its density :

$$s = \frac{1}{0.7} \sqrt[3]{\frac{e}{e_{max}}} s_{max} \quad (1)$$

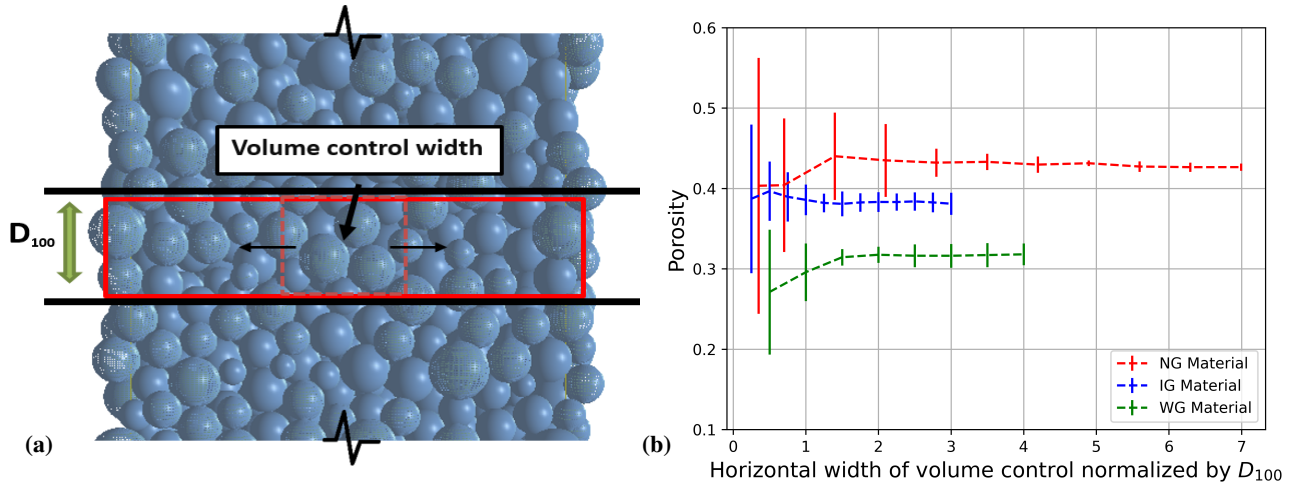


Figure 2 Determination of the representative elementary volume of the granular sample: (a)- Layers division within the sample, (b)- Porosity with increasing volume control width in each layer in the loosest state of NG filter

where e and e_{max} are the actual and maximum void ratios for the material. s_{max} is the mean pore diameter in the material for the loosest case. An estimate for s_{max} can be obtained by taking a value equal to $0.5D_{50SA}$, where D_{50SA} represents the particle diameter at which 50% of the material is smaller than from a particle size distribution (PSD) of the material defined in terms of surface area. The validity of this estimate for s_{max} can be found in³⁴.

In this work, a thickness of 12 cm and 30 cm for NG and IG filters, respectively, was assigned to ensure that at least 40 constrictions are encountered for both loose and dense cases. However, in the case of WG filter, the number of constrictions was deliberately reduced to 30, equivalent to a thickness of approximately 3 cm. This adjustment was made to restrict the particle count to 20,000, considering the substantial number of particles necessary for sample creation and the subsequent increase in computational time required for conducting filtration tests. The values of the physical properties of the obtained samples are given in Table 2.

Table 2 Characteristics of numerical samples

Material	Coefficient of uniformity (C_u)	Number of particles loose / dense	Void ratio e_{max}/e_{min}
NG	1.2	4,000 / 4,400	0.75/0.57
IG	2.2	10,000 / 11,000	0.62/0.43
WG	3.7	18,400 / 20,000	0.48/0.33

3 | NUMERICAL FILTRATION MODELS

A set of 200 mono-sized fine particles belonging to a given class of diameter were released randomly on top of the filters, and five repetitions of the same test for each fine particle class were processed where the initial position of the fine particles was different between repetitions. It allowed us to be sure that enough individual particle paths were explored through the filter and that the characterization of the filtration properties of the filters was representative. The release conditions and the chosen number of fine particles ensured that individual particle paths were explored with few interactions between fines, avoiding

group effects and bridging^{25,26}.

The range for the size classes of released fines was established between a size slightly smaller than the smallest constriction present in each material and the size at which complete blockage (100%) occurred. The minimum constriction size was approximated to be $D_0/6.5$, where D_0 represents the diameter of the smallest particle. This specific value arises from the mutual contact between the three smallest particles in the material. Only particles smaller than the smallest constrictions have a null probability of being blocked by a filter, irrespective of their thickness. Following this, the size of the smallest fines was set to 1 mm, 0.2 mm, and 0.01 mm for samples with NG material, IG material, and WG material, respectively. In subsequent tests, the size of the fine particles increased incrementally until complete blockage was observed.

During filtration tests, the inter-particle friction φ_c at the contact between filter particles and fine particles was set to zero. This "lubrication" allows the friction forces to be reduced and allows the fine particles to migrate more ahead in the filter³⁵. Moreover, the coefficient of restitution e was set to a low value of 0.3 in order to prevent the excessive bouncing of the fine particles on the surface of the filter particles^{23,25}.

Density scaling is a widely used technique to reduce the computational cost of DEM simulations, as demonstrated in previous studies^{36,37,38,39}. The influence of particle density on computational time varies depending on the specific type of numerical experiment conducted. In this study, the density of particles was increased by a factor of 10^3 to scale up the mass of particles. For the wet filtration model, the hydraulic pressure was also increased by a factor of 10^3 in order to conserve the same dragging forces on the fine particles and hence the same accelerations for them. In the dry filtration model, the acceleration of fines was introduced by gravity. Notably, variations in particle density do not influence the falling velocity since the free-falling movement of particles under gravity is independent of the particle mass. A parametric study was conducted to evaluate the effect of density scaling on filtration results. The results indicated that the technique had no significant impact on the final results, as tortuosity and coefficient of retention remained unaltered for dry and wet filtration with density amplification with respect to the case with no density scaling. Herein, the computational time was reduced by 10 to 25 times.

3.1 | Dry filtration

In the dry filtration model, the movement of the particles in the granular filter is only governed by the gravitational force. The use of this simplified model for filtration processes is generally motivated by a reduction of computational time but also allows the question of the fluid modelling together with the modelling of fluid-solid interactions to be bypassed. Herein, the procedure for modelling a dry filtration test is straightforward. Once the numerical sample had been prepared by gravity deposition and filter particles had been fixed, the set of 200 fines was released under gravity on the top sample.

3.2 | Wet filtration

There are several approaches available to simulate coupled fluid-solid systems. In this work, the pore finite volume method (PFV) was selected as it has been demonstrated to be an efficient tool for simulating the solid-fluid interaction in laminar fluid flows through spherical granular materials⁴⁰.

The PFV method performs a segmentation of the space in granular materials into locally connected pore bodies by means of a regular weighted Delaunay triangulation, where local pores are represented by the void inside the tetrahedral cells. Stokes flow equations are upscaled at the pore level and calculated using a finite volume scheme. By assuming constant fluid pressure in each pore and using the Darcy form to compute fluid fluxes, the flux q_{ij} between two connected pores i and j is computed as:

$$q_{ij} = g_{ij} \frac{p_i - p_j}{l_{ij}} \quad (2)$$

Where p_i and p_j are the pressure in pore i and j respectively, g_{ij} is the hydraulic conductance of the throat linking pores i and j , and l_{ij} is the length between the centres of the two connected pores.

To obtain the pressure field within the void space of granular material, the equation of continuity for the fluid is solved within each pore. It means that the rate of volume change for each pore can be calculated by summing up the fluxes through the four facets of the tetrahedral pore element. Considering pore i , the continuity equation is computed as follows:

$$\dot{V}_i^f = \sum_{j=1}^{j=4} q_{ij} \quad (3)$$

where \dot{V}_i^f is the volumetric strain rate of the pore i obtained from the particle velocities at the four vertices of the tetrahedron i . Finally, the dragging force applied by the hydraulic flow on each particle with a boundary surface $\partial\Gamma_k$, is calculated in function of pressure p and viscous shear stress τ (note that the gravitational forces are neglected):

$$\vec{F}^k = \int_{\partial\Gamma_k} p\vec{n}ds + \int_{\partial\Gamma_k} \tau\vec{n}ds \quad (4)$$

where \vec{n} is the unit normal to the surface $\partial\Gamma_k$.

The viscous shear force on the solid phase F_{ij}^V was calculated by integrating the momentum conservation equation over the volume of the throat connecting the two pores, which is defined as the space between the centres of the two pores and encompasses the centres of the three particles situated between them. From this integration, three terms emerge: viscous stress applied on the solid phase, viscous stress on the fluid part, and the sum of pressures between the two pores. By neglecting the second term (based on the assumption that pressure gradients are primarily balanced by the viscous stress on the solid phase), the viscous stress applied on the solid is directly proportional to the product of the throat's cross-sectional area A_{ij} and the pressure difference between the two pores:

$$F_{ij}^V = \int_{\partial\Gamma_k} \tau\vec{n}ds \approx A_{ij}(p_j - p_i)\vec{n}_{ij} \quad (5)$$

Then, the complexity of the fluid-solid problem is simplified and can be solved efficiently by the inversion of sparse matrix algorithms. More details can be found in²².

In this study, the granular filters were exposed to a hydraulic charge of 2,000 kPa imposed in the same direction as gravity. This high value actually takes into account the effects of the amplification of the particle density but actually corresponds to a hydraulic head of 20 cm. The fluid viscosity was set to 0.001 kg/ms. To isolate the effect of hydrodynamic forces on fine particle movement in fluid filtration processes, gravity was set to zero. In fact, when hydraulic gradients are high and relatively lightweight particles are involved, hydrodynamic forces dominate over gravitational forces, making the latter negligible.

3.3 | Cyclic wet filtration

Granular filters can experience variations in flow rates over time in many different applications. For instance, in hydraulic structures like dams and dikes, the water level can periodically rise and fall due to factors such as rainfall or tides. Similarly, in industrial engineering, such as water treatment, the flow of water through a granular filter is not always steady and may stop and start again periodically. Experimental results show that fine particles trapped in unstable sites, like "hydrodynamic" sites can be dislodged and mobilised again when subjected to transient conditions in the porous domain. Such transience can be induced by sharp changes in flow conditions, including complete flow cessation and subsequent relaunch or flow direction reversal. Note that a gradual increase or decrease in flow does not introduce this mobilisation⁵.

Herein, the possibility of cyclic conditions that may quantitatively modify the results in terms of retention coefficient was taken into account. Its objective was to consider trapped particles that could be released and subsequently migrate further into the filter and thus characterise the retention capacity of the filtration from a conservative perspective. The PFV method provides a solution in a fixed regime, making it ineffective for re-mobilizing blocked particles through a substantial increase in flow. Although reversing the flow direction can facilitate this re-mobilization, managing the movement of fine particles in

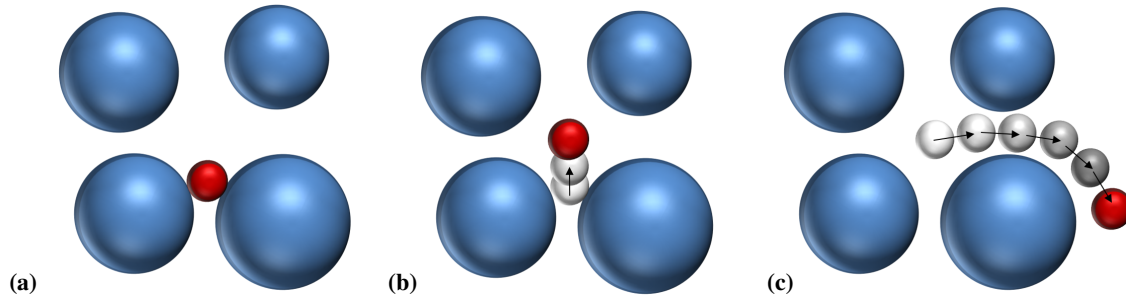


Figure 3 Schematic drawing of a trapped fine particle in cycle i being released from the pore in cycle $i + 1$: (a) - cycle i , phase 1: trapped particle at the end of the phase, (b) - cycle i , phase 2: relaunching phase, (c) - cycle $i + 1$, phase 1: reactivated flow and filtration test

the opposite direction poses a significant challenge. To successfully re-mobilise the trapped fines while preventing them from exiting the pore in the reverse direction or experiencing excessive movement in the opposite direction, we introduced a new model based on the PFV method.

The model is composed of two phases. The first phase consisted of applying the boundary conditions for the fluid using the PFV model and letting fines flow within the filter until a stable regime of blockages of fines was found. In the second stage, the possibility of re-mobilizing fine particles is introduced in an indirect way. The particles that passed the filter entirely were removed, while the particles that were found to be blocked were kept at their exact blockage site for the next cycle (Figure 3-a)). Then, the fluid flow was deactivated, and the blocked particles were subjected to a movement in the opposite direction of the flow in the previous stage. This movement is equal to half of the mean pore diameter (Figure 3-b)), which can be computed using equation (1). On average, it allowed fines to be shifted in a zone located at the centre of the pore, where the velocity in the fluid is supposed to be maximum. Finally, its velocity was set to zero. Then, in the next cycle, the flow was reactivated. This procedure offered blocked particles the possibility of being re-engaged with streamlines characterised by the highest flux towards the large sideways exits of the pore (Figure 3-c)). Cycles were repeated continuously until stabilised filtration results were obtained, indicating that no more fine particles could pass through the filter.

4 | RESULTS AND DISCUSSION

Since the trajectories of fine particles were tracked throughout the simulations, different filtration characteristics for the filter can be computed, including tortuosity and retention coefficient, among others.

4.1 | Tortuosity

Tortuosity, as initially conceptualised by Kozeny⁴¹, refers to the measure of how a flow path within a porous medium deviates from a direct straight-line path connecting the inlet and outlet points. Specifically, Kozeny's definition involves the ratio between the effective hydraulic flow path (L_{eff}) within the filter and the vertical straight depth penetration (L_0). This concept has found extensive applications in shedding light on the morphological attributes of granular materials, offering insights into how the arrangement of pores within these materials affects their transport properties⁴². In this study, tortuosity was associated with the pathways of fine particles percolating throughout the filter. These pathways reflect the existence of specific streamlines capable of carrying fine particles throughout the succession of pores and constrictions in the granular filter. Consequently, L_{eff} is defined as the mean of the lengths of all fine particle trajectories as they migrate through the filter. This approach serves two primary purposes. Firstly, it facilitates a comparison of the influence of the grading and porosity of different materials on the directional transportation mechanisms of fine particles. Secondly, it enables an examination of how the chosen filtration model

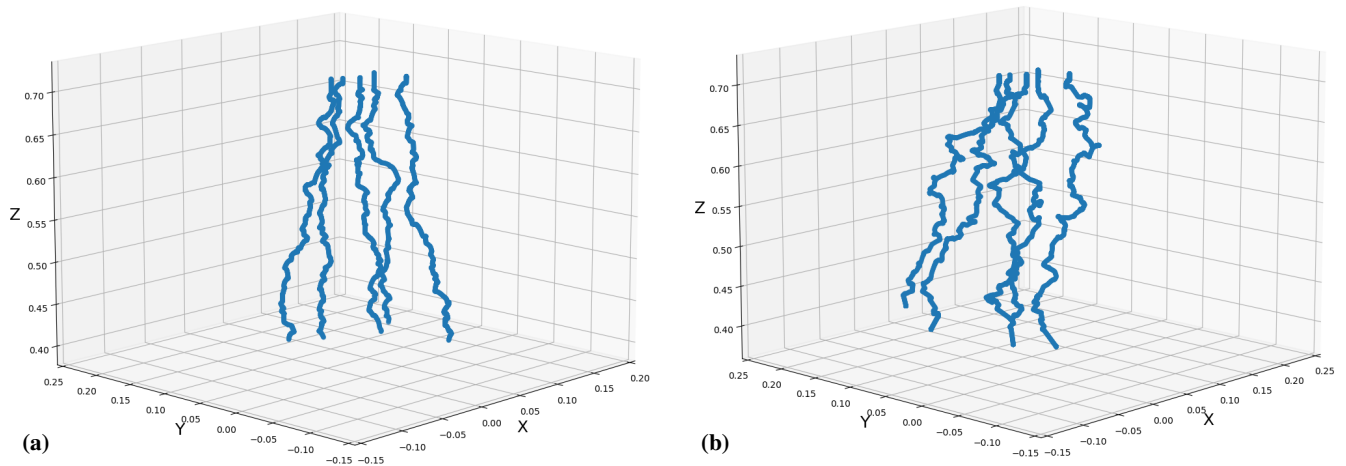


Figure 4 Path trajectory of fine particles in the densest state of NG filter: (a)- dry filtration, (b)-wet filtration

affects the transportation behaviour of fines within the same material.

Table 3 gives the average tortuosity computed for filters NG, IG, and WG in both the loosest and densest cases using fine particle sizes of 1 mm, 0.2 mm, and 0.1 mm respectively. Through dry filtration computation, the average tortuosity is about 1.34 irrespective of grading and porosity. In fact, when fine particles encounter a filter particle on their pathway, they are bound to follow the curvature of the filter particle until there is no more contact. Then, they resume falling straight until they encounter another filter particle. Since filters only involve spherical particles, the filter particles hold the same curvature, regardless of the filter grading or porosity, which explains why there is the same average tortuosity for all studied filters.

The results obtained for the wet filtration showed a different feature, revealing a consistently higher average tortuosity compared to the dry filtration model. The values are indeed 10% to 16% larger in this case than in the case of dry filtration. A higher tortuosity indicates that not all particles took direct pathways but explored sideways, increasing their average path length in the filter. It may be attributed to the influence of dragging forces that align fine particles with the direction of maximum fluxes between local pores, inciting them to explore sideways. As a consequence, path lengths are bound to be larger than those found in dry filtration tests. Figure 4 shows the path trajectory of the fine particles under dry and wet filtration models in the densest state of NG filter. It is clear that in wet filtration, the trajectory of the particles is more tortuous than what can be observed in dry filtration.

The results from wet filtration also indicated that the average tortuosity is greater in IG and NG materials than in WG materials. It may indicate that fine particles explore more sideways in NG and IG materials than in WG materials. Reboul *et al.*⁴³ and Seblany *et al.*⁴⁴ showed that the probability density function (PDF) of construction sizes in NG materials is more spread

Table 3 Average tortuosity for NG, IG WG filters in the loosest and densest state with the dry and wet filtration models.

Material	Size (mm)	Dry	Wet
NG L	1	1.34 ±(0.03)	1.60 ±(0.07)
NG D	1	1.34 ±(0.02)	1.60 ±(0.06)
IG L	0.2	1.34 ±(0.04)	1.61 ±(0.09)
IG D	0.2	1.34 ±(0.03)	1.59 ±(0.07)
WG L	0.1	1.35 ±(0.05)	1.51 ±(0.09)
WG D	0.1	1.34 ±(0.05)	1.48 ±(0.08)

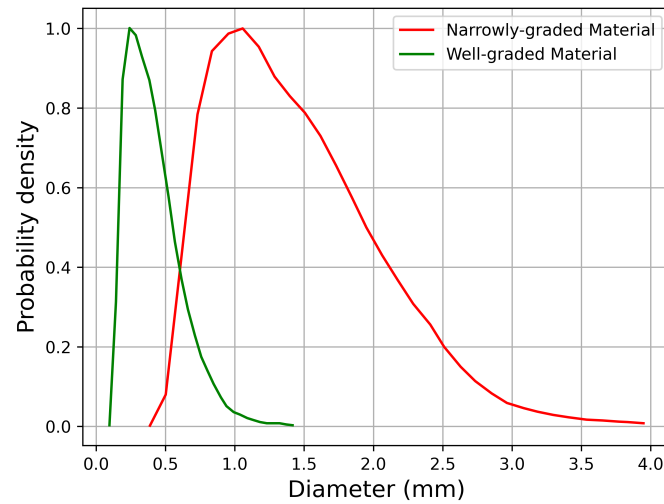


Figure 5 Probability density function of constriction sizes for a narrowly-graded and a well-graded material in a loose state⁴⁴.

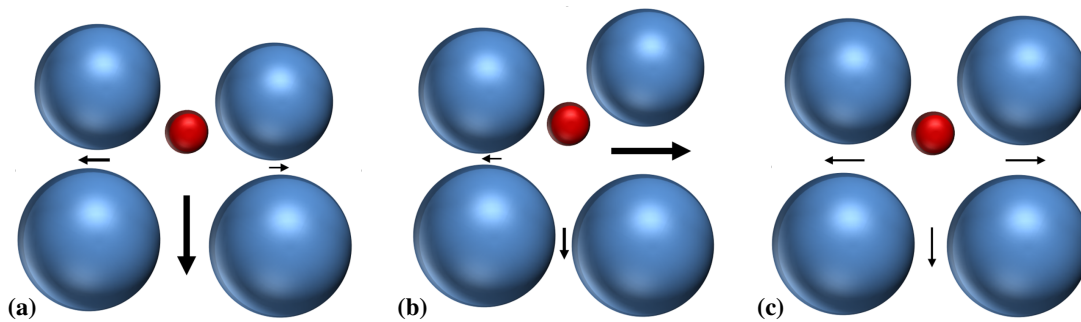


Figure 6 Effect of heterogeneity in constriction sizes on the values of local dragging fluxes at pore exits: (a) - constrictions with very different sizes resulting in a high flux parallel to the direction of the flow, (b) - constrictions with very different sizes resulting in a high flux perpendicular to the direction of the flow (c) - constrictions with similar sizes

than in well-graded materials. In Figure 5, this PDF is given for narrowly-graded material and well-graded materials with a coefficient of uniformity equal to 1.7 and 3.9, both in a loose state⁴⁴. The larger difference in constriction sizes within the same pore in NG materials may result in a greater variety of values for the fluxes exiting a pore and thus for the values of dragging forces (Figure 6 (a-b)). This may induce more opportunities for fines to explore sideways in NG materials when the maximal local flux is in the direction of the sideways (Figure 6-b). In WG material, the contrast between constriction sizes of a given pore may be smaller, providing fine particles with less chance to deviate from the average direction of the fluid flow (Figure 6-c).

4.2 | Coefficient of retention

The study investigated the retention capacity of different classes of fine particles by six numerical filters using dry, wet, and cyclic wet filtration models. Herein, the coefficient of retention is defined as the ratio of the number of fine particles retained by the filter to the initial number of fine particles released at the top of the filter. It is associated to the capability of the filter to capture a given class of fine particles. Comparing the wet and dry models, Figure 7 reveals a difference between the two models irrespective of grading and porosity. It seems that the filter is less efficient at stopping fines in wet filtration than in dry filtration. This difference can be attributed to the reorientation of some fines through constrictions corresponding to a condition of local

maximum flux in the wet filtration model. These particles deviate from a direct path using sideways with larger constrictions, which increases the opportunity for them to go ahead in the filter.

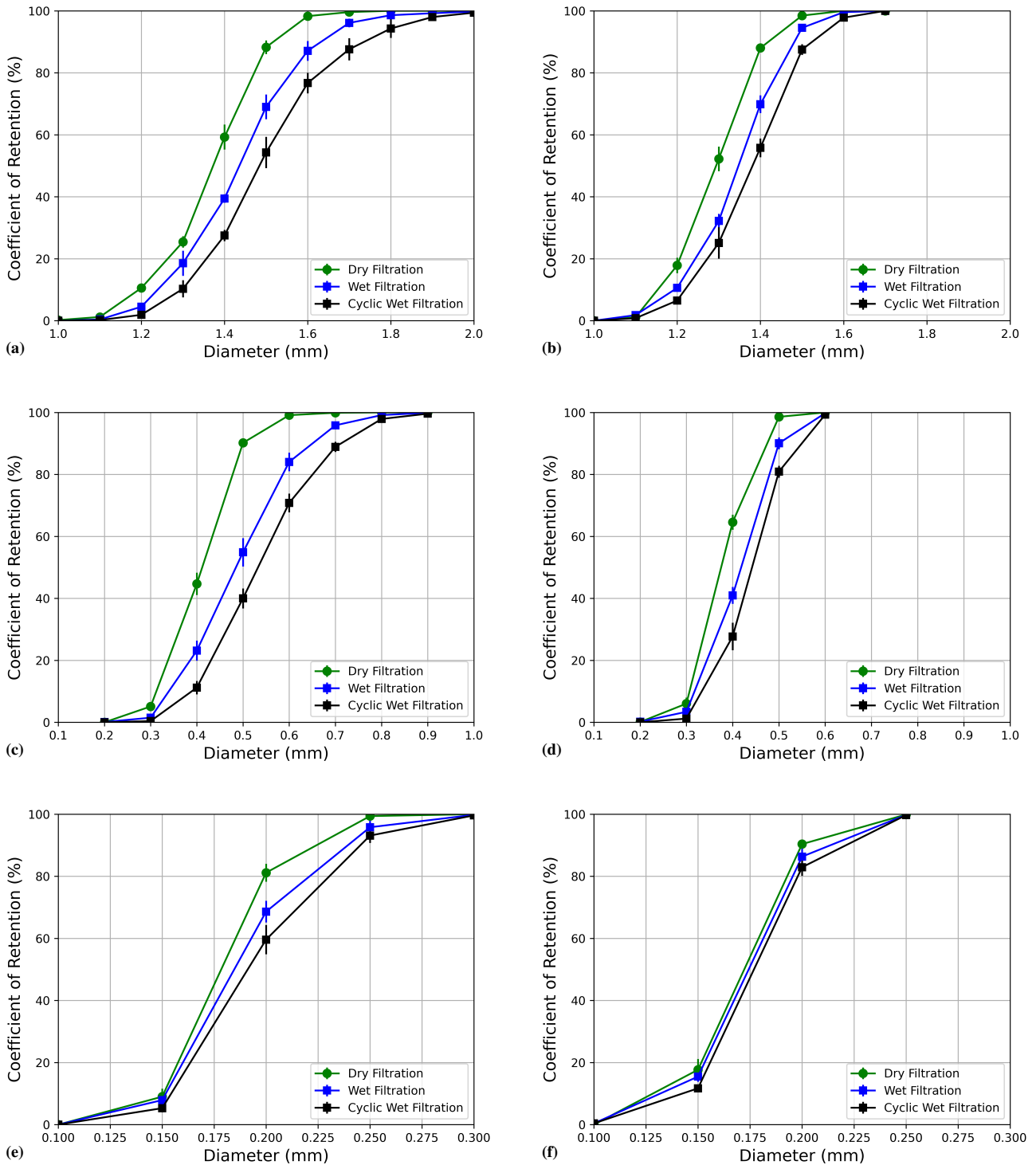


Figure 7 Coefficient of retention of fine particles in the filters according to their size and the kind of filtration model: (a-b): NG, (c-d): IG, (e-f): WG at loosest (a-c-e) and densest (b-d-f) states.

However, comparing NG and WG grading, one can note that in the latter case, there is little difference between dry and wet filtration in terms of retention whatever the particle size released in the filter. As mentioned before, in dry filtration, direct paths (parallel to gravity direction) throughout the filter are always favoured by fines. In the case of wet filtration, since WG materials hold a pore network that is more uniform than NG materials, there is not a great contrast between the constriction sizes of a given pore and thus the direct path (parallel to the average direction of flow) is also privileged in this case. In the case of NG grading, the contrast is greater and more fines are deviated towards sideways according to the condition of local maximum flux that involves larger constrictions. It results in a greater gap between the results provided by dry and wet filtration.

Porosity also influences the gap between the retention factors found in dry and wet filtrations. As density increases, the number of constrictions per pore increases, and as a consequence, the void space of the material becomes more connected²³. Then, it is easier for a fine particle of a given size to find a way across the filter with a movement close to the average direction of the fluid flow as in dry filtration. It explains why the difference in the retention factor between dry and wet filtration decreases when density increases.

The study also revealed that, regardless of the porosity, a similar value for the percolation size (i.e., the largest particle size associated with a 100% probability of completely passing through the filter) is observed for the same material type. This characteristic is supposed to also be equal to the minimum constriction size of the filter since fines smaller than its size have a 100% probability of crossing a filter irrespective of its thickness. For the three materials investigated in the study, the theoretical minimum constriction sizes were estimated to be 0.85 mm, 0.19 mm, and 0.1 mm for NG, IG, and WG materials, respectively. These values were found to be in close agreement with the obtained percolation sizes (Table 3).

However, when attempting to determine the equivalent opening size of the filter that corresponds to the size of fines with a 100% chance of being blocked during filtration tests (other choices can be taken for the percentage associated with the criterion of blockage²⁶), the dry filtration model always leads to a smaller value than the wet filtration model, especially for NG and IG materials (Figure 7(a-d)). One must remind that, in general, the wet filtration model always incites fine particles to select more open pathways than in dry filtration and the filter is envisioned as a less efficient filter. Then, it indicates that relying solely on dry filtration outcomes to design filter criteria may not be conservative, especially when dealing with IG and NG materials. In such cases, the chosen equivalent opening size may underestimate the size of larger fine particles that can entirely cross the filter. However, for WG materials, dry filtration tests may be sufficient to determine the filter opening size (Figure 7 (e-f)).

The cyclic wet filtration model enabled additional possibilities for fine particles to go ahead in the filter, in the range of explored fine sizes (Figure 7 (a-f)). In fact, some particles that were initially blocked at a constriction site in the wet filtration model seemed to have been relaunched into the filter and were bound to explore sideways in the case of the cyclic wet filtration model.

It seems that the existence of sideways paths associated with a local maximum flux (compared to other exits) does not always guarantee the migration of fine particles through the associated constriction. In reality, fine particles have an initial velocity and a dragging force acquired from their past trajectory when they enter a pore. If the value of the local dragging force at stake in a pore towards sideways is insufficiently high, the fine particle may be incited to choose the direct path with a constriction size smaller than its size instead, leading to its blockage. In cyclic wet filtration, fine particles are propelled towards the centre of the pore and subsequently follow the path of the highest dragging force from an initial zero velocity state. It enables fine particles to explore sideways more efficiently and to potentially follow a different path than in a previous cycle.

Regarding the difference in filtration mechanisms depending on porosity, our simulations demonstrate that cyclic filtration is more effective in releasing fine particles for loose filters compared to dense ones. This aspect is consistent with what was previously mentioned: a less-connected void space in loose material results in fewer pore exits, which limits the opportunities for particles to exit. Cyclic filtration provides these particles with more opportunities to exit pores by exploring sideways. In contrast, dense filters have more exits per pore, providing fine particles with greater opportunities to leave a pore through wet filtration. Therefore, cyclic wet filtration provides limited additional benefits for dense filters compared to loose filters.

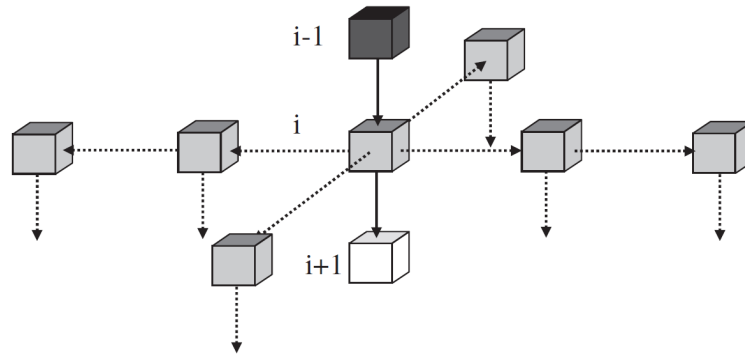


Figure 8 Cubic pore-network model³².

5 | EQUIVALENT CYCLIC WET FILTRATION

5.1 | Framework

Dry filtration tests have emerged as a cost-effective alternative to computationally expensive wet filtration models for studying the migration of fines in granular filters. However, our analysis revealed that the underlying physical phenomena involved in the simulations are very different in nature. For example, in dry filtration tests, the movement of fine particles in a filter is mainly driven by gravity forces. As a consequence, their trajectory is mainly parallel to the gravity direction. Fine particles are always bound to choose a direct path to exit a pore which tends to reduce their chance of finding an exit larger than their size. In a wet filtration test (PFV method), the condition of local maximal fluxes related to the existence of larger constrictions connecting adjacent pores seems to incite fine particles to also explore sideways to exit a pore. Then, in this case, more chances are given to fine particles to go ahead in the filter.

This study introduces a novel filtration model, called the equivalent cyclic wet filtration model (which is, in fact, an enhanced dry filtration), which overcomes the limitations of existing numerical dry filtration models. This model combines the advantages of both wet and dry filtration models by allowing fine particles in a dry filtration test to explore sideways.

More precisely, the proposed model is based on some features associated with the pore network model originally proposed by Schuler³². In this model, the pore space is envisioned as a regular network of void cubes, where each pore body is connected to others by constrictions on each face (Figure 8). When a fine particle enters a pore, it can exit the pore along a unidirectional path (direct path) or sideways (towards lateral pores on the same layer) through one of four possible exits. Building upon this framework, the proposed dry filtration model incorporates a combination of both gravity force and imposed sideways movements to enable fine particles to explore sideways as a possible successful exit.

To simplify the model, the constrictions are assumed to be centred on the cube face. Then, on average, a fine particle located at the centre of a pore can reach a sideways if it crosses a distance equal to half the average pore diameter. This assumption will be used hereafter to define the average sideways movements that are imposed on trapped fine particles. The computation of the average pore diameter for a filter composed of spherical material is based on equation (1).

The proposed methodology for the filtration test is decomposed into cycles. Each cycle has two different phases. Throughout the first phase, fine particles are gradually released under gravity until a steady state is obtained for their position (either passing through the filter or becoming trapped inside it). The penetration depth of each fine particle in the filter at the end of this first phase is computed. In the second phase, gravity is set to zero, and a movement towards sideways equal to half of the mean pore diameter is imposed on trapped fine particles with a random orientation. This movement is enforced by setting gravity in a random direction perpendicular to the natural one for a defined period of time to potentially allow blocked fine particles to move a distance equal to half of the mean pore diameter. At the end of the imposed movement, gravity's direction is set again to its natural direction, and the velocity of the fine particles is set to zero. Figure 9 shows a sketch of a trapped fine particle being

released after the imposed sideways movement.

In the next cycle, initially trapped particles that successfully exited a pore through a sideways migrate again ahead in the filter until further blockage occurs and a new steady state for the positions of all fine particles is obtained. These cycles are repeated until a stop criterion is achieved, which is defined based on the difference between the penetration depth of each blocked particle throughout two consecutive cycles. When this difference is less than half of the mean pore diameter for each blocked particle, the simulation is stopped. It means that no fine particle is able to leave the pore they have been trapped in.

5.2 | Validation

The proposed model was designed to address the high computation time associated with coupled DEM-fluid dynamics models in the context of the filtration of fine particles by granular filters. It is based on a dry filtration model, but also involves a combination of cycles where sideways imposed movements were applied to fines to give them the opportunity to explore sideways to exit pores. The number of cycles required to reach the stop criterion in cyclic filtration was not significantly influenced by the grading or porosity of the filter material. Instead, the primary factor affecting the number of cycles was the number of fine particles initially blocked in the filter. The maximum number of cycles observed in our study was found equal to about 30 cycles, which was observed for the class of fine particle diameters that were slightly smaller than the equivalent opening size of the filter.

Figure 10 (a-f) presents the results obtained from the new proposed model, denoted “Equivalent cyclic wet filtration model”, in comparison with the reference model it constitutes a substitute for, the cyclic wet filtration model. The findings indicate that the new model significantly narrows the disparity found between the dry filtration model and the cyclic wet filtration model across all materials and densities. For instance, with the loose NG sample, the disparity between the dry and cyclic wet filtration tests was nearly 30% ((Figure 7 (a)). However, this gap was diminished to roughly 5% using the newly proposed model. A similar trend was observed with the other samples as well. While the relative difference between the new model and the cyclic wet filtration varies between samples in the range of 0 to 15% based on the size of the fines, the new model consistently matches the equivalent opening size derived from the cyclic wet filtration model.

One must note that the equivalent cyclic wet filtration model has limitations as it only considers a few aspects of fluid flow through granular materials and the interaction between fluid and flowing particles. The filtration mechanism depends mainly on the value of the imposed lateral movement on the fine particles. Indeed, some fine particles are trapped in pores with a diameter larger than the imposed lateral movement, thus they will not be able to exit through the sideways. However, imposing high lateral movement may cause fine particles to exit the pore through a lateral exit and continue migrating laterally, which is unrealistic. To address this issue, we correlated the value of the imposed lateral movement with a physical parameter that has a tangible meaning such as the average pore diameter. It’s important to note that the empirical equation (eq. 1) used to estimate the average pore diameter is based on a probabilistic approach, which can result in either an overestimation or an underestimation of the

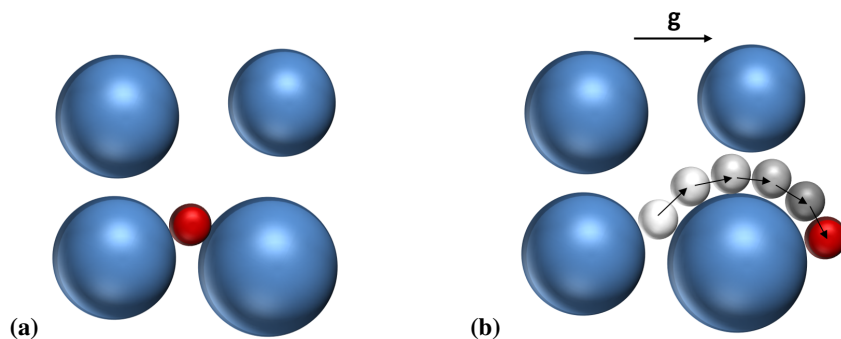


Figure 9 Schematic drawing of a trapped fine particle being released by the imposed sideways movement in the equivalent cyclic wet filtration model: (a) - blocked particle, (b) - imposed sideways movement

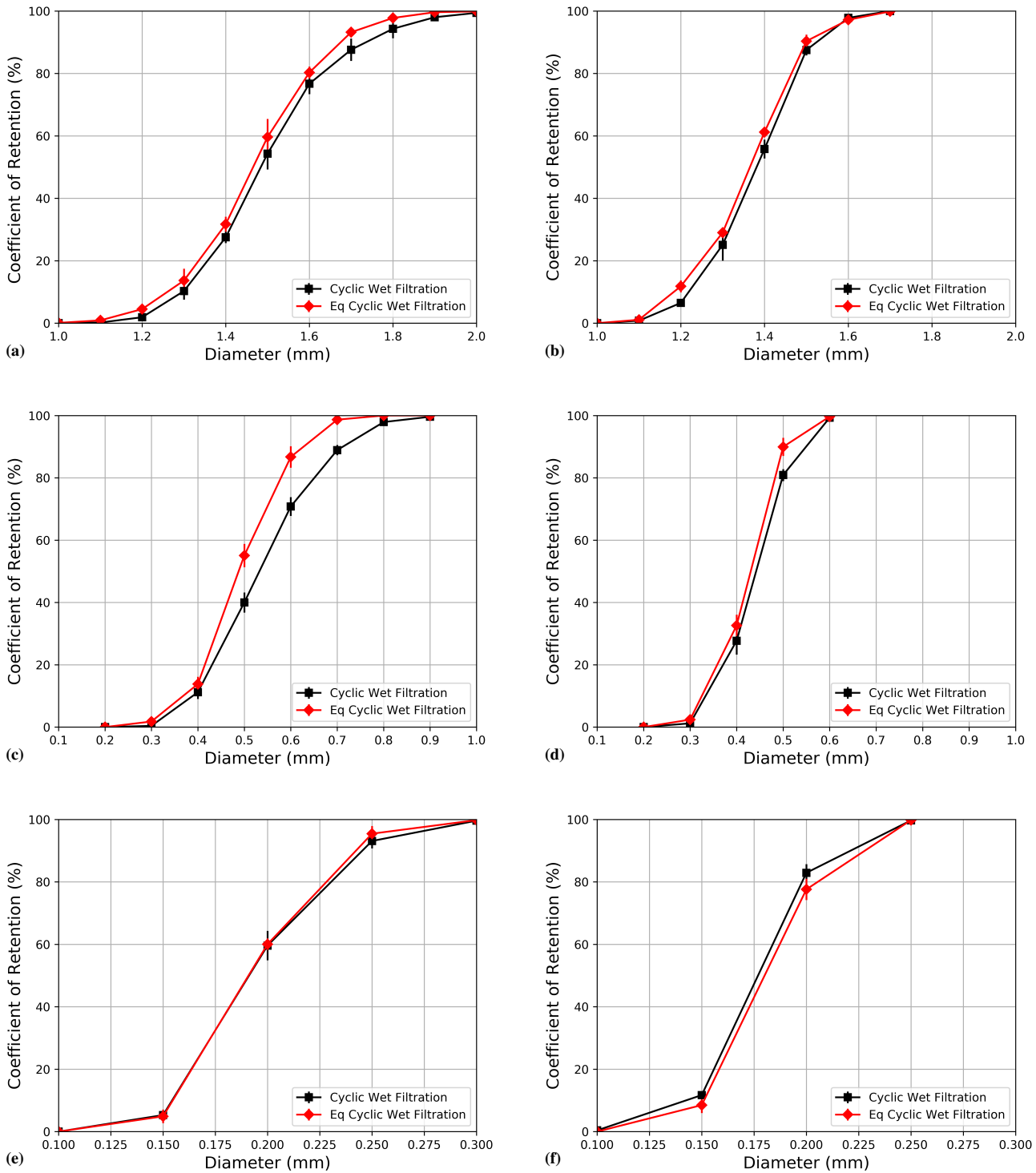


Figure 10 Coefficient of retention of fine particles against their size in the filtration tests under the new proposed model “Equivalent cyclic wet filtration model”, and the reference model “cyclic wet filtration” : (a-b): NG, (c-d): IG, (e-f): WG at loosest (a-c-e) and densest (b-d-f) states.

actual value. As a result, this may account for the fluctuation in results observed between wet and cyclic wet filtration curves.

The equivalent cyclic wet filtration model holds definitive advantages for quantifying the retention characteristics of a granular filter. First, it successfully retrieved the equivalent filter sieve opening size, which is a crucial property of the material for the design of a filter. Secondly, it allows faster computations than wet filtration in spite of the involved cycles, taking five to fifteen times less time. It is even more efficient when compared to the cyclic wet filtration model. For instance, in the case of WG sample, an equivalent cyclic wet filtration test can complete a filtration test in 12 hours, while a wet (non-cyclic) filtration test takes 103 hours. Compared to the PFV model, which is a simplified tool and faster than traditional CFD models for simulating wet filtration, the proposed model offers greater flexibility as it can be applied to any particle shape as long as the average pore diameter of the granular filter is known.

6 | CONCLUSION

Researchers have extensively used the discrete element method to explore and quantify the filtration mechanisms in granular filters. Two common approaches are generally chosen to model filtration tests including wet filtration models that couple DEM with fluid dynamic models and dry filtration models that apply gravity as the driving force for the movement of suspended particles. The latter approach is preferred because it circumvents the complexity associated with dynamic fluid models and has a favourable computation time. This paper aims to present a comprehensive numerical analysis that quantitatively compares these two approaches by conducting filtration tests on six distinct numerical filters involving various materials and porosities.

The filtration tests conducted using both wet and dry filtration models demonstrated that the former was associated with a higher tortuosity and a lower coefficient of retention of fine particles, compared to the latter. It was mainly due to the existence of dragging forces that aligned the fine particles with the direction of maximum fluxes between local pores, causing them to explore sideways alongside direct paths. This possibility is not given by the dry filtration model, where only pathways close to the gravity direction can be explored. Herein, different features were obtained according to the grading and porosity of the filters. For narrowly-graded materials, the larger heterogeneities in terms of constriction sizes created more opportunities for fines to explore sideways in the wet filtration model, leading to a higher difference in tortuosity and coefficient of retention compared to dry filtration. On the other hand, for dense materials, the high connectivity of the pore space allowed the fine particles to have equal chances of exiting the pores from all directions, resulting in reduced differences between the wet and dry filtration models in terms of coefficient of retention.

Regarding the efficiency of the filter in completely trapping a specific size of fine particles, the dry filtration model led to a smaller value for the equivalent opening size of the filter compared to the wet filtration model. The findings suggest that using only dry filtration results to establish filter criteria where this geometrical characteristic holds a key role may not be conservative, especially when dealing with narrowly-graded materials. However, for well-graded materials, conducting dry filtration tests alone could be adequate for determining the filter opening size.

Additionally, this investigation included cyclic wet filtration tests to examine how perturbations in the hydraulic head affect the stability of trapped particles in the filter. The simulations revealed that cycles allowed the initially trapped fine particles to migrate more through the filter. It means that in the wet filtration model, fines did not necessarily choose the larger constrictions in an attempt to exit a pore. It demonstrated that in the PFV model, there may exist a competition between the effects of the conditions of local maximum fluxes and the velocity vector of fine particles entering a pore on their dynamic movements. It may either lead fine particles to favour a direct path but with a constriction smaller than their size or to choose a sideways path.

Overall, this study provided valuable insights into the filtration mechanism in granular filters by comparing the wet and dry filtration models and investigating the effects of grading, porosity, and the perturbation of the hydraulic head effects on particle retention. The results highlighted the importance of dragging forces in wet filtration tests, especially when dealing with narrowly-graded materials but also granular filters in a loose state.

Finally, this study presented an enhanced dry filtration model called the "equivalent cyclic wet filtration model" that addresses the limitations of dry filtration tests as a model for wet (cyclic) filtration tests. This model, which allowed sideways exploration,

successfully retrieved the value for the equivalent filter opening size. Furthermore, the equivalent cyclic wet filtration model allowed significant gains in computation time. Highly flexible, it can be used irrespective of the involved particle shapes as long as the average pore diameter of the granular filter can be estimated.

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