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Feda Seblany, Eric Vincens, Christophe Picault. Simplified Estimation of Some Main Characteristics of Pores and Constrictions in Granular Materials. Internal Erosion in Earthdams, Dikes and Levees, 17, Springer International Publishing, pp.189-199, 2019, Lecture Notes in Civil Engineering, <10.1007/978-3-319-99423-9\_18>. <hal-04806712>

**HAL Id: hal-04806712**

**<https://hal.science/hal-04806712v1>**

Submitted on 27 Nov 2024

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# Simplified estimation of some main characteristics of pores and constrictions in granular material

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**Abstract:** The void space of granular materials can be considered as a collection of poly-sized pore bodies separated by narrow pore throats or constrictions. Pore network models have been developed to estimate the probable path length covered by fine particles flowing through a granular filter. For this calculation two pieces of information are required: the constriction size distribution and the mean void spacing between two constrictions which corresponds to the mean pore diameter. Different assumptions have been previously made in the literature to determine this void spacing. However, they all neglect the influence of the density and thus, the estimation of this quantity remains an open issue. This paper compares different definitions for the mean pore size based on statistical analyses performed on numerical samples composed of spheres by means of the Discrete Element Method. Different sphere packings with different gradings and density states were considered and a weighted Delaunay tessellation was applied to extract the main void characteristics. In a second part, a simple formula is proposed to quickly estimate the equivalent sieve opening size of the granular filter. This value is very close to the mode value of the constriction size distribution which is related to the most represented constriction size in a granular material.

**Keywords:** Void space, Spheres, Filter, Mean pore size.

## 1. Introduction

It is now well established that a better insight into the problems of internal erosion in earth structures can be provided by investigating the void space within granular materials and more specifically by determining their so called constriction size distribution (Silveira, 1965; Wittmann, 1979; Kenney et al., 1985; Witt, 1986; Soria et al., 1993).

Indeed, within a granular material, constrictions are the narrowest channels be-

tween the pores and, therefore, represent the main obstacles to overcome for a fine particle flowing under seepage through the filter.

Since constrictions govern the filtration performance of granular soils, different methods have been proposed to obtain the constriction size distribution (CSD) based on an analysis of the void space. Among these methods, we can cite: experimental techniques (Witt, 1986; Soria et al., 1993), numerical approaches (Reboul et al., 2008; Homberg et al., 2012) and analytical models (Silveira et al., 1975; Locke et al., 2001; Reboul et al., 2010; O'Sullivan et al., 2015). Vincens et al. (2015) provided a detailed review of these methods.

The CSD has been at the basis of new filter criteria for earth dams (Witt, 1993; Indraratna et al., 2007). Moreover, constrictions are increasingly being used in models to predict the transport of fine particles through filters (Locke et al., 2001; Sjah and Vincens, 2013). It has also been involved in the study of materials prone to suffusion (Scheuermann et al., 2010; To et al., 2015).

The CSD is intimately related to pore network models which idealise the void space in porous media by a lattice of pore bodies linked by pore throats following a respective size distribution. Such models have been widely used to simulate and understand the transport of particles in soils.

For example, Silveira (1965) and Kenney et al. (1985) proposed a multi-layered constriction network where fine particles can move from one layer to another if their size are smaller than the associated constriction sizes. Following Schuler (1996), many studies considered a regular cubic model where each pore is modelled by a void cube having six exits (constrictions), one on each cube face (Locke et al., 2001; Sjah and Vincens, 2013). In the case of a unidirectional movement of fine particles, the movement from one layer to another layer is directly related to the probability  $P$  of occurrence of a constriction (between two consecutive layers) smaller than the particle diameter. Therefore, the probability of passing  $n$  layers for a fine particle of a given diameter is equal to  $P^n$ . The probability  $P'$  to be captured after a path corresponding to  $n$  layers is equal to  $1-P^n$ . Consequently,  $n$  is given by  $\log(1-P^n)/\log(P)$ .

However, if one needs either to compute the CSD from the probabilistic models (experimental technique) or to estimate the maximal distance  $L$  travelled by fine particles through a filter using these models, a statement for the unit layer size  $s$  must be taken. Indeed,  $L$  is equal to  $n.s$ .

In the past, different estimates for this unit layer size which is generally associated to the mean pore size have been proposed but the authors did not always provide a clear justification for them or the influence of density was not taken into account. Since the characteristics of the void space including pores and constrictions are highly related to both the grading and the density of the filter, a relevant estimate for the void spacing should involve this information.

This paper compares different estimates for the mean pore size with statistical values extracted from numerical samples for different types of gradings at different density states. For this purpose, the Discrete Element Method (DEM) was used to generate packings of spheres, and a weighted Delaunay tessellation is applied to

discretize the whole space into tetrahedra. On the basis of this partition, the main void characteristics including the size of constrictions located on each tetrahedron face and the inscribed void sphere between the four particles vertices of each tetrahedron are deduced (Al-Raoush et al., 2003; Reboul et al., 2008).

In a second part, a quick estimate of the equivalent sieve opening size on the base of the CSD is proposed. This property that may be used for the design of filter criteria discriminates fine particles able to travel large distances through the filter from those that will be trapped after a rather short distance.

## 2. Samples of numerical spheres

Numerical packings were prepared by means of DEM-Yade (Šmilauer et al., 2010). Samples composed of spheres are generated by deposit under gravity within a box. Periodic boundary conditions are imposed on the lateral sides of the samples. The contact between spheres is ruled by an elastic-frictional model with a normal and a tangential stiffness ( $K_n$  and  $K_t$ ) and by a Coulomb friction angle ( $\phi$ ). The final equilibrium state is supposed to be reached when the unbalanced forces become negligible.

Table 1 summarizes the characteristics of the five samples studied herein. Different types of soils (Uniformly Graded, Widely Graded and Gap Graded) are tested. In addition to the materials studied by Reboul et al. (2010), samples with a higher coefficient of uniformity ( $C_u$ ) were tested. The labels of samples indicate the material type (for example, UG for uniformly graded material).

For each sample, two density states are evaluated (L and D denoting Loose and Dense respectively). For the loosest samples, the coefficient for inter-particles friction is set to 0.3 (0.15 for UG material) in order to reach the maximum void ratio obtained by experiments on similar actual materials (Biarez and Hicher, 1994). The densest samples were obtained by setting the friction value at contact to zero.

For these numerical samples, pores and constrictions were identified using a weighted Delaunay tessellation. The void space is then partitioned into tetrahedra whose vertices are formed by the centres of spherical particles.

**Table 1: Characteristics of DEM samples**

Sample	Number of particles	D <sub>0</sub> -D <sub>100</sub> (mm)	Coefficient of uniformity ( $C_u$ )	Maximum void ratio ( $e_{max}$ )	Minimum void ratio ( $e_{min}$ )
UG	2000	3-12	1.7	0.61	0.52
WG1	25000	0.7-10	3.9	0.49	0.33
GG1	25000	0.7-10	3.6	0.52	0.33
WG2	40000	0.7-14	6	0.41	0.27
GG2	40000	0.7-14	6	0.43	0.27

The pore sizes can be estimated by means of the maximum void sphere inscribed in each tetrahedron, whereas the constriction sizes are computed by considering the maximum inscribed disc that can fit between the three particles defining each face in the tetrahedral partition. This direct computation is referred to level 0 ( $L_0$ ) (Reboul et al., 2008).

According to Al-Raoush et al. (2003), this partition leads to an over-segmentation of the void space and thus, a pore merging must be applied. In this study, a level 1 ( $L_1$ ) of merging is adopted (Reboul et al., 2008, Seblany et al., 2018). A merging applies when the inscribed void spheres of two adjacent tetrahedra overlap.

### 3. Mean pore size

In this section, we give a brief summary of the previous estimates of the mean pore size considered in the literature, and we evaluate these assumptions by performing a statistical analysis over DEM samples.

#### 3.1 Previous estimates

Since actual granular soils are composed of polydisperse and mostly disordered particles and since direct measurements are difficult to carry out, estimating the mean pore size is not an easy task.

Witt (1986) deduced the mean pore size by defining the mean pore volume which is equal to the porosity divided by the mean grain volume. Silveira (1965) and Soria et al. (1993) used  $D_{50}$  by mass ( $D_{50M}$ ), i.e. the diameter corresponding to 50% finer by weight. Locke et al. (2001) proposed to take the mean diameter  $D_{50}$  by number ( $D_{50N}$ ). Referring to Indraratna et al. (2007), the  $D_{50}$  of the PSD by lateral surface area ( $D_{50SA}$ ) is adopted. Such values can be derived from the filter particle size distribution (PSD) by mass, by number or by surface area.

According to Humes et al. (1996), the PSD by mass may overestimate the role of coarser particles in the organisation of the void space, and conversely, the PSD by number may overestimate the role of the finer particles. The use of the PSD by surface area can be considered as the best compromise between these two choices. However, all these estimates were proposed without a clear proof of their validity or have been validated only for the tested soils at their densest state.

Sjah and Vincens (2013) showed that the diameter of the mean void sphere inscribed within the four particles forming the tetrahedra in a Delaunay partition can give a good estimate of the mean pore size. However, their study was restricted to a uniformly-graded granular filter at its loosest state and could not give a general

and convenient framework for the study of actual granular materials.

To provide a better estimate of the mean pore size  $s$  for an assembly of spheres at a given density, Wu et al. (2012) used a simplified equation stating that any change of the void ratio can be associated to a variation of the diameter of the mean inscribed void sphere:

$$s = \sqrt[3]{\frac{e}{e_{\max}} s_{\max}} \quad (1)$$

where  $e$  and  $e_{\max}$  are the actual and maximum void ratio of the filter respectively.  $s_{\max}$  is the mean pore size at the loosest state for the material and can be computed following an analytical methodology similar to the one used for the determination of the CSD (Locke et al., 2001). In this case,  $s_{\max} \approx 0.5D_{50SA}$ .

### 3.2 Statistics over DEM samples

In order to select the best estimate for the mean pore diameter from previous proposals, a comparison has been established with statistics over DEM samples including the mean inscribed void sphere for both loose and dense samples. They will be compared to the mean constriction spacing which may better reflect the distance crossed by a fine particle within a mean local pore. Knowing the number of constrictions in each sample, a simple estimate of the constriction spacing can be given by the mean distance between centers of adjacent pores computed from the initial statistics over the entire sample (non-directional approach) (Seblany et al., 2017).

Figure 1 shows the mean pore size computed from different approximations normalized by  $D_{50M}$  for both loose (Fig.1a) and dense states (Fig.1b). As  $C_u$  increases, the mean void spacing ( $L_0$  computation is considered herein) decreases which is in agreement with results found by Reboul (2008). The influence of density is less pronounced than that of grading which was expected (Reboul, 2008).

Secondly, the proposal by Sjah and Vincens (2013) to use the mean inscribed void sphere as an estimate of the mean pore spacing seems to be validated in this study since it led to the closest value to this latter. The other ones ( $D_{50N}$ ,  $D_{50SA}$ ,  $D_{50M}$ ) overestimate the mean pore spacing. In Figure 1,  $D_{50M}$  is systematically equal to 1 due to the chosen normalisation process. This result agrees with a recent work by Taylor et al. (2018) who estimate the constriction spacing by considering the constriction density in micro computed tomography images and DEM samples.

Finally, the formula (Eq. (1)) proposed by Wu et al. (2012) which allows the density to be taken into account is in good agreement with the mean pore spacing from DEM samples for both loose and dense samples.

For lower  $C_u$  values, particularly in the case of UG, the discrepancy between the mean pore size using Eq. (1) and the mean pore spacing from DEM samples

can be due to the imposed void ratio in this case. Indeed, in order to match the actual void ratio from experiments by Biarez and Hicher (1994), a lower friction value is used for generating the loosest sample (UGL) contrary to the other loosest samples. The error was found much lower if the same friction value as for the other samples is used to create the loosest state.

Seblany et al. (2017) highlighted that a consistent choice for the mean pore size must be associated to a given choice for the computation of the CSD, since when applying a merging of adjacent tetrahedra (case of  $L_1$  merging), the number of constrictions tends to reduce. As a consequence, the value for the mean constriction spacing tends to increase. In this case, as the number of constrictions decreases by approximately 30% from  $L_0$  to  $L_1$  regardless of the grading and the density state of the sample, a division factor of 0.7 can be suggested (we denote this formula as Equation 1\*).

The estimate of the mean void spacing (labelled  $L_1$ ) resulting from the modified formula (Eq. 1\*) is shown in Figure 2. It seems that the new estimate is close to the mean void spacing derived from DEM data considering  $L_1$  criterion. The lesser quality of the result provided by Equation 1\* in the case of  $C_u$  smaller than 2 arises from the same reason that was previously mentioned for  $L_0$ .

#### 4. Equivalent sieve opening size

It is well established that the CSD is a key factor for understanding the mechanisms of filtration in granular materials. Indeed, within the framework of the probabilistic theory, a flowing fine particle having a diameter greater than the largest constriction size in the filter cannot enter the filter and conversely, if this fine particle is smaller than the smallest constriction size, it can cross the entire filter. This statement is based on the assumption of a constant hydraulic loading independent from the porous media thickness.

Kenney et al. (1985) introduced the notion of controlling constriction size to design the maximum particle size that can cross the filter. In other words, it corresponds to the smallest constriction size that can be found along a filtration path. Sari et al. (2011) performed some numerical filtration tests and find that the finer part of the CSD curve and more precisely constrictions smaller than the mode are mainly responsible for trapping particles.

In fact, the mode of the constriction size distribution can hold an important physical meaning being the most represented constriction size within the sample. Fine particles smaller than this value can easily cross the filter since the probability of finding constrictions larger than their own sizes, is significant.

Herein, the study does not focus on determining the characteristic constriction size corresponding to the mode of the distribution but rather on a value slightly larger by 5%. It implies that we limit the possible travel distance within the granular filter (herein smaller than forty confrontations with constrictions) which may be

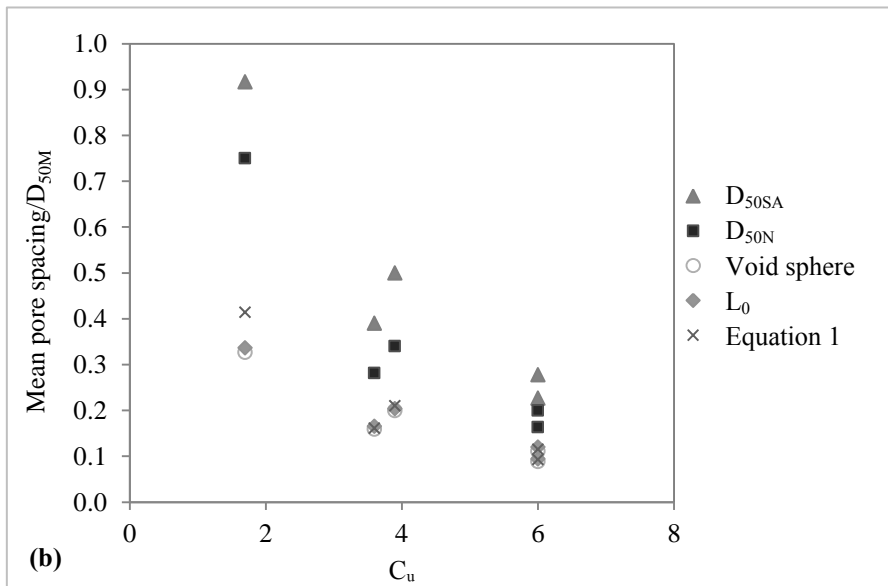
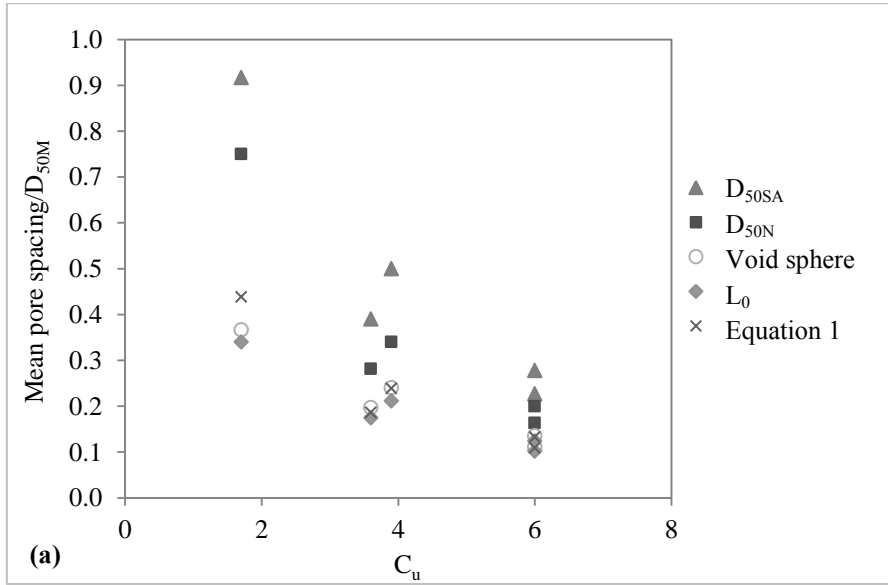
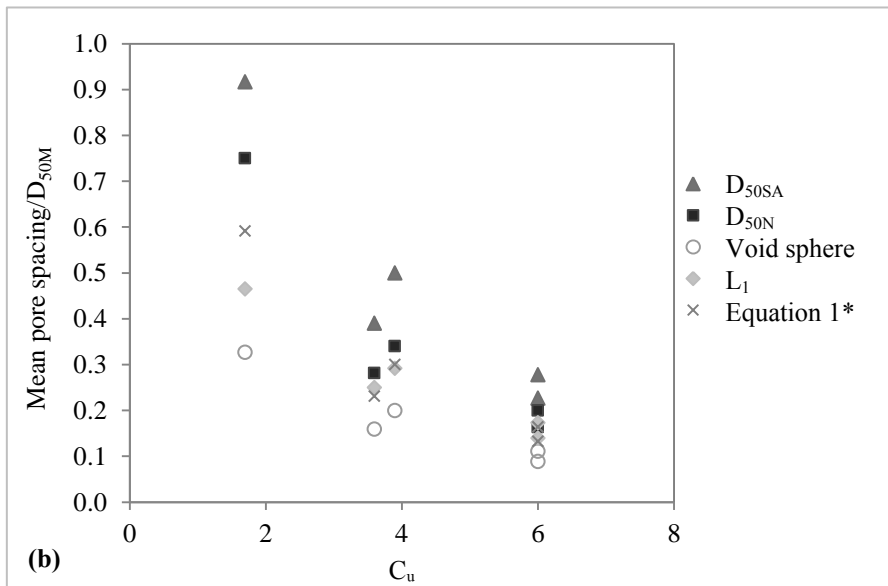
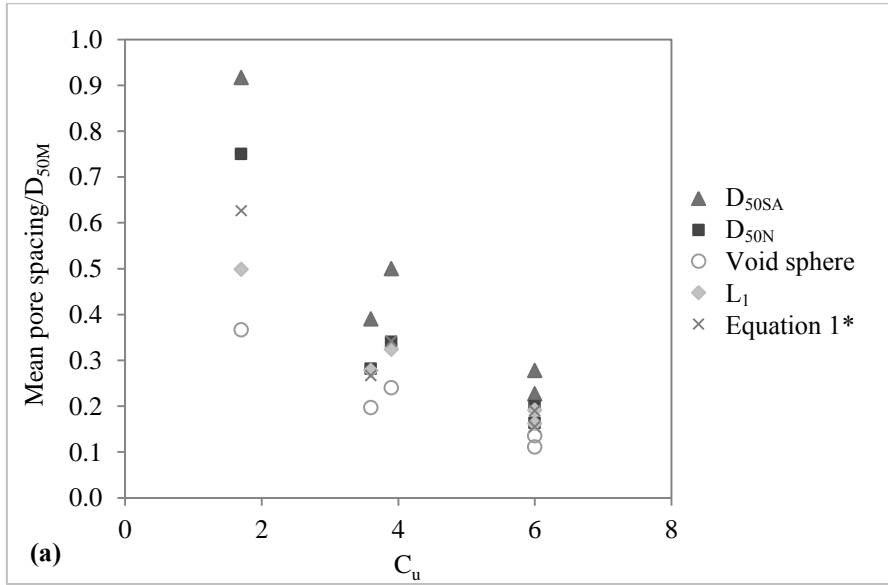


Figure 1: Comparison of the mean pore spacing based on DEM statistics ( $L_0$ ) with estimates proposed in the literature for (a): loose samples; (b): dense samples



**Figure 2: Comparison of the mean pore spacing based on DEM statistics ( $L_1$ ) with estimates proposed in the literature for (a): loose samples; (b): dense samples**

large if the flowing fine particle has a diameter equal to the mode of the CSD. Hereafter, we denote this characteristic the equivalent sieve opening size  $d_{OS}$ .

The knowledge of this value may be then useful to quickly evaluate the efficiency of a filter to retain a fine particle of a given diameter. A quick and general estimate of the equivalent sieve opening size can be given following the method that allowed building analytical models for CSD. Under this guidance, the constrictions within the sample can result from two main geometrical configurations for the solid particles generating constrictions: three mutually tangent spheres (densest state) and four tangent particles (loosest state). The size of the constriction formed by these arrangements ( $d_{c3}$  or  $d_{c4}$ ) corresponds to the diameter of the circle that can fit between the three or four spherical particles of diameter  $D$ .  $d_{c3}$  and  $d_{c4}$  can be computed analytically by using Eq. (2) and Eq. (3) respectively.

$$d_{c3} = \left(\frac{2}{\sqrt{3}} - 1\right)D \quad (2)$$

$$d_{c4} = (\sqrt{2} - 1)D \quad (3)$$

The equivalent sieve opening size, at the loosest state,  $d_{OS,L}$  associated to a cumulative probability  $P$  is eventually given by a weighted contribution of the two configurations within the sample (Locke et al., 2001):

$$d_{OS,L}(P) = Pd_{c4} + (1 - P)d_{c3} \quad (4)$$

For other densities, Reboul et al. (2010) proposed a 1-parameter model validated on the results obtained by the Delaunay approach. For gap-graded materials, they used a more sophisticated model. In this work, the 1-parameter model from Reboul et al. (2010) is considered sufficient since it gave a good match with the dense DEM samples irrespective of the grading (continuous or discontinuous grading). More precisely, the mean relative error was found to be less than 7% which is reasonable.

$$d_{OS}(P, e) = \frac{e}{e_{\max}}(d_{OS,L}(P) - d_{c\min}) + d_{c\min} \quad (5)$$

where  $d_{c\min}$  is the minimum constriction size formed between the three smallest particles of the sample in mutual contact (Eq. (2)).

Since the smaller particles are responsible for the formation of the finer fraction of the CSD curve, a trial and error process is applied to the lower range of the PSD by surface area to find the characteristic particle size that must be involved in these

calculations which allows a direct determination of equivalent sieve opening size. It was found that  $D_{35SA}$  as a characteristic diameter for the grading can give a good estimate of the equivalent sieve opening size for all materials whatever their density state except for GG1 material which represents a special case of gap-graded soils.

In fact, the distribution of constriction sizes for GG1 material exhibits two modal values even for  $L_1$  computation. This bimodal shape is related to a significant contribution of particles larger than the gap size to the CSD. In this special case, it would be better to find the second mode of the constriction size distribution; a larger particle diameter must be then involved in the analytical calculation. In this special case,  $D_{45SA}$  can be considered adequate for the characteristic diameter of the grading.

Table 2 gives the value of the equivalent sieve opening size derived from the numerical distribution of constriction sizes and the calculated value obtained by using Eq. (4) and Eq. (5) for loose and dense states, respectively. One can note that the analytical values are very close to the numerically computed equivalent sieve opening sizes with a relative error of about 5%.

**Table 2: Comparison of numerical and analytical values for the equivalent sieve opening size  $d_{OS}$  (in mm)**

Sample	Numerical value (Loose)	Analytical value (Loose)	Numerical value (Dense)	Analytical value (Dense)
UG	1.14	1.15	1.03	1.04
WG1	0.27	0.26	0.22	0.21
GG1	0.34	0.32	0.28	0.26
WG2	0.27	0.27	0.22	0.21
GG2	0.27	0.26	0.22	0.21

## 5. Conclusion

In this work, we proposed a technique to estimate two main characteristics of the void space within a granular material involving spherical particles. The first one is the mean pore size which is used in pore network models either to compute the CSD or to predict the longest path covered by a flowing fine particle through a granular filter. The second one is the equivalent sieve opening size defined on the basis of the mode of the constriction size distribution which can be used for the design of filter criteria.

The estimates of the mean pore size and of the equivalent sieve opening size are built on main physical properties of the granular material, namely the grading and

the density of the material. The results were validated by means of statistics over DEM samples involving different materials at their loosest or densest state. The proposed estimates were found of very good quality irrespective of the grading and of the density of the sample.

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