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A Preconditioned Inverse Scattering Scheme for High-Contrast Bone Imaging at Microwave Frequencies

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Abstract—This work presents a combined quasi-Helmholtz preconditioning strategy for the contrast source volume integral equation (J-VIE) with a Newton-Kantorovich method allowing 2D microwave imaging for high-contrast scatterers. We show that the preconditioning improves the convergence of the J-VIE, reducing the number of iterations required, even for highly inhomogeneous objects. We demonstrate the practical significance of this preconditioned inverse scattering scheme on a realistic human bone and thigh imaging problem.

I. INTRODUCTION

Microwave imaging (MWI) is a promising technique for imaging biological tissues and offers a low-cost alternative to standard imaging procedures such as magnetic resonance imaging (MRI) and X-ray. When the background medium is already known, a formulation employing volume integral equations (VIE) is advantageous since it allows for the reconstruction of inhomogeneous scatterers without discretizing the background. Among VIE formulations, the contrast source VIE (J-VIE) is the most adopted one since it yields a data equation in the form of an inverse source problem. However, the high permittivity of biological tissues with respect to the background introduces a severe ill-conditioning of the J-VIE and causes a slowdown of iterative solvers, which is often referred to as the high-contrast (HC) breakdown. A regularization technique based on quasi-Helmholtz projectors has been introduced in [1] to rescale the permittivity dependent spectrum of the J-VIE. In this work, we adapt these projectors to a 2D setting with the transverse electric J-VIE (TE J-VIE) and we apply this preconditioner to a Newton-Kantorovich method (NKM) [2] derived for the TE J-VIE. Numerical results will show that these projectors enable handling high-contrast settings and we additionally demonstrate the effectiveness of this novel preconditioned inverse scattering scheme in a realistic bone and thigh imaging problem.

II. NOTATION AND BACKGROUND

Consider a scattering problem in which a transverse electric (TE) time-harmonic incident wave with electric field \mathbf{E}_i and the angular frequency ω is scattered in a nonmagnetic inhomogeneous scatterer with arbitrary cross-sectional shape $\Omega \subset \mathbb{R}^2$ and complex permittivity $\epsilon(\mathbf{r})$ ($\mathbf{r} \in \mathbb{R}^2$) embedded in free-space of permittivity ϵ_0 and permeability μ_0 . The TE contrast source volume integral equation relates the in-plane contrast source \mathbf{J} and \mathbf{E}_i as $\mathbf{J} - \chi(\nabla\nabla + k_0^2) \cdot$

$\mathbf{S}(\mathbf{J}) = -i\omega\epsilon_0\chi\mathbf{E}_i$ where $\mathbf{S}(\mathbf{F})(\mathbf{r}) = \int_{\Omega} G_2(\mathbf{r}, \mathbf{r}')\mathbf{F}(\mathbf{r}') ds'$, $G_2(\mathbf{r}, \mathbf{r}') = i/4H_0^1(\mathbf{r}, \mathbf{r}')$, and k_0 is the wavenumber of \mathbf{E}_i .

We can numerically solve the TE J-VIE by discretizing Ω with N_T triangles having a constant permittivity and expanding the contrast current $\mathbf{J} = \mathbf{J}^x + \mathbf{J}^y$ as $\mathbf{J}^\alpha(\mathbf{r}) \approx \sum_{m=1}^{N_T} [\mathbf{J}_m^\alpha] \mathbf{f}_m^\alpha(\mathbf{r})$ in which $\hat{\alpha} = \hat{x}, \hat{y}$. The basis function \mathbf{f}_m^α , which lives in $L^2(\Omega)^2$, is defined as $\mathbf{f}_m^\alpha(\mathbf{r}) = \hat{\alpha}P_m(\mathbf{r})$ with $P_m \in L^2(\Omega)$ being a patch defined on triangle T_m . Following a Galerkin procedure as in [1], the continuous TE J-VIE is discretized as $(\mathbf{I} - \chi\mathbf{G}_S)\mathbf{J} = -i\omega\epsilon_0\chi\mathbf{E}_i$, where $\mathbf{G}_S \in \mathbb{C}^{2N_T \times 2N_T}$, $\mathbf{I} \in \mathbb{R}^{2N_T \times 2N_T}$, $\mathbf{J} \in \mathbb{C}^{2N_T \times 1}$, $\chi \in \mathbb{C}^{2N_T \times 2N_T}$, and $\mathbf{E}_i \in \mathbb{C}^{2N_T \times 1}$.

We then define the inverse scattering problem based on the discretized state and data equations

$$(\mathbf{I} - \chi\mathbf{G}_S)\mathbf{J} = -i\omega\epsilon_0\chi\mathbf{E}_i \quad (1)$$

$$\mathbf{G}_D\mathbf{J} = -i\omega\epsilon_0\mathbf{E}_s, \quad (2)$$

where $\mathbf{G}_D \in \mathbb{C}^{N_A \times 2N_T}$ is the discretized radiation operator and $\mathbf{E}_s \in \mathbb{C}^{N_A \times 1}$ is the measured scattered field with N_A being the number of independent measurements. The aim of this scattering problem is to estimate χ from the knowledge of \mathbf{E}_i and \mathbf{E}_s .

III. NEWTON-KANTOROVICH METHOD FOR THE PRECONDITIONED J-VIE

The first step of this contribution is the preconditioning of the forward problem. Following a similar approach as in [1], we define projection operators to the solenoidal, irrotational and harmonic subspaces as $\mathbf{P}_\Lambda = \mathbf{\Lambda}(\mathbf{\Lambda}^T\mathbf{\Lambda})^+\mathbf{\Lambda}^T$, $\mathbf{P}_\Sigma = \mathbf{\Sigma}(\mathbf{\Sigma}^T\mathbf{\Sigma})^+\mathbf{\Sigma}^T$, and $\mathbf{P}_H = \mathbf{I} - \mathbf{P}_\Lambda - \mathbf{P}_\Sigma$, respectively, where $^+$ denotes the Moore-Penrose pseudoinverse. The transformation matrices $\mathbf{\Sigma}$ and $\mathbf{\Lambda}$ are constructed from $[\mathbf{\Sigma}^\alpha]_{mn} = \langle \mathbf{f}_m^\alpha, \nabla b_n \rangle$, with b_n a pyramid function and $\langle \mathbf{f}, \mathbf{g} \rangle = \int_{\Omega} \mathbf{f}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{r}) ds$, and $[\mathbf{\Lambda}^\alpha]_{mn} = \langle \mathbf{f}_m^\alpha, \hat{z} \times \nabla b_n \rangle$, which are the transformation matrices from irrotational and solenoidal functions to the piecewise constant basis functions $\mathbf{f}_m^\alpha(\mathbf{r})$.

In the framework of the inverse scattering problems the imaging object is in general inhomogeneous and the details of its internal structure are unknown. Therefore we build a preconditioning operator as in [1], using instead the average relative permittivity $\tilde{\epsilon}$

$$\mathbf{P} = \mathbf{P}_\Lambda + \mathbf{P}_\Sigma \tilde{\epsilon}^{-1} + \mathbf{P}_H 2(\tilde{\epsilon} + 1)^{-1}, \quad (3)$$

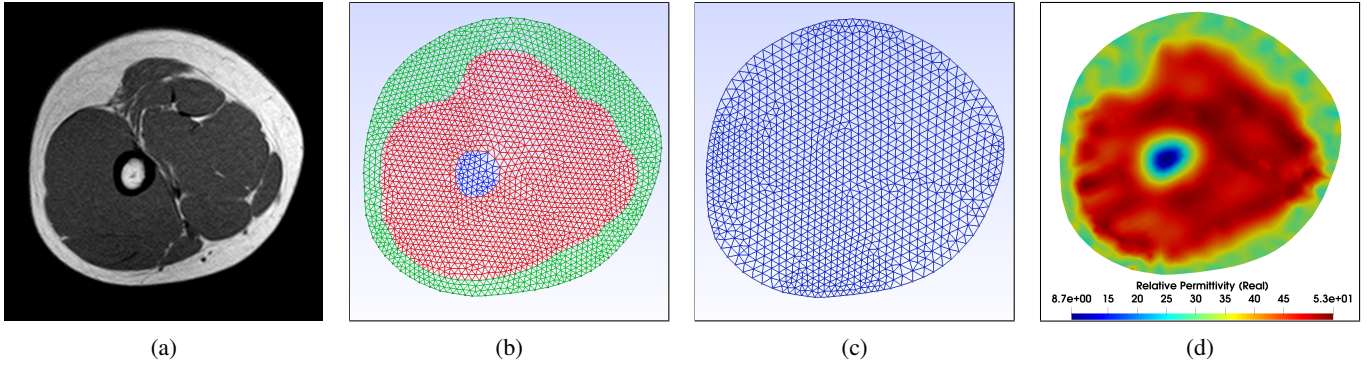


Fig. 1: (a) MRI image of a thigh [3]. (b) Mesh for generating the measured data (4570 elements). (c) Mesh used for imaging (2215 elements). (d) Resconstruction of the permittivity profile with a frequency of 0.5 GHz.

yielding the preconditioned equation

$$\mathbf{P}(\mathbf{I} - \chi \mathbf{G}_S) \mathbf{P} \mathbf{Y} = -i\omega\epsilon_0 \mathbf{P} \chi \mathbf{E}_i, \quad (4)$$

where $\mathbf{P} \mathbf{Y} = \mathbf{J}$. The reduction of the condition number enforced by this preconditioning operation allows for a faster convergence of the iterative method employed to solve (1), as shown in Fig.2 where the forward problem is solved on an inhomogeneous disk with ϵ_r randomly distributed in an interval $\Delta\epsilon = |\epsilon_{max} - \epsilon_{min}|$.

A. The Preconditioned Newton-Kantorovich Method

We then employ the NKM to solve the data and preconditioned state equations. This method is based on the minimization of a residual cost functional derived by applying a first order perturbation to the contrast χ . The equations that describe the preconditioned NKM are

$$\chi_{n+1} = \chi_n + \delta\chi \quad (5)$$

$$\delta\chi = \arg \min \|\rho_n - \mathbf{G}_S \mathbf{P} \mathbf{L}_n^{-1} \mathbf{P} \delta\chi \mathbf{P} \tilde{\mathbf{L}}_n^{-1} \mathbf{P} \mathbf{E}_i\|^2, \quad (6)$$

where χ_n is the contrast at the n -th step, χ_0 is the initial guess, ρ_n is the difference between \mathbf{E}_s and the computed scattered field at iteration n , $\mathbf{L}_n = \mathbf{P}(\mathbf{I} - \chi_n \mathbf{G}_D) \mathbf{P}$, and $\tilde{\mathbf{L}}_n = \mathbf{P}(\mathbf{I} - \mathbf{G}_D \chi_n) \mathbf{P}$. The preconditioning operator \mathbf{P} is computed using the initial homogeneous guess χ_0 . To reduce the ill-posedness of the problem, we further regularize it using a Tikhonov regularization, which adds an extra term proportional to $\|\delta\chi\|^2$ to (6), hence enforcing the minimization of the norm of $\delta\chi$.

IV. APPLICATION TO BONE IMAGING

In this final section we use our method on a realistic biomedical imaging problem in which the geometry is obtained from an MRI of a human thigh (Fig. 1a). The scattered field used as measured data is computed on a circle around the object ($N_A = 60$) using an incident field oscillating at 0.5 GHz from 10 independent sources and a mesh with three layers corresponding to the skin/fat, the muscle, and the bone with $\epsilon_r = \{30 - 25i, 50 - 29i, 10 - 4i\}$ (from the outer layer to the inner one), depicted in Fig. 1b. Subsequently,

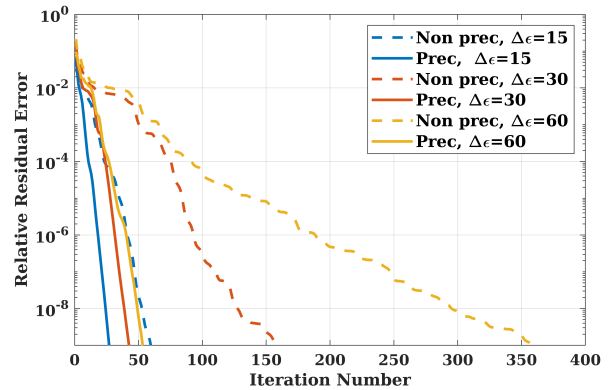


Fig. 2: Relative residual error obtained from the solution of the preconditioned and non-preconditioned TE J-VIE matrix system with $\Delta\epsilon = |\epsilon_{max} - \epsilon_{min}|$.

the reconstruction of the image is performed using the homogeneous coarser mesh shown in Fig.1c. We obtain a good reconstruction of the image both in shape and values of ϵ_r , as shown in Fig.1d. Moreover, the preconditioner reduces the total computational time by a factor of almost 4 with respect to the non-preconditioned case, showing the benefit of HC regularization in this realistic scenario.

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