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► To cite this version:

Alberto Scazzola, Adrien Merlini, Clément Henry, Francesco Andriulli. A Preconditioned Inverse Scattering Scheme for High-Contrast Bone Imaging at Microwave Frequencies. 2024 IEEE International Symposium on Antennas and Propagation and INC/USNC-URSI Radio Science Meeting (AP-S/INC-USNC-URSI), Jul 2024, Firenze, France. pp.429-430, <10.1109/AP-S/INC-USNC-URSI52054.2024.10686480>. <hal-04801323>

HAL Id: hal-04801323

<https://hal.science/hal-04801323v1>

Submitted on 25 Nov 2024

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A Preconditioned Inverse Scattering Scheme for High-Contrast Bone Imaging at Microwave Frequencies

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Abstract—This work presents a combined quasi-Helmholtz preconditioning strategy for the contrast source volume integral equation (J-VIE) with a Newton-Kantorovich method allowing 2D microwave imaging for high-contrast scatterers. We show that the preconditioning improves the convergence of the J-VIE, reducing the number of iterations required, even for highly inhomogeneous objects. We demonstrate the practical significance of this preconditioned inverse scattering scheme on a realistic human bone and thigh imaging problem.

I. INTRODUCTION

Microwave imaging (MWI) is a promising technique for imaging biological tissues and offers a low-cost alternative to standard imaging procedures such as magnetic resonance imaging (MRI) and X-ray. When the background medium is already known, a formulation employing volume integral equations (VIE) is advantageous since it allows for the reconstruction of inhomogeneous scatterers without discretizing the background. Among VIE formulations, the contrast source VIE (J-VIE) is the most adopted one since it yields a data equation in the form of an inverse source problem. However, the high permittivity of biological tissues with respect to the background introduces a severe ill-conditioning of the J-VIE and causes a slowdown of iterative solvers, which is often referred to as the high-contrast (HC) breakdown. A regularization technique based on quasi-Helmholtz projectors has been introduced in [1] to rescale the permittivity dependent spectrum of the J-VIE. In this work, we adapt these projectors to a 2D setting with the transverse electric J-VIE (TE J-VIE) and we apply this preconditioner to a Newton-Kantorovich method (NKM) [2] derived for the TE J-VIE. Numerical results will show that these projectors enable handling high-contrast settings and we additionally demonstrate the effectiveness of this novel preconditioned inverse scattering scheme in a realistic bone and thigh imaging problem.

II. NOTATION AND BACKGROUND

Consider a scattering problem in which a transverse electric (TE) time-harmonic incident wave with electric field \mathbf{E}_i and the angular frequency ω is scattered in a nonmagnetic inhomogeneous scatterer with arbitrary cross-sectional shape $\Omega \subset \mathbb{R}^2$ and complex permittivity $\epsilon(\mathbf{r})$ ($\mathbf{r} \in \mathbb{R}^2$) embedded in free-space of permittivity ϵ_0 and permeability μ_0 . The TE contrast source volume integral equation relates the in-plane contrast source \mathbf{J} and \mathbf{E}_i as $\mathbf{J} - \chi(\nabla\nabla + k_0^2) \cdot$

$\mathbf{S}(\mathbf{J}) = -i\omega\epsilon_0\chi\mathbf{E}_i$ where $\mathbf{S}(\mathbf{F})(\mathbf{r}) = \int_{\Omega} G_2(\mathbf{r}, \mathbf{r}')\mathbf{F}(\mathbf{r}') ds'$, $G_2(\mathbf{r}, \mathbf{r}') = i/4H_0^1(\mathbf{r}, \mathbf{r}')$, and k_0 is the wavenumber of \mathbf{E}_i .

We can numerically solve the TE J-VIE by discretizing Ω with N_T triangles having a constant permittivity and expanding the contrast current $\mathbf{J} = \mathbf{J}^x + \mathbf{J}^y$ as $\mathbf{J}^\alpha(\mathbf{r}) \approx \sum_{m=1}^{N_T} [\mathbf{J}_m^\alpha] \mathbf{f}_m^\alpha(\mathbf{r})$ in which $\hat{\alpha} = \hat{x}, \hat{y}$. The basis function \mathbf{f}_m^α , which lives in $L^2(\Omega)^2$, is defined as $\mathbf{f}_m^\alpha(\mathbf{r}) = \hat{\alpha}P_m(\mathbf{r})$ with $P_m \in L^2(\Omega)$ being a patch defined on triangle T_m . Following a Galerkin procedure as in [1], the continuous TE J-VIE is discretized as $(\mathbf{I} - \chi\mathbf{G}_S)\mathbf{J} = -i\omega\epsilon_0\chi\mathbf{E}_i$, where $\mathbf{G}_S \in \mathbb{C}^{2N_T \times 2N_T}$, $\mathbf{I} \in \mathbb{R}^{2N_T \times 2N_T}$, $\mathbf{J} \in \mathbb{C}^{2N_T \times 1}$, $\chi \in \mathbb{C}^{2N_T \times 2N_T}$, and $\mathbf{E}_i \in \mathbb{C}^{2N_T \times 1}$.

We then define the inverse scattering problem based on the discretized state and data equations

$$(\mathbf{I} - \chi\mathbf{G}_S)\mathbf{J} = -i\omega\epsilon_0\chi\mathbf{E}_i \quad (1)$$

$$\mathbf{G}_D\mathbf{J} = -i\omega\epsilon_0\mathbf{E}_s, \quad (2)$$

where $\mathbf{G}_D \in \mathbb{C}^{N_A \times 2N_T}$ is the discretized radiation operator and $\mathbf{E}_s \in \mathbb{C}^{N_A \times 1}$ is the measured scattered field with N_A being the number of independent measurements. The aim of this scattering problem is to estimate χ from the knowledge of \mathbf{E}_i and \mathbf{E}_s .

III. NEWTON-KANTOROVICH METHOD FOR THE PRECONDITIONED J-VIE

The first step of this contribution is the preconditioning of the forward problem. Following a similar approach as in [1], we define projection operators to the solenoidal, irrotational and harmonic subspaces as $\mathbf{P}_\Lambda = \mathbf{\Lambda}(\mathbf{\Lambda}^T\mathbf{\Lambda})^+\mathbf{\Lambda}^T$, $\mathbf{P}_\Sigma = \mathbf{\Sigma}(\mathbf{\Sigma}^T\mathbf{\Sigma})^+\mathbf{\Sigma}^T$, and $\mathbf{P}_H = \mathbf{I} - \mathbf{P}_\Lambda - \mathbf{P}_\Sigma$, respectively, where $^+$ denotes the Moore-Penrose pseudoinverse. The transformation matrices $\mathbf{\Sigma}$ and $\mathbf{\Lambda}$ are constructed from $[\mathbf{\Sigma}^\alpha]_{mn} = \langle \mathbf{f}_m^\alpha, \nabla b_n \rangle$, with b_n a pyramid function and $\langle \mathbf{f}, \mathbf{g} \rangle = \int_{\Omega} \mathbf{f}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{r}) ds$, and $[\mathbf{\Lambda}^\alpha]_{mn} = \langle \mathbf{f}_m^\alpha, \hat{z} \times \nabla b_n \rangle$, which are the transformation matrices from irrotational and solenoidal functions to the piecewise constant basis functions $\mathbf{f}_m^\alpha(\mathbf{r})$.

In the framework of the inverse scattering problems the imaging object is in general inhomogeneous and the details of its internal structure are unknown. Therefore we build a preconditioning operator as in [1], using instead the average relative permittivity $\tilde{\epsilon}$

$$\mathbf{P} = \mathbf{P}_\Lambda + \mathbf{P}_\Sigma \tilde{\epsilon}^{-1} + \mathbf{P}_H 2(\tilde{\epsilon} + 1)^{-1}, \quad (3)$$

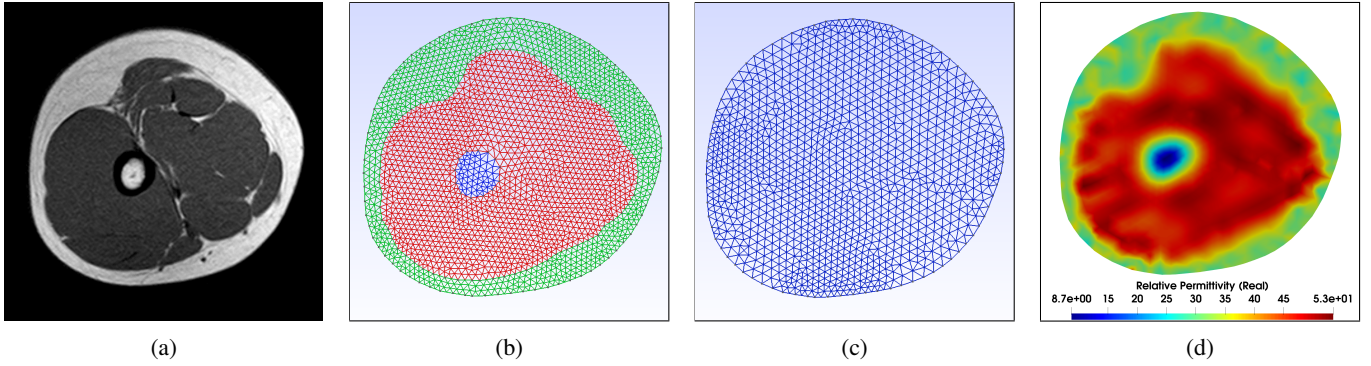


Fig. 1: (a) MRI image of a thigh [3]. (b) Mesh for generating the measured data (4570 elements). (c) Mesh used for imaging (2215 elements). (d) Resconstruction of the permittivity profile with a frequency of 0.5 GHz.

yielding the preconditioned equation

$$\mathbf{P}(\mathbf{I} - \chi \mathbf{G}_S) \mathbf{P} \mathbf{Y} = -i\omega\epsilon_0 \mathbf{P} \chi \mathbf{E}_i, \quad (4)$$

where $\mathbf{P} \mathbf{Y} = \mathbf{J}$. The reduction of the condition number enforced by this preconditioning operation allows for a faster convergence of the iterative method employed to solve (1), as shown in Fig.2 where the forward problem is solved on an inhomogeneous disk with ϵ_r randomly distributed in an interval $\Delta\epsilon = |\epsilon_{max} - \epsilon_{min}|$.

A. The Preconditioned Newton-Kantorovich Method

We then employ the NKM to solve the data and preconditioned state equations. This method is based on the minimization of a residual cost functional derived by applying a first order perturbation to the contrast χ . The equations that describe the preconditioned NKM are

$$\chi_{n+1} = \chi_n + \delta\chi \quad (5)$$

$$\delta\chi = \arg \min \|\rho_n - \mathbf{G}_S \mathbf{P} \mathbf{L}_n^{-1} \mathbf{P} \delta\chi \mathbf{P} \tilde{\mathbf{L}}_n^{-1} \mathbf{P} \mathbf{E}_i\|^2, \quad (6)$$

where χ_n is the contrast at the n -th step, χ_0 is the initial guess, ρ_n is the difference between \mathbf{E}_s and the computed scattered field at iteration n , $\mathbf{L}_n = \mathbf{P}(\mathbf{I} - \chi_n \mathbf{G}_D) \mathbf{P}$, and $\tilde{\mathbf{L}}_n = \mathbf{P}(\mathbf{I} - \mathbf{G}_D \chi_n) \mathbf{P}$. The preconditioning operator \mathbf{P} is computed using the initial homogeneous guess χ_0 . To reduce the ill-posedness of the problem, we further regularize it using a Tikhonov regularization, which adds an extra term proportional to $\|\delta\chi\|^2$ to (6), hence enforcing the minimization of the norm of $\delta\chi$.

IV. APPLICATION TO BONE IMAGING

In this final section we use our method on a realistic biomedical imaging problem in which the geometry is obtained from an MRI of a human thigh (Fig. 1a). The scattered field used as measured data is computed on a circle around the object ($N_A = 60$) using an incident field oscillating at 0.5 GHz from 10 independent sources and a mesh with three layers corresponding to the skin/fat, the muscle, and the bone with $\epsilon_r = \{30 - 25i, 50 - 29i, 10 - 4i\}$ (from the outer layer to the inner one), depicted in Fig. 1b. Subsequently,

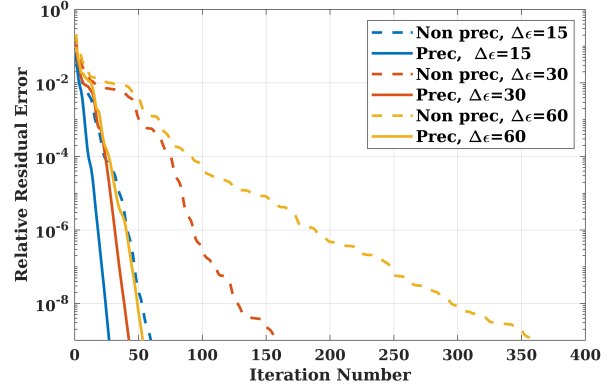


Fig. 2: Relative residual error obtained from the solution of the preconditioned and non-preconditioned TE J-VIE matrix system with $\Delta\epsilon = |\epsilon_{max} - \epsilon_{min}|$.

the reconstruction of the image is performed using the homogeneous coarser mesh shown in Fig.1c. We obtain a good reconstruction of the image both in shape and values of ϵ_r , as shown in Fig.1d. Moreover, the preconditioner reduces the total computational time by a factor of almost 4 with respect to the non-preconditioned case, showing the benefit of HC regularization in this realistic scenario.

ACKNOWLEDGEMENT

This work has received funding from the European Innovation Council (EIC) through the European Union's Horizon Europe research Programme under Grant 101046748 (Project CEREBRO) and from the ANR Labex CominLabs under the project "CYCLE".

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